# Excerpta ex Mss. R. Descartes

# EXCERPTA

Ex

MSS.

# R. DES-CARTES.

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ex

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## Ex quantitate linearum, quæ in dato Circulo inſcriptæ ſunt, *quantitatem arcus, cui datæ lineæ ſubtenduntur, agnoſcere.*

ASſumo generaliter circulum, cujus radius ſit unitas, in quo conſidero omnes inſcriptas, quarum habitudo ad partes circumferentiæ, quibus ſubtenduntur, eſt cognita hoc modo; ſubtenſa *mediæ* partis ſemicirculi eſt $\surd\ 2$, ſubtenſa $\frac{1}{4}$ eſt $\surd.2-\surd\ 2$, ſubtenſa $\frac{3}{4}$ eſt $\surd\ 2+\surd\ 2$. item $\frac{1}{8}$ eſt $\surd.2-\surd.2.+\surd\ 2$. $\frac{3}{8}$ eſt $\surd.2-\surd.2-\surd\ 2$. $\frac{5}{8}$ eſt $\surd.2+\surd.2-\surd\ 2$. $\frac{7}{8}$ eſt $\surd.2+\surd.2.+\surd\ 2$. $\frac{1}{16}$ eſt $\surd.2-\surd.2.+\surd.2+\surd\ 2$. $\frac{3}{16}$ eſt $\surd.2-\surd.2+\surd.2-\surd\ 2$. $\frac{5}{16}$ eſt $\surd.2-\surd\ 2-\surd.2-\surd\ 2$. $\frac{7}{16}$ eſt $\surd.2-\surd.2-\surd.2+\surd\ 2$. $\frac{9}{16}$ eſt $\surd.2+\surd.2-\surd.2+\surd\ 2$. $\frac{11}{16}$ eſt $\surd.2+\surd.2-\surd.2-\surd\ 2$. $\frac{13}{16}$ eſt $\surd.2+\surd.2+\surd.2-\surd\ 2$. $\frac{15}{16}$ eſt $\surd.2+\surd.2+\surd.2+\surd\ 2$. & ſic de cæteris. $\frac{1}{32}$ $+-+++$ $\frac{3}{32}$ $+-++-$ $\frac{7}{32}$ $+-+-+$ $\frac{9}{32}$ $+---+$ $\frac{11}{32}$ $+----$ $\frac{13}{32}$ $+--+-$ $\frac{15}{32}$ $+--++$ $\frac{17}{32}$ $++-++$ & ſic de cæteris.

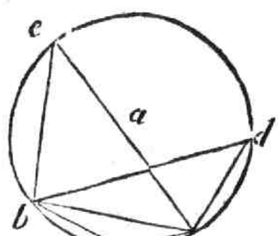
Subtenſa *tertiæ* partis ſemicirculi eſt unitas. Duarum tertiarum eſt $\surd\ 3$. $\frac{1}{6}$ eſt $\surd.2-\surd\ 3$. vel $\surd.\frac{3}{2}-\surd.\frac{1}{2}$. $\frac{5}{6}$ eſt $\surd.2+\surd\ 3$. vel $\surd\ \frac{3}{2}+\surd\ \frac{1}{2}$. $\frac{1}{12}$ eſt $\surd.2-\surd.2+\surd\ 3$. vel $\surd.2-\surd.\frac{3}{2}-\surd\ \frac{1}{2}$. $\frac{5}{12}$ eſt $\surd.2-\surd.2-\surd\ 3$. $\frac{7}{12}$ eſt $\surd.2+\surd.2-\surd\ 3$. $\frac{11}{12}$ eſt $\surd.2+\surd.2+\surd\ 3$. &c. eodem etiam ordine ponuntur notæ $+$ & $-$ ut ſupra.

Subtenſa *quintæ* partis Semicirculi eſt $\surd\ \frac{3}{2}-\surd\ \frac{5}{4}$. vel $\surd.\frac{5}{4}-\frac{1}{2}$ $\frac{2}{5}$ eſt $\surd.\frac{5}{2}-\surd\ \frac{5}{4}$. $\frac{3}{5}$ eſt $\surd.\frac{3}{2}+\surd\ \frac{5}{4}$. vel $\surd\ \frac{5}{4}+\frac{1}{2}$ $\frac{4}{5}$ eſt $\surd.\frac{5}{2}+\surd\ \frac{5}{4}$. $\frac{1}{10}$ eſt $\surd.2-\surd.\frac{5}{2}+\surd\ \frac{5}{4}$ $\frac{3}{10}$ eſt $\surd.2-\surd.\frac{5}{2}-\surd\ \frac{5}{4}$ $\frac{7}{10}$ eſt $\surd.2.$

$+\surd.\frac{5}{2}+\surd\frac{5}{4}.\ \frac{9}{10}$ eſt $\surd.2+\surd.\frac{5}{2}+\surd\frac{5}{4}\ \ \frac{1}{20}$ eſt $\surd.2-\surd.2.$
$+\surd.\frac{5}{2}+\surd\frac{5}{4}.\ \frac{3}{20}$ eſt $\surd.2.-\surd.2+\surd.\frac{5}{2}-\surd 2\frac{5}{4}.\ \frac{7}{20}.$ eſt $\surd.2$
$-\surd\frac{5}{2}-\surd\frac{5}{4}-\ \frac{9}{20}$ eſt $\surd.2-\surd.2-\surd\frac{5}{2}+\surd.\frac{5}{4}.\ \frac{11}{20}$ eſt
$\surd.2+\surd.2-\surd.\frac{5}{2}+\surd\frac{5}{4}.\ \frac{13}{20}$ eſt $\surd.2+\surd.2-\surd.\frac{5}{2}-\surd\frac{5}{4}.$
& ſic de cæteris in infinitum.

Item ſubtenſa *decimæ quintæ* partis Semicirculi eſt $\surd.\frac{9}{4}-\frac{1}{4}\surd.$
$5-\surd\frac{15}{8}+\frac{3}{8}\surd.5.\ \frac{2}{15}$ eſt $\surd.\frac{7}{4}\ \frac{1}{4}\surd\ 5-\surd\ \frac{15}{8}-\frac{3}{8}\surd 5\ \ \frac{4}{15}$ eſt
$\surd\frac{7}{4}+\frac{1}{4}\surd 5-\surd.\frac{15}{8}+\frac{3}{8}\surd.5.\ \frac{4}{15}$ eſt $\frac{9}{4}+\frac{1}{4}\surd.5-\surd.\frac{15}{8}-\frac{3}{8}$
$\surd 5.\ \frac{8}{15}$ eſt $\surd.\frac{9}{4}-\frac{1}{4}\surd.5.+\surd\ \frac{15}{8}-\frac{3}{8}\surd.\ 5.\ \frac{11}{15}$ eſt $\surd.\ \frac{9}{4}-\frac{1}{4}$
$\surd.5+\surd.\frac{15}{8}-\frac{3}{8}\surd 5\ \frac{13}{15}$ eſt $\surd\ \frac{9}{4}-\frac{1}{4}\surd.5+\surd.\frac{15}{8}-\frac{3}{8}\surd 5.$
$\frac{14}{15}$ eſt $\surd.\frac{7}{4}+\frac{1}{4}\surd.5+\surd\ \frac{15}{8}+\frac{3}{8}\surd 5.\ \frac{1}{30}\ \surd.2-\surd.\frac{9}{4}+\frac{1}{4}$
$\surd.5+\surd\ \frac{15}{8}+\frac{3}{8}\surd 5.$ Sed forte hi numeri paulo breviores eſſe poſſunt, ut pro $\frac{13}{15}$ poſſum ponere $\frac{1}{4}\surd.5+\surd.\frac{15}{8}-\frac{3}{8}\surd 5.$ & ſic de cæteris.

Atque hæc tabula in infinitum poteſt continuari, ſi ſemper ex ſubtenſa majoris partis circumferentiæ quæratur ſubtenſa mediæ partis: hoc modo, ſit $a$ ſubtenſa *unius* partis circumferentiæ, ſubtenſa *mediæ* partis erit $\surd.2-\surd.4-aa$, & complementum erit $\surd.2+\surd.4-aa$. Atque per hanc unam regulam omnes *Sinus*, quos Geometria poteſt invenire, numeris exhibentur.




Facto igitur hoc indice, ſi datum ſit aliquod $\triangle$lum, cujus anguli quærantur, deſcribo ſimile dato $\triangle$lo, in Circulo, cujus Radius unitas: deinde video, quibus numeris in noſtra tabula quælibet latera reſpondeant; quod ſi dati $\triangle$li latera nullis numeris noſtræ tabulæ æqualia ſint, tunc demonſtrative aſſeremus, nullos illius angulos in Geometria ſimplici poſſe inveniri. Vel alio modo.

Quæro differentiam inter potentiam baſis & potentias laterum, quæ niſi ſe habeat ad rectangulum ſub lateribus comprehenſum, ut aliquis ex numeris noſtri indicis ad unitatem, pro certo aſſeremus talem angulum in Geometria ſimplici non inveniri, vel $\square$tum $bd + \square\, dc \propto \square\, bc +$ producto ex $\sqsubset\!\sqsupset\, bdc$ in lineam $be$; cum $ae$ æquatur unitati.

Ex his poſſumus deducere *progreſſionem Pythagoricæ* ad omnes angulos; ſicut enim in Triangulo rectangulo baſis potentia æqualis eſt potentiis duorum laterum, ita in $\triangle$lo, ubi unus angulus eſt 60 grad. baſis potentia æquatur quadratis duorum laterum minus rectan-

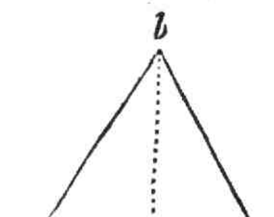
rectangulo ſub illis comprehenſo: & in △lo, in quo angulus unus eſt complementum ſuperioris ad duos rectos, nempe 120 grad. baſis potentia excedit potentiam laterum eodem rectangulo quia ſubtenſa complementi in noſtro indice eſt unitas.

Item in △lo, cujus angulus eſt 45 grad. baſis potentia minor eſt potentiâ laterum, quantitate media proportionali inter ▭lum ſub lateribus comprehenſum, & ejuſden ▭li duplum. In △ Complementi duorum rectorum, nempe 135 grad. baſis potentia major eſt potentiâ laterum eadem quantitate, quia ſubtenſa complementi eſt $\sqrt{.\,2}$.

Item in △lo, cujus angulus eſt 30 grad. baſis potentia minor eſt potentiâ laterum, quantitate media proportionali inter ▭lum ſub iis comprehenſum, & ejuſdem triplum. In △lo Complementi deficit laterum potentia eadem quantitate, quia ſubtenſa complementi eſt $\sqrt{.\,3}$.

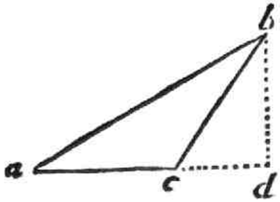
Et generaliter; in omnibus △lis oxygoniis baſis potentia minor eſt potentiâ laterum, ▭lo ſub lateribus comprehenſo ducto per numerum, qui exprimit ſubtenſam complementi in noſtro indice. Et generaliſſime;

△li *bdc* potentia baſis *bc* minor eſt potentia laterum, quantitate, quæ ſit ad ▭lum ſub illis comprehenſum, ut ▭lnm ſub lineis *be*, *ea* (quarum una eſt, nempe *be*, ſubtenſa complementi, & alia, nempe *ea*, eſt Semidiameter Circuli dato triangulo circumſcripti) ſe habet ad □tum lineæ *ea*. Vel ut *be* ad *ea*: hoc eſt, fiat, ut *ae* ad *be* ſic ▭lum *bdc* ad quantitatem, quæ vocetur A. Dico □ta *bd* + *dc* ∞ □to *bc* + quantitate A: (vide Fig. pag. 2.) E contrario in amblygonio △lo *bfc* potentia baſis *bc* major eſt potentia laterum eadem quantitate.



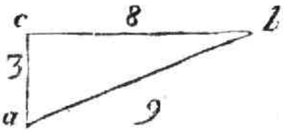

Dato autem △lo diameter Circuli, in quo inſcribitur, facile invenitur dicendo, ut *bd* perpendicularis ad unum latus, ita aliud ad quæſitam diametrum. NB. *ac* eſt baſis: *ab* & *bc* latera.

Atque ex ſuperioribus tale Theorema deducitur: Quotieſcumque in duobus △lis inæqualibus & diſſimilibus, baſis potentia unius differt à potentiis laterum, quantitate quæ habet eandem proportio-




nem cum ▭lo ſub lateribus comprehenſo, quam habet in altero △lo, tunc in utroque △lo anguli baſibus oppoſiti ſunt inter ſe æquales, (ſiquidem potentiæ laterum in utroque ſint majores potentiâ baſis, vel in utroque minores) ſed ſi in uno ſint minores, in altero majores, tunc duo illi anguli baſibus oppoſiti ſunt æquales duobus rectis.

Triangulorum, quorum omnia latera numeris rationalibus exprimuntur, poſſunt etiam omnes anguli numeris rationalibus exprimi, nempe ſumendo pro quantitate anguli proportionem, quæ eſt inter ▭lum ſub lateribus comprehenſum, & differentiam, quam baſis eidem angulo oppoſitæ potentia ſuperat vel ſuperatur a potentiis laterum ſimul junctis: ſuperat nim: ſi angulus quæſitus ſit major recto; vel ſuperatur, ſi ſit minor, & ad hoc judicandum aliqua nota eſt adhibenda, ut exempli cauſa, △li, cujus latera




ſunt 3, 8, 9, angulus $abc$ eſt $\frac{9}{17}$ angulus $cab$ eſt $\frac{27}{26}$ & $acb$ eſt $\frac{3}{1} + o$. Ubi notandum, me ſemper ponere numerum, qui oritur ex multiplicatione laterum ſupra, & inferius ponere illum, qui oritur ex differentia, quæ eſt inter baſim & laterum potentias: & cum potentia baſis excedit potentias laterum, me adhibere $+ o$, ut oſtendam, angulum eſſe majorem recto, hic enim $o$ eſt numerus exponens anguli recti.

Omnis numerus conſtat vel uno, vel duobus, vel tribus triangularibus; item vel uno, vel duobus, vel tribus, vel 4 □tis: item uno, vel duobus, vel tribus, vel 4, vel 5 pentagonis; item vel 1, vel 2, vel 3, vel 4, vel 5, vel 6 Hexagonis, & ſic in infinitum.

Omnis numeri triangularis octuplum plus unitate eſt □tum, quod facile demonſtratur. Eſt enim numerus △laris $\frac{x + xx}{2}$, ergo 8plum $\frac{8x + 8xx}{2}$ ſeu $4x + 4xx$, cui ſi addatur 1, fiet $1 + 4x + 4xx$, cujus radix $1 + 2x$.

## *De partibus aliquotis numerorum.*

AD ſolvendas quæſtiones circa numerorum partes aliquotas, imaginamur illos compoſitos, vel ex numeris primis inter ſe, vel ex iis, qui ex multiplicatione numeri cujuſdam primi ſæpius iteratâ, vel partim ex his, partim ex illis producuntur. Jam verò numerus aliquis primus nullas partes aliquotas habet, præter unitatem. Numerus autem primus ſæpius per ſe ipſum multiplicatus, ſicuti $a^n$, partes aliquotas habet $\frac{a^n - 1}{a - 1}$, hoc eſt ſe ipſum minus 1, diviſum ſuâ radice minus 1. Si reperire velimus partes aliquotas numeri cujuſdam primi, per alium numerum multiplicati, cujus jam habemus partes aliquotas, veluti ſi partes aliquotæ numeri $a$ ſunt $b$, & $x$ ſit numerus primus, partes aliquotæ numeri ſunt $bx + a + b$.

Quòd ſi deſideramus invenire partes aliquotas numeri cujuſdam primi ſæpius per ſe ipſum multiplicati, & denuò per alium multiplicati numerum, qui etiam ſæpius per ſe ipſum multiplicatus ſit, & ſi unus ex numeris ſit $+ a^n$, alius verò $c^o$, partes aliquotæ $a^n c^o$ erunt $a a^n c^o + a^n \frac{c c^o + c c^o - a a_n - a^n c^o + n.}{a c - a - c + n.}$

Si reperire cupimus partes aliquotas numeri cujuſdam primi ſæpius per ſe ipſum multiplicati, & cujus productum porrò multiplicatur per alium numerum, qui primus eſt reſpectu alterius, licet abſolutè primus non ſit, cujus partes aliquotæ datæ ſunt, ſi numerus per ſe ipſum multiplicatus ſit $x^n$, & alter numerus ſit $a$, ejuſque partes aliquotæ $b$, habemus $\frac{b x x^n + a x^n - a - b}{x - 1}$ partes aliquotas numeri $a x^n$.

Si habemus duos numeros primos inter ſe, eorumque partes aliquotas, habemus etiam partes aliquotas producti ipſorum; veluti ſi unus ſit $a$, ejuſque partes aliquotæ ſint $b$, alter verò ſit $c$, cujus aliquotæ partes ſint $d$, partes aliquotæ $ac$ erunt $ad + bc + bd$.

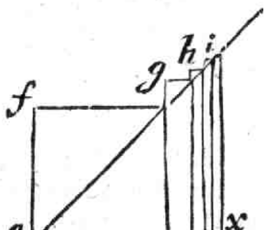
Nec profectò aliquid hac in materiâ novi, quod ope Theorematum, quæ hîc pono, reperiri non poſſit.

Si quæramus Cubum & Quadratum æqualia Quadrato, habemus 13824, 100 & 13924, quorum radices 24, 10, 118. item 27. 9. 36, aliaque infinita. NB. inveni ſolutionem facillimam $x^3 + xx \propto aaxx$. Ergò $x + 1 \propto aa$, & $x \propto aa - 1$. Hinc infiniti inveniuntur.

Ad *extrahendam radicem Cubicam binomii* $a + \sqrt{b}$ quæro radicem hujus æquationis $x^3 \propto \begin{matrix} 3aax + 2a^3 \\ -3bx - 2ab \end{matrix}$, quando $aa$ major eſt $b$, & triplo iſtius radicis, adjungo $2a$, & dimidium radicis cubicæ producti eſt primus terminus radicis quæſitæ: Quod ſi $aa$ minor eſt quàm $b$, quæro radicem hujus æquationis $x^3 \propto \begin{matrix} 3aax - 2a^3 \\ -3bx + 2ab \end{matrix}$ cujus triplum aufero ex $2a$, & dimidium radicis cubicæ reſidui iſtius eſt primus terminus quæſitus. Poſthæc aufero ex numero $a$ cubum iſtius primi termini, & poſtquam reliquum per triplum iſtius primi termini diviſero, radix quadrata quotientis ſecundus terminus eſt.

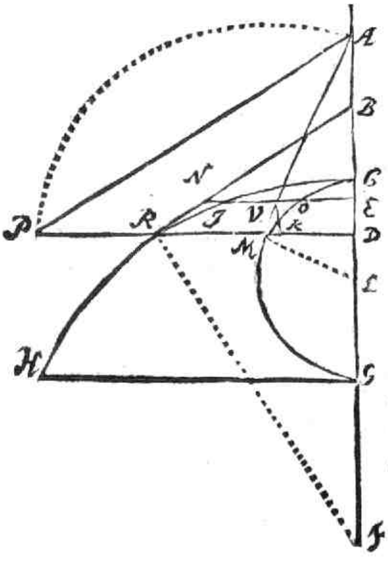
Pari modo, ſi velim invenire radicem Cubicam $10 + \sqrt{98}$, habeo $x^3 \propto 6x + 40$, cujus radix eſt 4, ejuſque triplo, quod eſt duodecim, addito 20, provenit 32, cujus radix Cubica eſt $\sqrt{c\,32}$, ejuſque dimidium eſt $\sqrt{c\,4}$ pro primo termino. Poſtea 4 ablato à 10, reſtat 6, quem divido per $3\sqrt{c\,4}$, provenit $\sqrt{c\,2}$, cujus radix quadrata eſt $\sqrt{Q\,c\,2}$ pro ſecundo termino. Et ad inveniendam radicem Cubicam $2 + \sqrt{5}$, habeo $x^3 \propto -3x + 4$, cujus radix eſt 1. Ejus autem triplo ſublato ex 4, reſtat 1, cujus radix cubica eſt 1, ejuſque dimidium $\frac{1}{2}$ pro primo termino. Poſtmodum ablato cubo $\frac{1}{2}$, qui eſt $\frac{1}{8}$ à 2, reſtat $\frac{15}{8}$, quem divido per $\frac{3}{2}$, provenitque $\frac{5}{4}$, cujus radix eſt $\sqrt{\frac{5}{4}}$ pro ſecundo termino, atque ita de reliquis.

Quin & in genere pro radice Cubicâ alicujus binomii, duarum iſtarum Cubi partium maximam $c$, & minimam $d$ appello, deinde extraho radicem hujus æquationis $x^3 \propto \begin{matrix} 3ccx + 2c^3 \\ -3ddx - 2cdd \end{matrix}$ & triplo iſtius radicis adjungo $2c$, & dimidium radicis Cubicæ producti eſt una ex partibus radicis quæſitæ. Poſtea divido $c$ per illam primam partem radicis, à quotiente aufero quadratum ejuſdem primæ partis, & tertia pars reſidui eſt altera pars radicis.




Ad *quadrandum Circulum* nihil aptius invenio, quam ſi dato Quadrato $bf$ adjungatur ▭lum $cg$ comprehenſum ſub lineis $ac$ & $cb$, quod ſit æquale quartæ parti Quadrati $bf$, item ▭lum $dh$, factum ex lineis $da$, $dc$ æquale quartæ parti præcedentis, & eodem modo ▭lum $ei$, atque alia infinita uſque ad $x$, quæ omnia ſimul æquabuntur tertiæ parti Quadrati

drati $bf$, & hæc linea $ax$ erit diameter Circuli, cujus circumferentia æqualis eſt circumferentiæ hujus □ti $bf$: eſt autem $ac$ diameter circuli, octogono quadrato $bf$ iſoperimetro, inſcripti: $ad$ diameter Circuli inſcripti figuræ 16 laterum, $ae$ diameter inſcripti figuræ 32 laterum, quadrato $bf$ iſoperimetræ; & ſic in infinitum.



*Lineæ curvæ*, in quibus *Tangentes* inquirimus, *proprietates* ſuas ſpecificas, vel per lineas tantum rectas abſolvunt, vel per curvas rectis aut aliis curvis quomodolibet implicatas. Exemplum ſit *curva* HRIC, cujus vertex C, axis CF, & deſcripto Semicirculo COMG ſumatur punctum quodlibet in curva ut R, à quo ducenda eſt *Tangens* RB. Ducatur à puncto R recta RMD perpendicularis ad CDF, quæ ſecet Semicirculum in M. Ea igitur curvæ *proprietas* eſto ſpecifica; ut recta RD ſit æqualis portioni Circuli CM & applicatæ DM. Ducatur in puncto M tangens MA ad Circulum, & à puncto E ducatur EOVIN, parallela rectæ RMD. Ponatur factum quod quæritur, & ſit recta DB quæſita $\infty\, a$, DA inventa ex conſtructione $\infty\, b$; MA $\infty\, d$, MD $\infty\, r$, RD $\infty\, z$, curva CM $\infty\, n$, DE $\infty\, e$.

DB BE RD

Fiat ut $a \,\pi\, a - e$. ita $z \,\pi\, \frac{az - ez}{a} \infty$. NE. Igitur recta $\frac{za - ze}{a}$ debet adæquari rectæ OE + CM — MO; ſi autem hi termini ad terminos analyticos reducantur pro recta OE, ad vitandam aſymmetriam ſupponatur recta EV applicata Tangenti, & pro curva MO ſumatur portio Tangentis MV, cui ipſa MO adjacet. Ad inveniendum autem EV in terminis analyticis, fiat ut

DA AE MD

$b \,\pi\, b - e$ ita $r \,\pi\, \frac{br - er}{b} \infty$ EV, ad inveniendum deinde MV,

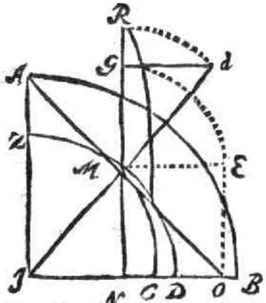
fiat

DA MA DE vel KV.

fiat ut $b \,\pi\, d$ ita $e \,\pi\, \frac{de}{b} \,\infty\,$ M V. Curva autem C M vocata eſt $n \,\infty\, z - r$, unde æquatio $z - \frac{ze}{a} \,\infty\, z - \frac{er}{b} - \frac{de}{b}$ & $bz \,\infty\, ar + ad$.

PD DA RD DB

Id eſt, $r + d \,\pi\, b$ ita $z \,\pi\, a$, & recta R B Tangens.

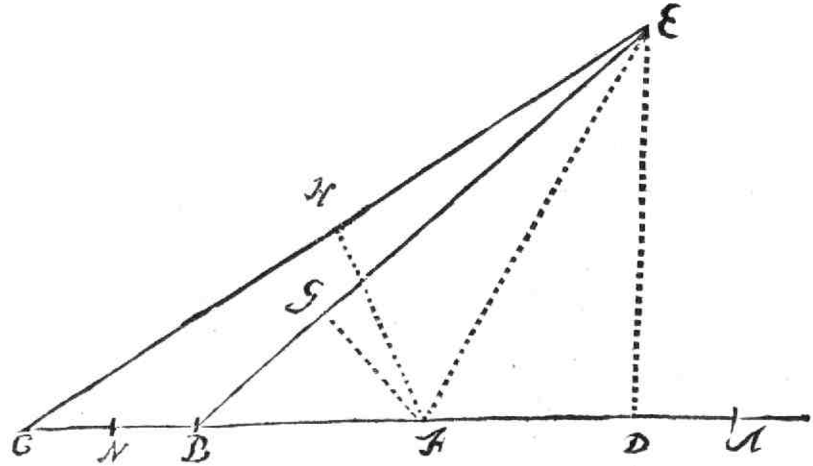



Sit quadrans Circuli A I B, *Quadrataria* A M C, in qua ad datum punctum M ducenda eſt *tangens*, juncta M I, centro I, intervallo I M quadrans Z M D deſcribatur, & ducta perpendiculari M N, fiat ut I M ad M N, ita portio quadrantis M D ad rectam N O. Juncta M O tanget Quadratariam.

Si dentur tales termini $\sqrt{a} + \sqrt{b} + \sqrt{c} \,\infty\, \sqrt{d}$ vel $\sqrt{a} + \sqrt{b} \,\infty\, \sqrt{c} + \sqrt{d}$ *aſymmetriâ liberandi* & ad æquationem ordinatam reducendi, facile hoc omnes poſſunt per 3 multiplicationes, ex quibus formatur talis canon $\underset{4}{a^4} - \underset{12}{4a^3b} + \underset{6}{6aabb} + \underset{12}{4aabc} - \underset{1}{40abcd} \,\infty\, 0$. Hic appoſitus eſt tantum unus terminus cujuſque ſpeciei brevitatis cauſa, & infra ipſum numerus individuorum ejuſdem ſpeciei. Jam ſi dentur tales termini $\sqrt{a} + \sqrt{b} + \sqrt{c} \,\infty\, \sqrt{d} + \sqrt{e}$ aſymmetria liberandi, difficile hoc videtur nonnullis, quia non advertunt per multiplicationem non augeri numerum aſymetriarum ac proinde omnes aſymetrias per multiplicationem tolli poſſe; compendioſius autem fieri poteſt per præcedentem æquationem, ſi tantum in illa pro $d$ ponatur ubique $d + 2\sqrt{de} + e$, & pro $dd$ hujus ſummæ □tum, pro $d^3$ ejuſdem cubum, &c. ac deinde omnes termini in quibus eſt $\sqrt{de}$, æquentur omnibus aliis; ut per multiplicationem quadratam cujuſque partis tollatur aſymmetria $\sqrt{de}$, vel etiam brevitatis cauſa ſufficit, ſi unus terminus cujuſque Speciei quæratur ad canonem conficiendum, qui eſt talis: $\underset{5}{a^8} - \underset{20}{8a^7b} + \underset{20}{28a^6bb} + \underset{30}{10a^6bc} - \underset{20}{56a^5b^3} - \underset{60}{72a^5bbc} - \underset{20}{76a^5bcd} + \underset{10}{70a^4b^4} + \underset{60}{40a^4b^3c} + \underset{30}{36a^4bbcc} + \underset{60}{344a^4bbcd} - \underset{5}{752a^4bcde} + \underset{30}{16a^3b^3cc} + \underset{30}{416a^3b^3}$

cd

$ed - 272\underset{60}{a^3}bbcc + 928\underset{20}{a^3}bbcde + 2008\underset{5}{aa}bbccdd$
$- 1520 aa\underset{10}{b}bccde \infty o.$ Ita ſunt terminorum ſpecies 18 & termini 495, nec refert qui termini prioris æquationis affecti fuerit nota + vel —. Hæc enim omnes continet.



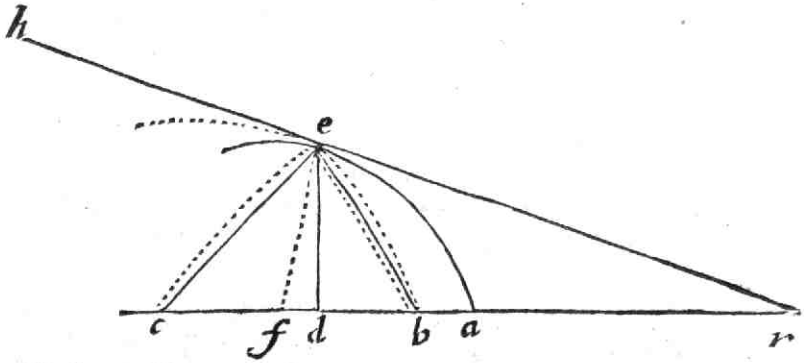

Datis punctis A B C in recta linea, invenire lineam *Curvam*, cujus vertex A, axis A B, & quæ ita ſit incurvata, ut radii à puncto B venientes, poſtquam in illa paſſi erunt *refractionem*, pergant ulterius, tanquam ſi veniſſent ex puncto C, vel contra.

Sumo N punctum medium inter B & C, ſitque N A $\infty$ $a$ & N B. $\infty$ $b$. C E + B E $\infty$ $2a - 2y$ & D A $\infty$ $x$, ſuntque $x$ & $y$ duæ quantitates indeterminatæ, quarum alterutra manens indeterminata, deſignabit omnia puncta lineæ curvæ, & altera determinabitur ex modo quo deſcribi debet linea curva. Qui modus ut inveniatur, quæro imprimis punctum F, à quo ut centro concipio deſcribi circulum, qui *tangit* curvam in puncto E. Deinde dico lineam B E ductam per F G eſſe ad C E ductam per H F, ut inclinatio radii refracti in uno medio tranſparenti ad ejuſdem inclinationem in alio. B D $\infty$ $a - b - x$ vel $\surd.\, xx + aa + bb - 2ax + 2bx - 2ab$. C D $\infty$ $a + b - x$ vel $\surd.\, xx + aa + bb - 2ax - 2bx + 2ab$. B E $\infty$ $yy\,\frac{- 2ay + aa + bx + ab}{a - y}$

$+ab.$ CE $\infty\ \frac{yy-2ay+aa-bx+ab}{a-y}$ & DE $\infty\ \sqrt{.}$

$$\frac{y^4-4ay^3 \left.\begin{array}{r}+5aa\\-bb\\-xx\\ \end{array}\right\} yy \left.\begin{array}{l}+2axx\\-4aax\\+2abb\\-2a^3\end{array}\right\} y \begin{array}{l}-aaxx\\+bbxx\\-2abbx\\+2a^3x\end{array}}{yy-2ay+aa}$$

Fiat nunc NF $\infty\ c$ & F E $\infty\ d$, quæ duæ $c$ & $d$ inveniendæ ſunt ex eo, quod æquatio, quam producit △lum ▭lum F D E, cujus latera ſunt determinata, debeat æquari huic $xx-2ex+ee$ faciendo ſolum differentiam $\infty\ x$ & ſimul $e\ \infty\ x^{\circ}$, FD $\infty\ a-c-x$ vel $\sqrt{.}\ xx+aa+cc-2ax+2cx-2ac$.




Datis punctis $ca\ \infty\ 5$, $ba\ \infty\ n$, & $ar\ \infty\ 5$, imagineris deſcribi curvam $ae$, à fune $af$ fixo foco $c$ & transeunte a $c$ ad $e$ ad $b$, & à $b$ redeunte ad $e$, ac deinde ſe extendente in infinitum verſus $b$, adeò ut longior fiat prout aperitur angulus $erc$ erit, ſemper $er\ \infty\ 5\ \infty\ 7y$, $eb\ \infty\ 1+5y$, $ec\ \infty\ 5-3y$, $da\ \infty\ 2yy+5y$, $de\ \infty\ \sqrt{.}\ -49y^4-20y^3+4yy+20y$, & deinde ſi fiat $fa\ \infty\ \frac{29y+10}{4y+5}$ centro $f$ circulus deſcriptus per $e$ tanget datam curvam, & ſi ducatur $fc\ \infty\ \frac{-9y+15}{4y+5}$ per $er\ \infty\ 5+7y$ productum erit ad $fr\ \infty\ \frac{49y+35}{4y+5}$ ductum per $ce\ \infty\ 5-3y$, ut 3 ad 7. Ergò ſi curva $ea$ contineat ſolidum corpus transparens, in quo refractio fiat ut 3 ad 7, omnes radii à puncto $r$ venientes tendent verſus $c$ poſt refractionem.

Sit nunc $ac\ \infty\ a$ & $ar\ \infty\ a$, $ab\ \infty\ b$, $be\ \infty\ b+y$ erit $re\ \infty\ \frac{2by}{a}+y+a$ & $ce\ \infty\ \frac{2by}{a}-y+a$, $ad\ \infty\ \frac{2byy}{c-a}+y$.

de

$de \infty \sqrt{.} - \frac{4bb}{a4} y4 - \frac{4by3}{aa} + \frac{4bb}{aa} yy + 4by, fa \infty \frac{4bby + 2baa + aay}{4by + aa}$

& $cf$ per $er$, eſt ad $fr$ per $ce$ ut $a - 2b$ ad $a + 2b$.

Sit nunc $ar \infty a$, $ab \infty b$, $ac \infty c$, $be \infty by$, $er \infty \frac{3ay - cy + 4by + aa + ac}{a + c}$, $ce \infty \frac{+ ay - 3cy + 4by + ac + cc}{a + c}$,

$da \infty \frac{4ayy - 4cyy + 8byy + 3aay + 3ccy - 2acy + 4aby - 4cby}{aa + 2ac + cc}$

$fa \infty \frac{4aab + 4abb - 4bbc + 4bcc + aay + 8aby + 16bby + 2acy + xy - 8bcy}{3aa + 3cc - 2ac + 4ab - 4bc + 8ay + 16by - 8cy}$.

Datis tribus punctis $a$, $b$, $c$, quæritur linea cujus ope radii omnes in vitro diſpoſiti, tanquam ſi venirent à puncto $a$, diſponantur egrediendo ejus ſuperficiem, cujux vertex ſit in puncto $c$, tanquam ſi venirent à puncto $b$, vel ſi tenderent verſus $b$; vel denique ut radii in aëre diſpoſiti, tanquam ſi venirent à puncto $a$, diſponantur in vitro, tanquam ſi venirent à puncto $b$.

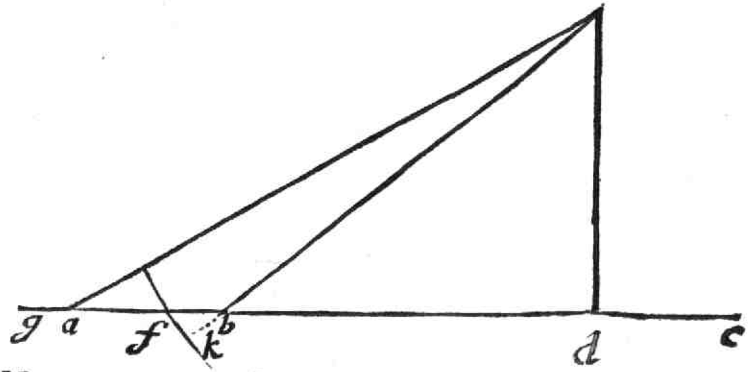
1. Cadat punctum $b$ inter $a$ & $c$, & $f$ centrum circuli Tangentis curvam cadat inter $a$ & $b$, ſi fiat $ae \infty a - y$ & $be \infty cy + b$, erit $fa$ ad $fl$ ut $+ y + a$ ad $ccy + bc$, hoc eſt inclinatio radii $ae$ in vitro ad inclinationem radii $be$ producti in aëre, ut $x$ ad $c$, idemque omnino continget ab aëre ad vitrum, ſi fiat 1. major quam $b$. Sed vero hic eſt error, valet enim tantum hæc linea, ad *reflexionem* inæqualem, non ad refractionem, quia punctum $f$ cadit inter $a$ & $b$.

Jam cadat punctum $a$ inter $b$ & $c$, eritque omnino idem genus lineæ, puncta enim $a$ & $b$ ſunt reciproca, & ſemper punctum $f$ erit inter $a$ & $b$, cum fiet $ae \infty a - y$ & $be \infty b + cy$. Sed fiat $ae \infty a + y$ & $be \infty b - cy$, tuncque punctum $f$ inter puncta $b$ & $c$, reperietur; ſed non videtur fieri poſſe, nec proinde hæc linea utilis eſt, ad regendas refractiones, ſed tantùm ad reflectiones, & redeundum ad alteram jam inventam, quæ tres habet focos. Imò punctum $f$ tunc poteſt cadere ultra punctum $a$ verſus $g$, & tunc pro certo linea ita deſcripta facit ut radii omnes tanquam à puncto $a$ venientes in vitro; poſt refractionem, quæ fit in ſuperficie, cujus vertex $c$, videantur veniſſe ex puncto $b$, vel contra ut in aëre radii à puncto $b$ venientes, ita refringantur in ſuperficie concava vitri, cujus vertex in $c$, ut videantur veniſſe ex puncto $a$.

Ponatur nunc $ae \infty a - y$ & $be \infty b - cy$, cadit $f$ inter $b$ & $c$ & tunc pro certo radii omnes ab $a$ venientes divaricantur in vitro, tan-

tanquam si venissent ex $b$, vel contra radii ab $b$ venientes in vitro, coguntur in aëre tanquam si venissent ex $a$.

$$ac \propto a,\ ae \propto a - y$$
$$bc \propto b,\ be \propto b + cy$$
$$dc \propto ayy - yy + 2ay + 2ay$$
$$de \propto \sqrt{.\ \dfrac{\begin{array}{l} -c^4 \\ +2cc \\ -1 \end{array} y^4 \begin{array}{l} -4bc^3 \\ -4acc \\ +4bc \\ +4a \end{array} y^3 \begin{array}{l} -4aa \\ +4bb \\ +4aacc \\ -4bbcc \end{array} yy \begin{array}{l} -4ab \\ -4abcc \\ -8abc \end{array} yy \begin{array}{l} +8aaby \\ -8abby \\ +8aabcy \\ -8abbcy \end{array}}{4aa - 8ab + 4bb}}$$




Nunc quæratur punctum $f$ quod sit centrum Circuli Tangentis curvam in puncto $e$, & fiat $fc \propto f$, $fd \propto \dfrac{\begin{array}{r} yy - cyy - 2bcy \\ -2ay + 2af \\ -2bf \end{array}}{2a - 2b}$

cujus $fd$ □tum si addatur □to $ed$, fit ▭lum

$$fe \propto \sqrt{\dfrac{\begin{array}{l} -4ab \\ +4bb \\ +4aacc \\ -4abcc \end{array} yy \begin{array}{l} +4af \\ +4bfcc \\ -4afcc \\ -4bf \end{array} yy \begin{array}{l} +8aab \\ -8abb \\ +8aabc \\ -8abbc \end{array} y \begin{array}{l} -8abcf \\ +8bbzf \\ -8aaf \\ +8abf \end{array} y \begin{array}{l} +4aaff \\ -8abff \\ +4bbff \end{array}}{4aa - 8ab + 4bb}}$$

Unde per generale Theorema ad inveniendas contingentes habeo

$$\begin{array}{l} -ab \\ +bb \\ +aacc \\ -abcc \end{array} y \begin{array}{l} +af \\ +bfcc \\ -afc \\ -bf \end{array} y \propto \begin{array}{l} -aab + abcf \\ +abb - bbcf \\ -aabc + aaf \\ +abbc - abf \end{array}$$

ac proinde linea $f$ sive quantitas lineæ $cf$ erit

$$cf \propto \dfrac{\begin{array}{c} -aby + bby + aaccy - abccy \\ +aab - aab + aabc - abbc \end{array}}{\begin{array}{c} -ay - bccy + accy + by \\ +aa - ab + abc - bbc. \end{array}}$$

$fa \propto$

$fa \infty -aay+2aby-bby+a^3-2aab+aabb$, dividendum ut supra.

$fb \infty aaccy-2abccy+bbccy+aabc-2abbc+b^3$ dividendum eodem modo, vel dividendo utrumque per $aa-2ab+bb$, fit $fa \infty -y+a$ & $fb \infty ccy+bc$, & ducendo $fa$ in $be$ fit $-cyy+acy-by+ab$ & ducendo $fb$ in $ae$ fit $-ccyy+accy-bcy+abc$. Ergo est $fb$ in $ae$ ad $fa$ in $be$ ut $c$ ad 1. hoc est ut $fk$ ad $fb$.

Cadat nunc $c$ inter $a$ & $b$ & $d$ inter $a$ & $c$, fieri potest ut $ae$ sit $a+y$ iterumque ut sit $a-y$ & $a^{\circ}$ sit $a-y$ & tunc una est ex lineis quæsitis, ponendo autem $ae \infty a+y$, punctum $f$ cadet ultra punctum $a$, nec proinde linea proderit ad hoc institutum, sed ad reflexiones inæquales.

Hic in secunda figura sit vertex lineæ curvæ $g$, ita ut $bg$ major sit quam $bd$, ponendo $ae \infty a+y$ & $be \infty b+cy$ fit $dg \infty \frac{ccyy+2bcy-yy-2ay}{2a-2b}$ cujus □tum brevitatis causa vocabitur $xx$ & fiet $de \infty \sqrt{-xx+\frac{ccayy-byy+2abcy-2aby}{a-b}}$ & sit punctum $h$, centrum circuli tangentis curvam in puncto $e$ fiet: $hg \infty \frac{accy-by+abc-ab}{ccy-y+bc-a}$ unde patet etiam quæsitum.

Nunc ex prima figura quæro duos alios focos curvæ inventæ, qui sint $g$ & $h$, & sumo $ge \infty g+cy-dy$, $he \infty h+y+dy$, $gd \infty g-x$, $hd \infty h-x$, unde quæro $x$ vel $dc$ & fit $dc \infty \frac{2dyy+yy+2cdyy-ccyy+2gdy-2gcy+2hdy+2hy}{2g-2h}$ quod æquatur cum priori $dc$, nempe $dc \infty \frac{ccyy-yy+2ay+2bcy}{2a+2b}$ ($d \infty c-n$, hoc est differentiæ quæ est inter proportionis terminos) & facio æquationem inter divisores, nempe $g \infty a+b+h$, deinde æquationem inter terminos $yy$, & denique inter terminos $y$, unde habeo $d \infty \frac{cc-1}{c+1}$ siquidem $c$ sit major unitate, ac deinde $g \infty \frac{acc+2bcc+2ac+2bc+a}{cc-1}$ vel linea $cg$ & $h \infty \frac{bcc+2ac+2bc}{cc-1}+2a+b$ vel linea $ch$ & linea $he \infty \frac{bcc+2ac+2bc+2a+b}{cc-1}+cy$ & linea $ge \infty \frac{acc+2bcc+2ac+2bc+a}{cc-1}+y$.

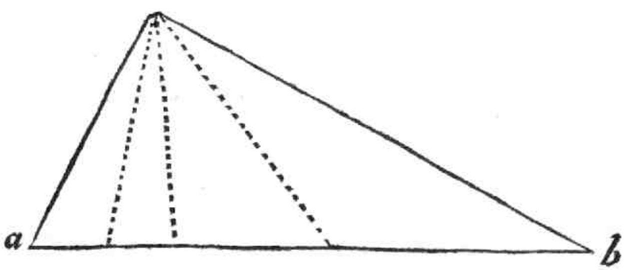
NB. $cg \infty \frac{ac+a+2bc}{c-1}$, $ch \infty \frac{2a+bc+b}{c-1}$ & tunc fit $gh \infty a+b$, si $a$ & $b$ sint æquales: fit $g \infty \frac{ac-a}{2c+2} \infty h$, $ac \infty a$, $ae$

$\infty a$

$\infty\ a+y,\ bc \infty b,\ bc \infty b+cy,\ dc \infty \frac{ccyy-yy+2ay+2bcy}{2a+2b}$

$$de \infty \frac{\sqrt{\begin{array}{llllllllll} -c^4 & & -4acc & & -4aa & & 4ab & & -8aab & \\ +2cc & y^4 & -4bc^3 & y^3 & -8abc & yy & 4bb & yy & -8abb & y \\ -1 & & +4a & & +4abcc & & 4aacc & & +8abbc & \\ & & +4bc & & -4bbcc & & & & +8aabc & \end{array}}}{4aa+8ab+4bb}$$

Iª. FIGURA.




Sitque $f$ in linea $acb$ inter $a$ & $c$ centrum circuli tangentis curvam, in puncto $e$, fit

$$fc \infty \frac{abccy+aby+bby+aaccy-aab-abb+abbc+aabc}{accy+bccy-ay-by+aa+ab+abc+bbc}$$

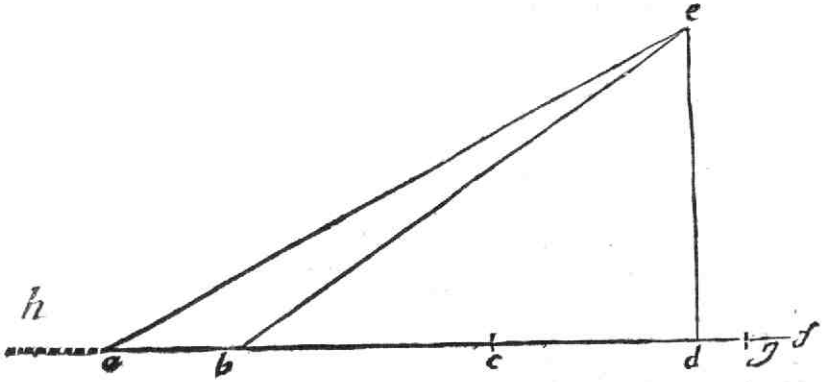
Unde clare demonſtratur omnes radios à puncto $b$ refractos in curva $ec$ tendere verſus $a$; vel contra tam in convexa, quam in concava figura; modo refractio corporis verſus $a$ ad corpus verſus $b$ ſit ut unitas ad $c$.

Fiat nunc $ae \infty a+y,\ be \infty b+cy,\ cd \infty \frac{yy-ccyy+2ay-2bcy}{2a-2b}$

$$de \infty \frac{\sqrt{\begin{array}{llllllllll} -c^4 & & -4bc^3 & & +4bbc & & +4aacc & & 8aabc & \\ +cc & y^4 & +4acc & y^3 & +8bc & yy & -4abcc & yy & -8abbc & y \\ -1 & & +4bc & & -4aa & & -4ab & & -8aab & \\ & & -4a & & & & +4bb & & +8abb & \end{array}}}{4aa-8ab+4bb}$$

IIda FI-

## II$^{da}$ FIGURA.




Et hic neceſſario punctum $d$ inter $f$ & $c$ vel $b$ cadit, atque habeo $fc \infty \frac{accy - by + abc - ab}{y - ccy + a - bc}$, $bf \infty \frac{accy - bccy + abc - bbc}{y - ccy + a - bc}$; $af \infty \frac{ay - by + aa - ab}{y - ccy + a - bc}$ quæ duo ſunt inter ſe ut $ccd + bc$ ad $y + a$; Porro ad enumerandas omnes ſpecies lineæ curvæ, quæ refractio nes ab uno puncto ad aliud diſponit, ſuppono ſemper $a$ majus, quam $b$, & $c$ quam $d$, & facio

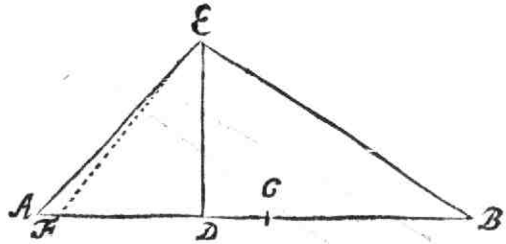
E $\infty\ b \overset{1}{+} cy$ vel $b \overset{2}{-} 1cy$. Deinde AE $\infty\ a + cy$ & BE $\infty\ b \overset{3}{+} dy$ vel $b \overset{4}{-} dy$. & AE $\infty\ a + dy$ & BE $\infty\ b \overset{5}{+} cy$ vel $b \overset{6}{-} cy$. Tandem E $\infty\ b + dy$ $\overset{7}{\text{vel}}$ $b - dy$.$^{8}$ Hic itaque ſunt 8 capita, ad quorum unumquodque conſiderandum an C vertex Curvæ ſit, vel B inter A & C, ac etiam an curvatura lineæ adſpiciat verſus A, vel contra.

C eſt inter A & B pro 1°. capite D cadet inter A & C, eritque DC $\infty \frac{ccyy - ddyy + 2bcy + 2ady}{2a + 2b}$ cujus □tum vocetur $xx$, eritque DE $\infty \frac{xx + accyy + bddyy + 2abcy + 2abdy}{a + b}$ & FC $\infty \frac{accy + bddy + abc - abd}{ccy - bdy + ad + bc}$

Pro 2° & 3° capite nihil hic reperitur, nec pro 6° & 8°, cum coïncidit cum primo; ſed permutatæ ſunt vices quantitatum $a$ & $b$.

Pro

Pro 5° capite linea est *Spiralis*, & 1°. quidem versus A curvatur, deinde versus $b$, nec utilis est refractioni, sed irregulari reflexioni tantum, imò clauditur.




Denique pro 7° capite figura quidem est oviformis, sed quia punctum F non cadit inter A & B, non est utilis ad refractiones, sed ad reflexiones irregulares tantum, & fit $CD \infty \frac{ccyy - ddyy + 2acy - 2bdy}{2a + 2b}$ $ED \infty \sqrt{-xx + \frac{addyy + bccyy + 2abcy + 2abdy}{a + b}}$

$CF \infty \frac{addy + bccy + abd + abc}{ccy - dy + ac - db}$ $AF \infty \frac{accy + bccy + acc + abc}{ccy - ddy + ac - bd}$

$BF \infty \frac{addy + bddy + abd + bbd}{ccy - ddy + ac - bd}$ $FC \infty \frac{bccy + addy + abc - abd}{ccy - ddy + bd + ac}$

Pro 5° capite, si D sit inter A & C fit $CD \infty \frac{ccyy - ddyy + 2bcy - 2ady}{2a + 2b}$ si inter B & C, fit $CD \infty \frac{ddyy - ccyy + 2ady - 2bcy}{2a + 2b}$ & in utroque est $DE \infty \sqrt{-xx + \frac{accyy + bddyy + 2abcy + 2abdy}{a + b}}$, ut in 7° capite.

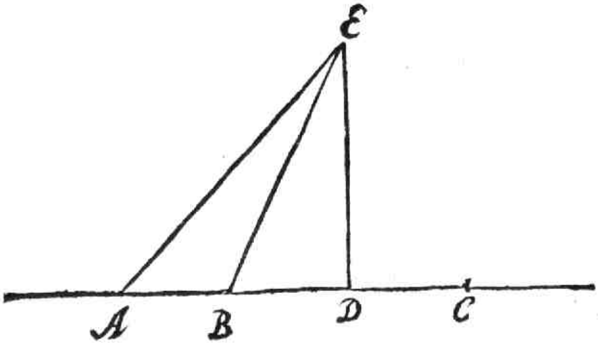
Pro 8° capite est $CD \infty \frac{ccyy - ddyy + 2acy + 2bdy}{2a + 2b}$

$DE \infty \sqrt{xx + \frac{bccyy + addyy + 2abcy - 2abdy}{a + b}}$.

Sit jam inter A & C in 1° capite est D inter B & C, estque $CD \infty \frac{ccyy - ddyy + 2ady + 2bcy}{2a - 2b}$ $DE \infty \sqrt{-xx + \frac{accyy - bddyy + 2abcy + 2abdy}{a - b}}$.

FC

$FC \infty \frac{accy - bddy + abd + abc}{ccy - ddy + ad + bc}$ & poteſt F eſſe inter A & B, vel A eſſe inter F & B.




Si primum ſit $AF \infty \frac{-addy + bddy + aad - abd}{ccy - ddy + ad + bc}$ nec eſt utilis niſi ad reflexiones.

Si ſecundum ſit $AF \infty \frac{addy - bddy - aad + abd}{ccy - ddy + ad + bc}$ & eſt ſemper $BF \infty \frac{accy - bccy - bbc + abc}{ccy - ddy + ad + bc}$. In tertio capite omnia ſunt ſimilia huic præterquam quod permutatæ ſint vices quantitatum $c$ & $d$, ſecundum autem deeſt, item $6^{tum}$, 7. & 8.

In 4° capite D eſt inter B & C & ſit $CD \infty \frac{ddyy - ccyy + 2acy - 2bdy}{2a - 2b}$ $DE \infty \sqrt{-xx + \frac{addyy - bccyy + 2abcy - 2abdy}{a - b}}$ F poteſt eſſe inter B & C, eſtque $FC \infty \frac{bccy - addy + abd - abc}{ccy - ddy - ac + bd}$ & $BF \infty \frac{-bddy + addy + bbd - abd}{ccy - ddy - ac + bd}$ $AF \infty \frac{accy - bccy - aac + abc}{ccy - ddy - ac + bd}$ vel A & B ſunt inter F & C, eſtque $FC \infty \frac{-bccy + addy - abd + abc}{-ccy + ddy + ac - bd}$ $BF \infty \frac{addy - bddy + bbd - abd}{-ccy + ddy + ac - bd}$ $AF \infty \frac{accy - bccy - aac + abc}{-ccy + ddy + ac + bd}$.

*FINIS.*

# INDEX EXCERPTORUM.



## *Corrig.*

In figuris *pag.* 7. Recta V K ducenda eſt parallela ipſi E D. & *pag.* 8. Curva à puncto R deſcendens, in punctum D terminari debet; tangenſque M O cadere in punctum M, in quo Quadratrix A M C Quadrantem Z M D ſecat.