Photographic measures of close double stars

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D. REUIJL.

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PROEFSCHRIFT

TER VERKRIJGING VAN DEN GRAAD VAN DOCTOR IN DE WIS- EN NATUURKUNDE AAN DE RIJKS-UNIVERSITEIT TE UTRECHT, OP GEZAG VAN DEN RECTOR MAGNIFICUS DR. L. S. ORNSTEIN, HOOG-LEERAAR IN DE FACULTEIT DER WIS- EN NATUUR-KUNDE, VOLGENS BESLUIT VAN DEN SENAAT DER UNIVERSITEIT TEGEN DE BEDENKINGEN VAN DE FACULTEIT DER WIS- EN NATUURKUNDE TE VER-DEDIGEN OP

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AAN MIJN VADER EN MOEDER
AAN MIJN VROUW

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§ 1.

INTRODUCTION.

In the past 35 years photography has become one of the most powerful aids to the study of astronomy. It has been developed to such an extent that at the present time its application covers almost all fields of astronomical research.

One of its last conquests is double star astronomy.

The earliest photographic observation of a double star was made by Bond in 1858 with a 15 inch equatorial telescope at the Harvard College Observatory. From a number of plates the distance and position angle of Mizar were derived.

Since then there has been relatively little progress. At the present time the number of photographic measures is still small as compared with visual observations at the micrometer. This may partly be ascribed to the fact that with the same telescope the limiting separation is greater for the photographic plate than for the micrometer.

In the observation of photographically close doubles peculiar difficulties arise as is also the case in the visual observation of micrometrically close pairs.

The great advantage of photographic measures over visual ones has clearly been shown by the results of Hertzsprung [1]) (1920) and others. The accuracy of the distance obtained from one image only is about the same as the mean of micrometer observations on 3 to 4 nights.

Micrometer observations very often are seriously affected by systematic or personal errors. In photographic work errors of this kind — at least for the wide pairs — are practically negligible.

However this absence of systematic errors holds only for pairs of separation larger than about $^{mm}.15$ on the photographic plate. Below this limit the measures are affected by errors due to the action of repulsive and attracting forces.

The matter has been studied or mentioned in a number of publication [2]) K o s t i n k s k y states the presence of systematic publication K o s t i n s k y states the presence of systematic errors occurring in measures of satellites. Quoting from page 150:

„.... pour deux images *très voisines* et *très différentes* en dimension, une sorte d'influence d'une image sur l'autre .... doit affecter spécialement la distance mutuelle apparente entre le satellite et la planète."

In a second publication [3]) (1909) Kostinsky states that measures of close doubles taken with different exposure times show an increase in distance with growing exposure time. For images overlapping $^{mm}.014$ the repulsion amounts to + $^{mm}.020$. Expressed as a function of Di, the distance between the inner borders, the repulsion is given by:

$$7.9\,(1 - e^{\frac{80-Di}{100}})\text{, the unit being one micron.}$$

Kostinsky puts forward three hypotheses, finally combining two of them as the most probable explanation of the phenomenon:

„....dass die Centra infolge einer *reellen* Deformation der Bilder von unserem Auge unwillkürlich in entgegengesetzten Richtungen verrückt werden,...." (p. 27, 28).

According to R o s s [4]) (1924) three effects are likely to occur:

1. turbidity effect,
2. gelatine effect,
3. „Kostinsky" effect.

The first two will make the distance too small, the third one causes a repulsion.

1. The turbidity effect is due to the interaction of the light of the two images. This causes an elongation of the images at their adjacent sides. The distance of the centers will thus be measured too small when bisections on the pear shaped areas are made.

The amount of the attraction is dependent upon

a) the distance Di of the inner borders, the attraction increasing with decreasing Di,

b) a factor $\triangle$, this being the growth $\triangle$ (in microns) of the diameter of the image with doubling of the exposure time.

Assuming that Scheiner's equation holds for all exposure ranges in question, $\triangle$ is defined by

$$\triangle = \Gamma \log 2$$

the term astrogamma Γ being proposed by Ross to denote the coefficient of log E in

$$\text{diameter} = a + \Gamma \log E.$$

Ross has shown that the equation of Scheiner can be readily deduced from Bouguer's law of intensity variation

$$I = I_0 e^{-\varkappa x}$$

$\varkappa$ being
$$\varkappa = \frac{2 \log 2}{\text{Mod.}} \cdot \frac{1}{\triangle}.$$

Thus $\triangle$ is a measure of the seeing or definition. Its influence on the turbidity attraction is such as to have an increase in attraction with growing $\triangle$, i. e. with declining seeing.

The attraction reaches a maximum value amounting to ½ $\triangle$ in the case of contact. Under average conditions $\triangle$ amounts to 30 microns. The corresponding maximum attraction in the case of contact is —^{mm}.015.

2. The attraction caused by the gelatine effect is due to the contraction during the process of drying, the adjacent images behaving as one single image. Excluding pyro metol, Ross finds the amount of contraction to be independent of the class of developer. Moreover the contraction proves to be independent of the separation within the limits of the experiment. For doubles of separation ^{mm}.1 or less the effect amounts to —^{mm}.0016 in the average.

3. The repulsive action of the Kostinsky effect must be ascribed to a peculiar reaction of the developer. The reaction products spreading outwards from the centers of

the images will evidently be twice as concentrated, roughly speaking, in between the two images as in the neighbourhood of the outer borders.

Hence the inner borders will suffer from this deficiency of the developer in the intermediate space and the images will get flattened at the adjacent sides. This flattening is very often apparent to the eye. It causes an increase in distance when bisections on both images are made.

It is reasonable to assume that the Kostinsky repulsion increases with increasing size and density of the image and with decreasing distance Di. Thus for one and the same star the repulsion will grow with increasing exposure time. The assumption proves to be justified in view of the results of Ross and others.

It will be readily seen that the Kostinsky effect may be considered a consequence of the Eberhard effect. According to Eberhard [5]) (1926) the developer affects the density of an image. Of two images of equal surface intensity but of unequal size the larger will be of smaller density. Assuming the same chemical or physical process to occur in the development of the double star image, the distortion of the inner borders can thus be readily accounted for.

As the small gelatine effect is practically constant and the turbidity action depends on the distance of the inner borders and the definition only, Ross is able to apply corrections for both. The corrected distances will then only be affected by the Kostinsky repulsion, assuming Ross' results and theory to be correct.

In some of his experiments Ross finds the Kostinsky repulsion to be + ^{mm}.015.

For further information concerning the above mentioned effects the reader is referred to the quoted publications and to § 6 of the present publication.

In connection herewith attention is called to possible further effects of physiological nature.

a) There is little doubt that the bisection of the component of a double image will be influenced by the presence of the neighbouring image. It is a difficult matter to decide to which extent the bisections will be affected, the sign of the error being even uncertain.

b) Moreover the question is put forward whether bisection errors of a pear shaped object occur in the sense as mentioned under (1).

Evidently we must discriminate between two effects in this case. The first is the already mentioned turbidity effect, the addition of blackened grains in the intermediate space, causing the images to extend towards each other. The second one is the physiological error, varying from one observer to the other. For a certain observer it may be such as to have bisections made outside of the centers, the boundary lines being a family of lemniscates as indicated by Ross. In view of various results it seems that generally the turbidity attraction overshadows this particular type of physiological error.

For the sake of completeness and convenience some other researches concerning the problem will shortly be mentioned.

Lau [6]) (1912) has measured the artificially produced double stars of a region which was exposed twice with a shift of 11″ — i. e. $^{mm}.177$ on the plate. He finds an apparent repulsion for greater intensity, the quantitative agreement with Kostinsky's measures being very close also.

Bellamy [7]) (1917) has investigated 436 doubles of the Oxford Astrographic Catalogue by comparing the distances with the visually determined values in the Burnham General Catalogue. He finds the distances too small, the errors growing with decreasing distance.

Mitchell and Olivier [8]) (1920) have measured the distance of Krueger 60 on 27 plates, extending over a period of 3 years, the distance decreasing from 2″.4 to 1″.8. Comparison with Barnard's micrometer measures shows neither attraction nor repulsion.

Hertzsprung [1]) (1920) finds that for close pairs both attraction and repulsion may occur. The first in the case of faint fuzzy images, the latter for sharp well exposed images.

Van den Bos [9]) (1923) finds negative differences photographic minus visual below 4″.5 — i. e. ^{mm}.114 on his plates — increasing with diminishing distance of the components.

From laboratory experiments Przybyllok and Labitzke [10]) (1929) find neither attracting nor repulsing effects.

The results of Shajn [11]) (1927) are in close agreement with the laboratory data obtained by Ross. All images being overexposed, the effect of the Kostinsky action on the distance measurements is very pronounced.

It may be mentioned in connection with this particular problem concerning the distance between double star components that similar results were obtained for close spectral lines. In some cases the distances turn out too large, in others attracting forces must have acted upon the images.

Measuring réseau lines intersecting at small angles Turner [12]) (1917) finds attraction near the point of intersection.

An attempt is made (§ 7) to interpret the results mentioned above by closer examination of the methods applied to obtain the data and discussion thereof.

§ 2.

PROGRAM.

The double star program in progress at the Leander McCormick Observatory consists mainly in photographing double stars with the purpose to determine absolute positions, i. e. the positions with respect to sets of comparison stars. In this program the number of close pairs was relatively small owing mainly to the observational difficulties, e. g. the dependence on the seeing and the streneous guiding. Moreover in view of the knowledge of possible systematic errors the closer pairs were avoided for this reason also.

The extension of the program towards the closer pairs, suggested to me by Dr. van de Kamp, seemed a suitable subject for a dissertation. The work involves additional labour as has been mentioned and requires much patience from the observer.

Thus the plan originated to photograph and measure these close pairs and to investigate the measures for systematic errors.

During the summer of 1929 a number of plates was taken under such exposure times as to secure a suitable set of comparison stars on the plate. By means of a rotating sector the brightness of the double was cut down to the mean magnitude of the comparison stars.

The intercomparison of the measures of this first series showed discordances in the distance measures, the latter evidently being affected by the Kostinsky effect. Comparison with micrometer measures revealed an other kind of errors, the photographic distances being smaller for the closer pairs. An analogous treatment of Hertzsprung's measures yielded similar results, in good quantitative agreement with my own measures. The preliminary results were presented at the forty-third meeting of the American Astronomical Society in a paper, the abstract of which appeared in Popular Astronomy **38**, 406; 1930.

The following results were derived:

Table 1. McCormick material.

ρ_{Micr}	$\rho_{Ph} - \rho_{Micr}$	N
$1''.5 - 2''.4$	$-''.10$	16
$> 2''.4$	$+''.01$	4

$1^{mm} = 20''.8$

N is the number of stars. The total number of images amounts to 116.

Table 2. Potsdam material.

ρ_{Micr}	$\rho_{Ph} - \rho_{Micr}$	N
$1''.0 - 2''.0$	$-''.09$	19
$> 2''.0$	$+''.02$	30

$1^{mm} = 16''.4$

The McCormick material was devided into underexposed, normal and overexposed images. Thus the following values of $\rho_{Ph} - \rho_{Micr}$ were derived:

Table 3. McCormick material.

	$\rho_M < 2''.4$	$\rho_M > 2''.4$
1. underexposed	$-''.14$	$+''.02$
2. normal	$-''.02$	$-''.04$
3. overexposed	$+''.14$	$+''.05$

Group (1) shows the influence of the attracting effects only, as the Kostinsky effect is expected to be small for weak images.

The increase of $\rho_{Ph} - \rho_M$ with increasing exposure time (Kostinsky effect) — particularly in the case of the closer pairs — is very pronounced.

In view of these results it was decided to secure a set of plates on close pairs exposed in the same manner as performed by Hertzsprung [1]) (1920). In connection with this work extensive measurements in the laboratory should be performed in order to derive the absolute amount of possible photographic effects, either repulsing or attracting. A description of the method and some preliminary results thereof may be found in paragraph 8. Due to a course of circumstances this work had to be stopped for the time being. Hence the material obtained at the telescope had to be treated in a somewhat different manner than was intended. Instead of correcting the double star distances by means of the corrections derived from experiments on artificial pairs the corrections had to be found from the telescopic material itself, comparing the latter with micrometric determinations.

It will be readily seen that this method has a fundamental drawback compared with the laboratory method. In the latter the „true" distance between the components can be determined and used. From double star photographs in most cases no such true value can be found as all images, whether under- or overexposed, may be affected by systematic errors. Therefore the value adopted as the true distance is the one determined from micrometer observations. It is known that the latter are affected by accidental errors of considerable size and often by systematic or personal errors of the same order of magnitude. The probable error of the mean of distance observations made on 3 to 4 nights is of the order of ″.1. This error may be decreased by increasing the number of nights, but exhibits only the internal agreement of the measures as was e.g. pointed out by van den Bos [13]) (1925).

The accidental plate error, i.e. the error of the distance of a single image, is about ″.1 on McCormick plates. Therefore the accuracy of the mean distance derived from a large number of images will be very high as compared with the accuracy of a micrometric distance as determined by one and the same observer.

Up to the present time our knowledge of the systematic errors of micrometer observations, particularly of the recent ones, is relatively small. It was therefore decided to treat all observations as if affected by accidental errors only and use a large number of determinations by different observers. It proved to be advisable to plot both recent and earlier observations as most pairs, even those marked „fixed" by Burnham, show some change in distance in the course of time. Extrapolating to the epoch of the photographic observation gives a value for the distance which may be used as the „standard" value for comparison with the photographic distances. (see p. 32).

These standard distances are free from errors of the kind which may affect the positions of close photographic images.

The number of comparisons must be sufficiently large. In that case it is assumed that the standard values will be free from systematic errors.

In the selection of stars for the observing program I have limited myself to doubles whose components are of about the same brightness. For nearly all stars the difference in brightness between the components is less than $^{m}.5$. To exceed this limit would tend to introduce a magnitude error, which on account of its often very troublesome influence should be avoided by any means.

In connection with the exposure times stars of about the seventh magnitude (photovisual) are the most suitable. Hence stars of magnitude 9^{m} requiring exposure times about ten times as large were taken in exceptional cases only. For a few stars brighter than 7^{m} photographs were obtained with the aid of the rotating sector.

Thus the program was made up applying the above mentioned criteria in the first place. Secondly preference was given to stars on which photographic measures had been made by others and to those for which a large number of recent micrometer observations was available.

As no special attention was paid to such properties as orbital motion, common proper motion, etc. not all stars on the program are interesting ones from this point of view.

For reasons which will be mentioned in the next paragraph the selection favors stars of high declination.

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§ 3.

OBSERVATIONAL MATERIAL.

a) The 1930 material.

The plates were taken with the 26 inch visual refractor of the Leander McCormick Observatory of the University of Virginia. A description of the instrument may be found in „Parallaxes of 260 Stars", by S. A. Mitchell (Columbia University Press, 1920). A colour filter Wratten No. 12, minus blue, in front of the plate admits light of wave lengths between 5300 Å and 5900 Å to the plate, the maximum sensitivity being at about 5550 Å, when Cramer Isochromatic plates are used. All plates were taken as near to the meridian as possible, the greatest hour angle being smaller than one hour East or West. As the mounting of the telescope on the pier is fairly light the former is rather sensitive to wind. Therefore preference was given to stars of high declination. This selection furthermore offers the advantage of making the necessary guiding on account of irregularities of the driving clockwork less troublesome.

Table 4 gives the number of stars for different declinations.

Table 4.

decl.	number of stars
< 0°	5
0 — 20	19
20 — 40	19
40 — 60	15
60 — 80	3

The geographic latitude of the observatory is $+ 38° 2' 1''$.

The table shows that 56 % of the doubles, exposed on 51 % of the total number of plates, were taken within about 20 degrees of the Zenith. The telescope is in constant use during all clear nights for various classes of research. The time available for the present work was limited to hours during the middle part of the night and had to be made as profitable as possible. Therefore a card system was made, arranged so as to give the necessary information at the telescope. After having pointed the latter approximately the field in the finder was compared with the field as copied from the Bonner Durchmusterung charts and the direction accordingly adjusted.

Next the double star was set near the centre of the field of the large telescope and a suitable guiding star selected.

Most of the exposures were made on Cramer Iso Presto plates as shown by Table 5.

Table 5.

Plate numbers	Emulsion
1 — 9	Cramer Isochromatic 12394
10 — 56	„ Iso Presto 27801
57 — 123	„ „ „ 27821

The exposures were made by giving the telescope small shifts in declination and after completion of a row of 10 to 12 images shifting in right ascension, and so forth. The equipment of the telescope does not include a shutter. Therefore the beginning and the end of each exposure were performed respectively by quickly putting the guiding star on the cross of wires in the guiding eyepiece and moving it off.

The time required for either manipulation is less than half a second and therefore small compared even with the shortest exposure time (12 sec.). In fact most images do not show any guiding error at all. Images having tails whether due to this cause or to poor guiding were rejected before measurement. Or, if measurable yet, the measures were given smaller weight. Although the instrumental part of the work was performed with the utmost care, this does not prevent however certain images from being affected by atmospheric influences, errors of telescope and plates or anomalous action of the developer. Yet one should remember that the measurable images among all exposures were selected after development.

Finding as a rule that the images did not suffer from the above mentioned way of exposing I applied this method without hesitation for all exposures.

Having made the last exposure the clock work of the telescope was stopped and a bright star allowed to trail over the plate.

For exposure times the geometric series 12—15—19—24—30—38—48—60—... sec. (Hertzsprung [1]); 1920) was used. The initial exposure time is dependent upon the star's apparent magnitude, the transparency of the atmosphere and the seeing, whereas the total range was chosen according to distance and seeing. The numbers of doubles, plates and images for different distances and qualities are grouped in Table 6.

The qualities of the plates indicate to which extent the plates answer the purpose to which they were taken. In a few cases for instance plates were purposely taken under rather bad seeing (see p. 36). Therefore the seeing is not the main factor determining the qualities of plates and images.

In this connection it seems worth mentioning that most plates were taken under good and excellent or even „perfect" conditions of seeing (see p. 40).

The average number of images per plate is about 25, the maximum and minimum numbers are 146 and 1 respectively.

The qualification of the plates was obtained by examining the plates after development, using a magnifying glass of low power. The numbers of images were taken from the sheets with the measurements.

An attempt was made to obtain fairly equal numbers of doubles, plates and images for the different distance ranges in the first column. In view of the dependence of the observer on the atmospheric conditions the result is satisfactory.

Table 6.

Distance	Stars	Plates				Images			
		e	g	fg	total	e	g	fg	total
< 2″.0	7	4	6	3	13	79	154	129	362
2.1 — 2.5	10	12	3	1	16	51	128	150	329
2.6 — 3.0	9	13	5	0	18	111	166	162	439
3.1 — 3.5	6	10	1	2	13	169	98	122	389
3.6 — 4.0	8	12	3	2	17	49	125	193	367
4.1 — 4.5	4	7	4	1	12	66	136	136	338
4.6 — 5.0	5	5	1	1	7	116	56	52	224
> 5.1	9	9	5	3	17	75	159	152	386
Total	58	72	28	13	113	716	1022	1096	2834

e = excellent.
g = good.
fg = fairly good.

As developer *Carbonal* (Hauff) was used. Plates 1—6 were developed in a tank, the solution being

Carbonal	½ oz.
Water	64 „
Potassium bromide . . .	30 grains

Time: 30 minutes at 65° F.

Plates 7—123 were developed in a tray, the formula used being as follows:

Carbonal	¼ oz.
Water	8 „
Potassium bromide . . .	30 grains

Time: 10 minutes at 65° F.

The formula of the fixing bath is:

A.	Water	102	oz.
	Hypo	26	„
B.	Water	26	„
	Sodium sulphite (dry) . .	$2^3/_8$	„
	Sulphuric acid C.P. . . .	$^3/_8$	„
	Chrom alum	$1^5/_8$	„

A and B mixed in given proportion and rotation, B poured in A while stirring so as to avoid precipitation.

b) The 1929 material.

The plates used in the preliminary investigation, mentioned in § 2 have been taken along in the present discussion. The total number of images is small, the numbers of plates and stars are relatively large.

Cramer Isochromatic plates were used throughout.

The arrangement of Table 7 is similar to that of Table 6. The table is selfexplanatory.

Table 7.

Distance	Stars	Plates				Images			
		e	g	fg	total	e	g	fg	total
$< 2''.0$	12	4	19	10	33	12	30	34	76
2.1 — 2.5	5	2	10	3	15	7	21	11	39
> 2.6	4	7	2	3	12	13	6	6	25
Total	21	13	31	16	60	32	57	51	140

Nearly all plates of this set were taken under excellent seeing.

Various developers were used:

Developer	Plate
Hydrochinone	25922—3—4
(Determinations of Stellar Parallax, diss. Sten Asklöf)	25946—7—8
	25968—9
A. { Water . . . . 35 oz.	26008—9
Hydrochinone . . . . $1\frac{1}{2}$ „	26011
Sodium sulphite . . . 1 „	26026—7—8—9
	26030—1—2
B. { Water . . . . 35 „	26044—5—6
Sodium carbonate (dry) 1 „	26060—1—2—3—4
Potassium carbonate . 3 „	26150—1
Potassium bromide . . $\frac{1}{4}$ „	26181
Sodium sulphite . . . 3 „	26198—9
1A and 1B.	
Elon Quinol (Eastman Kodak) 2 tubes Water 8 oz. 70° F.	25849—50
Elon Quinol (E. K.) 1 tube Water 8 oz. 70° F.	25851—2
"Special cartridge" (E. K.) 2 tubes Water 8 oz. 65° F.	25861—2
Elon Quinol (E. K.) 1 tube Water 8 oz. Potassium bromide 1 : 16 60 drops 62° F.	25879
Elon Quinol (E. K.) 1 tube Water 8 oz. Potassium bromide 1 : 16 60 drops 63° F.	25880

Elon Quinol (E. K.) 1 tube		25881
Water 8 oz.		
Potassium bromide 1 : 16 15 drops		
64° F.		

Moffitt's developer			25926
A.	Elon	52 grains	
	Sodium sulphite (dry) .	270 "	
	Distilled water to make	120 oz.	
B.	Sodium carbonate (dry)	85 grains	
	Potassium bromide . .	5 " *	
	Distilled water to make	120 oz.	
1A and 1B.			

* It proved necessary to use 20 grains in order to prevent fogging.

Hauff's tank developer			24004
Water	32 oz.		24223
Metol	1/8 "		24371—2
Hydrochinone . . .	5/8 "		24416—7
Sodium sulphite (dry)	4 "		24452—3
Sodium carbonate . .	1½ "		25085
Pyro	½ "		
Potassium bromide .	1/16 "	(30 grains)	
Proportion 1 : 20.			
30 min. at 65° F.			

Carbonal (tank)	25808—9
	25830
	25847—8
	26023
	26129

§ 4.

MEASURES.

a) The 1930 material.

The plates have been measured on a Repsold measuring engine. This instrument, belonging to the Observatory of Leiden and temporarily used at the Astronomical Institute of Amsterdam, was kindly put at my disposal through the courtesy of Professors Dr. W. de Sitter and Dr. A. Pannekoek.

A description of the instrument may be found in

Sande Bakhuijzen, van de. Mesure des Clichés d'après la méthode des coordonnées rectangulaires. Bulletin du Comité international permanent pour l'exécution photographique de la Carte du Ciel. Tome I, p. 164—204. 1892.

Nijland, A. A. Uitmeting van den Sterrenhoop G.C. 4410 (dissertation, 1897; Dutch).

The periodic errors of the micrometer screw have been investigated by measuring the distance of two specks on a plate about $^{rev}.1$ apart. Table 8 gives the results expressed in thousandths of a revolution as a unit ($1^{rev} = {}^{mm}.085$). The first column contains the starting points on the screw, the second one the corresponding distances, whereas the differences with the mean appear in the last column. The probable error of a distance is 2.6 expressed in the same unit.

Each value in the second column is the mean of 2 distance measurements. In all cases 5 successive settings on a speck have been made.

It is evident that the periodic screw errors can be neglected, in agreement with the results obtained by Nijland [14]). As to the progressive error, judging by Nijland's measures and taking into account the small distances to be measured, a redetermination of this error seemed not necessary.

All measures were performed at Utrecht during the months

Table 8.

0	97	−2
1	99	0
2	97	−2
3	102	+3
4	102	+3
5	102	+3
6	100	+1
7	98	−1
8	98	−1
9	98	−1
Mean	99	

March till July of 1931. To illuminate the plate artificial light was used only, the light source being an ordinary electric bulb. Underneath the plate a piece of lense paper was placed so as to diffuse the light reaching the plate. The magnifying power of the microscope was about 35 times.

The plates numbered R 2, 10, 13, 17, 24, 28, 33, 40, 41, 43, 54, 83 and 89 were measured on a Gaertner measuring machine in Virginia also. This long screw engine is described in „Parallaxes of 260 Stars" by Mitchell, whereas the description of a slightly different type may be found in Publications of the Allegheny Observatory, Vol 3, No. 11; 1916.

The examination of the screw resulted in finding both periodic and progressive errors to be negligible. The measures were made from January till June of 1930, with the purpose of getting some preliminary results. It was then found that a magnifying power of about 30 times is the most suitable. The grain structure of the image is well visible, whereas the image is not too large for bisection.

As regards the Repsold machine its circle carrying the plate was investigated for excentricity. The accuracy required in the determination of position angles as an additional result is relatively small. Hence excentricity errors must be large to be of any influence. As this was not the case small errors, if at all present, were neglected and one microscope read only.

The measurements were performed as follows.

Firstly the circle positions for the trail parallel to the horizontal and vertical wire were read. After substracting 90 degrees from the vertical reading the mean was taken and called trail reading.

Then the plate was oriented in such a manner as to have the line joining the centers of the images parallel to the horizontal wire. The advantage of this method is that the distances are always measured in a similar manner which is not the case when measurements are made in rectangular coordinates, e.g. in the direction of right ascension and declination. From a comparison of the residuals Hoff [18]) (1929) found the measurements in distance to be more accurate than the measurements made in rectangular coordinates.

For every suitable image the plate was oriented by hand and the circle read. This was repeated after reversing the plate 180 degrees. The mean of all these readings was taken as the orientation of the plate for the distance measurements of all images. The distances run from 1″.5 to 5″.5, 1 mm. corresponding to 20″.8 on the plate. The error of an orientation amounts to 0°.5 as determined from good normally exposed images. Therefore any errors introduced in the distances by applying this method are negligible.

Next the microscope was pointed at the millimeter scale and the distance between the lines 61 and 60 determined in thousandths of a screw revolution. The distance between the components of the double thread is slightly larger than the width of a line. This assures an accurate setting. On each line 3 settings were made. In the average 1 mm corresponds to $11^{rev}.75$.

Next the images were examined and estimated excellent, good or fairly good. As to the shape of the double image a scale was used according to which the images were classified as follows:

0 — no blackened grains between the images.
a — some " " " " " .
b — a bridge of " " " " " .
c — the images in „contact".
d — " " „blended", i.e. overlapping.

Then all suitable images were measured. Each image was bisected twice with the single thread and the screw was read in thousandths of a revolution. This was repeated with the plate rotated 180°. Furthermore in both positions of the plate single settings were made on all 4 borders of the double image. These measures were recorded in hundredths of a revolution.

The temperature in the room was kept at about 67° F. during all measurements.

All images having been measured the distance between the lines 61 and 60 was measured once more.

Information concerning the error of bisection and the error of a setting on a line of the scale will be found in the next paragraph.

b) The 1929 material.

This series of plates was measured on the above mentioned Gaertner machine. The method of measurement was the same as that applied to the 1930 material.

§ 5.

ERRORS.

1. The errors of *measurement* are classified as follows:

a) The error of bisection ε_{bi}.
Investigation of the material gives for the groups of excellent, good and fairly good images as the probable error of one bisection .54, .65 and .74 μ respectively, the weighted average being .66 μ.

Thus in the distance measurement of one image probable errors are introduced amounting to .38, .46 and .52 μ respectively, the weighted average being .47 μ.

b) The error of setting on a border ε_{bo}.
This error has been determined from measures of the distance between the outer borders Do. The error introduced in the measurement of one Do by the error of one setting on a border ε_{bo} is equal to the latter. The probable values determined are 2.5, 2.7 and 3.4 μ respectively, the weighted average being 2.9 μ.

c) The error of setting on a line of the millimeter scale ε_{l}.
The probable error of one setting on a millimeter line amounts to ^{rev}.0061. Thus a probable error of ^{rev}.0035 or .3 % is introduced in the value of a 1 mm interval as expressed in screw revolutions.

The unpublished results of a determination of the scale value of the McCormick plates were kindly put at my disposal by Dr. Vyssotsky.

The value used is 1 mm = 20″.748, the value of the probable error being about ″.0005.

2. *Image* and *plate* errors.

The systematic errors are discussed in full in the next paragraph. A table of corrections has been derived by investigating all available material. Application of the corrections for the images of a same plate will introduce an error ε_c which may affect both image errors ε_I and plate error ε_P. This will be readily seen as the corrections C have been derived from all plates.

Regarding the remaining errors after correction as purely accidental the error of one image may be expressed by

$$\varepsilon_1 = \sqrt{\varepsilon_I^2 + \varepsilon_P^2 + \varepsilon_c^2 + \left(\frac{\varepsilon_{bi}}{\sqrt{2}}\right)^2 + \varepsilon_v^2},$$

ε_v representing the other errors of measurement namely ε_1 and the errors of the micrometer screw.

A comparison of the corrected distances on a same plate gives the internal error

$$\varepsilon_i = \sqrt{\varepsilon_I^2 + \varepsilon_c^2 + \left(\frac{\varepsilon_{bi}}{\sqrt{2}}\right)^2 + \varepsilon_v^2}.$$

ε_v is small with respect to the other errors and can be neglected.

Thus $\sqrt{\varepsilon_i^2 - \left(\frac{\varepsilon_{bi}}{\sqrt{2}}\right)^2} = \sqrt{\varepsilon_I^2 + \varepsilon_c^2}$ represents the image error after correction in which expression ε_c is small as compared with ε_I, the pure image error.

As to the plate error, for its determination a large number of plates on one and the same star is needed. However the largest number occurring is 4. Therefore neither ε_P, the pure plate error, nor the plate error which is inherent in the corrected measures can be determined with sufficient accuracy.

The intercomparison of curves of different plates on a same star has been made relative to the corresponding values of $\triangle$ (see p. 37). A marked dependence on $\triangle$ has been shown.

Correcting the curves by means of this relation offers the opportunity to derive the order of magnitude of the pure plate error ε_P.

For the construction of the table of corrections in § 6, e the material has been devided into 3 groups of $\triangle$. Thus slight errors are introduced which are partly responsible for the errors ε_c.

Finally the order of magnitude of the pure image error ε_I may be determined with sufficient accuracy from the distances between individual points and curve on the plots mentioned on page 27.

§ 6.

DISCUSSION OF THE MEASURES.

a) The relation between ρ and Di.

If there is such a phenomenon as the interaction of two neighbouring images, a reasonable *assumption is that this* interaction depends upon

1. the distance of the inner borders Di,
2. the distance between the centers ρ,
3. the sharpness of the image.

The third factor is governed mainly by the seeing or definition and may be expressed numerically, e.g. by using a personal scale from 0 to 5, 5 representing the condition of perfect seeing, i.e. maximum sharpness. Since however not the seeing itself but its effect on the plate is the essential criterium, preference must be given to the $\triangle$ scale, $\triangle$ being the growth in microns of the diameter of an image with doubling of the exposure time (see p. 3).

For one and the same plate $\triangle$ is assumed constant. Hence it seemed advisable to investigate graphically the relation between ρ and Di, each plate giving a plot.

As has been stated the presence of at least three influences may be expected. The attracting forces are the turbidity and gelatine effects, respectively due to the interaction of the light of the two images and to the contraction of the image during its drying process. The first increases with decreasing Di, the second is independent thereof. The repulsing force is caused by the developer, the latter being deficient in between the two images. This repulsion grows also with decreasing Di.

As to the developer effect one seems justified to assume that it will be negligible in the case of underexposed images. Hence for these images systematic errors, if any, will be negative — i.e. $\rho_{Ph} - \rho_M < o$. As a matter of fact the plots show this phenomenon in many cases. It appears that already for fairly blackened images the developer repulsion outgrows the attractions and this difference continues to increase with increasing exposure time, i. e. with approaching adjacent borders. However, when the exposure time is such that the condition of contact or slight overlap is reached, the rate of increase of the measured distance diminishes and the curve, instead of keeping its resemblance to an exponential function, starts to bend down in the direction of the Di axis.

It will be readily seen that not every plate gives a curve like the sinuous one just mentioned. In fact many different types have been found. If the number of images and the range of exposure times are sufficiently large all close pairs show the above mentioned relative increase of ρ with decreasing Di. This may then in the simplest case be a linear relation. As to the absolute dimensions of the errors these are uncertain for a single plate as a consequence of the uncertainty in the extrapolated micrometer distance.

In Fig. 1 the plot of R 61 is shown. This plate on 44 Bootis was taken on the 26th of April, 1930.

The seeing was estimated to be 4—5, the transparency of the atmosphere was called 3 on a scale on which 5 denotes maximum transparency. The sky was called „clear" and the thermometer read at 51° F.

The total number of images amounts to 146 of which 42 images are excellent, 56 good and 48 fairly good.

The probable error of a single image, derived from the deviations of the plotted points from the curve, amounts to 3.0 μ. The exposure times have a range from 12 to 120 sec. The value of $\triangle$ derived from 67 images is

$$\triangle = 28\,\mu.$$

In both coordinates the unit is 1 micron = ″.02075.

The distance determined from micrometer observations is

$$\rho_M = 3''.06 \pm ''.014$$

or ^{mm}.1475 on the plate.

The excellent, good and fairly good images are represented by black, half black and open circles. The relative increase of

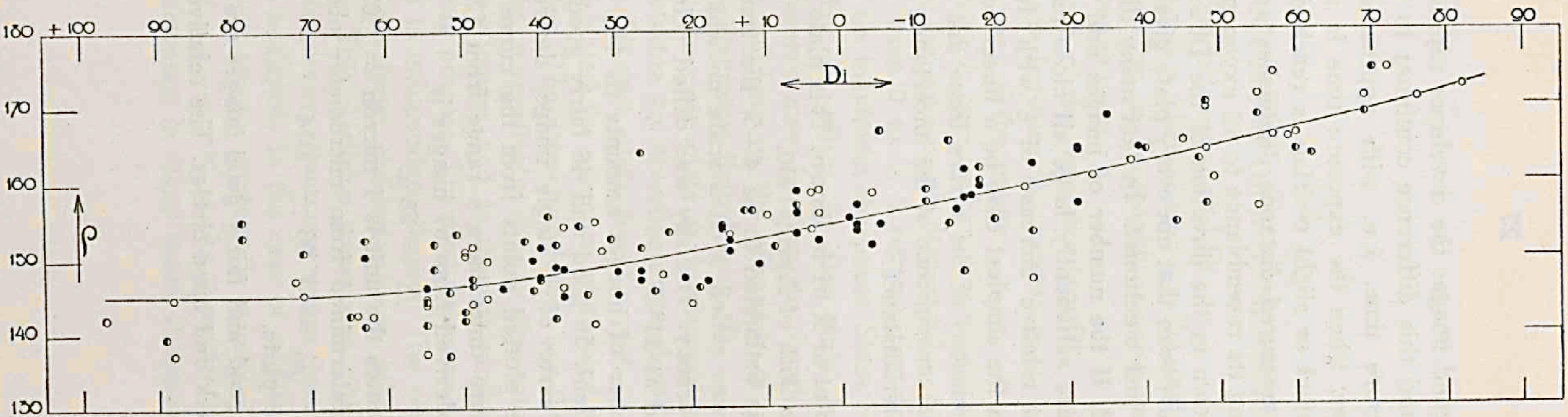

Fig. 1.

ρ_{Ph} with decreasing Di is very evident and is numerically shown by Table 9.

The values of Di have been computed by means of the relation

$$Di = 2\ \rho_M - Do.$$

Table 9.

Di	ρ_{Ph}	$\rho_{Ph} - \rho_M$
+ 90	145.0	− 2.5
+ 80	145.0	− 2.5
+ 70	145.1	− 2.4
+ 60	145.6	− 1.9
+ 50	146.5	− 1.0
+ 40	147.8	+ 0.3
+ 30	149.4	+ 1.9
+ 20	151.1	+ 3.6
+ 10	152.8	+ 5.3
0	154.7	+ 7.2
− 10	156.7	+ 9.2
− 20	158.5	+ 11.0
− 30	160.4	+ 12.9
− 40	162.5	+ 15.0
− 50	164.7	+ 17.2
− 60	166.9	+ 19.4
− 70	169.3	+ 21.8
− 80	172.0	+ 24.5

Unit: 1 micron.

An attempt will now be made to give a qualitative explanation of the most complicated type of ρ-Di curve found. The explanation is tentative only, owing to the mixing up with personal errors which very likely will be dependent upon Di also.

Starting with the underexposed images it is evident that if the borders are sufficiently close, e.g. Di $< 80\,\mu$ for $\triangle = 30\,\mu$, the images will be elongated at the inside. Bisections will be made on two areas being reinforced at their adjacent sides and appearing of homogeneous density to the eye. Hence the distances will be measured too small.

With an increase of exposure time the images grow, involving a growing attraction. At the same time the increase of intensity will result in having a gradual decrease of the density from the centre outwards. To the eye the image appears to have a nucleus of maximum density. In view of the fact that bisections are made on such a nucleus, if present, the measures will get freed from the turbidity influence. Moreover the latter is strongly decreased by the behaviour of the developer which is deficient in between the two images.

Continued increase of exposure time involves a steady growth of the images, i.e. a decrease of Di, and at the same time an increase in size and density of the nucleus.

Assuming the developer effect to increase with increasing exposure time the adjacent sides of the images will suffer more and more from this deficiency of the developer. Several observers have noted the apparent flattening of the images. The result in the measures is shown by a steadily growing distance.

Suppose the exposure time be pushed that far that the maximum density on the plate is reached. Having passed this stage the gap between the flattened borders will gradually be filled. Thus the rate of increase of ρ will gradually diminish and the curve will bend towards the Di axis.

As has been stated no attempt has been made to give a quantitative analysis of the ρ-Di curve. The table of corrections to be given (§ 6, e) has been derived by comparing the curves with standard values (see p. 32, 33). Due to the uncertainty in the measurement of Di, the inner borders being moreover systematically displaced, in practice ρ has been plotted against Do and finally for each plate each Do has been converted into Di by means of the standard value ρ_M.

The greatest range in ρ_h on a single plate is 31.6 microns, corresponding to a range in Di of 160 microns, or to a range in exposure time of 1 to 45.

b) $\triangle$. The growth $\triangle$ (in microns) of the diameter of an image with doubling of the exposure time has been chosen for a classification of the plates according to seeing or definition. The values of $\triangle$ have been derived for all plates on which the number of images is sufficiently large. The results are unreliable for such plates as were taken through clouds or under progressively changing seeing.

The average $\triangle$ amounts to 29 microns, the maximum and minimum $\triangle$ are 69 and 12 microns respectively. There is a pronounced correlation between $\triangle$ and the estimated seeing at the telescope, which is shown by Table 10 and its graphical representation in Fig. 2.

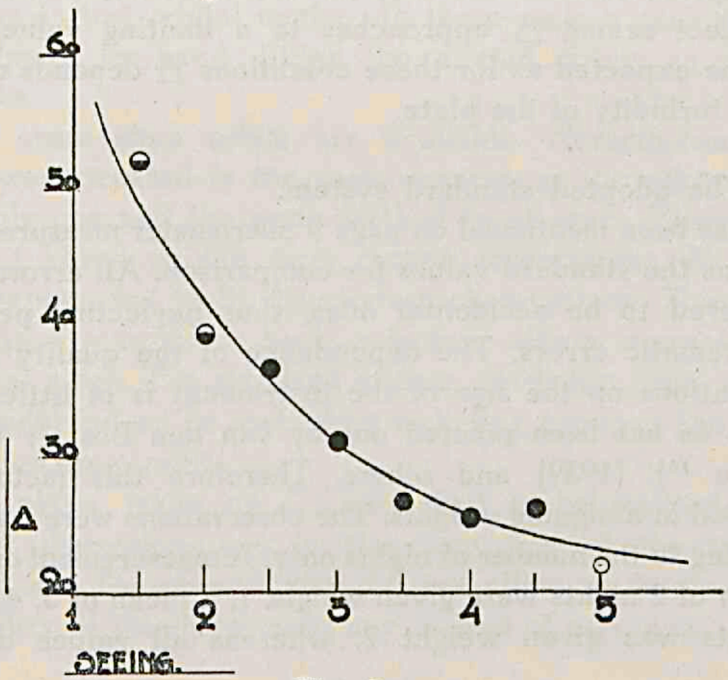

Fig. 2.

Table 10.

Seeing	△	n
5	22	1
4 — 5	26.2	20
4	25.6	23
3 — 4	26.8	19
3	31.2	25
2 — 3	36.3	7
2	39.3	3
1 — 2	52.3	3
Average 3 — 4	29.2	(101)

n is the number of plates.

It appears furthermore that for the condition of excellent or perfect seeing △ approaches to a limiting value. This could be expected as for these conditions △ depends mainly on the turbidity of the plate.

c) The adopted standard system.

As has been mentioned on page 9 micrometer measures were taken as the standard values for comparison. All errors were considered to be accidental ones, thus neglecting personal or systematic errors. The dependence of the quality of the observations on the size of the instrument is of little significance as has been pointed out by van den Bos [13]) (1925), Baize [15]) (1929) and others. Therefore this factor was neglected in assigning weights. The observations were weighted according to the number of nights only. A measurement on 1 and a mean of 2 nights were given weight 1, a mean of 3, 4, 5 and 6 nights was given weight 2, whereas all values derived from measures on more than 6 nights were given weight 3. As a rule the measures were collected by going backwards chronologically, starting with the very recent ones. Thus an average of 25 observations per star was obtained. For most cases the number of measures made during the last 15 to 20 years was sufficiently large, for the remaining part measures had to be taken back to 1900 or still further back yet. For the latter Lewis' Catalogue of Σ-stars and Burnham's General Catalogue were used. In a few cases 25 observations could not be found, even in going back as far as William Herschel's measures. As to the modern observations these were collected from a large number of different publications, some of them listed on pages 77 and 78. Though completeness is not claimed the number of overlooked recent observations will probably be small.

For each star the measures were plotted chronologically and the extrapolated value determined on the assumption of a linear relation. This seems justified as the interval of time is short for the rapidly changing — and therefore frequently observed — pairs, whereas in the cases of a long interval of time the distance changes very slowly. An exception was made for a few stars showing clearly a curved relation due to fast orbital motion. In those cases a smooth curve was drawn by hand, fitting the plotted points as well as possible.

For some stars orbits are available. Nevertheless these stars were treated in the same manner as the others so as to apply one and the same method to all stars. Their orbits are not based on the most recent observations. Moreover they happen not to fit the modern observations. Thirdly observations may have been rejected, others corrected for personal error. The standard system which has been adopted here might indeed be spoiled by including one or a few values computed from orbits.

The plates taken on ζ Cancri had to be omitted in the general discussion, i. e. in the derivation of the table of corrections. The observations did not allow to determine the variability of the distance to any degree of accuracy.

The total number of observations collected is 1843 on 73 stars, an average of 25 observations per star.

The average probable error of the extrapolated value amounts to ".025, the average probable error of a single observation is ".11.

d) Photographic effects.

A very elaborate discussion has been given by Ross [4]) (1924) whereas several other observers have considered the matter to some extent. Plate errors rather than physiological effects are the cause of the occurrence of systematic errors in the photographic distance measures of close pairs.

Owing to the change in the original working plan the investigation of artificial pairs (§ 8) has reached a preliminary stage only. It is evident that photographic effects should be studied in the laboratory. It is suggested to investigate photometrically the intensity distribution of enlarged images. This would yield the relative effect of the developer and would furthermore show to which extent the erroneous bisections must be ascribed to the personality of the observer.

The theory of the turbidity action as given by Ross seems to be essentially correct. There is some doubt however whether its influence on the bisections is as simple as assumed. Particularly in those cases for wich a notable density has been reached it seems doubtful whether the Kostinsky repulsion can be calculated by simply correcting the observed difference by means of the computed turbidity effect. The effect of the gelatine contraction as determined empirically on the other hand may readily be accounted for.

The turbidity effect is dependent upon $\triangle$ and Di. For the latter Ross uses the measured distance between adjacent borders. For large values of $\triangle$ the borders get more and more diffuse. Moreover the values of Di obtained in this manner are too large due to the incomplete development at the adjacent sides of the images.

The second one of the three effects considered by Ross is the gelatine effect, a shrinkage of the image as a whole during the process of drying. With the exception of pyro metol the same numerical result was obtained for all classes of developer, within the errors of the experiment. Furthermore the contraction appears to be constant between Di = 0 and Di = 80 μ, amounting to — 1.6 μ for doubles of separation of $^{mm}.1$ and less.

As to the above mentioned Kostinsky effect this is very likely caused by the developer reaction known as the Eberhard effect. Whatever be its chemical or physical explanation the existence of the Eberhard effect merely is sufficient to account for the Kostinsky effect observed in measures of close doubles.

The results obtained for a possible dependence upon the class of developer are not conclusive. From 11 plates developed in various solutions Ross finds a pronounced retrocession of centers amounting to + 9.6 μ for Di = + 10 μ. The object measured was an artificial double star of $^{mm}.17$ separation. The ranges in exposure time, Di and image diameter were 1 to 240 sec., 96 to 10 μ and 76 to 171 μ respectively, the corresponding $\triangle$ being 12.5 μ.

Taking into account the influence of both gelatine contraction and turbidity attraction the repulsion due to the developer alone is even larger than the value given above.

From Table 13a (p. 41) it appears that the systematic error as derived from the telescopic material for $\triangle_1$, $\rho_M = 175$ μ and Di = 0 amounts to + 4.8 μ, in good agreement with the results obtained by Ross.

In § 8 a comparison between Ross' and my laboratory results is given.

e) The systematic errors.

For a statistical discussion the material was treated in the following manner. Each curve may be represented by

1. its slope with respect to the Di axis,
2. the amount of $\rho_{Ph} - \rho_M$ for Di = 0, i.e. the case of contact, to be denoted by „zeropoint".

For most plates the plots are such as to allow a straight line to be drawn through the points. In those cases in which a marked curvature is present it was decided to use that part of the plot for which the curvature is smallest. Thus either the underexposed or the overexposed images or both were neglected and a straight line was drawn fitting the remaining points as closely as possible.

The dependence of the slope and zeropoint on ρ_M and $\triangle$ was investigated. Moreover the coefficients for the correlation between $\triangle$ and ρ_M and the one between slope and zeropoint were computed.

The dependence of $\triangle$ on ρ_M was investigated with the only purpose to determine quantitatively the value of the correlation coefficient. As a matter of fact the dependence is qualitatively a mere consequence of the observational selection. The closest pairs cannot be photographed unless the seeing is excellent. On the other hand for the wider pairs a wide range of seeing is allowable. Moreover in view of the dependence of photographic effects on $\triangle$ a number of plates on wide pairs was purposely taken under rather poor seeing.

The following correlation coefficients were found:

1. zeropoint and ρ_M.	$r = + .600 \pm .048$	(p. e.)
2. zeropoint and $\triangle$.	$r = + .234 \pm .075$	,,
3. slope and ρ_M.	$r = - .362 \pm .061$	,,
4. slope and $\triangle$.	$r = + .074 \pm .068$	,,
5. $\triangle$ and ρ_M.	$r = + .592 \pm .052$	,,
6. slope and zeropoint. .	$r = + .059 \pm .086$	,, .

1. The first result shows the marked increase of zeropoint with ρ_M or the increase with increasing diameter. This is a manifestation of the Kostinsky effect which assumedly increases with increasing diameter.

3. The third correlation coefficient indicates the tendency of the slope to decrease with growing ρ_M. This result can be readily explained by the fact that for the larger distances the curves in the average do not extend to such small values of Di as are reached for the closer pairs. Hence photographic effects will be smaller for the first.

2. In the second case there seems to be no pronounced correlation. This is a remarkable fact since the intercomparison of plates on a same star indicates a pronounced dependence as will be shown later (p. 37). The correlation coefficient

may be statistically explained as follows. The correlation (5) shows the prevalence of large values of $\triangle$ for the larger distances, which could be expected a priori (see p. 36). Combining this with the coefficient found in case (1) a correlation coefficient for case (2), if of any significance, must be of positive sign.

4. No correlation between slope and $\triangle$ was found. In view of the correlation between $\triangle$ and ρ_M, relative to the correlation (3) one might expect to find a slight correlation between slope and $\triangle$. This would then be such as to show a tendency of the slope to be smaller for larger $\triangle$.

6. No correlation is present between zeropoint and slope. The first is clearly related to ρ_M as has been shown in (1), the relation being such as to have zeropoint increasing with increasing ρ_M. The less pronounced correlation between slope and ρ_M shows a tendency of the slope to diminish with increasing ρ_M. Therefore this result does no need any further comment either.

5. This correlation has been mentioned in connection with case (2).

The dependence of ρ_{Ph} on $\triangle$ was investigated as follows. The curves representing different plates on a same star were compared relative to their corresponding values of $\triangle$. This intercomparison is free from the errors of the standard values. An examination showed at first sight that ρ_{Ph} decreases with increasing $\triangle$ in the average.

Denoting $d\rho$ as the change in ρ_{Ph} corresponding to an increase in $\triangle$ of 1 μ, it is evident that $d\rho$ must be governed by two factors acting in the same direction. The first is the differential Kostinsky effect. Let $\triangle_1$ and $\triangle_2$ belong to plates (1) and (2) on a same star, $\triangle_2$ being larger than $\triangle_1$. The images on (1) and (2) with equal diameters — i. e. equal values of Di — are differently acted upon by the Kostinsky effect. The images on (2) are of smaller density than the corresponding ones on (1). Hence they will be less influenced by the repulsive developer reaction.

Thus the differential Kostinsky effect causes:

$$\rho_2 - \rho_1 < 0 \qquad \text{for} \qquad \triangle_2 > \triangle_1.$$

The second factor is the differential turbidity action. The turbidity attraction increases with $\triangle$. In the case of contact it amounts to ½ $\triangle$ (see p. 3).

Thus e.g.:

$$\rho_2 - \rho_1 = -5\,\mu \quad \text{for} \quad \triangle_2 - \triangle_1 = 10\,\mu.$$

For increasing Di this difference decreases, the rate of decrease being dependent upon $\triangle$.

A value of $d\rho$ has been determined from each set of — i.e. 2 to 4 — plates on a same star.

Next the dependence of $d\rho$ on Di has been investigated. The individual values of $d\rho$ have been found from the differences between corresponding parts of the curves, i.e. overlapping in Di. Thus each $d\rho$ is related to a certain Di, for which the average Di for the overlapping parts of the curves was taken.

Disregarding the weights the large dispersion among the individual values of $d\rho$ can be accounted for by the cases for which the range in $\triangle$ is relatively small. Assigning weights however allows a fair determination of the relation between $d\rho$ and Di. $d\rho$ is found to be negative, $|d\rho|$ decreasing with increasing Di, in excellent agreement with the dependence of both differential Kostinsky action and turbidity effect on Di. Table 11 contains the values of $d\rho$, dt and $dK = d\rho - dt$.

Table 11.
Unit : 1 micron.

Di	$d\rho$	dt	dK
0	—.90	—.50	—.40
10	—.75	—.47	—.28
20	—.60	—.43	—.17
40	—.40	—.28	—.12
80	—.15	—.08	—.07

The order of magnitude of the probable error of each d_ρ in the second column is .05 μ.

The third column contains the differential turbidity effect at $\triangle = 30\,\mu$ as a function of Di. The last column contains the differences between the values in columns (2) and (3).

In view of this result and the coefficient found for the correlation between zeropoint and ρ_M the assumption made on p. 26 proves to be justified. Thus the work was carried out on this principle — i.e. grouping the plates according to ρ_M and $\triangle$.

Each curve was read at points differing 10 μ in Di. Thus for each plate a set of differences $\rho_{Ph} - \rho_M$ was obtained relative to their corresponding values of Di. The differences were weighted taking into account the error of the standard value and the error of the curve, derived from the deviations of the individual points from the curve.

Weighting according to $\frac{1}{\varepsilon_M^2 + \varepsilon_{Ph}^2}$ would perhaps be preferable from a theoretical standpoint. It would however cause the curves of smallest weight, though significant as separate observations, to be of practically no influence. Therefore it was decided to assign weights proportional to $\frac{1}{\sqrt{\varepsilon_M^2 + \varepsilon_{Ph}^2}}$.

The numbers of plates for different values of $\triangle$ are given in Fig. 3.

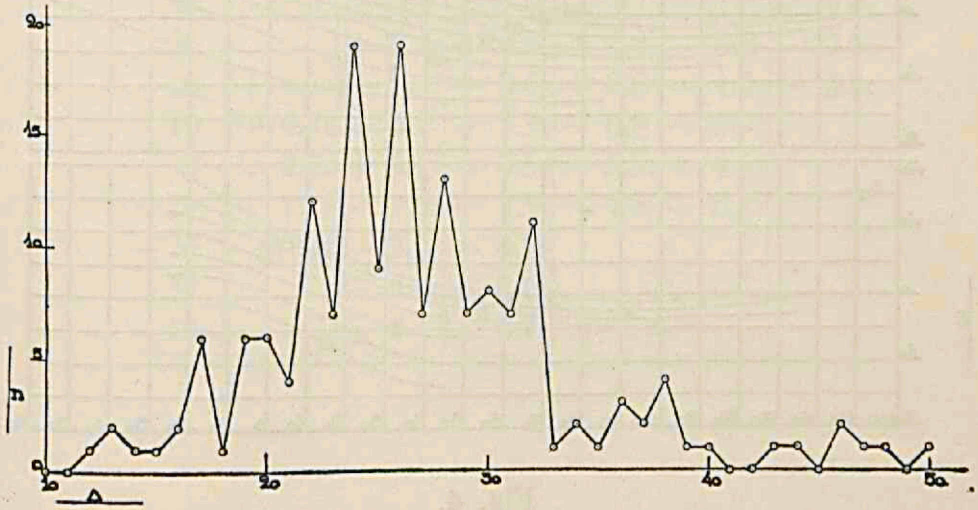

Fig. 3.

The grouping according to $\triangle$ is shown by the following Table.

Table 12.

	Δ	seeing	number of		average Δ
			plates	stars	
Δ_1	< 25	> 4	68	36	20.9
Δ_2	25 — 32	3 — 4	81	58	28.2
Δ_3	> 32	< 3	22	19	39.7

In Fig. 4 the results are plotted with respect to Di, each curve belonging to a certain ρ_M (see Table 13). The unit is 1 micron.

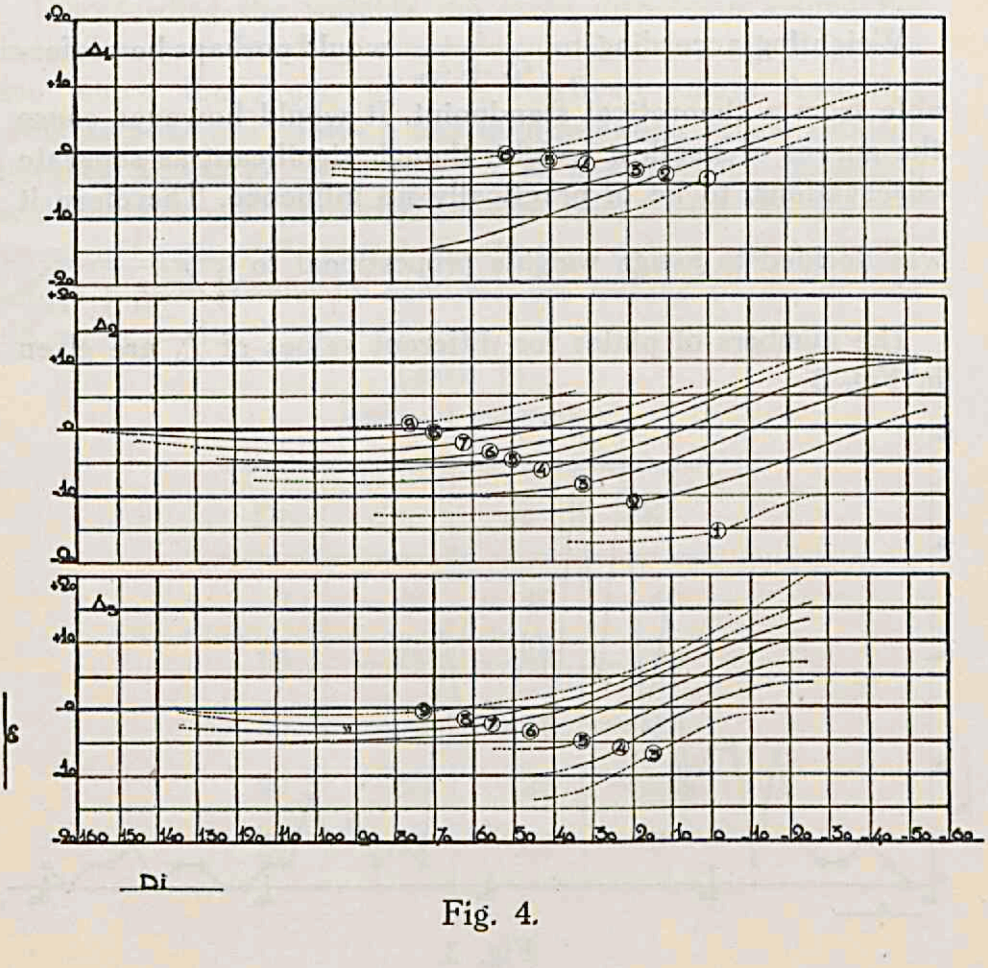

Fig. 4.

Fig. 4 shows that on any vertical line for corresponding curves, i. e. curves belonging to the same ρ_M

$$_\varepsilon\triangle_1 > {_\varepsilon\triangle_2} > {_\varepsilon\triangle_3}$$ (see p. 37).

Converting the errors into corrections by reading the curves and reversing the sign yields 3 sets of corrections, given in Table 13 a, b, c.

The values marked : are less accurate than the others. As to the latter the value of their probable error is about .5 μ.

TABLE 13a.

$\triangle_1$ Unit : 1 micron.

ρ_M / Di	1 75	2 100	3 125	4 150	5 175	6 200
90				+ 3.8:	+ 2.7:	+ 1.7:
80				+ 3.9	+ 2.8:	+ 1.8:
70		+14.8	+ 8.4	+ 4.0	+ 2.6	+ 1.5:
60		+13.4	+ 8.0	+ 3.8	+ 2.3	+ 1.1:
50		+11.6	+ 7.0	+ 3.4	+ 1.8	+ 0.5:
40		+ 9.5	+ 5.8	+ 2.7	+ 1.2	— 0.4:
30		+ 7.4	+ 4.5	+ 1.8	+ 0.2	— 2.0:
20	+ 9.2:	+ 5.3	+ 3.0	+ 0.6	— 1.2	— 3.8:
10	+ 6.7:	+ 3.2	+ 1.3:	— 0.8	— 2.9:	
0	+ 4.2:	+ 1.1	— 0.5:	— 2.5:	— 4.8:	
—10	+ 1.7:	— 1.0:	— 2.5:	— 4.6:	— 6.8:	
—20	— 0.7:	— 3.0:	— 4.7:			
—30		— 5.0:	— 7.0:			
—40			— 8.7:			

TABLE 13b.

$\triangle_2$

Unit : 1 micron.

ρ_M \ Di	1 75	2 100	3 125	4 150	5 175	6 200	7 225	8 250	9 275
150								+ 0.5:	
140							+ 2.0:	+ 0.8	0:
130							+ 2.6:	+ 1.1	0:
120						+ 4.6:	+ 3.0	+ 1.4	+ 0.1:
110				+ 7.8:	+ 6.2:	+ 4.7	+ 3.2	+ 1.7	+ 0.2:
100				+ 7.9:	+ 6.3:	+ 4.8	+ 3.3	+ 1.7	0:
90				+ 8.0:	+ 6.2	+ 5.0	+ 3.3	+ 1.5	— 0.2:
80		+13.0:	+10.0:	+ 7.9	+ 6.0	+ 4.8	+ 3.0	+ 1.1	— 0.5:
70		+12.9:	+ 9.9:	+ 7.7	+ 5.7	+ 4.4	+ 2.5	+ 0.5	— 1.2:
60		+12.8:	+ 9.8:	+ 7.3	+ 5.2	+ 3.8	+ 1.8	— 0.3	— 2.2:
50		+12.6	+ 9.6	+ 6.7	+ 4.4	+ 2.8	+ 0.8	— 1.3	— 3.4:
40		+12.3	+ 9.0	+ 5.8	+ 3.3	+ 1.6	— 0.4	— 2.5:	— 4.8:
30	+17.0:	+11.8	+ 8.0	+ 4.5	+ 1.9	+ 0.2	— 1.8	— 4.0:	
20	+16.8:	+11.0	+ 6.6	+ 2.8	+ 0.2	— 1.4	— 3.4:		
10	+16.5:	+ 9.9	+ 4.8	+ 0.8	— 1.8	— 3.2	— 5.2:		
0	+15.8:	+ 8.5	+ 2.6	— 1.4	— 4.0	— 5.2:			
—10	+13.6:	+ 6.8	+ 0.2	— 3.8	— 6.4	— 7.4:			
—20	+11.2:	+ 4.7	— 2.0	— 6.0	— 8.5:	— 9.8:			
—30		+ 2.5	— 4.0	— 8.0	—10.0:	—11.0:			
—40		+ 0.3	— 5.8	— 9.2	—10.2:	—10.8:			
—50		— 1.8:	— 7.4:	— 9.8	—10.0:	—10.6:			

TABLE 13c.

$\triangle_3$ Unit: 1 micron.

Di \ ρ_M	3 125	4 150	5 175	6 200	7 225	8 250	9 275
130					+ 3.0:	+ 1.0:	+ 0.5:
120					+ 3.2	+ 1.7	+ 0.7:
110				+ 5.0:	+ 3.4	+ 2.3	+ 0.9:
100				+ 4.9:	+ 3.6	+ 2.4	+ 1.0:
90				+ 4.8	+ 3.8	+ 2.4	+ 0.9:
80				+ 4.7	+ 3.5	+ 2.2	+ 0.8:
70				+ 4.4	+ 3.2	+ 1.9	+ 0.5:
60				+ 4.1	+ 2.8	+ 1.3	— 0.4:
50			+ 6.2:	+ 3.8	+ 2.0	+ 0.4	— 1.0:
40	+13.5:	+ 9.7:	+ 6.0	+ 2.8	+ 0.8	— 0.8	— 2.0:
30	+11.6:	+ 8.0	+ 4.3	+ 1.0	— 1.0	— 2.3	— 3.5:
20	+ 8.5:	+ 4.9	+ 1.6	— 1.0	— 3.4	— 4.6	— 5.5:
10	+ 5.5:	+ 1.8	— 0.9	— 3.2	— 5.9	— 7.0	— 8.0:
0	+ 2.6:	— 0.7	— 3.2	— 5.6	— 8.2	— 9.5	—11.0:
—10	+ 1.2:	— 2.6	— 5.3	— 8.0	—10.3	—12.1:	—14.4:
—20		— 3.0:	— 6.5:	— 9.8:	—12.3	—14.8:	—18.1:

§ 7.

RESEARCHES BY OTHERS.

Some of the investigations dealing with the subject have been mentioned in § 1.

It may be of interest to give the results of a comparison with the „standard" visual system. Only those investigations were considered for which the number of coincidences with the present series is sufficiently large.

Hertzsprung's measures [1]) yield the following result:
For $\rho_M < 3''.0$ $\rho_{Ph} - \rho_M = -''.076 \pm ''.010$ (p.e.) from 11 stars.
For $\rho_M > 3''.0$ $\rho_{Ph} - \rho_M = -''.001 \pm ''.011$ (p.e.) from 16 stars.

The probable errors of one difference are ".033 and ".045 respectively. From Hertzsprung's published errors and the errors of the micrometer measures as derived by me the average probable errors of one difference amount to ".022 and ".027 respectively.

The discrepancy is easily explained by the fact that both Hertzsprung's errors and the errors of the standard values are but measures of the internal agreement in the two cases. It is probable however that the difference is mainly due to systematic errors of the standard values.

The result shows that in the average the distances of Hertzsprung's close pairs are too small. This is in good agreement with the results published in Pop. Astr. 38, 406; 1930.

The scale value of the Potsdam plates is 1 mm = 16".39.

Osgood's measures [16]) do not show a pronounced negative difference „photographic minus standard" for the close pairs. This is not surprising as the scale value of his plates is 1 mm = 8".24, whereas all pairs considered have distances larger than 2".3 excepting one double of separation 1".85.

The differences for 25 stars yield:

For $\rho_M < 6''.0$ $\rho_{Ph} - \rho_M = +''.007 \pm ''.013$ (p.e.),

the probable error of one difference being ".065.

As derived from Osgood's errors and the errors of the standard values the average probable error of one difference is ".030.

The measures of Przybyllok and Labitzke [10]) treated in a similar manner yield:

For $\rho_M < 4''.0$ $\rho_{Ph} - \rho_M = -''.096 \pm ''.030$ (p.e.) from 7 stars.

For $\rho_M > 4''.0$ $\rho_{Ph} - \rho_M = -''.011 \pm ''.030$ (p.e.) from 9 stars.

The probable errors of one difference are ".080 and ".068 respectively. The order of magnitude of the errors of the photographic measures was derived. Combined with the errors of the standard values the average probable errors of one difference were found to be ".023 and ".027 respectively.

The scale value of the plates is 1 mm = 41".84.

In this connection the laboratory experiments of the authors must be mentioned. No evidence of photographic errors was found. It must be kept in mind however that this may have partly been caused by the difference in magnitude of the components and the relatively large distances of the centers. It is likely that in this case photographic effects are small, maximum effects occurring for components of equal magnitude.

Van den Bos [9]) has shown from comparisons with visual measures that for close pairs his distances are measured too small.

A comparison with the standard system yields:

For $\rho_M < 6''.0$ $\rho_{Ph} - \rho_M = -''.180 \pm ''.029$ (p.e.) from 8 stars,

the probable error of one difference being ".082.

As derived from van den Bos' errors and the errors of the standard values the average probable error of one difference amounts to ".057.

The scale value of the plates is 1 mm = 39".37.

The comparison of the measures of Marriott and Pitman [17]) with the standard values gives:

For $\rho_M < 5''.5$ $\rho_{Ph} - \rho_M = -''.183 \pm ''.034$ (p.e.)

from 10 stars, the probable error of one difference being ".108.

For the photographic measures the order of magnitude of the errors was determined. Combining with the errors of the standard values the average probable error of one difference was found to be ".057.

The scale value of the Sproul plates is 1 mm = 18".74.

The coincidences of Hoff's measures [18]) with the standard values give the following result:

For $\rho_M < 5''.0$ $\rho_{Ph} - \rho_M = -''.150 \pm ''.033$ (p.e.) from 9 stars, the probable error of one difference being ".098.

The average probable error of one difference was derived from Hoff's errors as estimated by me and the errors of the standard values. It was found to be ".029 approximately.

The scale value is 1 mm = 29".26.

Shajn [11]) has compared his measures of doubles taken with the normal astrograph of the Poulkovo Observatory with visual observations. The images were overexposed. The results are in good agreement with Ross' laboratory data. Arranged according to distance ρ the repulsion is largest for the closest pairs. The differences decrease when the images taken with the longest exposures are omitted. The Kostinsky effect is shown most clearly by grouping the differences with respect to Di.

Shajn finds a repulsion of + 4.5 μ in the case of contact and a maximum effect of + 9.7 μ for Di about — 60 μ. If the relative decrease for still more overlapping images shown by a repulsion of + 7.2 μ for Di = — 120 μ is real, it would be in excellent agreement with the frequently occurring type of curve found in the present investigation and the qualitative explanation attempted thereof.

It was suggested by Shajn and others to cut down the exposure times as much as possible. As far as reduction of the troublesome Kostinsky effect is concerned this is undoubtedly advisable. It must be remembered however that in this manner one is likely to introduce the errors due to attracting causes to their full extent. They may be of considerable size too, particularly so under mediocre conditions of seeing.

The results of Kostinsky [3]) were the first chronologically, so far as I am aware; the repulsions noted by several other observers later on were named after him.

In view of all recent results his measures can easily be explained. Relative to the unequal magnitudes of the components the question is put forward to which extent his measures may have been affected by a magnitude error. Assuming both Kostinsky effect and turbidity action to reach a maximum for components of equal magnitude, Kostinsky's results may loose slightly as to their quantitative importance. Yet the order of magnitude of the effect agrees very well with the data of others. The repulsion reached a maximum value of $+ 17 \mu$ for Di $= - 40 \mu$, being zero for Di larger than $+ 80 \mu$ and amounting to $+ 10 \mu$ in the case of contact.

The results obtained by Lau [6]) were found by a different method. Possible errors inherent in the method were already mentioned by Ross but thought to be negligible as to their probable size.

The measures show a good agreement with those of Kostinsky.

The complicated combination of repulsive and attracting forces offers the possibility of observing the distances unaffected by systematic errors. This case seems to be present in the measures of Krueger 60 by Mitchell and Olivier [8]). The distances were compared with Barnard's visual measures. Yet one objection seems worth being raised namely as regards the comparison with one visual observer only. Lately the accuracy of Barnard's micrometric work has been doubted by van den Bosch [57]). Aside from possible personal errors the accidental errors render the detection of small systematic errors of the photographic measures impossible.

As to the measures of Olivier[19]) on parallax plates and so-called D-plates, the number of comparisons with the standard system is 6.

The result is:

For $\rho_M > 2''.5$ $\rho_{Ph} - \rho_M = +''.15 \pm ''.067$ (p.e.) from 5 stars.
For $\rho_M < 2''.5$ $\rho_{Ph} - \rho_M = -''.19$ from 1 star.

the probable error of one difference for the first comparison being ".12. From the estimated photographic error and the error of a standard value this error is calculated to be ".06.

The number of comparisons is too small particularly for the closer distances to have much confidence in the values derived. It would therefore be of interest to have a detailed account of the author's statement regarding the absence of systematic errors in his measures found from comparison with visual ones. Unfortunately this was not given. A critical discussion of systematic errors present in some of Olivier's measures has been given by van den Bos [20]) (1927).

Vyssotsky's data [21]) obtained from measures of Espin doubles as compared with Espin's measures show strong evidence as to the effect of the turbidity action for weak images. The manner of comparison being similar to that applied in the case of Krueger 60 the results are subject to the same objection as has been raised therefor.

The discordance shown by the investigation of Bellamy [7]), namely the apparent attraction for overlapping images seems more difficult to explain. The cause must be sought for either in the photographic measures, as regards their accuracy, or in the micrometer distances used for comparison. Yet a third possibility must be considered also, i. e. the images being weakly exposed in spite of their blended shape.

The other photographic determinations will not be discussed here, the number of comparisons being too small, or the epoch of the observations being too early. As to King's measures [22]) e.g. there are many stars in common with the present list. However, owing to the early epoch of King's plates, the number of stars suited for a comparison is too small.

As has been shown in view of recent data various results, in fact nearly all, though disagreeing among each other at first sight, can be plausibly accounted for.

A discussion of data obtained from similar investigations on spectral lines will not be given, notwithstanding the subject possibly being equally interesting as the one considered here. It seemed preferable to limit myself to the case of double stars exclusively. A limitation which proved justified a posteriori since so much work remains to be done on this subject.

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§ 8.

ARTIFICIAL DOUBLE STARS.

There is little doubt that in order to obtain good knowledge about the effects governing the behaviour of close images on a photographic plate, artificial star images must be investigated. The true distance being known one is able to find the absolute amount of attraction or repulsion, whereas from ordinary double star exposures relative effects are found only.

An apparatus for producing artificial double stars need not be complicated. Therefore it seems advisable for the double star observer in the case of photographing close pairs to make some additional experiments on artificial pairs, using the same observing conditions as regards plates, developer etc.

With the purpose of making a more or less extensive series of measurements on artificial double stars an apparatus was constructed in the Physical Laboratory of Utrecht under careful supervision of Dr. M. G. J. Minnaert. I am greatly indebted to him for his help and interest in the work and wish to express my gratitude to the director Prof. Dr. L. S. Ornstein for giving me the opportunity of making use of the splendid equipment of his Laboratory.

In spite of the fact that no definite results were obtained, due to the unavoidable discontinuation of the work, it seems worth while to give at least a description of the apparatus constructed.

Firstly it will be readily understood that the experimental conditions in the laboratory should approach the observational ones at the telescope as closely as possible. The starlike images have to be produced in such a manner as to obtain the same distribution of intensity as in the case of the star images. Since in the formation of the latter the scintillation is the main factor the light distribution must be Gaussian (M. G. J. Minnaert, Onregelmatige Straalkromming; diss., Dutch).

As light source L (Fig. 6) a large Philips Argenta bulb (300 Watts, 220 Volts) was used. A brass plate S (Fig. 6), 15 cm. square, placed close to the lamp, was pierced with holes arranged in such a way as to have the number of holes from the centre towards the edges decreasing as a Gaussian function. To obtain this distribution concentric circles were drawn at equal distances and for each ring between two successive circles the computed number of holes bored. The final shape of the holes is indicated by the section view in Fig. 5.

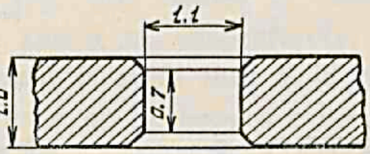

Fig. 5.

The holes were coned out so as to reduce the ,,shadow effect" of the holes near the edges. The total number of holes in all 12 rings, each of which has a width of 5.5 mm., amounts to 1024.

Before entering into any more details a schematic view of the mounting is shown by Fig. 6.

The thin brass plate A is pierced with circular holes of 0.7 mm. diameter arranged as indicated in Fig. 7 (Section view in Fig. 8), each pair representing a double star. Each hole gives a pinhole image of the diaphragm S on the piece of ground glass d. These pinhole images resemble diffuse star images. They are sharpest for d close to A, the sharpness decreasing with increasing distance d-A. The latter may be read on a scale. By means of this device any condition of seeing can be imitated, the reproducibility making this method preferable above others.

The images on the ground glass are photographed with the

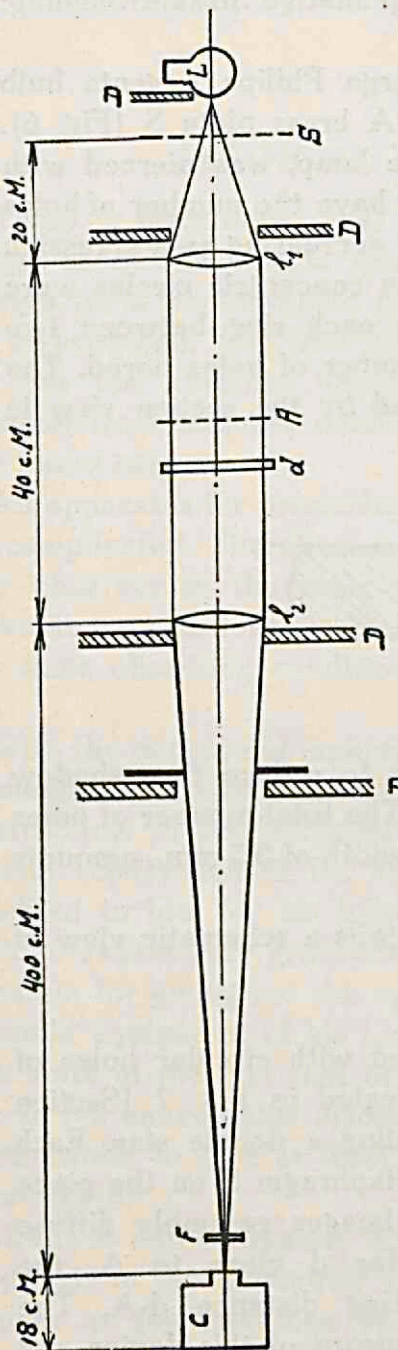

Fig. 6.

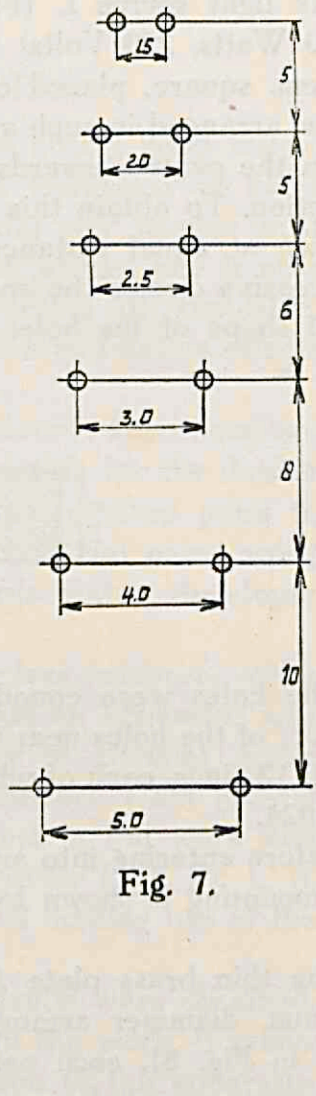

Fig. 7.

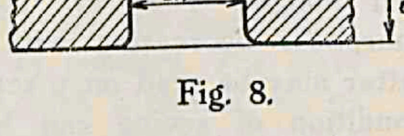

Fig. 8.

camera C and reduced in the ratio 20 to 1. Hence the size of the components and their mutual distance are comparable with those of the telescopic images.

To this simple apparatus some improvements had to be made so as to be sure of obtaining a light distribution in the image strictly comparable with the theoretical one.

1). The first requirement is the following.

Let „corresponding points" of two images on d, forming a pair, be two points at equal height and having exactly the same mutual horizontal distance as the two holes in A. As seen from the camera objective which is pointed at d, two such corresponding points must have the same quantity of light when closing either one of the two holes in A. This was achieved by placing the lenses l_1 and l_2, having focal lengths of 20 cm. and 4 m. respectively, in the path of the rays. The adjustment is such that S is placed at the focus of l_1. Thus each hole of S throws a beam of parallel rays on to A, two parallel beams of which will reach the corresponding points on d. The place of l_2 is such that its focus coincides with the camera objective. Hence the two beams reaching the objective from corresponding points will have left d in precisely the same direction.

2). Secondly care must be taken to have in each image the light distribution as computed theoretically.

In the first place it is necessary to investigate the intensity distribution of the light coming from the left hand side of L. Some photographs were made providing an intensity scale also, the densities measured on an Ica Sensitometer and the intensity determined for each point on the surface. The measures resulted in finding the surface intensity distribution, some areas having an intensity of only 50 % of the maximum. This wide range is more than could be expected from a visual examination only. It seems probable that this unequal distribution is caused by local irregularities in the thickness of of the glass. The numbers of holes in S were then corrected so as to neutralize the effect of the heterogeneous light giving disc.

Furthermore it is readily understood that the holes of S, as seen from A, are not equally large but so much the smaller the larger their distance from the centre. Therefore allowance should be made for the remaining „shadow effect" (Fig. 9).

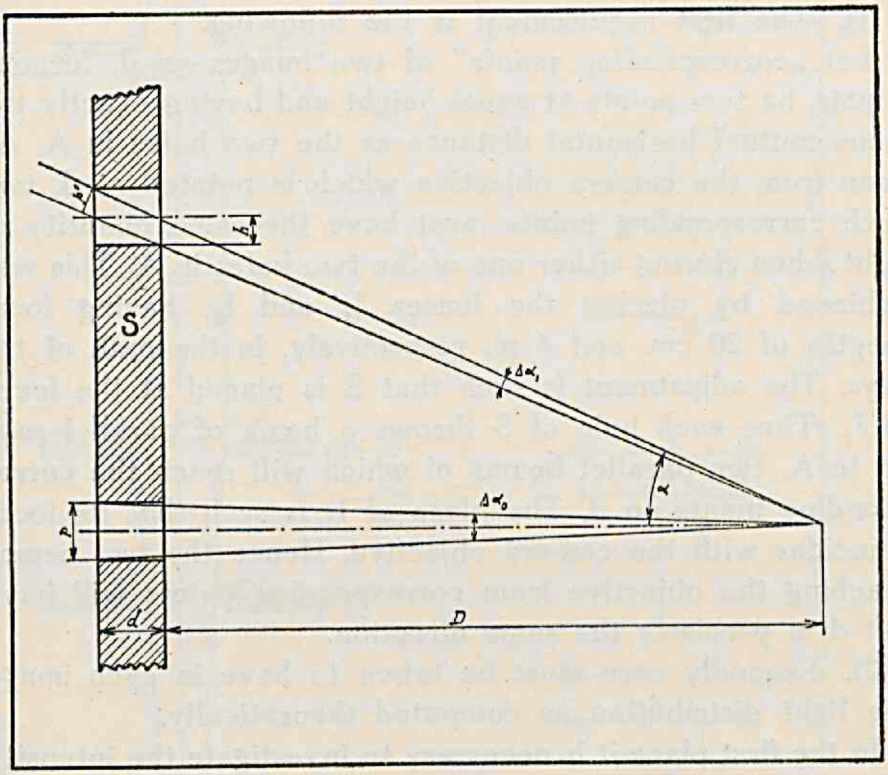

Fig. 9.

For a central hole let the angle be $\Delta\alpha_0$, for one off centre $\Delta\alpha_1$, then we have:

$$\Delta\alpha_0 = \frac{p}{D + d} \quad \text{and} \quad \Delta\alpha_1 = \frac{p' \cos\alpha}{D + d},$$

$$\text{hence} \quad \frac{\Delta\alpha_1}{\Delta\alpha_0} = \frac{p' \cos\alpha}{p} \quad \ldots\ldots \quad (1)$$

As $h = d\,\mathrm{tg}\,\alpha$,

$p' = (p - d\,\mathrm{tg}\,\alpha) \cos\alpha$.

Thus, substituting in (1):

$$\frac{\Delta\alpha_1}{\Delta\alpha_0} = \frac{(p - d\,tg\alpha)\cos^2\alpha}{p} = \cos^2\alpha - \frac{d}{p}\sin\alpha\cos\alpha.$$

The distance D is 200 mm., the width of the holes 1.1 mm. Let the effective depth of a hole be 0.7 mm. in the average, then we get correction factors running from 0.98 between $r_1 = 5.5$ mm. and $r_2 = 11.0$ mm. down to 0.72 for the ring between $r_{11} = 60.5$ mm. and $r_{12} = 66.0$ mm. The arrangement of the holes in S was altered accordingly.

There still remains one effect to be corrected for, i. e. the scattering of the light by passing through the piece of ground glass d. The light distribution of the images on d is altered by the peculiar law of scattering of the ground glass which transmits less of the incident light the larger the angle between the transmitted and the infalling beam. The effect was determined photographically and corrected for.

In order to shield the camera from stray light a set of diaphragms was used indicated by D in Fig. 6. F is a yellow filter.

With this arrangement a number of plates was taken. Ilford Rapid Chromatic plates were used throughout. The formula of the developer is

Carbonal	2 cc.
Water	64 „
Potassium bromide 10 % . .	5 „

Time: 15 min. at 65° F.

The formula of the fixing bath used is the same as given on page 16.

Alltogether 34 plates were selected for measurement, the total number of images being 102.

The „true" distances were derived from photographs taken with d close to A. The following values (in microns) were obtained:

1. 83.7.
2. 114.1.
3. 133.2.
4. 154.8.
5. 204.2.
6. 265.6.

As to the other plates, these were taken with 6 different distances d-A, thus reproducing 6 different conditions of seeing. The corresponding values of $\triangle$ were found to be 15, 16, 19, 24, 30 and 37 microns.

The differences $\rho_{Ph} - \rho_{true}$ are given in Table 14.

The values marked : are uncertain.

Notwithstanding the small number of images the qualitative agreement with the previous results (see p. 40) is very good. With the exception of a few values Table 14 exhibits the same characteristics as shown by Fig. 4. For instance the decrease of $\rho_{Ph} - \rho_{true}$ with increasing $\triangle$ for corresponding values of ρ_{true} and for the same Di is pronounced.

As to a quantitative comparison, herefor the average values of $\triangle_1$, $\triangle_2$ and $\triangle_3$, as given in Table 12, may be used. It will then be seen that the numerical agreement is satisfactory.

The agreement with Ross' results is very good. This may be seen by comparing Ross' measures [4]) (Table 30, p. 179) with the values in columns 4 and 5 of Table 14 a, b, c of the present publication.

It may be remarked in this connection that the artificial images are of high quality compared with the telescopic ones.

It is my intention to continue the experiments in the way as indicated above and to obtain an extensive series of measures. A matter of great importance is the separation of photographic and physiological errors. It has been suggested to investigate photometrically the enlarged double images.

The results may be of interest to those applying photography in the determination of positions of double stars.

TABLE 14.

Unit : 1 micron.

ρ_{true} / Di	a) Δ = 15				b) Δ = 16					c) Δ = 19					d) Δ = 24				e) Δ = 30				f) Δ = 37		
	1.	2.	3.	4.	1.	2	3.	4.	5.	1.	2.	3.	4.	5.	3.	4.	5.	6.	3.	4.	5.	6.	4.	5.	6.
+100				0																					
+ 90				0										+1.5				0				−3.5			
+ 80			0	0				0	+2.0:					+1.5			+0.5					−3.0			
+ 70			0	0			−6.0	−1.5					0	+1.5			+2.0				0	−2.0			−6.0
+ 60		−1.0	+0.5	0			−6.0	−2.5					0	+1.5			+3.5				0	−1.5			−3.0
+ 50		−1.0	+2.0				−5.0	−2.5					−0.5	+1.5			+5.0			−27.0:	+1.0	−0.5			0
+ 40	−6.5:	−0.5				−7.5	−3.5	−1.5			−16.5:	−7.5:	−1.5			−13.0	+7.0			−24.0:	+2.0				+2.5
+ 30	−4.0:	0				−6.5	−2.0	+3.0			−15.0:	−5.0	−1.5			−12.0	+9.5			−21.0:	+3.0			−15.0	+5.0
+ 20	−1.0					−4.0	0				−13.0	−3.5	0		−19.5:	−11.0				−18.5:	+4.0			− 7.0	+7.5
+ 10	+1.5					0	+2.5				−10.0	−2.0	+1.5		−15.5:	− 9.0				−16.0	+6.0			− 2.0	+8.5
0	+4.0					+5.0					− 4.5	+1.0			−12.0:	− 7.5				−13.5	+8.0			+ 3.0	+9.0
− 10	+5.5				−6.5:						+ 2.5				− 8.0:	− 5.0			−20.0:	−11.0				+ 6.5:	+8.0
− 20					−3.5:						+10.5				− 4.5:				−17.0:	− 8.0			0:	+10.0:	+6.5
− 30					0:					−10.0:					0:					− 5.0				+12.5:	
− 40					+3.5:										+ 4.5:					− 3.0				+14.5:	
− 50																				− 1.0				+16.0:	
− 60																				+ 1.5				+17.5:	
− 70																								+19.0:	
− 80																								+20.0:	

§ 9.

CATALOGUE.

Tables 15a and 15b contain some information for both series of plates, respectively for the R-series (1930 material) and the π-series (1929 material).

Column (2) gives the range in exposure time.

Columns (3), (4) and (5) contain the numbers of measured images.

Column (6) gives the values of $\triangle$. Some of these were derived with the aid of the curve given in Fig. 2 (p. 31).

Column (10) of Table 15b contains the openings of the rotating sector.

For instance for sector 5 the angle of the opening amounts to $\frac{5}{100} \times 360°$, i. e. 5 % of the infalling light is transmitted.

REMARKS (Table 15a).

1. through clouds.
2. some haze.
3. thick.
4. clouds; high wind.
5. passing clouds.
6. hazy.
7. haze.
8. clouds at the end.
9. thin clouds.
10. seeing sometimes 2.
11. some wind.
12. wind.
13. seeing variable.
14. sector 17.
15. „ 16.
16. „ 6½.
17. „ 3.
18. „ 30.
19. „ 15.
20. faint guiding star.
21. partly cloudy.
22. lightning.
23. image unsteady.

TABLE 15a.

Plate R	Exp. time range 1 to	Images e	Images g	Images fg	△	Seeing	Transp.	Temp. °F	Remarks
1	8	—	—	4	32	3^-	4	47	
2	8	63	11	2	47	2—3	4	47	
5	25	11	13	16	28	3	4	37	
7	20	—	4	20	28	4^+	5	37	
8	10	—	1	5	25	$4^\pm$	5	37	
9	20	—	8	12	22	4^+	5	36	
10	40	6	5	2	31	2—3	5—0	41	1.
12	20	—	2	12	43	$3^\pm$	4	42	
13	20	4	5	7	30	3^+	4	51	
14	20	—	2	14	39	3^-	4	51	
16	40	—	—	5	31	3—4	3	57	2.
17	20	1	7	7	29	4—5	2—1	55	
19	20	13	11	20	30	{2—3 / 3—4}	2—3	55	
20	30	1	4	3	25	4—5	2—1	55	3.
21	20	1	2	6	24	4—5	2—1	55	
23	35	—	3	7	30	3^-	3—0	60	4.
24	35	—	7	4	50	1—2	4—5	59	5.
25	13	20	12	5	30	3—4	3—4	43	2.
26	25	1	7	3	26	4^+	3 ?	43	
27	8	1	—	4	26	4—5	?	43	
28	40	11	3	3	32	4—5	3—4	43	6.
29	10	9	29	4	26	4—5	3	43	
30	10	—	—	33	69	$1\text{-}2^+$	2	46	7., 12.
31	20	10	49	31	30	{(5) / $3\text{-}4^+$}	{3—2 / 2—0}	52	6., 8.
32	20	—	1	8	32	3^+	2^-	50	3.
33	60	—	—	17	26	{2—3 / 3—4}	{3—2 / 2—0}	54	1.
34	30	4	11	1	14	4	2—3	43	
35	25	8	12	5	24	4—5	2—3	43	
36	40	1	8	10	26	4	3	42	
37	20	6	10	7	27	3—4	2—3	53	9.

Plate R	Exp. time range 1 to	Images e	g	fg	△	Seeing	Transp.	Temp. °F	Remarks
38	25	—	6	8	24	3—4 / 4—5	3—2	52	9.
39	40	2	2	2	31	3	5	47	
40	40	17	8	7	40	4—3	4	43	10.
41	8	3	1	1	38	2—3	4	42	11.
42	32	—	—	1	37	2—3	3	45	6.
43	20	—	1	2	37	2—3 / var.	?	43	1.
44	32	—	—	2	46	2—3	4	48	
45	32	—	—	8	26	3—4	4	51	
46	10	—	7	25	20	4	4	51	
47	6	—	2	7	22	4—5	4	51	11.
48	20	2	8	6	20	4	4	50	
49	20	—	1	4	18	4—5	4	50	11.
50	20	1	5	6	22	5^-	4^-	49	
52	40	3	11	11	30	4—3 / 3-2-3	3	68	12.
53	20	—	4	16	38	3-2-0	3^-	68	7.
54	20	55	23	9	28	3-4-3	3—4	55	
55	20	22	13	17	31	3	5	51	13.
56	20	—	1	9	38	2—1	5	50	
57	20	17	34	23	23	$3\text{-}4^+$	3	50	
58	20	—	3	8	22	4^-	3	49	8.
59	10	—	6	9	20	4—3	3	51	
60	20	1	4	3	29	4	3	50	
61	10	42	56	48	28	4—5	3^-	51	
62	20	18	18	11	30	4—5	3^-	50	
63	20	—	5	4	29	4—5	3	68	
64	10	—	1	10	25	$4^\pm$	3	67	
65	10	4	12	24	26	4	3^-	66	14.
66	10	17	25	14	35	3 / $3\text{-}4^+$	4—5	68	15.
67	10	4	22	16	32	$3\text{-}4^+$	4	67	16.

Plate R	Exp. time range 1 to	Images e	Images g	Images fg	△	Seeing	Transp.	Temp. °F	Remarks
68	10	—	1	3	32	3±	4—5	67	
69	20	5	6	7	46	3-2-1	4—5	66	
70	32	1	3	3	36	3±	4—5	65	
71	20	—	18	19	24	3—4	3	67	2.
72	16	—	7	10	22	3—4	3	66	17
73(*a*)	20	—(-)	—(-)	13(4)	26	3—4	3	70	1., 7. 10., 13.
74	25	—	—	8	26	4-3⁻	3	69	1., 13.
75	20	—	13	14	31	2—4	var.	68	
76	32	3	1	2	30	2—3	4	61	
78	20	—	6	4	48	3—2 / 4—2	4—0	60	8., 13.
79	10	7	8	21	29	3—5	4—3	61	
80	20	—	6	6	27	4—5	3—4	60	
83	5	11	12	6	31	4—5	3—0	58	1., 18.
84	5	—	1	4	31	2-3⁺	var.	58	13.
85	20	—	—	5	44	1-2-3	var.	65	1.
86	6	10	20	14	36	4	4	59	
87	10	13	24	18	24	4—5	4	58	
89	5	2	6	17	38	4—5	3—4	66	
91	24	5	4	3	32	3⁻	4	71	
92	16	2	7	12	32	2—3	4—3	75	21.
93	10	8	16	13	32	3-4-5	3	73	21.
94	3	1	2	5	26	3—4	2—0	76	5., 22.
95	5	3	21	13	25	3—4	4	77	18.
96	5	—	1	2	36	2	?	76	5.
97	16	—	11	11	33	3±	4	75	
98	8	2	3	2	34	3—4	4	75	
99	8	—	3	5	32	3—4	4	74	
100	2	—	2	—	26	3	?	77	1.
ϵ_1 (ϵ_2) 101	2	14(-)	9(8)	2(11)	21	4	4—5	79	19.
102	10	1	12	12	34	4±	4—5	78	13.

Plate R	Exp. time range 1 to	Images e	Images g	Images fg	Δ	Seeing	Transp.	Temp. °F	Remarks
103	5	—	3	3	28	4±	4	77	13.
104	3	—	—	3	26	4⁻	4	76	11.
105	4	1	8	14	25	3—4	4	75	11.
106	10	—	1	5	24	4—5	2—0	74	1.
107	5	28	28	16	25	4—5	2—0	75	1., 3.
ϵ_1 (ϵ_2) 108	4	17(2)	11(17)	7(10)	27	4-3-4	4	79	19.
109	10	11	32	28	24	4—5 / 5—4	4	77	
110	20	40	34	8	15	5—4	?	77	1.
111	40	2	14	12	28	3⁻-4 / 4—5	3—4	81	13.
112	10	—	11	11	29	3—2	?	80	5., 13.
113	20	3	18	18	24	3-4-3	3—2	79	7., 9.
114	10	32	18	13	26	3—4	2—0	75	7., 3.
115	4	—	12	29	27	3-4-3	4	69	
116	16	7	10	10	29	3±	4	77	23.
117	20	6	13	14	30	2—3	4	75	
118	10	26	12	16	26	3—4	?	81	1.
119	20	4	5	6	26	4—5	3	88	1., 6.
120	10	5	6	8	20	4?	?	86	20., 1.
121	20	2	9	5	23	3—4	2—3	84	1.
122	32	3	6	4	28	3⁻-3±	3	83	
123	16	9	19	9	25	4	3	85	6.

TABLE 15b.

Plate π	Exp. time range 1 to	Images e	g	fg	Δ	Seeing	Transp.	Temp. °F	Sector	Remarks
24004	1	2	—	—	24	4—5	3⁻	—	5	k
223	1	1	1	—	28	3—4	2	—	50	A/R
371	1	2	—	—	20	5	—	—	35	M
372	1.2	2	—	—	„	5	—	—	35	M
416	1	2	—	—	32	3	—	—	3	M
417	1	2	—	—	„	3	—	—	3	M
452	1.1	2	—	—	28	3—4	4—5	48	3	A/k
453	1	—	2	—	„	3—4	4—5	48	3	k
25085	1	—	1	1	25	4	2—3	—	6	M
808	1	1	1	—	24	4—5	—	68	20	
809	1	2	1	—	„	4—5	—	68	20	1.
830	2	—	2	—	22	4—5	3	76	½	k
847	1	—	—	1	24	4—5	4	—	½	k
848	1	—	—	1	„	4—5	4	—	½	k
849	3	—	2	2	29	4—5	4	69	2	
850	3	3	1	—	22	4—5	4	69	2	
851	3	—	—	2	28	4—5	4	69	2	
852	6	1	1	2	22	5	?	69	2	2.
861	3	—	1	2	16	4—5	4	68	20	
862	3	—	2	—	„	5-4-3	4	68	20	3.
879	4	—	1	2	22	4—5	3	67	10	
880	4	—	1	2	28	4—5	2—3	67	10	
881	2	1	4	1	24	4	2—3	66	20	
922	3	—	2	—	22	4 3—4	3	73	3	
923	2	—	—	3	„	3—4 4	3	73	3	
924	3	—	—	1	27	4 3—4	2—1	73	3	4.
926	2	—	1	2	12	4—5	3?	72	35	
946	1.5	—	3	1	22	$4^{\pm}$	3	73	20	
947	1.1	2	2	—	19	4—5	3	73	20	

Plate π	Exp. time range 1 to	Images e	Images g	Images fg	Δ	Seeing	Transp.	Temp. °F	Sector	Remarks
25948	1.1	—	2	1	27	4±	3—4	73	20	
968	1.5	—	2	1	21	5	3	75	2	5.
969	2	—	2	1	,,	5	3	75	2½	6., 7.
26008	4	—	—	1	24	4—5	—	78	2	
009	2	—	—	3	,,	4—5	—	78	3	
011	4	—	—	2	28	5	—	77	3	
023	1	—	—	2	25	4	2—3	74	¾	V
026	4	—	—	1	19	4—5	3	72	12	
027	4	—	1	—	,,	4—5	3—4	72	20	
028	4	1	—	1	,,	4—5	4	72	35	
029	2	—	—	1	26	4—5	3	71	20	8.
030	2	—	1	1	,,	4—5	3	71	33	8.
031	2	—	1	1	,,	4—5	3	71	50	8.
032	1	—	1	1	24	4—5	3	72	11	
044	4	—	1	—	24	4—5	2—3	78	3	9.
045	4	—	2	—	13	5	3	78	5	9.
046	4	2	1	—	,,	5	3	77	8	
060	4	—	1	1	17	4—5	3	76	12	15.
		1	1	1						16.
061	4	1	—	2	,,	4—5	3	75	18	15.
		1	1	1						16.
062	4	2	—	—	,,	4—5	3—4	75	30	15.
		—	1	1						16.
063	1	—	2	—	23	5	3	74	11	
064	1	1	—	1	,,	5	2?	74	11	10.
129	1	—	1	1	24	4—5	3	73	B_1 40	k, 11.
150	1.5	—	—	1	23	5	—	68	4	12.
151	1.5	—	1	—	,,	5	—	68	6	13.
181	1.1	—	—	1	23	5	—	64	30	14.
198	2	—	3	—	19	4—5	4	72	30	
199	5	—	3	1	19	4—5	4	72	50	

REMARKS (Table 15b).

k taken by van de Kamp.
A „ „ Asklöf.
M „ „ Mitchell.
V „ „ Vyssotsky.
R „ „ Reuyl.

1. passing clouds.
2. objective fogged.
3. seeing unsteady.
4. thick haze.
5. scattered clouds.
6. „ „
7. spider in guiding eyepiece.
8. tailpiece rotated 2°.
9. thin clouds.
10. clouds.
11. B_1-filter reduces about 5^m.
12. through clouds.
13. clear?
14. clouds.
15. B.G.C. 11233.
16. Anon.

Table 16 contains the results of the measures.

Column 1. Number in Burnham's General Catalogue.
2. Right ascension for 1900.
3. Declination for 1900.
4. Photovisual magnitude taken from the Henry Draper Catalogue. The values in brackets were taken from other sources.
5. Plate number.
6. Epoch of the observation. When marked .. the plate was taken on the same night as the previous one.
7. Position angle.
8. Probable error of (7).
9. Mean distance before correcting.
10. Probable error of (9).
11. Mean distance after correcting.
12. Probable error of (11).
13. Number of images.
14. Remarks.

For those stars on which 2 or more plates were taken weighted means are given. Weights were given proportional to $\sqrt{n}$ rounded off for the sake of convenience.

It will be readily seen that the probable error is decreased in the average by applying the corrections. This could be expected a priori.

The probable errors given have been derived from the deviations of the individual measures from the plate mean. Therefore the values exhibit only the internal agreement of the individual measures. They are of little significance for small values of n.

TABLE 16.

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
	h m	° ′			y	°		″		″			
941	1 43.3	+47 24	5.99	26150	29.65	213.0	—	1.834	—	1.952	—	1	
				26151	„	212.0	—	1.862	—	2.031	—	1	
					29.65	212.50		1.848		1.992			
1799	3 33.8	+33 48	6.85	1	29.99	86.12	.19	1.999	.094	2.141	.072	4	
2751	5 23.1	+25 4	5.44	2	29.99	203.84	.07	4.968	.017	4.986	.014	76	1.
4477	8 6.5	+17 57	5.56	30	30.18	104.03	.11	5.524	.033	5.276	.027	33	2.
				31	30.19	104.93	.06	5.553	.009	5.521	.008	90	
					30.19	104.57		5.541		5.423			
4601	20.7	+27 16	6.30	25	30.17	216.20	.05	5.023	.008	5.040	.008	37	3.
4798	44.4	+35 26	6.73	7	30.11	97.30	.16	3.658	.012	3.739	.011	24	4.
4890	55.8	+15 41	8.6	34	30.20	192.74	.11	5.178	.009	5.178	.009	16	
5003	9 11.5	+24 4	7.20	26	30.17	22.84	.94	5.806	.011	5.806	.011	11	
5071	19.2	+ 6 47	6.81	8	30.11	318.65	.13	1.797	.040	1.913	.029	6	5.
				16	30.14	318.38	.48	1.527	.017	1.660	.022	5	
					30.12	318.52		1.662		1.786			
5136	28.4	+73 32	6.43	5	30.11	129.69	.08	4.967	.014	4.977	.013	40	
				19	30.14	129.82	.08	4.947	.010	4.973	.010	44	
					30.13	129.76		4.956		4.975			
5422	10 19.7	+53 8	7.36	10	30.12	87.58	.07	3.290	.025	3.336	.029	13	
				13	30.13	87.42	.20	3.104	.037	3.220	.031	16	
					30.13	87.50		3.197		3.278			
5448	24.7	+21 20	8.4	25085	29.12	100.80	—	1.468	—	1.788	—	2	6.
5516	34.8	+42 40	7.8	20	30.14	165.03	.21	3.468	.025	3.554	.029	8	
				23	30.15	164.51	.43	3.321	.039	3.218	.034	10	
				35	30.20	164.27	.08	3.507	.009	3.548	.009	25	
					30.17	164.54		3.446		3.460			
5533	37.3	+45 7	(9.6)	27	30.17	8.93	.41	1.892	.046	2.019	.055	5	

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
5720	11 10.0	+28 7	7.13	9	30.11	96.13	.16	3.783	.020	3.822	.018	20	
				12	30.13	96.55	.22	3.606	.033	3.683	.031	14	
				14	30.13	95.70	.19	3.650	.023	3.706	.021	16	
				37	30.20	96.13	.14	3.688	.022	3.762	.019	23	
					30.15	96.13		3.682		3.744			
5817	27.2	+60 37	(9.0)	39	30.22	53.50	.65	2.542	.030	2.691	.025	6	
5842	31.2	+56 43	7.40	24	30.15	167.73	.36	5.774	.024	5.751	.022	11	
				28	30.17	168.42	.07	5.861	.008	5.861	.008	17	
				33	30.19	167.23	.19	5.959	.020	5.948	.022	17	
					30.18	167.80		5.873		5.863			
6106	12 11.7	+30 37	(9.0)	44	30.26	180.70	—	2.600	—	2.766	—	2	
				45	30.27	179.50	.16	2.670	.036	2.820	.025	8	
					30.27	179.80		2.652		2.806			
6108	12.0	+70 42	8.2	32	30.19	5.28	.80	1.905	.027	2.104	.029	9	
				36	30.20	4.03	.62	2.057	.018	2.270	.014	19	
				38	30.20	1.42	.99	1.972	.027	2.067	.027	14	
					30.20	3.42		1.985		2.151			
6216	31.0	+11 57	8.3	17	30.14	242.80	.48	2.331	.020	2.504	.021	15	7.
6236	35.9	+ 9 23	7.12	21	30.14	191.74	.42	1.350	.031	1.413	.024	9	
				59	30.31	193.69	.83	1.367	.024	1.403	.017	15	
					30.24	192.85		1.360		1.407			
6364	58.4	+56 57	(9.0)	42	30.24	336.0	—	3.506	—	3.612	—	1	8.
6415	13 7.3	+32 39	6.66	29	30.17	343.00	.33	1.874	.011	2.102	.008	42	9.
				46	30.27	342.10	.27	1.813	.017	1.886	.018	32	
				57	30.31	343.30	.23	1.861	.009	1.981	.007	74	
					30.26	342.87		1.851		1.999			
6494	23.6	+16 14	7.9	63	30.33	347.63	.46	1.643	.043	1.820	.030	9	10.

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
6527	13 28.8	+49 39	8.12	40	30.22	301.17	.06	3.731	.013	3.816	.011	32	
				52	30.27	301.56	.16	3.828	.024	3.867	.020	25	
					30.24	301.35		3.775		3.839			
6558	32.3	−7 22	7.11	68	30.33	221.18	.13	2.511	.051	2.529	.040	4	11.
6762	14 8.4	+5 51	7.7	41	30.22	192.43	.08	4.763	.022	4.793	.031	5	
				43	30.24	192.90	—	4.663	.026	4.728	.028	3	
					30.23	192.66		4.713		4.760			
6776	9.7	+55 48	8.2	64	30.33	90.02	.29	1.883	.023	2.033	.022	11	12.
6837	17.3	−7 19	6.80	47	30.27	169.99	.20	5.602	.019	5.602	.019	9	13.
				69	30.33	170.62	.10	5.599	.016	5.573	.014	18	
					30.30	170.35		5.600		5.585			
6875	22.0	+ 4 8	8.5	48	30.27	14.95	.66	2.753	.014	2.867	.013	16	
				58	30.31	16.33	.43	2.692	.023	2.818	.024	11	
				60	30.31	15.50	.37	2.679	.025	2.838	.023	8	
					30.29	15.53		2.712		2.844			
6954	36.0	+16 51	4.94 5.81	66	30.33	105.21	.05	5.720	.010	5.708	.008	56	14.
7031	46.3	+49 7	5.64	53	30.27	45.15	.24	3.101	.028	3.100	.012	20	15.
				54	30.30	44.55	.11	3.251	.009	3.293	.006	87	
				79	30.40	44.37	.06	3.258	.022	3.332	.014	36	
					30.33	44.62		3.222		3.265			
7120	15 0.5	+48 3	4.86	61	30.31	246.90	.07	3.195	.010	3.197	.006	146	16.
				65	30.33	246.52	.18	3.079	.014	3.123	.014	40	
					30.32	246.77		3.156		3.172			
7127	2.8	+ 9 37	6.69	55	30.30	210.72	.09	4.328	.007	4.390	.006	52	
				71	30.34	210.96	.10	4.356	.011	4.334	.008	37	
					30.32	210.83		4.341		4.364			

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
7160	15 7.9	+15 46	8.9	49	30.27	235.43	.36	2.225	.047	2.303	.052	5	
				80	30.40	235.50	.70	2.287	.023	2.471	.030	12	
					30.35	235.47		2.262		2.404			
7318	30.0	+10 53	4.23 5.16	72	30.34	181.58	.20	3.860	.013	3.882	.013	17	17.
				85	30.40	182.28	—	3.644	.055	3.743	.055	5	
				92	30.42	181.59	.43	3.768	.021	3.789	.018	21	
					30.39	181.71		3.779		3.814			
7314	33.8	+30 26	8.2	70	30.33	294.43	.20	4.968	.026	4.948	.010	7	
7389	42.6	— 5 1	8.85	74	30.34	80.57	.26	3.529	.025	3.486	.028	8	
7396	43.9	— 2 56	8.3	73	30.34	160.64	.37	2.450	.032	2.446	.039	13	
7398	45.1	— 2 54	8.7	73*a*	30.34	282.35	—	2.256	.036	2.232	.030	4	
7461	54.6	+22 5	8.7	50	30.27	207.54	.28	3.958	.015	3.984	.014	12	
7551	16 8.6	+13 48	6.84	56	30.30	339.53	.24	3.888	.046	3.890	.040	10	18.
				62	30.31	339.79	.10	4.090	.006	4.166	.005	47	
				75	30.34	339.49	.09	4.155	.028	4.079	.021	27	
				86	30.41	339.78	.08	4.090	.009	4.162	.008	44	
					30.35	339.68		4.077		4.107			
7703	33.9	+53 8	5.56 6.58	94	30.47	108.54	.36	3.435	.032	3.450	.030	8	19.
7878	17 3.2	+54 36	5.80 5.83	25830	29.46	114.65	—	2.150	—	2.338	—	2	20.
				25847	29.48	112.5	—	2.156	—	2.266	—	1	
				25848	,,	113.1	—	2.010	—	2.224	—	1	
				83	30.40	111.93	.22	2.189	.023	2.351	.018	29	
				89	30.41	111.60	.16	2.195	.024	2.355	.017	25	
				95	30.47	112.65	.26	2.168	.019	2.330	.017	37	
					29.47	113.42		2.105		2.276			
					30.43	112.10		2.183		2.344			

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
8003	17 20.2	+37 14	4.52	67	30.33	313.58	.12	3.983	.012	4.019	.011	42	21.
			5.47	93	30.42	313.76	.11	3.952	.012	4.013	.010	37	
					30.38	313.67		3.968		4.016			
8243	52.1	+18 21	6.59	84	30.40	113.25	—	2.404	.083	2.533	.069	5	
				87	30.41	112.63	.18	2.576	.012	2.664	.010	55	
				96	30.47	115.16	—	2.547	—	2.724	—	3	
					30.42	113.20		2.539		2.651			
8303	57.6	−8 11	5.34	26023	29.60	266.85	—	1.956	—	2.162	—	2	22.
			6.04	26129	29.64	267.55	—	2.012	—	2.152	—	2	
					29.62	267.20		1.984		2.157			
8579	18 23.4	+19 13	7.6	76	30.38	190.82	.24	5.471	.048	5.432	.044	6	
				78	30.39	190.18	.25	5.455	.030	5.467	.031	10	
				116	30.54	190.44	.10	5.435	.012	5.444	.012	27	
					30.46	190.44		5.488		5.448			
8663	31.4	+16 54	6.17	25849	29.48	181.32	—	1.695	.087	1.909	.065	4	23.
				25850	„	179.70	—	1.794	.038	2.042	.033	4	
				25851	„	182.40	—	1.538	—	1.809	—	2	
				25852	„	180.71	—	1.745	.152	2.013	.134	4	
				109	30.51	183.00	.36	1.854	.010	2.006	.008	71	
				110	30.51	183.13	.17	1.986	.009	2.131	.005	82	
					29.48	180.83		1.715		1.962			
					30.51	183.07		1.924		2.072			
8684	33.0	+41 12	6.86	91	30.42	339.40	.10	5.210	.021	5.197	.017	12	
				97	30.47	339.44	.15	5.158	.013	5.191	.013	22	
				111	30.52	339.39	.05	5.178	.012	5.189	.013	28	
					30.48	339.41		5.178		5.192			

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
8705	18 34.8	+30 39	(9.0)	104	30.49	219.94	—	2.342	—	2.463	—	3	
8732	36.6	+52 14	6.86	103	30.49	323.43	.83	1.633	.029	1.737	.026	6	
				106	30.50	324.92	—	1.640	.034	1.646	.027	6	
					30.50	324.18		1.636		1.692			
8783	41.0	+39 34	5.06	101	30.48	6.15	.08	2.967	.014	3.035	.014	25	24.
			6.02	108	30.51	6.17	.17	2.941	.009	3.085	.009	35	
					30.50	6.16		2.953		3.062			
8785	41.1	+39 30	5.14	101	30.48	115.15	.12	2.334	.020	2.369	.017	19	25.
			5.37	108	30.51	113.67	.26	2.226	.016	2.374	.016	29	
					30.50	114.33		2.274		2.372			
9023	19 0.6	+51 26	(8.7)	25808	29.45	74.80	—	2.322	—	2.318	—	2	
				25809	,,	75.23	—	2.113	—	2.170	—	3	
				25861	29.49	77.08	—	1.818	—	1.971	—	3	
				25862	,,	71.90	—	1.926	—	2.118	—	2	
				25879	29.50	76.70	—	1.766	—	2.017	—	3	
				25880	,,	75.59	—	1.810	—	2.073	—	3	
					29.48	75.22		1.959		2.111			
9114	7.7	+38 37	7.51	98	30.47	214.64	.22	4.315	.022	4.356	.016	7	26.
				100	30.48	214.75	—	4.375	—	4.380	—	2	
					30.47	214.68		4.335		4.364			
9693	48.2	+25 36	7.41	99	30.47	138.28	.17	3.735	.021	3.791	.028	8	
				105	30.49	138.53	.29	3.866	.019	3.930	.018	23	
				115	30.53	139.09	.22	3.806	.015	3.888	.014	41	
					30.50	138.72		3.812		3.882			
9808	55.9	+47 5	7.61	122	30.56	386.84	.09	5.248	.014	5.268	.014	13	

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
9818	19 56.6	+10 28	6.89	102	30.48	350.68	.17	4.095	.016	4.170	.017	25	
				113	30.52	350.85	.14	4.088	.009	4.100	.009	39	
				117	30.54	351.05	.18	4.097	.013	4.121	.011	33	
				119	30.55	350.52	.20	4.110	.022	4.193	.022	15	
					30.52	350.80		4.096		4.140			
9868	59.8	+35 45	6.69	26044	29.61	172.5	—	1.851	—	1.957	—	1	
				26045	,,	174.35	—	1.695	—	1.795	—	2	
				26046	,,	171.11	—	1.855	—	1.950	—	3	
				107	30.51	173.58	.16	1.942	.007	2.083	.007	72	
					29.61	172.27		1.814		1.913			
					30.51	173.58		1.942		2.083			
9944	20 4.2	+63 25	8.8	25881	29.50	180.52	—	1.905	.023	1.925	.024	6	
				25946	29.54	177.04	—	2.087	.082	2.120	.082	4	
				25947	,,	178.40	—	2.006	.028	2.044	.026	4	
				25948	,,	178.94	—	1.982	—	2.062	—	3	
					29.53	178.73		1.995		2.038			
9982	7.5	+0 34	6.23	112	30.52	209.47	.20	2.951	.014	3.069	.016	22	
				114	30.53	209.03	.10	2.996	.006	3.139	.005	63	
				118	30.54	208.63	.12	2.989	.010	3.110	.008	54	
					30.53	209.00		2.982		3.111			
10669	56.3	+49 49	8.9	26026	29.60	152.4	—	1.727	—	1.882	—	1	
				26027	,,	152.6	—	1.717	—	1.869	—	1	
				26028	,,	147.85	—	1.834	—	1.965	—	2	
					29.60	150.95		1.759		1.905			
10676	57.3	+6 48	6.64	25922	29.53	222.35	—	2.739	—	2.863	—	2	27.
				25923	,,	222.05	—	2.844	—	2.977	—	3	
				25924	,,	219.3	—	2.592	—	2.772	—	1	
				120	30.55	219.90	.34	2.659	.011	2.730	.015	19	
				121	30.55	219.80	.28	2.707	.020	2.762	.018	16	
				123	30.57	218.85	.13	2.758	.016	2.882	.011	37	
					29.53	221.44		2.746		2.888			
					30.56	219.42		2.715		2.804			

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
10713	20 59.4	+56 16	5.74	26008	29.58	351.9	—	1.602	—	1.732	—	1	
				26009	„	351.07	—	1.359	—	1.535	—	3	
					29.58	351.38		1.450		1.609			
11233	21 41.0	+21 29	(8.8)	26060	29.61	355.55	—	1.747	—	1.971	—	2	
				26061	„	356.67	—	1.818	—	1.994	—	3	
				26062	„	355.25	—	1.954	—	2.083	—	2	
					29.61	355.82		1.840		2.016			
—	41.4	+21 31	(10.0)	26060	29.61	309.03	—	1.935	—	2.120	—	3	
				26061	„	305.56	—	2.024	—	2.164	—	3	
				26062	„	305.40	—	1.906	—	2.091	—	2	
					29.61	306.66		1.955		2.125			
11334	49.1	+65 17	6.41	26011	29.58	104.50	—	1.440	—	1.739	—	2	
13606	57.9	+62 21	(9.7)	26029	29.60	43.9	—	1.290	—	1.622	—	1	
				26030	„	43.25	—	1.382	—	1.722	—	2	
				26031	„	43.45	—	1.306	—	1.618	—	2	
					29.60	43.53		1.326		1.654			
11968	22 42.7	—4 45	6.75	24004	28.70	266.05	—	3.148	—	3.185	—	2	28.
				24452	28.86	267.95	—	2.904	—	3.071	—	2	
				24453	„	266.10	—	2.892	—	3.058	—	2	
					28.79	266.70		2.981		3.105			
11997	45.6	+68 2	6.39	24416	28.85	62.85	—	3.978	—	4.094	—	2	29.
				24417	„	62.25	—	3.872	—	3.975	—	2	
					28.85	62.55		3.925		4.034			
12076	52.6	+62 19	7.76	26032	29.60	348.90	—	1.648	—	1.903	—	2	
				26063	29.61	349.50	—	1.778	—	1.969	—	2	
				26064	„	348.20	—	1.796	—	1.961	—	2	
					29.61	348.87		1.741		1.944			

B.G.C.	R.A. (1900)	Decl.	m	Plate	Epoch 1900 +	Θ	ϵ_Θ	ρ	ϵ_ρ	ρ_c	ϵ_{ρ_c}	n	Remarks
12378	23 22.6	+16 5	8.2	24223	28.79	195.70	—	2.646	—	2.789	—	2	
			9.3	24371	28.83	194.45	—	2.810	—	2.892	—	2	
				24372	„	193.05	—	2.836	—	2.921	—	2	
				25926	29.53	195.07	—	2.883	—	3.006	—	3	
					28.99	194.57		2.794		2.902			
12652	52.5	+47 31	8.5	26181	29.66	193.3	—	1.904	—	2.062	—	1	
				26198	29.67	194.05	—	1.737	—	1.880	—	3	
				26199	„	191.98	—	1.744	.036	1.853	.016	4	
					29.67	193.07		1.773		1.906			
12675	54.4	+33 11	6.58	25968	29.57	240.62	—	1.711	—	1.838	—	3	30.
			6.58	25969	„	242.03	—	1.654	—	1.793	—	3	
					29.57	241.32		1.682		1.816			

REMARKS (Table 16).

1. Σ 716 = 118 Tauri.
2. ζ Cancri.
3. ϕ_2 Cancri = Σ 1223.
4. 130 Lyncis = Σ 1282.
5. Σ 1348.
6. Σ 1439.
7. Σ 1661.
8. The difference in magnitude of the components is about $1^m.1$.
9. OΣ 261.
10. OΣ 266.
11. 81 Virginis = Σ 1763.
12. Σ 1820.
13. Σ 1833.
14. π Bootis.
15. 39 Bootis = Σ 1890.
16. 44 Bootis.
17. δ Serpentis.
18. 49 Serpentis = Σ 2021.
19. 17 Draconis = Σ 2078.
20. μ Draconis.
21. ρ Herculis.
22. τ Ophiuchi.
23. OΣ 358.
24. ε_1 Lyrae.
25. ε_2 Lyrae.
26. Σ 2481.
27. λ Equulei.
28. Σ 2944.
29. Σ 2947.
30. 37 Andromedae = Σ 3050.

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STELLINGEN.

1. De methoden voor het verkrijgen van kunstmatige sterren, zooals die aangewend zijn door Ross, Przybyllok en Labitzke en Swings en Legros, zijn niet doeltreffend.
(Mon. Th. Phot. Eastman Kod. Comp. No. 5; 1924,
Publ. Königsberg 45 (4); 1929,
Liège, Inst. Astr. Géod. No. 41; 1930).

2. Ten onrechte geeft Osgood gewicht „oneindig" aan zijn fotografische metingen.
(M. W. Contr. 371, 4; 1928).

3. Ten onrechte meent Lau dat de normaal belichte beelden vrij zijn van stelselmatige fouten.
(A. N. 192, 183; 1912).

4. Het onderzoek naar stelselmatige fouten in micrometerwaarnemingen van dubbelsterren is dringend gewenscht.

5. Voor de bepaling van trigonometrische ster-parallaxen geniet de refractor de voorkeur boven den reflector.

6. De conclusies, welke Zinner op grond van zijn resultaten, betreffende de toenemende helderheidsverandering van δ Cephei-sterren trekt, zijn twijfelachtig.
(Sitz. Ber. Preuss. Ak. Wiss., Phys. Math. 9; 1931).

7. De nieuwste bepalingen van de helderheid van Sirius B ontnemen aan dit bewijs voor de juistheid der relativiteitstheorie alle overtuigende kracht.
(P. A. S. P. 42, 155; 1930, P. A. A. S. 7 (1), 20; 1931).

8. Helderheden van objecten als Sirius B dienen met den microfotometer van Moll te worden bepaald.

9. Als men de lichtsterkte in een punt eener lijn van Fraunhofer berekent met de eenvoudige formule van Schuster, moet men de „laagdikte" H interpreteeren als $\sqrt{\frac{p'}{g\varkappa}}$, waarin p' het quotient van totalen en electronendruk, $\varkappa$ de coëfficient der algemeene absorptie en g de gravitatieconstante voorstellen.

10. Ten onrechte meent Wurm dat de constanten in de bandenserieformule niet veranderen, wanneer deze overeenkomstig de moderne quantentheorie als een quadratische functie van n + ½, in plaats van n, geschreven wordt.
(Handbuch der Astr. Phys. 3 (2), 750; 1930).

11. De bewering van Mecke, dat een electronen „Umlagerung" in een molecule een sterker energieverbruik beteekent dan in een atoom, is onjuist.
(Forts. Chem., Phys., phys. Chem. 20 (3), 37; 1929).

12. De uitbreiding van het onderricht in het practisch gebruik van grafische methoden, bij het middelbaar en voorbereidend hooger onderwijs, is gewenscht.