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Reflectivity and emissivity of tungsten : with a description of a new method to determine the total reflectivity of any surface in a simple and accurate way

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REFLECTIVITY AND EMISSIVITY OF TUNGSTEN

WITH A DESCRIPTION OF A NEW METHOD TO DETERMINE THE TOTAL REFLECTIVITY OF ANY SURFACE IN A SIMPLE AND ACCURATE WAY

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PROEFSCHRIFT

TER VERKRIJGING VAN DEN GRAAD VAN
DOCTOR IN DE WIS- EN NATUURKUNDE AAN
DE RIJKSUNIVERSITEIT TE UTRECHT, OP GEZAG
VAN DEN RECTOR MAGNIFICUS DR. C. W.
STAR BUSMANN, HOOGLEERAAR IN DE FACULTEIT DER RECHTSGELEERDHEID, VOLGENS
BESLUIT VAN DEN SENAAT DER UNIVERSITEIT
TE VERDEDIGEN TEGEN DE BEDENKINGEN VAN
DE FACULTEIT DER WIS- EN NATUURKUNDE
OP MAANDAG 4 JUNI 1934 DES NAMIDDAGS
TE VIER UUR

DOOR

HUGO CHRISTIAAN HAMAKER

GEBOREN TE BROEK OP LANGENDIJK

AMSTERDAM — 1934
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Aan mijn Ouders

VOORWOORD.

Gaarne maak ik van deze gelegenheid gebruik om U, Hooggeleerden ORNSTEIN, Hooggeachten Promotor, te danken voor het vele, waarvoor ik U erkentelijk mag zijn. De onbaatzuchtige wijze, waarop gij Uwe gaven stelt in dienst der samenleving, zal mij steeds tot voorbeeld strekken. Indien ik met dit proefschrift een bijdrage heb mogen leveren tot een van de belangrijke onderzoekingen, welke in Uw laboratorium worden verricht, schenkt mij dat een bijzondere voldoening van mijn werk.

De jaren, dat ik in Utrecht student ben geweest, zullen mij steeds een aangename herinnering blijven.

Dat het mij daarenboven gegeven is geweest aan boord van Hr. Ms. „Willebrord Snellius" een zoo boeiende en interessante reis naar en door Nederlandsch Indië mede te maken, acht ik een zeldzaam voorrecht.

Ook de maanden, die ik in Noorwegen mocht doorbrengen, zal ik niet licht vergeten.

Tenslotte betuig ik mijn dank aan VERMEULEN voor vriendschappelijken raad bij het onderzoek en aan SCHOUTEN, BEEZHOLD en VAN DER VEEN, die allen eenigen tijd bij het werk behulpzaam waren.

CONTENTS.

INTRODUCTION.

In researches on the relative intensities of spectral lines a standardised source of light, for which the spectral distribution of energy is known, is an indispensible instrument (1); as such electric lamps with filaments or strips of tungsten are now in general use.

To calibrate a standard lamp two independent methods have been worked out and they were found in mutual agreement to within 2 or 3 % in the region from 0.4 μ to 0.7 μ (2). The method which is most readily carried out in practice consists of a determination of the temperature of the tungsten by means of an optical pyrometer and of calculating the energy distribution from PLANCK's formula and the emissivities [1] of tungsten.

Though many investigations have been made on the emissivities, their results have not in every respect been satisfactory. In the ultraviolet the values given by different authors differ by no less than 15 % (3). In the visible region WORTHING has carried out determinations of the emissivity but at two wave-lengths, 0.665 μ and 0.467 μ, only (4). Further COBLENTZ and EMERSON (5) have made observations of the reflectivity [2] at room temperature and WENIGER and PFUND (6) investigated the change of reflectivity with temperature, from which the emissivities can be computed. These observations however were made with pieces of tungsten which were carefully polished and it is open to question whether their results are also valid for tungsten strips as used in standard lamps, which often have very rough surfaces.

At the Physical Institute of Utrecht University new researches on the subject were started some years ago in order to supply a better basis for the calibration of standard lamps by the above

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[1] With "emissivity" is meant the ratio of the radiation from the tungsten to the radiation from a black body at the same temperature. This quantity is also often called the "emission factor".

[2] The "reflectivity" is the ratio of the intensity of the reflected to that of the incident light. In other papers this quantity is frequently termed the "reflecting power" or the "reflection factor".

method. The results of these investigations will be dealt with in this thesis.

General remarks and summary of results.

According to KIRCHHOFF's law the emissivity e equals the coefficient of absorbtion a; and a again is equal to $1-r$ where r indicates the total reflectivity. From determinations of the reflectivity r the emissivity can be computed and this method we have adopted.

The experimental researches were made exclusively on the reflectivity of tungsten and the value of the observed reflectivities will be given by the accuracy of the experiments. It should however not be concluded that the emissivities derived from the reflectivities have the same value; for the relation $e+r=1$ holds only under certain special conditions and it is very probable that these conditions will not be fulfilled in all practical cases.

We will return to these theoretical considerations in chapter 5; in chapter 1 to 4, in which the experiments have been described, we will deal with the reflectivity only.

The strips investigated.

All determinations were made on tungsten strips such as are actually used in the tungsten strip lamps manufactured by PHILIPS at *Eindhoven*. The tungsten contains 1 % of thorium oxide and the strips are about 2.5 mm broad and 25 μ thick.

When tungsten is heated to incandescent temperatures for the first time the emission is known to change during the first two or three hours but then to remain practically constant. This process is called the "aging" of the tungsten and is explained by a process of recristallisation.

The tungsten strips used in the experiments had all been aged for 12 hours at least, at a temperature of 2000 or 2500° K and in an argon atmosphere of 50 cm mercury.

Angle of incidence.

Generally the reflectivity is dependent on the angle of incidence and the reflected light is polarised, the reflectivities for both components being unequal. In most cases however this effect is negligible as long as the angle of incidence does not exceed 20°. In the case of tungsten this follows more specially from researches in which the emissivity was determined as a function of the angle with the normal (12).

In our experiments the angle of incidence varied from 10 to 20°.

Summary of results.

In the determination of the total reflectivity difficulties arose from the facts

1. that the tungsten strips had no perfectly plane surfaces and
2. that they had no perfectly polished surfaces either. A part of the light is diffusely reflected and this part it is, according to my own experience, easy to underestimate.

In the course of the experiments the difficulties mentioned were overcome in a satisfactory way; a method was developed by which the total reflectivity could be determined in one single experiment and independently of the form and condition of the surface (Chapter 1). By this method the total reflectivity was measured in the region from 0.45 μ to 1.05 μ and for strips outside as well as inside the bulb. Though in appearance the surfaces of the strips varied from fairly polished to very rough, their total relectivities were found to agree with each other; real differences between different strips could not definitely be demonstrated. The following numbers may convey a general idea of the results arrived at.

Wave-length 0.7 μ Number of observations 12.
Maximum 56.4 %
Minimum 54.2 %
Average 55.0 %
Standard deviation from the average 0.7 %.

In addition to the determinations of total reflectivity experiments were made giving relative values in different overlapping regions of the spectrum. By appropriately adjusting these relative measurements the curve of total reflectivity could be extended into the ultra-violet down to 0.23 μ. Beyond 1.05 μ the curve could likewise be extended by observations of Coblentz and Emerson (5).

The work was completed by researches on the change of reflectivity with temperature made from 0.25 μ to 1.0 μ and up to 2400° K.

From the data thus collected the emissivity of tungsten was computed between 0.23 μ and 1.05 μ and from 1000 to 3000° K. The final results are given in table 18 on page 64 and some curves have been drawn in figure 21.

A few comparisons with the results of other investigators have been made in § 3 of Chapter 5, but a full discussion of the subject has not been attempted.

In a concluding chapter a discussion is given of the accuracy of a standard lamp calibrated by means of an optical pyrometer and the conception of a "color temperature" is critisised.

CHAPTER 1.

Determinations of total reflectivity at room temperature.

§ 1. *Principle of the method.*

The total reflectivity of a surface is defined as the ratio of the intensity of the total amount of light reflected to the intensity of the incident light.

We will put this definition into a more mathematical form. Suppose (fig. 1) the intensity of the light incident along A on a unit of area and within a solid angle $d\omega_A$ to be.

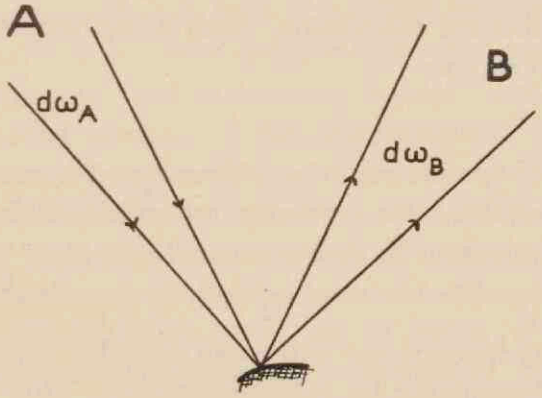

FIG. 1

$$I \, . \, d\omega_A$$

then in another direction B and within a solid angle $d\omega_B$ the amount

$$I \, . \, d\omega_A \, . \, r_{AB} \, . \, d\omega_B$$

will be reflected; r_{AB} is the specific reflectivity from A to B. The total amount of light reflected by the unit of area will be

$$I \, . \, d\omega_A \int r_{AB} \, . \, d\omega_B$$

where the integration has to be extended over a hemisphere and consequently the total reflectivity for direction A is

$$r_A = \int r_{AB} \cdot d\omega_B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The usual way in which r_A is determined by experiment consists of a practical application of formula 1, viz. of measuring the ratio of the sum of the intensities of the light reflected in different directions B to that of the incident light. In the following this method will be denoted as "the old method".

If the reflecting surface is curved and if the reflection is partly diffuse and partly specular, r_{AB} will be an irregular function of the direction B and it will therefore be almost impossible to arrive at accurate results by using the old method. In such cases we need a method in which the integration is made experimentally and not by means of a numerical calculation. Such a method can be based on the following theorem:

Theorem:

If a surface of opaque material is illuminated homogeneously from all sides, the ratio of the intensity reflected within a solid angle $d\omega_A$ *and in a direction* A *to the intensity incident within an equal solid angle is equal to the total reflectivity for direction* A.

To prove this theorem we will make use of a general optical law of reciprocity formulated by HELMHOLTZ (7). This law states that if from light incident along A a fraction r_{AB} falls in a direction B, either by transmission, reflection, refraction, diffuse scattering or by a combination of these processes, from light incident along B the same fraction $r_{BA} = r_{AB}$ will fall in direction A.

Applying this law to a reflecting surface we at once see that the specific reflectivity from A to B, r_{AB} must be equal to the specific reflectivity from B to A, r_{BA}.

Let the amount $Id\omega_B$ be incident along B, then a quantity $I \cdot d\omega_B \cdot r_{BA} \cdot d\omega_A$ is reflected in direction A and within a solid angle d $p\omega_A$ and if the surface is illuminated homogeneously, the total amount reflected towards A will be

$$I \cdot d\omega_A \cdot \int r_{BA} \cdot d\omega_B \, .$$

The intensity incident within a solid angle $d\omega_A$ will be $I . d\omega_A$ and dividing by this amount we get

$$r'_A = \int r_{BA} . d\omega_B \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

r'_A is the ratio with which the above theorem is concerned and which I will call the reflectivity for homogeneous illumination. But since $r_{AB} = r_{BA}$ we have

$$r'_A = \int r_{BA} . d\omega_B = \int r_{AB} . d\omega_B = r_A . \quad . \quad . \quad . \quad (3)$$

and the theorem is proved.

We thus arrive at a "new method" for the determination of a total reflectivity viz: by illuminating the surface homogeneously and measuring the reflected light in one direction only.

This new method has very great advantages compared with the old method; for the total reflectivity is now found in one single experiment and the accuracy of the observations is entirely independent of the form and the condition of the surface. In using the old method we have, on the contrary, to carry out a whole series of experiments in each separate case and when the reflection is very irregular, we can never expect to arrive at accurate results at all.

The investigations carried out on 12 strips of tungsten with widely different surfaces give full evidence of the value of the proposed new method; maximum and minimum of total reflectivity do not differ by more than 3 or 4 % (See § 6 of this Chapter).

The reasonings given above also apply to a more general problem. Let a beam of light be incident on a piece of milkglass or on any other piece of turbid material. The light will then be partly transmitted, partly reflected and partly scattered in all directions. The total fraction of the incident light that is not absorbed by the material can in this case again be represented by an integral as in formula 1

$$r_A = \int r_{AB} . d\omega_B$$

where the integration is now to be extended over a whole sphere.

And again HELMHOLTZ's law will give us an equation $r_{AB} = r_{BA}$ etc. etc.

We thus arrive easily at the following generalisation of the theorem given above.

Theorem:

If a piece of any material is illuminated homogeneously the ratio of the intensity of the light leaving the material in a particular direction A to the intensity of the incident light is equal to $1 - a_A$ *where* a_A *represents the total absorbtion for light incident along A.*

Under a homogeneous illumination it is thus possible to measure the total absorbtion of any material in one single experiment. Special applications of this proposition have not been made in this paper but I have given the theorem here, since its application may be of use to others.

We still have to investigate what errors will arise in the proposed experiments if the illumination is not perfectly homogeneous. In that case we do not measure r_A as given by formula 3 but

$$r_A + dr_A = \int (1 + d_B) \cdot r_{AB} \cdot d\omega_B$$

where d_B represents the errors in the homogeneity of illumination.

The total error will consequently be given by

$$dr_A = \int d_B \cdot r_{AB} \cdot d\omega_B$$

The value of d_B can be determined by experiment for different directions B (See § 4) and in most cases it will be easy roughly to estimate r_{BA}, or better r_{AB}, from a preliminary experiment. Since we are dealing with a correction only we need not know r_{AB} with great accuracy.

The discussion of errors due to inhomogeneities of the illumination for the experiments to be described will be found on page 17.

Finally it must here be said that the method to determine a total reflectivity by homogeneous illumination has already been proposed in 1920 by SHARP and LITTLE (8). These authors however did not give a sufficient proof of the general validity of the principle and the method has not been used except by themselves.

§ 2. *The experimental method.*

In order to realise a homogeneous illumination the tungsten strip was mounted at the centre of a cylinder of tin-plated iron, 45 cm in diameter and 52 cm high (fig. 2). This cylinder and all objects inside it were painted white with a zinc-white paint, made after a

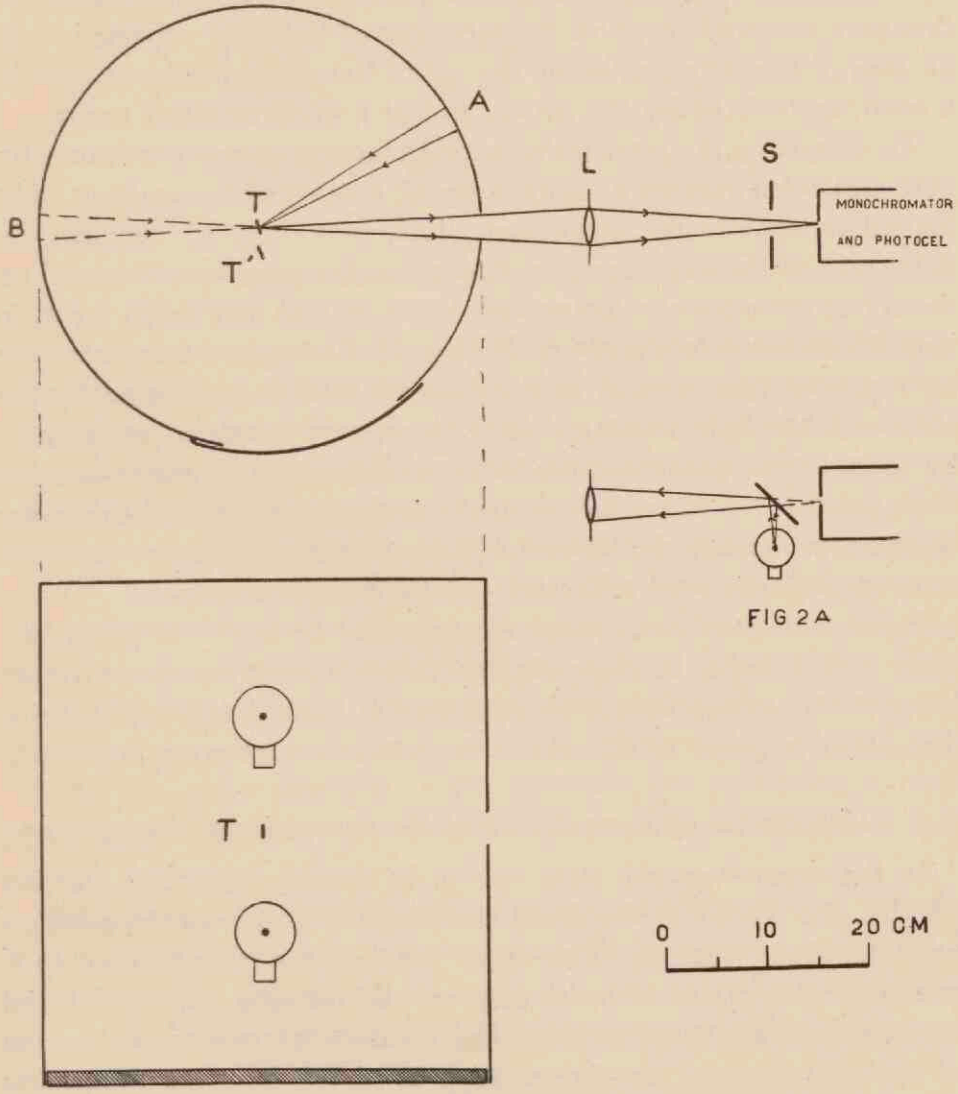

FIG. 2.

receipt of the Bureau of Standards (9). The interior was illuminated by two 100 watt electric lamps arranged as shown in figure 2, one above and one underneath the tungsten strip. The arrangement was such that no light could fall directly from the lamps on the strip ; the latter was illuminated by the light diffusely reflected from the walls of the cylinder only and it was proved by experiment (§ 4) that this illumination was sufficiently homogeneous.

Through a hole in the cylinder the strip was focussed on the slit of a double monochromator by a lens L. The radiation was measured by a photocel and amplifier placed behind the monochromator. In /w this way ~~b~~e could measure the intensity of the light reflected from an area of the strip limited by the slit of the monochromator, within a solid angle fixed by the lens L and of a given spectral range.

To determine the total reflectivity we have to compare the intensity of the reflected light with the intensity of a beam of the incident light limited in exactly the same way. This was done by reading the deflection of the galvanometer first when the strip was focussed on the slit of the monochromator and then a second time when the strip was drawn away into position T' (fig. 2). Doing so we compare the light coming from part A of the cylinder wall and reflected by the strip with the light coming directly from part B and it was proved by special experiment that the luminous intensities of parts A and B were equal (§ 4). Since moreover the deflection of the galvanometer was proportional to the intensity (§ 4) the ratio of the two readings at once gave the total reflectivity of the tungsten strip.

From this short description the principal features of the experiment will, I hope, be clear. In the following section the different parts of the apparatus will be described in some detail and in § 4 a discussion is given of the check experiments that were necessary.

§ 3. *Description of some details of the experimental arrangement.*

In the experiment the strip was to be drawn away into position T' (fig. 2). To do so the stand carrying the strip was mounted on hole-slot-plane fittings. The three legs of the stand were each provided with pieces of brass one with a hole, one with a slot and one plane ; these fitted on three brass points screwed to the bottom of the cylinder and so the position of the stand was accurately fixed. A similar arrangement has often been used throughout the experiments. To draw the strip away the stand was slightly tilted (turning on hole and slot) by means of a cord passing through a small hole in the cylinder.

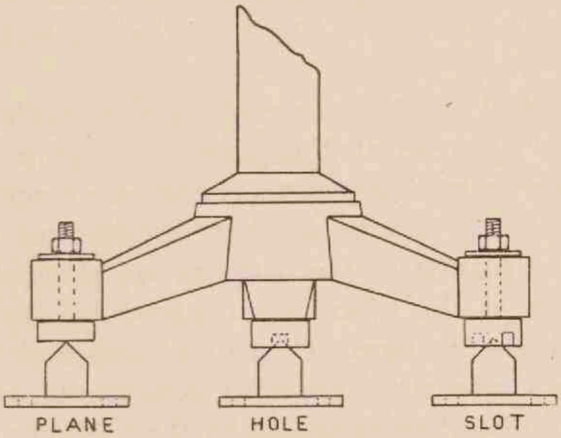

FIG. 3.
Stand mounted on hole-slot-plane fittings.

The stand itself was of a somewhat complicated construction so that the tungsten strip could easily be adjusted to whatever position desired. All strips examined were put in the same position and this was done by the following contrivance. The stand carrying the shutter *S* (fig. 2) was mounted on hole-slot-plane fittings and could be exchanged with another stand carrying a mirror and a small electric lamp with straight helical filament. The reflected image of the filament fell in the slit of the monochromator and in this way light was thrown backwards from the slit on the tungsten strip; from this the light was reflected to some part of the cylinder wall and all strips were mounted so that the centre of the reflected light fell in the same direction. This direction was fixed by a circle on a screen outside the cylinder. The angle of incidence was ± 12°.

To arrange the experiments in a convenient way the cylinder mantle could be entirely removed from its base which consisted of a wooden disc screwed to the table. On one side the cylinder had also a large door through which the interior could be inspected and slight alterations could be made if necessary.

For the lens *L* we used a Zeiss Tessar, the best achromate existing and the monochromator was a quartz double monochromator after v. CITTERT (10). Especially in the infra-red this instrument has a very low dispersion and consequently the spectral range transmitted was large, up to 0.05 μ. Happily however the reflectivity of tungsten only varies slowly with wave-length so that no serious errors are to be feared.

The photocel and amplifier were arranged according to a scheme devised by Dr. CUSTERS at *Eindhoven* [1]; in this amplifier two thermionic tubes with high grid insulation are coupled in parallel in a balanced bridge circuit so that disturbances due to small variations in the tension of the batteries are eliminated. The grid potential of one of the tubes is influenced by the photoelectric current through the photocel in the usual way and the difference between the plate currents of the tubes is measured with a galvanometer. To avoid disturbances caused by the humidity of the atmosphere the photocel and the amplifying tubes are placed in a vacuum chamber. Fuller details cannot be given here but they may be found in Dr. CUSTERS' publication (11) and in the literature cited there. The apparatus proved satisfactory though, of course, it cost some time and trouble before everything functioned satisfactorily.

The experiments were carried out on the second floor and in order to avoid mechanical disturbances we had to hang the galvanometer in a JULIUS' suspension, a measure which proved very effective.

The galvanometer used was a SIEMENS and HALSKE moving-coil galvanometer with soft iron core. The deflections of this instrument were found to be by no means always proportional to the current; this proportionality existed only when the axis of the galvanometer was adjusted to a strictly vertical position, but if the axis slightly deviated from the vertical the errors soon became great and irregular as may be seen from table 1.

The figures given apply to a set of experiments in which the current through the galvanometer could be commutated and the deflections both to the right and to the left were read off.

Consider for instance the left hand side of table 1; in the corresponding experiment the axis of the galvanometer deviated

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[1] We are indebted to Dr. CUSTERS for much advice on the use and construction of the amplifier.

TABLE 1.

Deflections of galvanometer under different circumstances.

Current 10^{-8} Amp.	Axis of the galvanometer deviating from the vertical			
	to the left		to the right	
	Deflections		Deflections	
	to left	to right	to left	to right
0.4	6.2	6.3	6.2	6.0
0.8	11.8	12.7	12.9	11.8
2.0	26.7	31.0	33.3	28.2
4.0	61.0	62.1	64.0	60.3
5.0	76.8	78.0	78.0	77.1

Deflections in mm.

from the vertical towards the left and under these circumstances the deflections to the right and to the left are unequal by no less than 15 % when the deflection is about 30 mm, but when the deflection is increased to 60 mm the difference decreases to 2 %. If the position of the galvanometer is altered so that the axis deviates towards the right from the vertical, the strange inequalities of the deflections of 30 mm also reverses as is shown by the right hand side of table 1 ; whereas in case the galvanometer was properly adjusted the errors did not exceed 3 pro mille for deflections up to 120 mm. In the experiments the distance of the reading scale from the galvanometer was about 120 cm.

How these great deviations can be explained is uncertain ; the results given in table 1 were strictly reproducible and the possibility that the coil of the galvanometer touched the core is therefore excluded.

These results may however give warning never to assume the linearity of a galvanometer but always to test it by experiment. During our observations such tests were regularly repeated.

§ 4. *Check experiments.*

As explained in § 2 the deflection of the galvanometer was read off first when the strip was focussed on the slit of the monochromator and then a second time when the strip was drawn away into position T' (fig. 2) ; the ratio of these deflections at once gave the total reflectivity.

In order that this be exactly true three important conditions must be satisfied viz.

1. The deflections of the galvanometer must be proportional to the intensities.

2. The luminous intensities of parts A and B (fig. 2) of the cylinder wall must be equal.

3. The illumination of the strip must be sufficiently homogeneous.

Each of these conditions was verified by special experiment and such verifications can, of course, only be made within certain observation errors. These errors will influence the reliability of the observations, so a detailed account of the verification experiments and their accuracy will now be given.

1. The proportionality between deflection and intensity was checked in the following way. The reflectivity of a strip was measured first when the total deflection of the galvanometer was about 10 cm; then the current through the electric lamps illuminating the interior of the cylinder was decreased so that the deflection of the galvanometer reduced to 3 cm or so and a second determination of the reflectivity was made. If the two determinations agree with each other, the proportionality between deflection and intensity is proved.

The great advantage of this method lies in the fact that the check could be repeated at any time during the observations, without making any changes in the experimental arrangement. The errors possible with the galvanometer (page 12) necessitate repeated testing of the proportionality. In the course of time quite a number of observations were made, a few of which are given in the following table.

TABLE 2.

Check on proportionality between intensity and deflection of galvanometer.

Wavelength in μ	Deflections in cm		R in $^0/_0$
	Direct	Reflected	
0.55	8.33	4.38	52.6
	3.51	1.85	62.8
0.75	9.90	4.87	49.3
	6.32	3.13	49.5
	1.59	0.78	49.1
0.90	9.74	5.21	53.5
	4.43	2.38	53.8

The deflections were read off to 0.1 mm and the differences in the reflectivities entered in the last column of the table are not greater than might be expected from reading errors. Perhaps there is some tendency for the reflectivity to increase with decreasing deflections. For instance a combinations of 9 different observations yielded the result that with a decrease of the average deflections from 10 to 3.5 cm, corresponded an average increase in the absolute value of the total reflectivity of 0.15 %, fixed with a mean square error of 0.07 %. The proportionality between deflection and intensity will be better for the smaller deflections. Since all observations were made with deflections of 5 to 10 cm we may conclude that the observed reflectivities are probably somewhat too low. The absolute error will be of the order of —0.15 %, to which corresponds a relative error of —0.3 %, the reflectivity always being about 50 %.

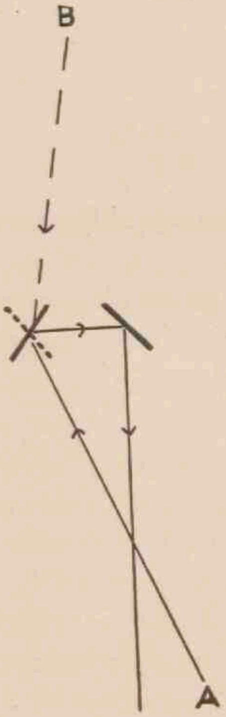


FIG. 4

2. Two small silver mirrors which could be rotated on vertical spindles 2 cm apart were placed in the cylinder instead of the tungsten strip. These mirrors were put in turn in two positions as shown in figure 4 and the luminous intensities of A and B were thus directly compared.

Sometimes a difference of 1 or 2 % was observed but such errors were easily corrected by slightly altering the position of the two light sources in the cylinder. To illustrate the results of these experiments, the data for one special case are given in table 3.

TABLE 3.

Comparison between the luminous intensities of parts A and B (fig. 2) of the cylinder walls.

Wavelength in μ	A	B	Relative difference in %
0.5	83.9	83.9	0.0
0.6	92.6	91.7	+1.0
0.7	94.2	94.1	+0.1
0.8	95.8	95.4	+0.4
0.9	96.2	95.4	+0.8

Average difference = +0.45 %
Mean square error of average = 0.2 „

The case represented in this table shows an average difference of +0.45 %.

The comparison between *A* and *B* was made 5 times altogether during the experiments. Calculating the average differences, as done in table 3, the following values were obtained

	Average differences $A - B$	
	−0.7	±0.45
	+0.5	0.2
	−0.1	0.1
	+0.8	0.15
	0.0	0.3
Total average	+0.1	0.25

The differences between the values resulting from different verifications may be due to the fact that the position of the light sources in the cylinder was not always sufficiently carefully adjusted. Upon the whole however the difference between *A* and *B* is small and in the average reflectivity calculated from all observations no serious systematic error is to be expected. An error of about 0.3 % either positive or negative is not impossible.

3. To verify the homogeneity of the illumination a silver mirror of high reflecting power (97 % at 0.7 μ) was mounted in the cylinder. For such a mirror the reflectivity is practically independent of the angle of incidence and the polarisation of the reflected light is very small (13). Starting from the normal position this mirror was turned through angles of 10, 20, 30, and 40° both horizontally to the left and to the right and vertically upward and downward. In this way the intensities incident at 20, 40, 60, and 80° were measured in 4 different quadrants. I found the following averages

Angle of incidence	Intensity
0°	1.001
20°	1.004
40°	0.994
60°	0.960
80°	0.960

Up to 40° the illumination was practically homogeneous, but at greater angles the intensity was about 4 % too low.

It is easily calculated that the error caused by this inhomogeneity will be —1.6 % for a perfect diffusing surface, following the cosine law. Experiments described later show that the diffuse reflection of a tungsten strip is only about 10 % of the total reflectivity. Consequently the error due to the inhomogeneity of illumination is not greater than —0.16 % and is probably still smaller, since the diffuse reflection of a tungsten strip does not follow the cosine law.

The error caused by the defect of radiation from the hole in the cylinder was by a similar reasoning estimated to —0.5 % for a perfect diffuser and less than —0.05 % for a tungsten strip.

In addition to these observations the total reflectivity of the white paint was determined. In the region from 0.45 μ to 1.0 μ its value varied irregularly between 90.6 and 93.1 %, but beyond 0.45 μ the reflectivity rapidly decreases; at 0.4 μ the value is 70 %. Thus the check on homogeneity carried out at 0.7 μ will be valid from 0.45 μ to $1.0\,\mu$. Occasionally the reflectivity of the tungsten was also observed at $0.4\,\mu$ but these values are not perfectly reliable.

Finally diffuse scattering or double reflections of the light in the lens L may cause a systematic error in the observations. To verify this point the total reflectivity of a "black body" was determined. If the body is perfectly black and if there is no diffuse scattering of light in the lens L, the total reflectivity must be zero. I observed a value of 0.5 % instead and this will be the maximum amount of light scattered by L. Suppose the total reflectivity of a tungsten strip is 50 %, the deflections for the direct and the reflected light being 100 and 50 mm respectively, then by scattering in the lens L a constant amount not greater than 0.5 mm will be added to both deflections. If so, we observe a reflectivity of $50.5 : 100.5 = 50.25$ %; the maximum error is 0.25 % in absolute or 0.5 % in relative value.

The black body used in these experiments consisted of a small cylinder 2 cm in diameter and 2 cm long with a hole in it and painted dead black on the inner side. If this object was not perfectly black, the error will have been smaller than the value computed above.

A summary of the systematic errors treated in this section and of their origin is given in the following list.

Sources of systematic errors.	Probable amount of the errors.
1. Proportionality between deflection and intensity.	-0.3 %.
2. Comparison between parts A and B (fig. 2) of the cylinder wall.	About 0.3 %, sign uncertain.
3. Inhomogeneity of illumination and hole in the cylinder.	Negative error less than -0.2 %.
4. Scatter of light in the lens L.	Positive error less than $+0.5$ %.

Altogether some of the errors are positive and some negative and they will partly cancel each other; the systematic error in the reflectivities will certainly not be greater than 0.5 % in relative or 0.25 % in absolute value.

It should also be noted that the systematic errors summed up above are all of the same order of magnitude. If we wish to increase the accuracy of the observations we should have to refine the experiment in every detail.

§ 5. *Determinations on strips inside a glass or quartz bulb.*

By the method described in the foregoing sections we may even determine the total reflectivity of a tungsten strip inside a glass or quartz bulb. We can determine the correction, necessary for the presence of the bulb, experimentally as will now be shown.

Suppose the bulb to reflect a fraction r, to transmit a fraction t and to absorb a fraction a of the incident light. To the fraction t entering the bulb an amount $r \, . \, t$ is added by a first reflection, $r^2 \, . \, t$ by a second reflection etc. (See fig. 5*A*). The intensity incident on the strip will thus be

$$t + r \, . \, t + r^2 \, . \, t + \ldots . = \frac{t}{1-r} \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

the intensity outside being unity.

The amount $\frac{R \, . \, t}{1-r}$ is reflected by the strip (R=reflectivity) and a fraction t of this reflected light will be transmitted by the bulb, to which again a quantity r is added by a reflection on the outer side (fig. 5*A*). The total intensity falling in direction M will thus amount to

$$R' = \frac{R \, . \, t^2}{1-r} + r \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

and this quantity is measured in exactly the same way as the total reflectivity of strips outside the bulb.

In order to find R we must know r and t. To determine r the bulb was wrapped in a sheet of black paper and a small hole in the paper mantle was focussed on the slit of the monochromator (See fig. 5*D*). For all bulbs examined in this way the reflectivity r was almost invariably found to be 7 % in the whole region from 0.45 μ to 1.0 μ.

To determine t the bulb was lowered to the position shown in figure 5C. We then measure the quantity

$$T = \frac{t^2}{1-r} + r \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

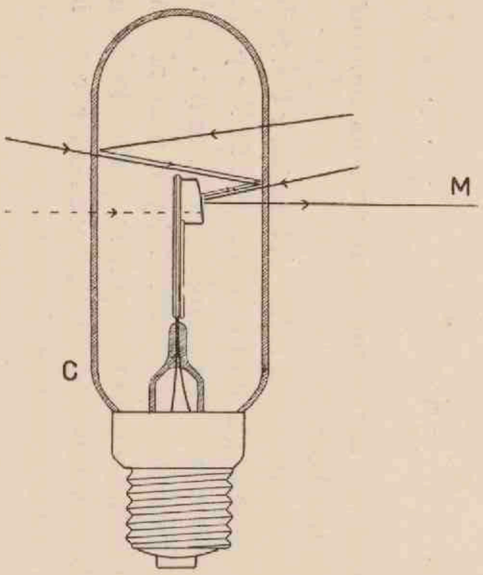
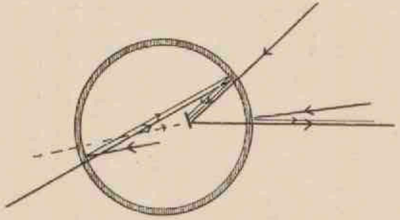


FIG.5A



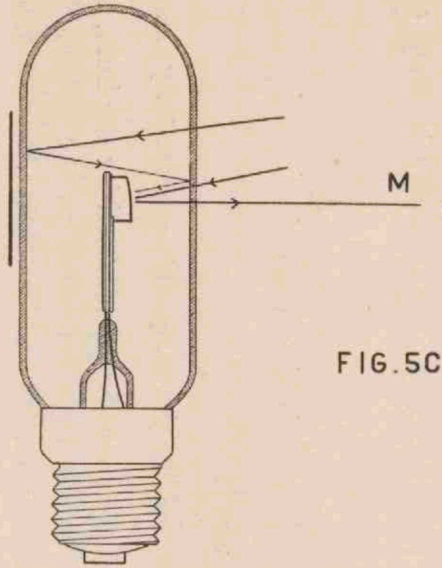
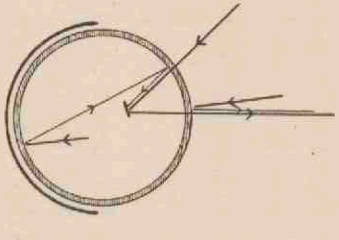


FIG.5B



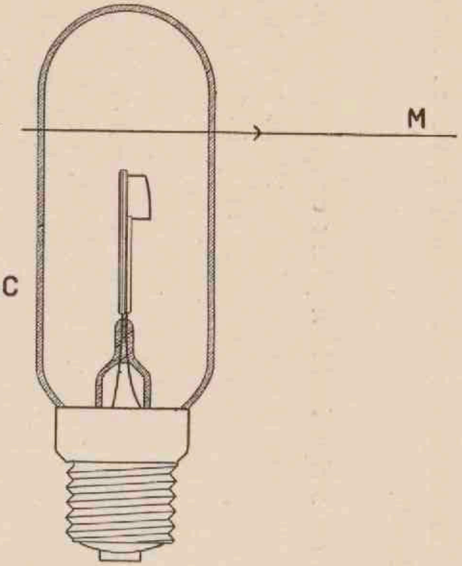


FIG.5C

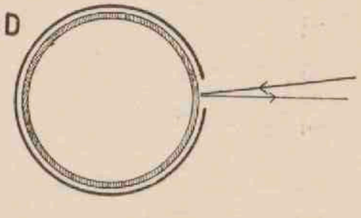


FIG.5D

as is easily understood. Since r is known already, t can be calculated and since $t+r+a=1$, the absorbtion a is also found by the same method.

Some values for the absorbtion of different bulbs are represented graphically in figure 6. In a strip lamp the tungsten slightly evaporates in the course of time, forming a thin coating on the glass.

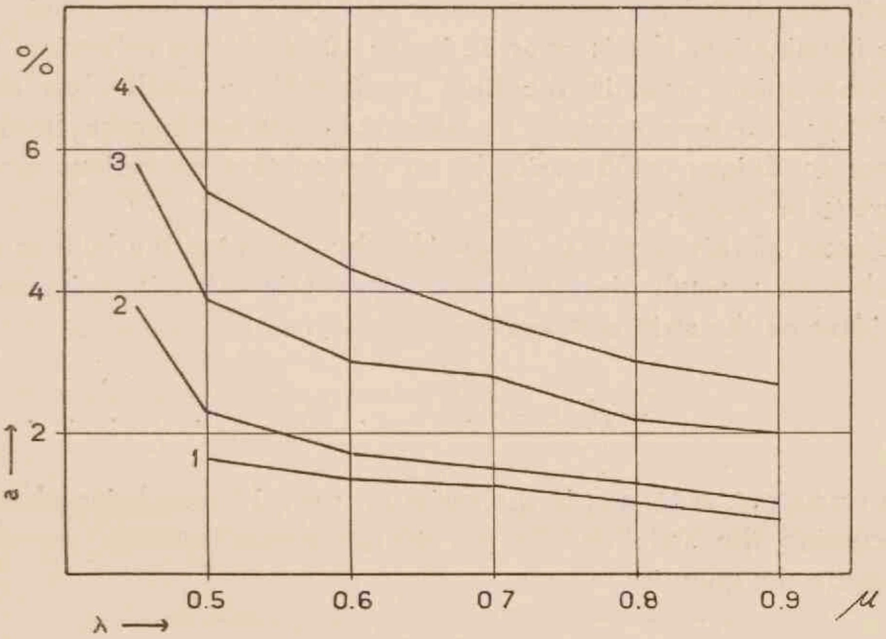


FIG. 6.

Absorbtion curves for different bulbs.

1. a clean glass bulb.
4. a quartz bulb with a heavy coating of evaporated tungsten.

The more a lamp has been used, the greater will be the absorbtion as is clearly demonstrated by the different curves in the figure. From 0.5 μ to 0.45 μ the absorbtion rapidly increases ; in this region and beyond 0.45 μ a standardised lamp cannot safely be used unless the absorbtion of the bulb is determined now and again. This is not the place, however, to discuss this question in detail (See also page 73).

The accuracy of a determination inside the bulb will, of course, be less than of a determination outside the bulb. The absorbtion of the glass may be unevenly distributed, thus causing systematic errors in the observations. Comparing measurements made inside and outside the bulb (tables 6*A* and 6*B* on page 26) we see, however, that errors of this kind, if any, have not been of importance.

Another error will arise from the fact that the tungsten strip will intercept a small fraction of the light which would after reflection in the glass form part of the incident light (See dotted lines in figure 5*A*). Consequently the presence of the strip will, for some particular directions, cause an error of 7 % in the homogeneity of the illumination. If this error be in the 10 % diffuse reflection, the corresponding error in the final result will be small (less than 0.7 %) ; care however must be taken that such be the case. In most cases conditions could easily be so chosen that no serious errors were to be feared.

Errors of this type may be avoided by screening the bulb at the back with a small piece of black paper (fig. 5*B*). The intensity incident on the strip will then be

$$t + r^2 . t + r^4 . t + \dots . = \frac{t}{1 - r^2} \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

the terms $r . t$, $r^3 . t$, etc. in the series on page 19 now being absent. Focussing the strip on the slit of the monochromator we now measure the quantity

$$R'' = \frac{R . t^2}{1 - r^2} + r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

from which R can be calculated as before.

In many cases I have used both methods, with and without a screen and the results were always found to agree very well with each other (See table 4).

I also had at my disposal a discarded strip lamp, the bulb of which was cut along the line *C* in figure 5*A*. A tungsten strip was investigated first with the bulb in position and afterwards when the bulb was removed. In this way it was verified that the presence of the bulb did not introduce serious systematic errors in the determinations of total reflectivity (See table 5).

In tables 4 and 5 a few data have been collected to illustrate the above.

TABLE 4.

Observations inside the bulb for strip lamp 128.

λ μ	R'	R''	T	r	t	a	R_1	R_2
0.5	51.3	50.5	93.7	7.0	89.8	3.2	51.1	53.8
0.6	55.2	51.9	95.1	7.2	90.3	2.5	55.0	54.8
0.7	54.5	52.0	95.8	7.2	90.7	2.1	53.5	54.2
0.8	55.5	52.8	97.0	6.7	91.7	1.6	54.1	54.5
0.9	56.5	54.0	97.5	6.7	92.0	1.3	54.9	55.5

r is the reflectivity, t the transmission factor and a the absorbtion factor for the bulb.

R', T. and R'' are defined by formulae 2, 3, and 5 of this section respectively.

R_1 is the value of the reflectivity calculated from R' and R_2 the value calculated from R''.

All quantities have been expressed in $^0/_0$.

TABLE 5.

Observations on the same strip inside and outside the bulb.

λ	R_1	R_2	R_3
0.5	49.2	50.9	50.6
0.6	49.7	50.2	(54.4)
0.7	53.6	54.6	53.3
0.8	51.4	52.5	52.5
0.9	52.9	52 6	53.5

R_1 and R_2 have the same meaning as in table 4.

R_3 is the value found outside the bulb.

When starting the observations on strips inside the bulb the author felt serious misgivings as to the results to be expected. As will be seen in the next section, however, the determinations inside and outside the bulb agree so well that the trustworthyness of the above methods can no longer be doubted.

§ 6. *Discussion of the observations.*

The strips investigated.

Determinations of total reflectivity were made on 12 different strips of tungsten which I will number from 1 to 12.

N⁰. 1 and N⁰. 2 were two different parts of the same strip, selected for having a smooth surface and aged for 24 hours at a temperature of 2500° K.

N⁰. 3 was a strip from an old strip lamp that had been used for years in this institute.

N⁰. 4 was also a strip from a discarded lamp. This strip had been heated to melting temperature, thereby getting a well polished surface.

Strips 1 to 4 had all been exposed to the air for about half a year before the final measurements were made.

In addition observations were carried out on two strips (N⁰. 5 and N⁰. 6) instantly after the bulb had been removed[1]). These strips had been aged for 14 hours at 2000° K; N⁰. 5 had an almost perfectly polished surface.

N⁰. 7, 8, and 9 were three strips inside a glass, and 10, 11 and 12 three inside a quartz bulb. Most of these lamps had been used for some time. Strip N⁰. 9 had an extremely rough surface, the roughest of all the strips investigated.

Observation errors.

The observations were made in the region from 0.45 μ to 1.05 μ for intervals of 0.05 μ. A series of observations was always started at 0.7 μ and was concluded by a second determination at the same wave-length.

Let d denote the difference between these two observations then, according to the theory of errors, the mean square error of a single observation is given by the formula

$$\varepsilon = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{\Sigma\, d^2}{n}}\,.$$

From 72 observations I computed $\varepsilon = 0.4\ \%$.

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[1]) I here wish to express my thanks to Prof. G. HOLST at *Eindhoven* for offering the two strip lamps from which these strips were taken.

At 0.05 μ and 1.0 μ the errors may have been somewhat greater, the photocel being less sensitive to these wave-lengths.

The observations.

To reproduce the complete material would be of little value but in tables 6*A*, *B*, and *C* a few statistics have been compiled to illustrate the results arrived at. As will be seen from these data, the observations made on 12 strips with widely different surfaces are in almost perfect agreement with each other; a better evidence of the reliability of the method used can hardly be imagined.

The spread of the observations has been expressed by the standard deviation $\sqrt{\frac{\Sigma d_k^2}{n}}$, d_k being the deviations from the average. The mean value of the standard deviation is 0.7 % and is therefore considerably greater than the mean square error of a single observation. It follows that the differences between the reflectivities of the various strips cannot be explained by observation errors only, and the question arises whether the strips differ in absolute value of reflectivity only or whether they also differ in the relative change of reflectivity with wave-length. The observations were numerous enough to settle this question by stastistical methods in the following way.

For each strip I have calculated the average reflectivity in the region from 0.45 μ to 1.0 μ; each value entered in table 7 is the mean of 12 observations and will therefore be practically free from observation errors.

Indeed we see from this table that the strips differ in absolute value but the deviations are so small that they can hardly be considered as being real; the standard deviation is 0.5 % only. In the determinations inside the bulb systematic errors of this order of magnitude may certainly be expected and 4 of the strips measured outside the bulb had been exposed to air for half a year; it is not impossible that their surfaces were somewhat dusty or slightly oxidized.

Comparing table 7 with the data, given at the beginning of this section, it is apparent that no clear relation exists between the absolute value of the reflectivity and the condition of the surface:

TABLE 6.

Statistics derived from the observations.

Wavelength μ	Maximum %	Minimum %	Average %	Standard deviation	M. S. E. of average
6 A. Observations outside the bulb.					
0.5	51.0	53.0	51.9	0.6	0.2^{5}
0.6	53.4	54.5	53.7	0.4	0.1^{5}
0.7	54.3	55.6	54.9	0.5	0.2
0.8	52.8	54.5	53.5	0.6	0.2^{5}
0.9	53.4	55.0	54.2	0.6	0.2^{5}
1.0	55.8	59.4	57.5	1.1	0.4
6 B. Observations inside the bulb.					
0.5	52.0	52.9	52.5	0.3	0.1
0.6	53.5	54.9	54.1	0.6	0.2^{5}
0.7	54.2	56.4	55.1	0.9	0.4
0.8	51.8	54.6	53.2	1.1	0.4
0.9	53.4	54.5	53.9	0.4	0.1^{5}
1.0	55.5	57.5	56.7	0.8	0.3
6 C. All observations combined.					
0.5	51.0	53.0	52.2	0.6	0.1^{5}
0.6	53.4	54.9	53.9	0.5	0.1^{5}
0.7	54.2	56.4	55.0	0.7	0.2
0.8	51.8	54.6	53.4	0.9	0.2^{5}
0.9	53.4	55.0	54.1	0.5	0.1^{5}
1.0	55.5	59.4	57.1	1.1	0.3

M. S. E. = Mean square error.

N⁰. 1 and 2 are two different parts of the same strip ; N⁰. 4 and 5 had very smooth surfaces and N⁰. 9 had a specially rough one.

TABLE 7.
Average value of reflectivity in the region 0.45 μ to 1.0 μ for different strips of tungsten.

Strip N⁰.	R ⁰/₀	Strip N⁰.	R ⁰/₀
1	53.6	7	53.3
2	54.5	8	54.4
3	53.5	9	54.8
4	54.7	10	53.4
5	53.6	11	54.2
6	53.5	12	53.6

Average = 53.9 ⁰/₀
Standard deviation from the average = 0.5 ⁰/₀

To see whether the strips also differed in the relative change of reflectivity with wave-length, the observations for each strip were multiplied by such factors that the averages of table 7 were reduced all to the same value. Data applying to the observations, after these reductions had been effected, are given in table 8.

TABLE 8.
Data applying to the observations on 12 different strips after reduction to the same average value.

Wavelength μ	Maximum ⁰/₀	Minimum ⁰/₀	Average ⁰/₀	Standard deviation
0.5	52.9	51.4	52.1	0.5
0.6	54.3	52.9	53.9	0.3^5
0.7	55.5	54.4	55.0	0.3^5
0.8	53.8	52.4	53.3	0.4
0.9	55.0	53.6	54.1	0.4
1.0	58.8	55.8	56.1	0.8

We see that the standard deviations are now reduced to about 0.4 %, that is to the mean square error of a single observation. We may draw the conclusion that differences in the relative curves, if any, are much smaller than the observation errors and can therefore not be demonstrated.

As final values for the total reflectivity of tungsten I have adopted the averages of all observations on 12 strips. According to the last column in table 6C these averages are fixed with a mean square error of about 0.2 % in absolute or 0.4 % in relative value. By the results of § 4 a relative systematic error of 0.5 % may also exist.

The final results of the observations will be found in table 13 on page 42. In figure 7 the reflectivities of 4 different strips have been plotted to give a further demonstration of the mutual accordance of the observations.

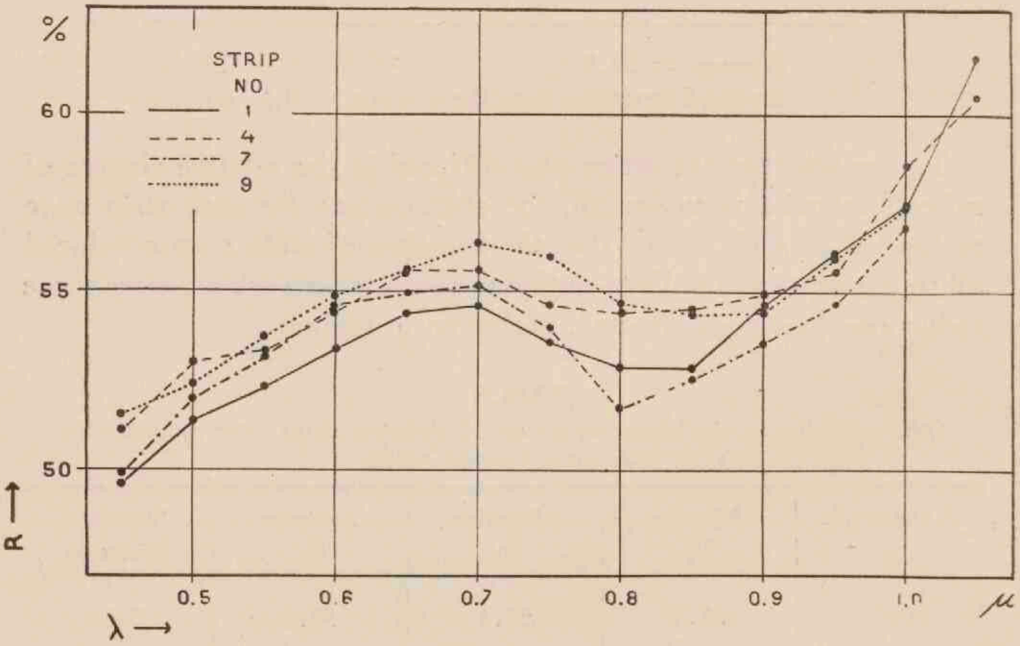

FIG. 7.
Total reflectivities for 4 different strips of tungsten.

CHAPTER 2.

Relative observations in different overlapping regions of the spectrum.

§ 1. *Photographic determinations in the visible and ultra-violet region.*

A. The experimental arrangement.

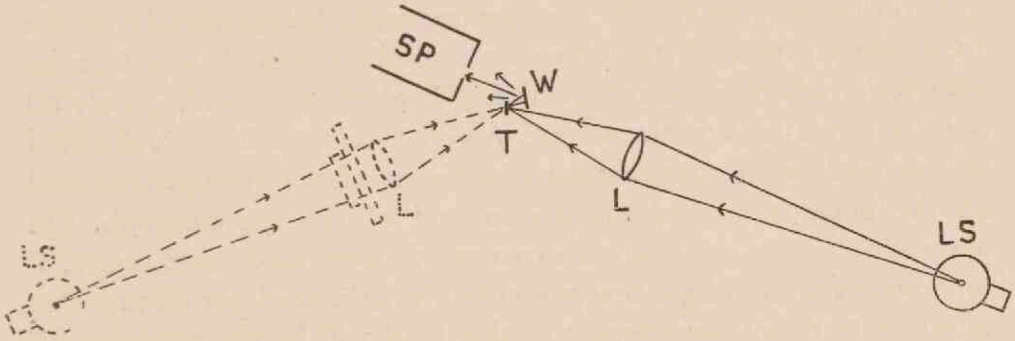

FIG. 8.

Experimental arrangement for photographic determinations.

The figure is not in true proportion; in reality the measures of the arrangement were as follows:

Distance *LS* to *L*	100 to 150 cm.
Distance *L* to *T*	± 16 cm.

The screen *W* had a diameter of 4.5 mm. only and the distance *T*—*W* was very small 8 to 10 mm.

The distance from *W* to the slit of the spectrograph was about 4 cm.

Angle of incidence 20°.

The light source *LS* (fig. 8) is focussed by the lens *L* on the tungsten strip *T*, and the reflected light is projected on a diffusing screen *W* (smoked magnesium oxide). The intensity of the light diffusely reflected by *W* is measured photographically in the spectrograph *SP*.

To compare the reflected light with the incident light the strip *T* is removed and the source *LS* and lens *L* are swung round together to the position indicated by dotted lines. In the ultra-violet the light source was a water-cooled hydrogen tube (14) that could not easily be moved. In that case the spectrograph, the tungsten strip and the diffuser were mounted on a board and swung round together instead. All parts that had to be put in two different positions or were to be removable were mounted on hole-slot-plane fittings (See page 11).

In the visible region the light source was an electric lamp and an ordinary Fuess glass spectrograph was used. The observations in the ultra-violet were made with a small Fuess quartz spectrograph of low dispersion but high luminosity.

The principle idea of using the diffusing screen *W* is that, if a certain constant amount of light falls on the screen, it will always give the same intensity in the spectrograph, independently of the form and position of the light patch on the screen. It will then be immaterial whether the tungsten strip is plane or curved, if only all light reflected falls on the screen. Whether this condition is satisfied will depend on the geometrical dimensions of the arrangement and on the homogeneity of the screen and of the spectrograph. On several occasions verification experiments were made and errors, if any, were found to be a few percent only. We need not discuss this question in detail since errors of this kind will be the same for all wave-lengths and they will not affect the relative value of the determinations aimed at.

Since the reflection of the strips is partly diffuse a small fraction of the reflected light will always fall beside the screen *W*. For this reason the observations now under consideration have only relative and no absolute value.

B. The accuracy of the photographic method.

Special attention was paid to the accuracy of the photographic method. To compare the incident with the reflected radiation we need a method to cut down the intensities in some well defined ratio. This was done by placing sector diaphragms before the lens *L* (fig. 9); inhomogeneities of the lens etc. were eliminated by rotating the diaphragms with a velocity of about one revolution in 6 seconds. The result was that the intensity on the photographic plate fluctuated by say 5 or 10 % of its total value and with a period of 3 seconds. It has been assumed that these fluctuations do not affect the density on the plate. The times of exposure varied from 30 seconds to 8 minutes.

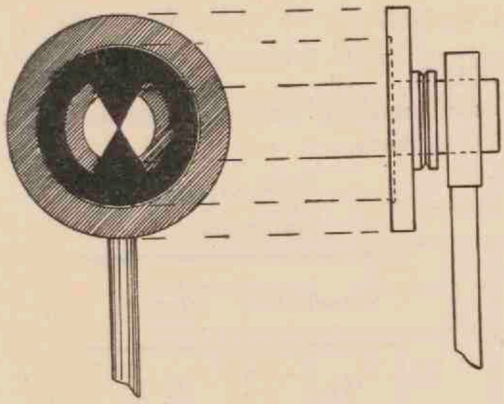

FIG. 9.
Rotating sector diaphragm.

The rotating sector diaphragms were used as standard method, but besides this the spectrograph was fitted with a wedge shaped slit. The intensity on the plate then gradually increases from one side of the spectrum to the other, and if we measure the density with a self recording microphotometer we get a record as shown in figure 10.

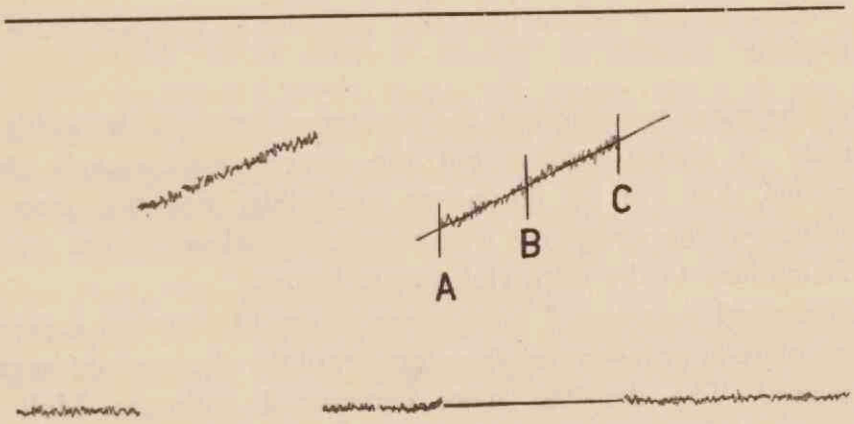

FIG. 10.

Drawing a smooth curve the irregularities due to the grains of the plate average out, while at the same time the densities are found for a set of different intensities. In this way the most efficient use is made of the space available on a photographic plate.

The intensities for different points (*A*, *B*, *C*) of the curve will be approximately given by the corresponding widths of the wedge slit. It depends however on the quality of the spectrograph and many other factors to what degree this approximation is exact. I have therefore preferred to calibrate the wedge slit by means of the rotating sectors. The results of some of these observations are given in the following table.

TABLE 9.

Comparison of slit-widths with the corresponding intensities determined by photographic methods.

Glass Spectrograph.		Small Quartz Spectrograph.	
Slit-width	Intensity	Slit-width	Intensity
13.8	13.5	24.2	22.7
21.7^5	21.7	33.4	31.2
28.3	28.1	42.5	46.2
36.2	36.7		

The values in each column have been multiplied by such factors that their sum is 100.0.

For the glass spectrograph the agreement between slit-width and intensity is almost perfect but the quartz spectrograph shows deviations that are by no means negligible. For this reason a calibration of the wedge slit was always carried out before a set of determinations in the ultra-violet were begun.

On several occasions I have taken 10 to 13 identical exposures on one plate in order to test the accuracy of the photographic method in general. The densities were determined with the Moll self recording microphotometer of the Utrecht Institute. From the data thus obtained the mean square error of a single density was computed by the known formula

$$\varepsilon = \sqrt{\frac{\Sigma d^2}{n-1}} .$$

I found

for Ilford Empress plates $\varepsilon = 0.5\ \%$

for Ilford Process Panchromatic plates $\varepsilon = 1.0\ \%$

These figures are valid for a density of about 50 %.

The above results were reproducible at different times and the errors in density were observed to be independent of wave-length.

Since the slope of the density-intensity curve varies with wave-length, to a constant error in the density different errors in the intensity will correspond, as is shown in the following table.

TABLE 10.

Errors in the intensity corresponding to an error of 1 % in the density.

Wave-length.	Error.	Wave-length.	Error.
0.6 μ	1.1 %	0.4 μ	1.9 %
0.5 „	1.3 %	0.3 „	3.0 %

On an Ilford Process Panchromatic plate and by a single point of the curve (fig. 10) the corresponding intensity is fixed with the mean square errors given in this table; for Empress plates the errors are about half as great. If we have to compare two different intensities, the errors will be a factor $\sqrt{2}$ greater, but if on the other hand the curve be measured in different points and if the number of exposures be increased, the accuracy will increase correspondingly. From these data it will be understood that under suitable circumstances, viz. continuous spectra and constant light sources, photographic plates may yield accurate results.

The errors so far dealt with are the combined effect of inconstancies of the light source, inaccuracies of the plate and errors in the density determination. The last have been separately investigated by taking two identical records of the same plate at the same wave-length directly after one another. Denoting the differences between corresponding densities read off from both records by d_k, the mean square error of a single density will be given by $\frac{1}{\sqrt{2}} \cdot \sqrt{\frac{\Sigma d_k^2}{n}}$. The error computed in this way was 0.5 to 0.6 % for the Empress plates. Comparing this with the value given on page 32, we see that the error in the density determination and the total error of photographic method are equal to each other. Probably the most important error is made in drawing the smooth curve (fig. 10) and if so, these errors are due to the graininess of the plate. The above result then demonstrates that the errors of the photographic method are mainly caused by the graininess of the plates but not by inhomogeneity of the plate at large.

Whether these conclusions are wholly justified I cannot say, but all results given above were found reproducible at least in two independent sets of experiments. Perhaps some of these questions are worth a further study.

As has already been said Ilford Process Panchromatic plates and Ilford Empress plates were used, the former in the visible and the latter in the ultra-violet region. The plates were developed with Rodinal $^1/_{20}$ or with a Metol Hydrochinon Borax developer (Wellington Handbook) ; fog on the plates was carefully avoided ; the plates were generally rocked also during fixation and after thorough rinsing with fresh water, they were finally rinsed for 10 minutes in distilled water. Whether these measures have actually contributed to the accuracy of the determinations, I feel unable to say with certainty. However it was found as a general rule that the greater the attention paid to the treatment of the plates, the greater the accuracy arrived at.

C. The observations.

In the visible region three exposures were made, two of the direct light and in between these one of the reflected radiation. The intensities of the direct light were cut down by a rotating sector diaphragm to 50 % ; the intensities to be compared in the spectrograph then stood almost in the ratio 1 : 1 to each other. By using the density-intensity relation given by the wedge slit the accurate value was easily determined.

In the ultra-violet the number of exposures was increased to 9, 5 of the direct alternating with 4 of the reflected radiation. The direct exposures were taken alternately with sectors of 50 and 40 % so that an independent check of the wedge slit was effected. Sometimes the hydrogen tube was not perfectly constant, but by the combined use of sectors and wedge slit the reflectivities could still be computed in these cases, though not always without some difficulty.

The observations in the visible part of the spectrum lost their importance after the more effective methods of Chapter 1 were developed. In figure 11 the measurements on one of the strips are given mainly as a demonstration of the accuracy of the methods adopted.

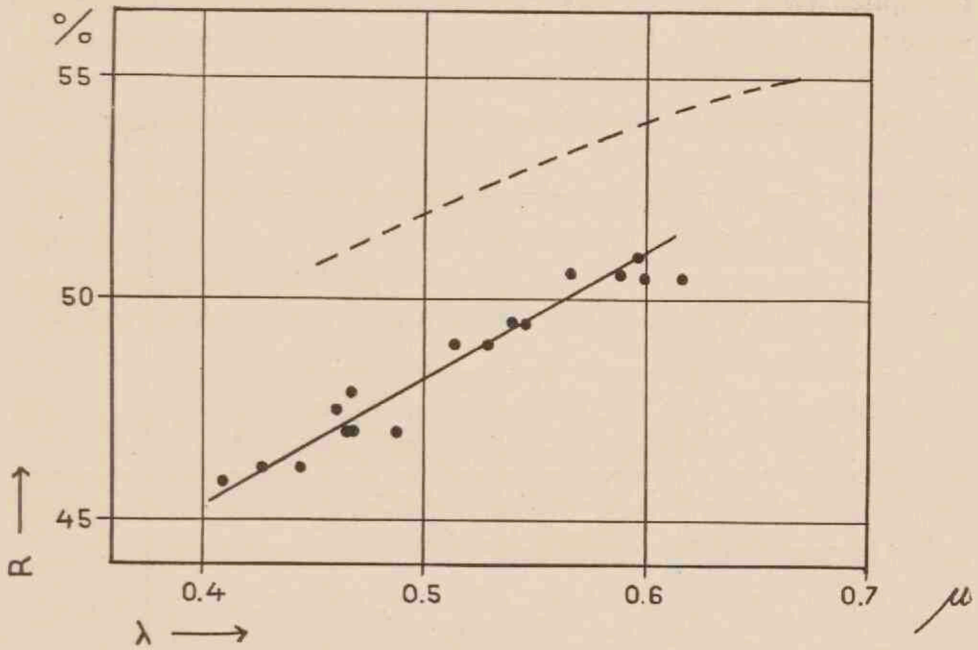

FIG. 11.

Relative photographic observations in the visible region compared with the total reflectivities of Chapter 1.

Comparing the photographic determinations with the total reflectivities of Chapter 1 we see that the difference is about 8 %. This then must be the amount of light that is not caught on the diffusing screen *W* (fig. 8), being lost by diffuse reflection on the strip.

In the ultra-violet the observations have greater importance. Two sets of experiments were carried out. In October 1932 5 series of observations were made on 4 different parts of the same strip of tungsten and these are represented by the dotted lines in figure 12. This strip had been exposed to the air for about 16 months.

The second series of observations was made in June 1933 on three different strips but directly after they had been taken from the bulb in which they had been mounted. In figure 12 these determinations are given by drawn lines.

The values for the different strips being only relative, we may multiply them with certain factors and so try to bring them into closer agreement. From the curves the reflectivities at 0.35, 0.325, 0.30, 0.275, and 0.25 μ were read off and the average of these 5 values was calculated; then the reflectivities for each strip were multiplied with such factors that the averages reduced all to the same value.

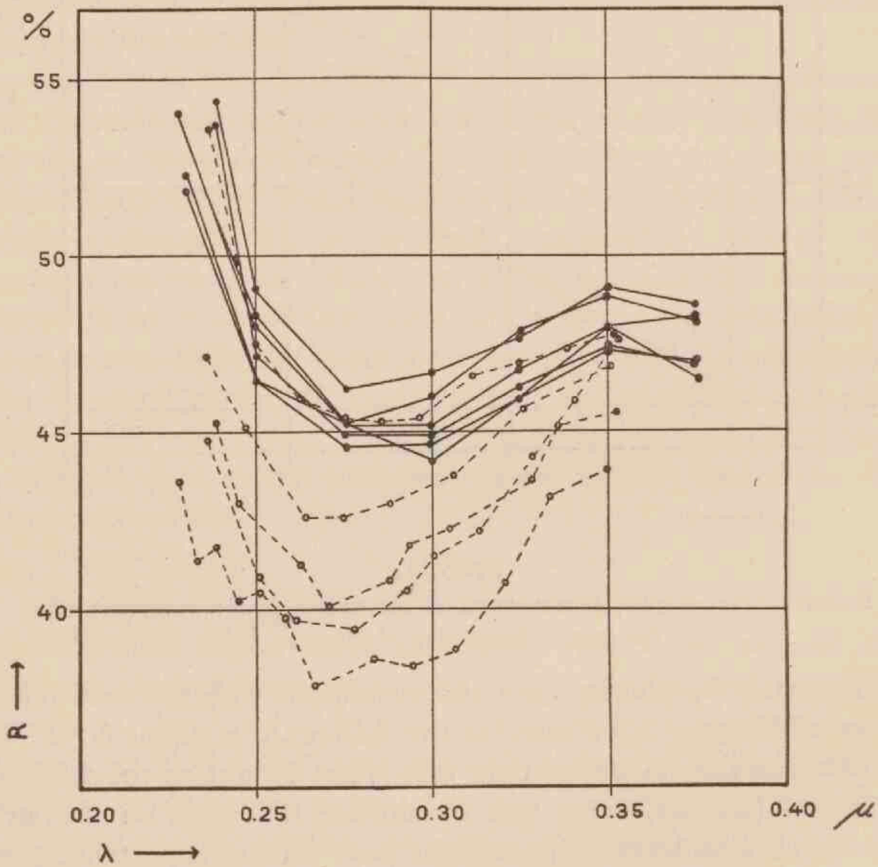

FIG. 12.

Original relative observations in the ultraviolet.

○ - - - ○ Observations made in October 1932 on strips which had been exposed to the atmosphere for 16 months.

● —— ● Observations made in June 1933 on strips which had recently been taken from their bulb.

These reductions being made it was observed that the first set of observations showed systematic deviations from each other as well as from the second set; the latter however are in almost perfect agreement as will be understood from figure 12. It is quite possible that the strips used in the first experiments were slightly oxidized or were spoiled in some other way by being exposed to the air for so long a period. Otherwise the observed discrepancies are difficult to explain. Moreover the deviations are so large that the mean square error of the average calculated for all observations together is greater than the same error calculated for the second set only. This being so I have thought it justifiable to discard the first set of observations entirely.

Data applying to the second set after reducing them to the same average are given in table 11. Being photographic determination the agreement is very striking.

A further discussion of the results will be given in Chapter 3.

TABLE 11.

Data applying to the relative observations in the ultra-violet region a reduction to the same average.

Wave-length μ.	Maximum %	Minimum %	Average %	Standard deviation	M. S. E. of average.
0.375	48.1	47.0	47.6	0.3	0.1
0.350	48.6	47.8	48.1	0.3	0.1
0.325	47.1	46.5	46.8	0.2	0.1
0.300	45.5	44.9	45.3	0.2	0.1
0.275	45.9	44.7	45.0	0.4	0.2
0.250	48.0	47.0	47.5	0.3	0.1
0.240	52.5	49.3	50.8	1.3	0.6
0.230	53.2	52.6	52.9	0.2	0.1

§ 2. *Relative determinations in the region from 0.578 μ to 0.313 μ with photocel and amplifier.*

The observations dealt with above do not reach beyond 0.375 μ and the total reflectivities of Chapter 1 were not extended below 0.45 μ. We still need some determinations in the intermediate part of the spectrum and overlapping both the former regions.

Such observations were made with an arrangement similar to that described in § 2 of Chapter 1. The cylinder now used was 30 cm in diameter and 30 cm high; the inner side was smoked with magnesium oxide which has a high reflecting power far into the ultra-violet (15).

As before, the tungsten strip was mounted at the centre of the cylinder; the light source was a quartz mercury arc lamp hung from the lid of the cylinder above the strip. This lamp proved sufficiently constant without special precautions. The intensities were measured with monochromator and photocel in the same way as in the experiments of Chapter 1.

Dimensions and arrangement of the present experiment were such that condition 2 and 3 of page 14 no longer held and consequently these determinations have no absolute but relative value only.

In all 8 series of observations were made on 5 different strips and at 6 wave-lengths. Again as in the foregoing section the data for each strip were multiplied by such factors as to reduce all strips to the same average value. In table 12 data concerning the observations, after the reduction had been made, are given; as we see from the table the mutual agreement is very satisfactory.

The mutual relation of these with the other observations, so far dealt with, will be discussed in the following Chapter.

TABLE 12.

Data applying to the relative reflectivities from 0.578 μ to 0.313 μ after reduction to the same average.

Wavelength μ	Maximum %	Minimum %	Average %	Standard deviation	M. S. E. of Average
0.578	59.0	54.9	56.4	1.2	0.4
0.546	56.5	54.2	54.9	0.9	0.3
0.435	53.5	52.6	52.9	0.4	0.1^5
0.405	53.0	51.6	52.4	0.5	0.2
0.365	54.5	53.0	54.0	0.5	0.2
0.313	54.4	50.5	53.0	0.9	0.3

CHAPTER 3.

The total reflectivity of tungsten at room temperature.

By means of the relative determinations the curve of total reflectivity of Chapter 1 can easily be extended into the ultra-violet part of the spectrum, as has been done in figure 13. The dotted lines represent the original relative observations of tables 11 and 12. By

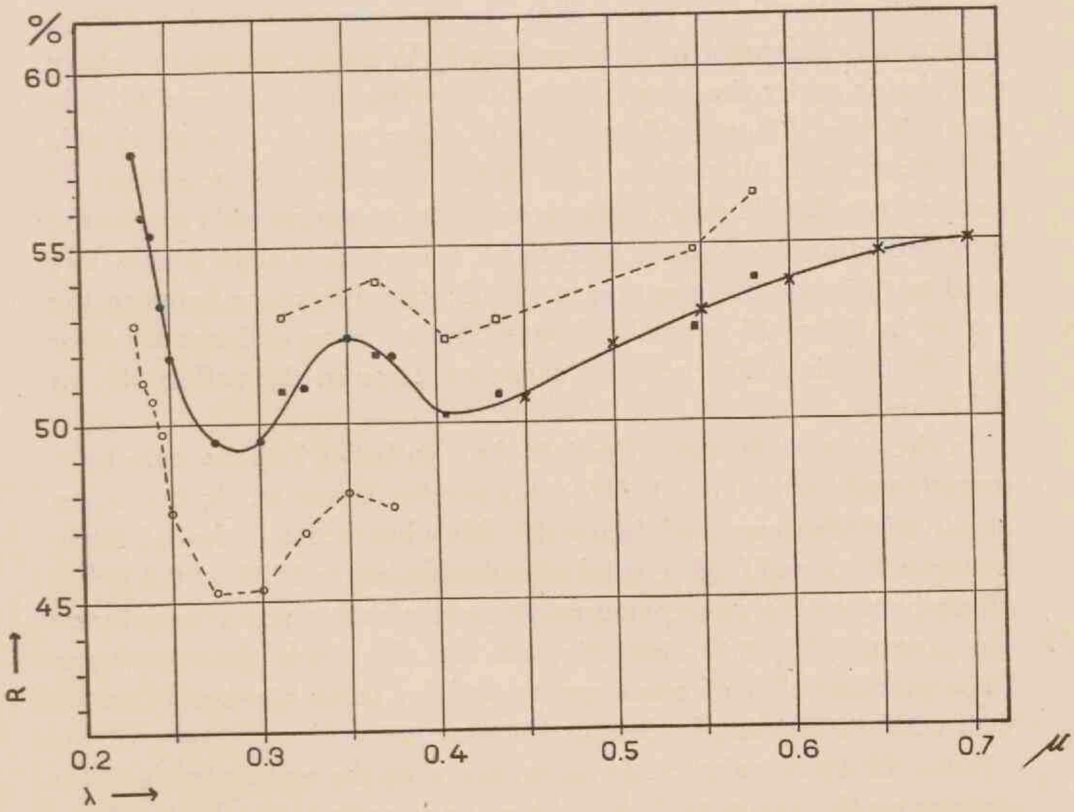

FIG. 13.

Extension of the curve of total reflectivity into the ultra-violet.

○ □ Original relative observations from table 11 on page 37 and table 12 on page 38.

● ■ The relative observations adjusted to each other and to the curve of total reflectivity.

× Total reflectivities of Chapter 1.

multiplying these relative curves each with a certain constant factor they were brought in accordance with each other and with the curve of total reflectivity.

As is seen from the figure all observations fit well together; the separate points do not deviate from the smooth curve by more than 0.5 % and it is improbable that the error made in adjusting the relative curves exceeds this amount.

The relative measurements made by the photographic method lie somewhat lower as the adjusted curve. As has already been pointed out on page 30 this is due to the fact that in the experiment of figure 8 a small part of the diffusely reflected light does not fall on the diffusing screen *W*. In adjusting the curve as above, the assumption has been made that the percentage loss by diffuse reflection is independent of wave-length. We have reason to believe that this is really the case. Dependance of wave-length would only occur if the diffuse reflection was either partly due to double reflections or to diffraction, both rather improbable suppositions. If for instance double reflections occurred to an appreciable extent the total reflection would have been much more diffuse than it was. We need not discuss the question in detail; the difference between the curves in figure 13 is small, about 4 %, and even as large an error as 5 % in this difference will cause an error in the reflectivity of 0.2 % only.

In the ultra-violet the reflectivity has, so far as I know, only been investigated by HULBURT (16); his results, however, do not agree at all with the curve of figure 13. He observed a value of 30 % between 0.4 μ and 0.35 μ, gradually decreasing to 15 % at 0.2 μ. To all probability the discrepancy must be ascribed to some impurity of the tungsten HULBURT investigated. He also made determinations for a great number of other metals and for these his data often fit well with the results of other investigators.

Later HULBURT also made observations of the emissivity (17) by comparing the radiation from a black body with the radiation from incandescent tungsten. The total reflectivities calculated from his emissivities agree fairly well with the curve of figure 13 (See also figure 23 on page 67).

As is seen from figure 13 the reflectivity rapidly increases from 0.25 μ to 0.23 μ; an extrapolation beyond 0.23 μ cannot be made with any degree of certainty and has therefore not been attempted.

We may now turn our attention to the visible and infra-red part of the spectrum. In these regions determinations at room temperature have been made by COBLENTZ and EMERSON (5) and these have been plotted in figure 14 together with my own observations.

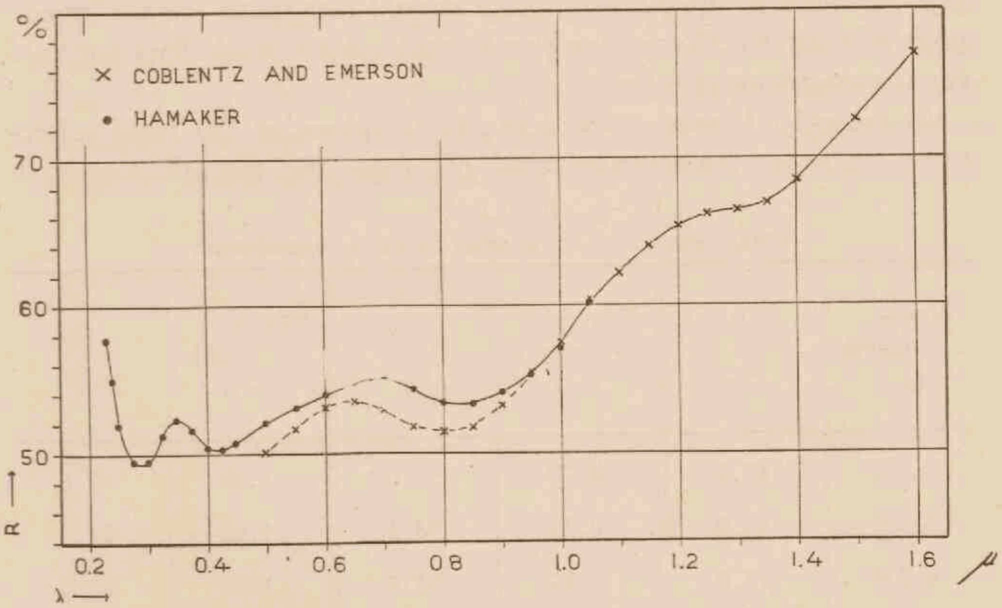

FIG. 14.

Total reflectivity of tungsten at room temperature.

From 0.5 μ to 0.9 μ the values of COBLENTZ and EMERSON lie somewhat lower, but never by more than 2 %. These differences may be due to the fact that they have used pieces of tungsten that were carefully polished but had not been aged. Moreover they have denoted their tungsten as "pure", from which might been understood that this tungsten did not contain thorium oxide. One piece of tungsten denoted as "impure" did not show the bend at 1.3 μ and the observations on this piece were not taken into account in calculating the averages.

At 0.95, 1.0, and 1.05 μ my observations are practically the same as those of COBLENTZ and EMERSON and with their data it is therefore possible to extend my own curve without any further adjustment. It is, of course, open to question whether this extension is quite safe; such differences as were found to exist between 0.5 μ and 0.9 μ may also occur in other parts of the spectrum, but since my observations do not reach beyond 1.05 μ nothing can be ascertained on this point.

In a paper on the change of reflectivity with temperature WENIGER and PFUND (6) have made the remark that the minimum at 0.8 μ and the bend in the curve at 1.3 μ should disappear for an "aged" piece of tungsten. According to my results this remark does not hold for the minimum at 0.8 μ.

In table 13 I have entered the final values of the total reflectivity at room temperature read off from the smooth curves in

TABLE 13.

Total reflectivity of tungsten at room temperature.

Wave-length in μ	R %	Wave-length in μ	R %	Wave-length in μ	R %
0.230	57.7	0.70	55.0	1.50	72.6
0.240	55.0	0.75	54.3	1.60	77.0
0.250	52.0	0.80	53.4	1.70	81.0
0.275	49.5	0.85	53.3	1.80	84.5
0.300	49.5	0.90	54.1	1.90	87.5
0.325	51.3	0.95	55.3	2.00	90.0
0.350	52.4	1.00	57.6	2.10	91.8
0.375	51.7	1.05	60.1	2.25	93.0
0.400	50.4	1.10	62.2	2.50	93.8
0.425	50.4	1.15	64.0	2.75	94.0
0.450	50.8	1.20	65.5	3.00	94.3
0.50	52.1	1.25	66.2	3.50	94.5
0.55	53.1	1.30	66.5	4.00	94.8
0.60	54.0	1.35	67.0	5.00	95.3
0.65	54.7	1.40	68.5	6.00	95.8

figures 13 and 14 and completed in the infra-red with the data of COBLENTZ and EMERSON. In using the values given in this table it must always be born in mind that I made my observations only on strips made by the PHILIPS' *Manufactories* at *Eindhoven*. It is not impossible that strips of other origin will have slightly different reflectivities.

According to what has been said on page 28 the relative mean square error of the values given in table 13 is 0.5 % in the region from 0.45 μ to 1.05 μ; perhaps the errors are somewhat greater in the ultra-violet part of the curve, but in virtue of the figures given in the last columns of tables 11 and 12 serious errors cannot exist.

CHAPTER 4.

The change of reflectivity with temperature.

§ 1. *A preliminary experiment.*

First experiments were made with an arrangement as shown in figure 15. By the lens L_1 the light source LS is focussed on the strip T of a tungsten strip lamp; a part of the reflected light is caught by the lens L_3 and concentrated on a small piece of ground glass G. The total intensity of the light falling on G is measured by means of the monochromator and photocel placed a short distance behind. By way of a reflecting prism P and the lens L_2 the source LS is also focussed directly on G. S_1 and S_2 are two shutters.

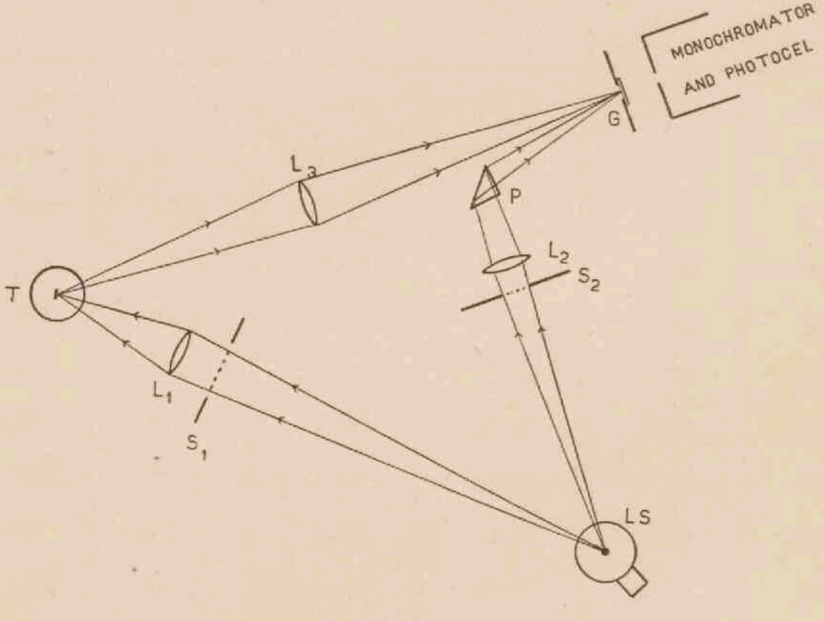

FIG. 15.

The observations were made in the following way. First T was kept at room temperature and by opening the shutters S_1 and S_2 in turn the ratio I_r/I_d of reflected to direct light was determined. As a rule this ratio was made about 1 : 1 by placing a diaphragm before the lens L_2.

Next the tungsten strip was heated by an electric current to a constant temperature. A constant amount of light emitted by T then falls on G and causes a constant deflection of the galvanometer. This constant deflection was simply considered as a change of zero and from the new zero point the ratio between reflected and direct light was again determined. The change of this ratio with temperature will then be equal to the change of reflectivity.

In case the radiation from the strip T should become very strong, the zero was kept on the reading scale by sending a compensating current through the galvanometer circuit.

It may happen, of course, that the shift of zero will slightly influence the sensitivity of the photocel arrangement, but any such changes are accounted for, since we observe the ratio of the deflections caused by two different intensities one of which (the direct light) is constant, the other varying with the reflectivity of the strip.

The great advantage of the method is that the radiation from T will not influence the accuracy of the observations, if the temperature of the strip can be kept sufficiently constant. It was possible to carry out measurements up to a temperature of 2500° K without any difficulties.

Nevertheless the experiments with this arrangement did not answer to our expectations; observations made with different strip lamps gave results differing by 10 or 20 %, amounts that could not possibly be real. These discrepancies were found to be due to warping of the tungsten strip when heated. The geometric dimensions of the arrangement were such that, in case both ends of the strip should move over 0.1 mm with respect to each other, this would cause a displacement of 6 mm of the beam of reflected light over the lens L_3 and we observed indeed that experiments on the same strip lamp yielded quite different results, when half of the lens L_3 was screened off with either a horizontal or a vertical screen. By projecting an enlarged image of the strip on a screen and varying the heating current, movements of about 0.1 mm could also directly be demonstrated.

Using ordinary strip lamps it will therefore be impossible to obtain reliable results with the experiment of figure 15 or any similar arrangement.

Note.

During the above experiments I had another opportunity to test the proportionality between the intensity of the incident light and the deflection of the galvanometer. On the ground glas G light falls along two different ways viz. the direct light via L_2 and P and the reflected light via L_1, T, and L_3. The check simply consisted of observing whether *the sum of the deflections caused by the separate intensities was equal to the deflection caused by the sum of the intensities*; the shutters S_1 and S_2 were operated first one after the other and then both simultaneously.

A check based on this principle can be effected in a very simple way by the arrangement shown in figure 16.

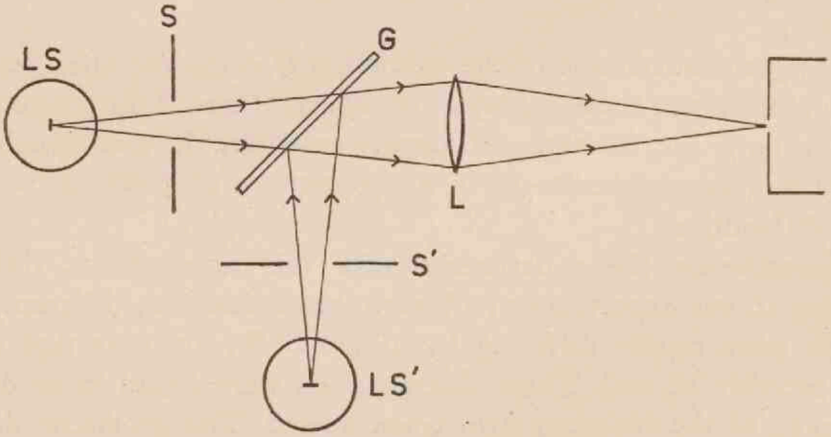

FIG. 16.

By one lens L two constant light sources LS and LS' are focussed on the slit of the monochromator, LS directly and LS' by way of a reflecting glass plate G; S and S' are two shutters.

The advantage of this method is that we need not make any changes in the experimental arrangement at all; the openings of the lenses etc. remain unaltered and all we have to do is to use the shutters and vary the electric currents through the light sources LS

and *LS′*. An accurate check of the proportionality between a deflection and an intensity is often a matter of some difficulty; perhaps the method indicated in figure 16 is one of the best experiments for the purpose.

The proportionality of deflection and intensity has already been dealt with on page 14; the experiments now under consideration confirmed the results given there. Up to a deflection of 10 to 15 cm the errors did not exceed 2 or 3 pro mille.

§ 2. *The definite experiments.*

In a similar investigation the difficulties caused by warping of the tungsten strip were also experienced by WENIGER and PFUND (6). In the final experiments they have used a strip lamp of special construction, but in their paper it is not said how this lamp was made nor how it was ascertained that the lamp came up to the requirements. This is to be regretted, as the results I obtained by a different method do not agree with their observations and a satisfactory explanation of the discrepancies cannot be given.

Using the method of Chapter 1 in which the tungsten strip is illuminated homogeneously, warping of the strip will no longer cause serious errors; we then measure the total reflectivity independently of the form and condition of the surface (page 7) and displacements of 0.1 mm or so will not influence the results.

A disadvantage of this method is that the incident radiation is weak in comparison with the radiation emitted by the tungsten strip itself at higher temperatures and it is now impossible to eliminate the radiation from the strip, as was done in the experiment of the foregoing section. The measurements could therefore not be made up to high temperatures, except in the ultra-violet region where the emission of the strip is very weak.

The experiments were made in exactly the same way as described in § 5 of Chapter 1. To increase the energy of the incident light as much as possible, the two light sources in figure 2 were replaced by a 100 watt sodium vapor lamp or a 500 watt electric lamp as used in projection apparatus. These light sources were placed somewhere in the cylinder and the illumination of the strip will therefore not have been quite as homogeneous as in the experiments of Chapter 1. We have however no reason to expect serious errors, since the displacements of the strip due to warping are very small, 0.1 mm only.

In the ultra-violet the cylinder of § 2 Chapter 2 (smoked with magnesium oxide) and a mercury arc lamp were used.

Since we are here dealing only with the change of reflectivity with temperature, the different check experiments of § 4 Chapter 1 are now superfluous and they were not repeated.

When the tungsten strip is heated the temperature of the bulb will increase too, and errors in the observations might arise if this increase of temperature causes changes in the transmission *t* or the reflectivity *r* (See formula 2 on page 19). Special experiments were made but no such changes could be detected, as was to be expected. I need not discuss these experiments in detail.

In the observations at high temperatures we have to correct for the radiation emitted by the tungsten strip itself. To determine this correction the true source of light (sodium vapor lamp etc.) was switched off so that the radiation from the strip could be measured. Here another error may arise. As long as the light source is burning, the tungsten strip will recieve radiation from all sides and it is not impossible that the strip is heated a few degrees by this radiation. If so, the temperature of the strip will slightly decrease as soon as the source is switched off and for the radiation from the strip too low a value will be observed; the change of reflectivity will consequently be found too high.

Using the sodium vapor lamp or mercury arc lamp, it was easy to check whether any errors of the above kind occurred. The radiation from the strip was measured at a wave-length not emitted by the source and we observed whether the radiation was influenced by switching the light source off and on. No such influences could be detected at all.

Using the 500 watt lamp as light source the errors were, on the contrary, quite considerable. This was first deduced from improbably high values found for the increase of reflectivity with temperature (45 %); later a fall of the temperature of the tungsten strip after switching off the light source was also distinctly observed.

To avoid these difficulties the observations with the 500 watt lamp were made at so low a temperature that the radiation from the strip itself was hardly perceptible. The temperature did not then exceed 1100° K and to higher temperatures an extrapolation has been made. How this was done will be discussed in detail when dealing with the results (page 53).

In the observations made with a sodium vapor lamp or mercury arc lamps too, the temperature of the strip was not raised above the value at which the emission from the strip was about equal in intensity to the reflected light. With a further rise of temperature the radiation from the strip rapidly increases and accurate determinations will soon become difficult.

The temperature of the strip was determined by means of a HOLBORN-KURLBAUM pyrometer calibrated in this institute at 0.65 μ. To calculate the temperature the emissivities at room temperature, computed from the total reflectivities of Chapter 1, were used. At 0.65 μ the reflectivity only increases slowly with temperature and the error caused by a slightly erroneous value for the emissivity has not been greater than 10° K. The corresponding error in the change of reflectivity is entirely negligible.

§ 3. *The observations.*

In the ultra-violet region the emission from the strip itself is very weak and here the observations could easily be extended up to 2400° K. Measurements were made on 5 different strip lamps each at different temperatures. To reduce the observations to the same temperatures a linear interpolation was made; the original data were plotted in a graph and connected by straight lines, and from the curves the values were read off at given temperatures. For these temperatures the averages and the mean square errors of the averages were calculated; the final results have been entered in table 14 and a set of curves are reproduced in figure 17.

As we see from the figure, the dispersion of the observations was large, the separate points spreading from 2 to 6 %. The question arises whether these differences are due to observation errors or to systematic differences of the strip lamps. At 0.313 μ two strip lamps were examined twice (two drawn and two dotted lines in figure 17) and these repeated observations seem to indicate systematic differences. It is, however, very likely that these differences did not really exist. We may better treat this question by a statistical method.

If d_k be the difference between two observations on one strip

TABLE 14.

Average percentage increase of reflectivity with temperature.

Temp. ° *K*	Number of obs.	Increase %	M. S. E.
0.254 μ.			
1600	10	3.7	0.2
1800	10	4.2	0.3
2000	10	4.8	0.4
2200	10	5.2	0.4
2400	10	5.8	0.4
0.313 μ.			
1600	7	2.5	0.4
1800	7	2.7	0.5^5
2000	7	3.5	0.6
2200	7	4.0	0.5^5
2400	7	4.3	0.4
0.365 μ.			
1600	5	0.9	0.4
1800	5	1.3	0.1
2000	5	1.4	0.2
2200	5	1.4	0.4^5
0.405 μ.			
1600	5	2.9	0.5
1800	5	3.4	0.4
2000	5	4.0	0.4
0.435 μ,			
1600	3	3.5	0.4
1800	3	4.4	0.2
2000	3	5.3	0.2

lamp, the root mean square difference $\sqrt{\frac{\Sigma d_k^2}{n}}$ will be equal to $\sqrt{2}$ times the standard deviation for a set of repeated observations. In

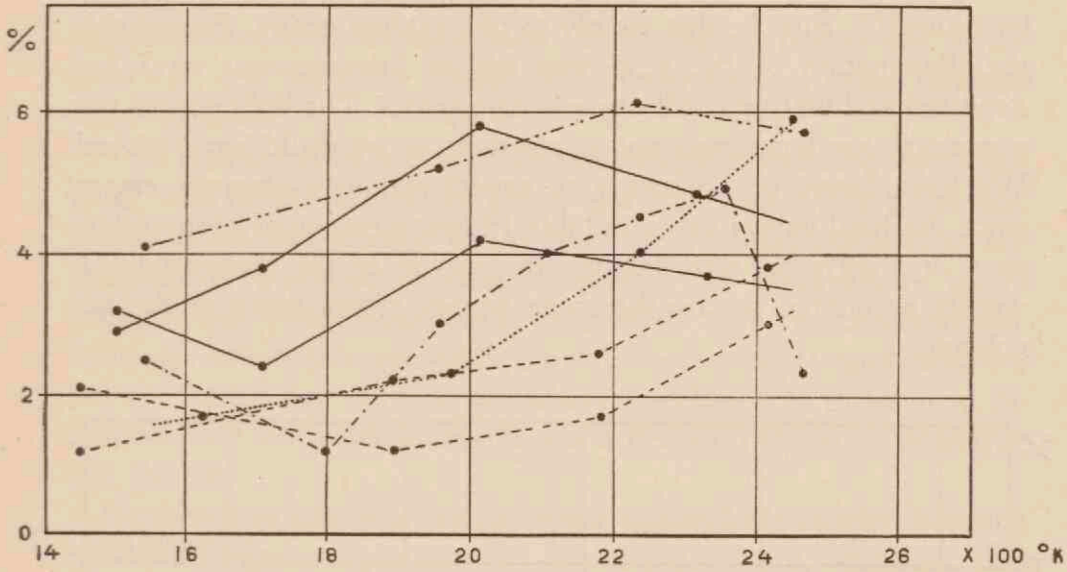

FIG. 17.

Percentage increase of reflectivity with temperature. The figure shows 7 series of observations made on 5 different strip lamps. Wavelength 0.313 μ.

this way we may calculate the standard deviation for repeated observations on one strip lamp and compare it with the standard deviation calculated from observations on different strip lamps. If the latter is considerably greater than the former, it may be deduced that systematic differences between the strip lamps actually existed.

At 0.254 μ repeated observations were made on 4 strip lamps and here we have the best chance of arriving at a definite result.

The calculations have yielded the following results.

Standard deviation for	Temperature		
observations	1600	2000	2400 °K
on 1 strip lamp	0.8^5	1.6	1.6
on different lamps	0.7	1.2	1.3

The standard deviation for observations on one strip lamp is even somewhat greater than for observations on different lamps and systematic differences are almost excluded by this result.

As has been stated above the observations at 0.313 μ seem to point in another direction, but they are too few (only 2) to allow any definite conclusion.

According to the above the spread of the observations, as shown by figure 17, must be due mainly to observation errors. Presumably the illumination of the strip was not as homogeneous as I had expected and warping still caused some errors. The light source too, a mercury arc lamp, was not as constant as we could have wished. We have, however, no reason to expect serious systematic errors, since the final values given in table 14 are averages of observations on 4 or 5 different strip lamps; the spread of the observations is wholly accounted for by the mean square errors given in the last column.

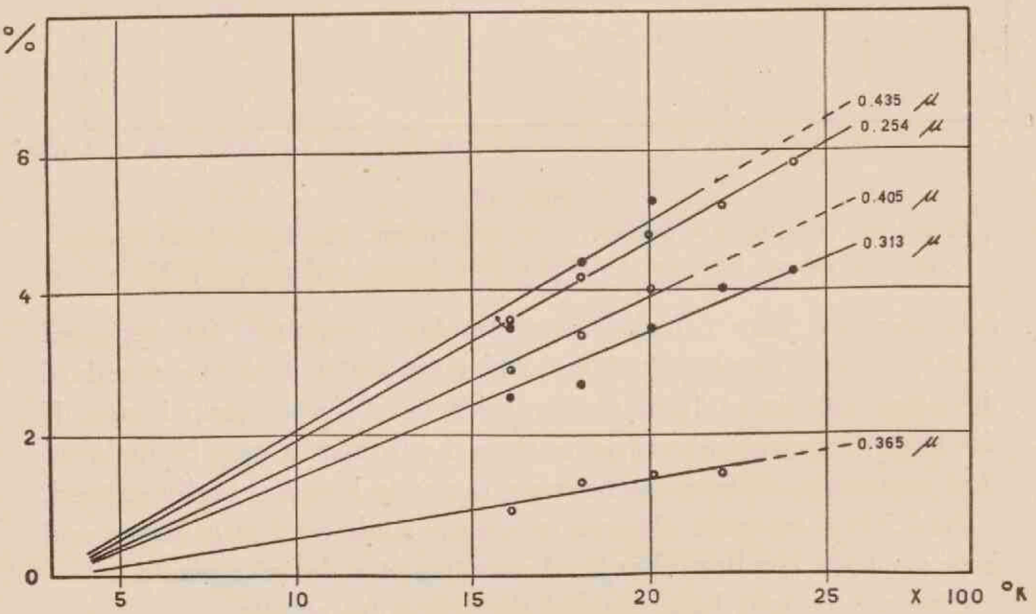

FIG. 18.

Percentage increase of reflectivity with temperature. Average of the observations in the ultra-violet showing a linear increase with temperature.

Plotting the percentage increase of reflectivity against temperature (fig. 18) we see that our observations within possible errors can be represented by straight lines through the origin. (The origin has been taken at room temperature = 300° K.)

In the visible and infra-red the same result has been found by WENIGER and PFUND (6), and WORTHING too, in a study on the emissivity of tungsten (4), observed a linear dependence of temperature at 0.665 μ and 0.467 μ.

From these facts we may draw the conclusion *that in the region from 0.25 μ to 4.0 μ the reflectivity of tungsten is a linear function of temperature.*

This is of great importance. Of the straight lines in figure 18 one point, the origin, is always given and only one other point will suffice to fix the position of the lines. In the infra-red part of the spectrum it was not possible to extend the observations above 1100° K, but we are now able to extrapolate to higher temperatures. From its nature this extrapolation will be somewhat uncertain, the more so since the value of the above conclusion is diminished by the fact that my observations do not agree with those of WENIGER and PFUND.

Nevertheless I have in the following assumed the relation between reflectivity and temperature to be always linear. Without this assumption I should have had to refrain from computations of the emissivity for temperatures above 1500° K, except in the ultra-violet region.

We have now only to study the slope of the straight lines of figure 18 as a function of wave-length; this slope I have expressed by the percentage increase of reflectivity corresponding to a rise of temperature of 1000° K. From each single observation this quantity can be calculated and we are therefore able directly to compare measurements made at different temperatures. The averages calculated from all observations are given in tables 15 and 16.

Table 15*A* contains the observations in the ultra-violet converted according to our present method. In table 15*B* the determinations, made with the 500 watt lamp as light source in the region from 0.4 μ to 0.65 μ and up to about 1400° K, are given. A difficulty experienced when making these observations was that the white painted cylinder and all objects inside were heated considerably by the radiation from the light source. The determination of the temperature of the tungsten strip was made outside the cylinder and it is doubtful whether the values found are also valid under the actual conditions of the observations; the increase of temperature caused by

TABLE 15.

Percentage increase of reflectivity per 1000 °K increase of temperature.

λ in μ	N	dR	M. S. E. 1.	M. S. E. 2.
A. Observations with mercury arc lamp up to 2400 °K.				
0.254	52	2.8	0.7	0.1
0.313	31	2.1	0.7	0.1^5
0.365	16	0.8	0.8	0.2
0.405	12	2.3	0.6	0.2
0.435	6	2.9	0.4	0.1^5
B. Observations with 500 watt lamp up to 1400 °K.				
0.40	4	3.1	0.7	0.3^5
0.45	4	3.1	0.4	0.2
0.50	4	2.8	0.5	0.2^5
0.55	4	2.5	0.7	0.3^5
0.60	4	2.3	0.2	0.1
0.65	4	1.6	0.7	0.3^5
C. Observations with sodium vapor lamp up to 1600 °K.				
0.58	12	2.3	0.7	0.2

N = Total number of observations.
dR = Increase of reflectivity in %.
M. S. E. 1. = Mean square error of 1 observation.
M. S. E. 2. = Mean square error of average.

the radiation from the light source may not have been the same at room temperature as at 1400° K. It is improbable however that the error exceeded 30° K, to which a relative error of 3 % in the observed

TABLE 16.

Relative change of reflectivity with temperature between 0.4 μ and 1.0 μ.

λ	N	dR	M. S. E.	$dR \times 1.42$
0.4	5	2.4	0.2^5	3.4
0.45	4	2.6	0.2	3.7
0.50	5	2.2	0.2	3.1
0.55	4	1.4	0.2	2.0
0.60	5	1.2	0.2^5	1.7
0.65	4	1.5	0.3	2.1
0.70	5	1.4	0.2^5	2.0
0.75	5	2.8	0.4	3.8
0.80	5	4.7	0.3^5	6.7
0.85	5	5.8	0.3^5	8.2
0.90	5	6.2	0.3^5	8.8
0.95	5	5.8	0.5^5	8.2
1.00	5	4.9	0.2^5	7.0

Notations as in table 15.
M. S. E. = Mean square error of average.

change of reflectivity corresponds. This error we may safely neglect, since the observation errors were many times as large.

In table 15C the result of the observations made with a sodium vapor lamp is given; these determinations agree very well with the data of table 15*B*.

In the infra-red region the radiation emitted by the strip is very strong and the observations could not be made for temperatures higher than 1000 or 1100° K. At these temperatures the strip is only glowing very faintly and I did not succeed in determining the temperature by means of a pyrometer with sufficient accuracy. Thus the observations given in table 16 have a relative value only, but we may adjust them to the data of table 15 by multiplying with a certain factor. This factor can be determined at 6 different wavelengths (from 0.4 μ to 0.65 μ) ; the average I found to be

$$1.42 \pm 0.14.$$

We see that in adjusting the values of table 16 a relative mean square error of 10 % is made. The influence this error has on the final results will be discussed at the end of this section.

In figure 19 the observations of table 15 and the adjusted values of table 16 have been plotted together. From 0.4 μ to 0.65 μ the determinations do not perfectly agree with each other, but on the whole the deviations from the smooth curve are not of much consequence.

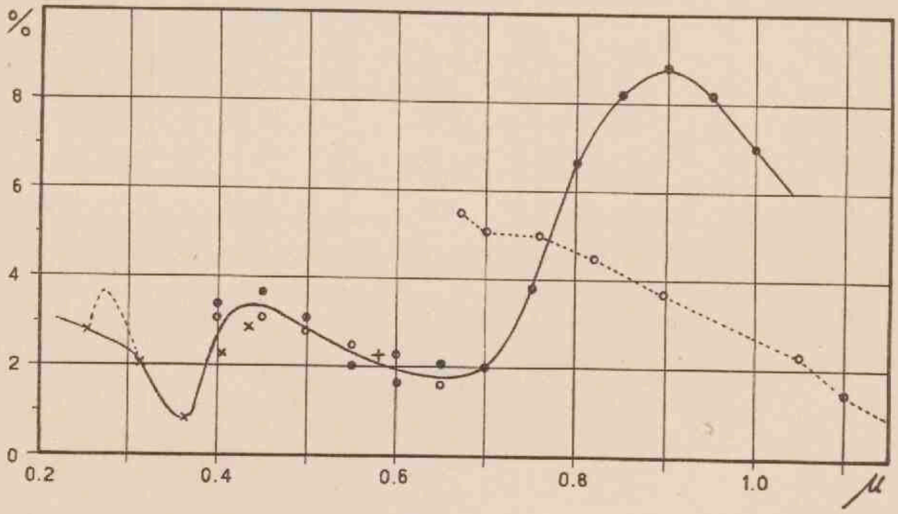

FIG. 19.

Percentage increase of reflectivity with temperature.

× Observations with mercury arc lamp. (Table 15 A.)
○ Observations with 500 watt lamp. (Table 15 B.)
+ Observations with sodium vapor lamp. (Table 15 C.)
● Adjusted relative observations with 500 watt lamp. (Table 16.)
○·········○ Weniger and Pfund's observations.

As is seen from figure 19 the change of reflectivity with temperature shows pronounced maxima and minima and comparing with figure 14 on page 41, we are struck by the fact that maxima and minima in both figures coincide. Where the reflectivity at room temperature has a minimum, the increase of reflectivity with temperature shows a maximum and reversely.

One apparent exception to this rule occurs at 0.28 μ. But in figure

19 we have no observation between 0.254 μ and 0.313 μ and it is quite possible that the curve must be drawn as indicated by the dotted line. In this region an interpolation will be somewhat uncertain. In the following table I have compared the values read off from the drawn and from the dotted curve.

λ in μ	Drawn curve	Dotted curve
0.23 μ	3.0 %	1.5 %
0.24 „	2.9 „	2.0 „
0.25 „	2.8 „	2.5 „
0.275 „	2.7 „	3.6 „
0.30 „	2.4 „	2.8 „

The final values in table 17 are the averages of these two

TABLE 17.

Percentage increase of reflectivity per 1000° K. increase of temperature. Final values read off from the curve in figure 19.

λ in μ	*dR* %	M. S. E.	λ in μ	*dR* %	M. S. E.
0.23	2.3	0.3	0.55	2.3	0.3
0.24	2.5	0.3	0.60	1.9	0.3
0.25	2.7	0.3	0.65	1.8	0.3
0.275	3.2	0.3	0.70	2.0	0.4
0.30	2.6	0.3	0.75	3.8	0.6
0.325	1.7	0.3	0.80	6.7	0.9
0.35	1.0	0.3	0.85	8.2	1.0
0.375	1.1	0.3	0.90	8.8	1.0
0.40	2.6	0.3	0.95	8.2	1.0
0.425	3.4	0.3	1.00	7.0	0.8
0.45	3.4	0.3	1.05	5.7	—
0.50	2.9	0.3			

readings; in all probability the error will not exceed 0.5 % except perhaps at 0.23 μ.

In figure 19 the observations of WENIGER and PFUND (6) have also been plotted and they are seen to disagree with my results completely. As has already been stated on page 47 the data published by WENIGER and PFUND do not suffice to judge of the reliability of their determinations so that it cannot be decided to what causes the discrepancy must be ascribed. It has been mentioned on page 42 that our observations disagree also in quite another respect.

The final results read off from the smooth curve of figure 19 (except below 0.3 μ) are given in table 17. With these data and the reflectivities of table 13 on page 42 the emissivities have been calculated (See Chapter 5).

In the last column of table 17 the mean square errors have been entered. In the region from 0.25 μ to 0.7 μ I have uniformly adopted an error of 0.3 % in virtue of the 5th and 4th column in tables 15 and 16 respectively and of figure 19; the *spread* $\sqrt{\frac{\Sigma d^2}{n}}$ of the separate points with respect to the smooth curve is 0.4 % and we may suppose the error in the curve itself to be somewhat less. Perhaps the value of 0.3 % is still too high.

In the infra-red region two sources of error co-operate viz.

1. Observation errors found by multiplying the values in the 4th column of table 16 by 1.42.
2. The relative error of 10 % made in adjusting the relative observations of table 16 to the absolute data of table 15.

Both are mean square errors and expressed in the same way they will combine to a total error according to the equation

$$E^2 = e_1{}^2 + e_2{}^2.$$

For instance at 0.8 μ we have
an observation error of $1.42 \times 0.35 = 0.5$ %
an error in the adjustment of $0.10 \times 7.0 = 0.7$ %
and consequently a total error

$$E = \sqrt{0.5^2 + 0.7^2} = 0.9\,\%.$$

The errors given in table 17 between 0.7 μ and 1.05 μ have been computed in this way.

Appendix.

A research, similar to that now carried out for tungsten, has previously been made with platinum (18).

This metal was then found to recristallize incessantly; whenever it was heated to incandescent temperatures, the reflectivity at room temperature changed its value and this process never came to a stop.

In making the experiments with tungsten I had a good opportunity of checking whether this metal shows any such properties. The reflectivity at room temperature before and after the strip had been heated, was determined repeatedly. In the ultra-violet 24 observations of this kind are available and from these I computed the root mean square difference to 0.7 %. On page 24 the mean square error of a single observation was deduced to be 0.4 % and accordingly we might expect a root mean square difference of $0.4 \times \sqrt{2} = 0.55$ %, if the differences are due to observation errors only. The above value of 0.7 % is somewhat greater but the difference is not of much consequence. Moreover the present observations were made with a mercury arc lamp, which was not perfectly constant, and it is quite possible that they were less accurate than in the case treated on page 24.

Anyhow we may conclude that recristallisation, if it existed, only caused minor changes in the reflectivity. In other regions of the spectrum I have obtained the same result, but there the observations were less numerous and I will not treat them in detail.

CHAPTER 5.

The spectral emissivity of tungsten.

§ 1. *Theoretical principles.*

From the data in table 13 and table 17 we can calculate the total reflectivity as a function of temperature and wave-length. I shall not give the results of these calculations here, but proceed at once to the emissivity which is connected to the reflectivity by the equation

$$e_{\lambda T} + r_{\lambda T} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is KIRCHHOFF's law, which we may only apply when a few special conditions are fulfilled. KIRCHHOFF's law is always derived from a system in thermodynamic equilibrium, in which all absorbed radiation is transformed into heat and in which loss of heat takes place from radiation only.

A burning strip lamp, however, is by no means in thermodynamic equilibrium, and to apply KIRCHHOFF's law we must introduce two hypotheses viz.

1. *that the emission of the tungsten strip is only determined by the temperature and is not dependent upon its surroundings*;
2. *that the same applies to the reflectivity.*

It is not my intention here to discuss whether these conditions actually are fulfilled, but it is advisable to point out that these conditions are essential if we wish to calculate the emissivity from the total reflectivity.

Besides this yet a 3rd condition must be fulfilled viz.

3. *that the emitting surface must have one definite temperature.*

The light emitted will not actually be sent out by a mathematical surface but from a layer, however thin. It is also possible that a thin gas layer close to the tungsten surface takes part in the emission. It is then essential that all material that contributes to the emission shall be of the same temperature.

Theoretically the 3rd condition is certainly not fulfilled, but it is impossible to make an estimation of how great the differences in temperature may be and whether they will cause important errors.

In § 1 of Chapter 1 a method was developed of measuring the total reflectivity by homogeneous illumination of the surface. This method we may also regard from a different theoretical point of view.

By a fundamental principle of the theory of radiation a closed space with black walls in thermodynamic equilibrium is always filled with homogeneous black body radiation $E_{\lambda T}$, irrespective of the nature of bodies which may be contained in the space.

The light sent out by an opaque body M in direction A (fig. 20) will consist of

1. a portion emitted by $M = e_{\lambda T} \cdot E_{\lambda T}$
2. a portion reflected by $M = r'_{\lambda T} \cdot E_{\lambda T}$

in which $r'_{\lambda T}$ is the reflectivity for homogeneous illumination as defined on page 7. The intensity of the homogeneous illumination is here of course $E_{\lambda T}$.

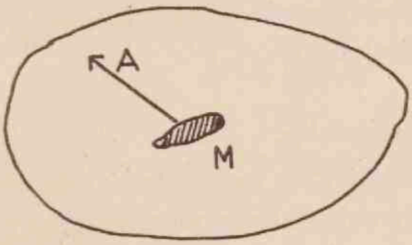

FIG. 20.

According to the above principle we now have

$$e_{\lambda T} \cdot E_{\lambda T} + r'_{\lambda T} \cdot E_{\lambda T} = E_{\lambda T}$$

from which follows

$$e_{\lambda T} + r'_{\lambda T} = 1 \quad \ldots \ldots \ldots \quad (2)$$

$r'_{\lambda T}$ must be taken in this formula as a separately defined quantity of which we do not know that it is equal to the total reflectivity. We only know this if we either apply HELMHOLTZ's law of reciprocity or KIRCHHOFF's law (See formula 1 on page 60).

Here we have thus three different laws.

1. The law of an enclosed space in thermodynamic equilibrium

filled with homogeneous black body radiation, from which we found

$$e_{\lambda T} + r'_{\lambda T} = 1$$

2. KIRCHHOFF's law giving

$$e_{\lambda T} + r_{\lambda T} = 1$$

3. The equation

$$r_{\lambda T} = r'_{\lambda T}$$

deduced from HELMHOLTZ's general principle of reciprocity.

Two of these laws are sufficient to prove the third; in deducing KIRCHHOFF's law however we have to make use of HELMHOLTZ's principle. Hence in my opinion the laws 1 and 3 should be regarded as the fundamental principles, KIRCHHOFF's law being a direct result of these two. This being so, I do not altogether think it right to treat KIRCHHOFF's law in the first place, as it is usually done in text books on the theory of radiation.

§ 2. *The spectral emissivity.*

In table 18 the emissivities calculated from the reflectivities are brought together. The calculations were made by means of the data from tables 13 and 17 and do not need any further elucidation.

In a separate table 19 the mean square errors are given. The relative error in the total reflectivity at room temperature was 0.5 % (page 43) and the errors in the increase of reflectivity with temperature are given in table 17. The last increase with temperature.

Both errors are mean square errors and will therefore give a total error found from the formula already used, $E^2 = e_1^2 + e_2^2$.

E being the relative error in the reflectivity, we find for the corresponding error in the emissivity

$$E' = \frac{E \, . \, R}{1 - R}.$$

This yields the values given in table 19.

The error in the ratio of the emissivity for two different wavelengths will be found by combining the figures in table 19 according to the law of squares. In the infra-red region this is, however, not

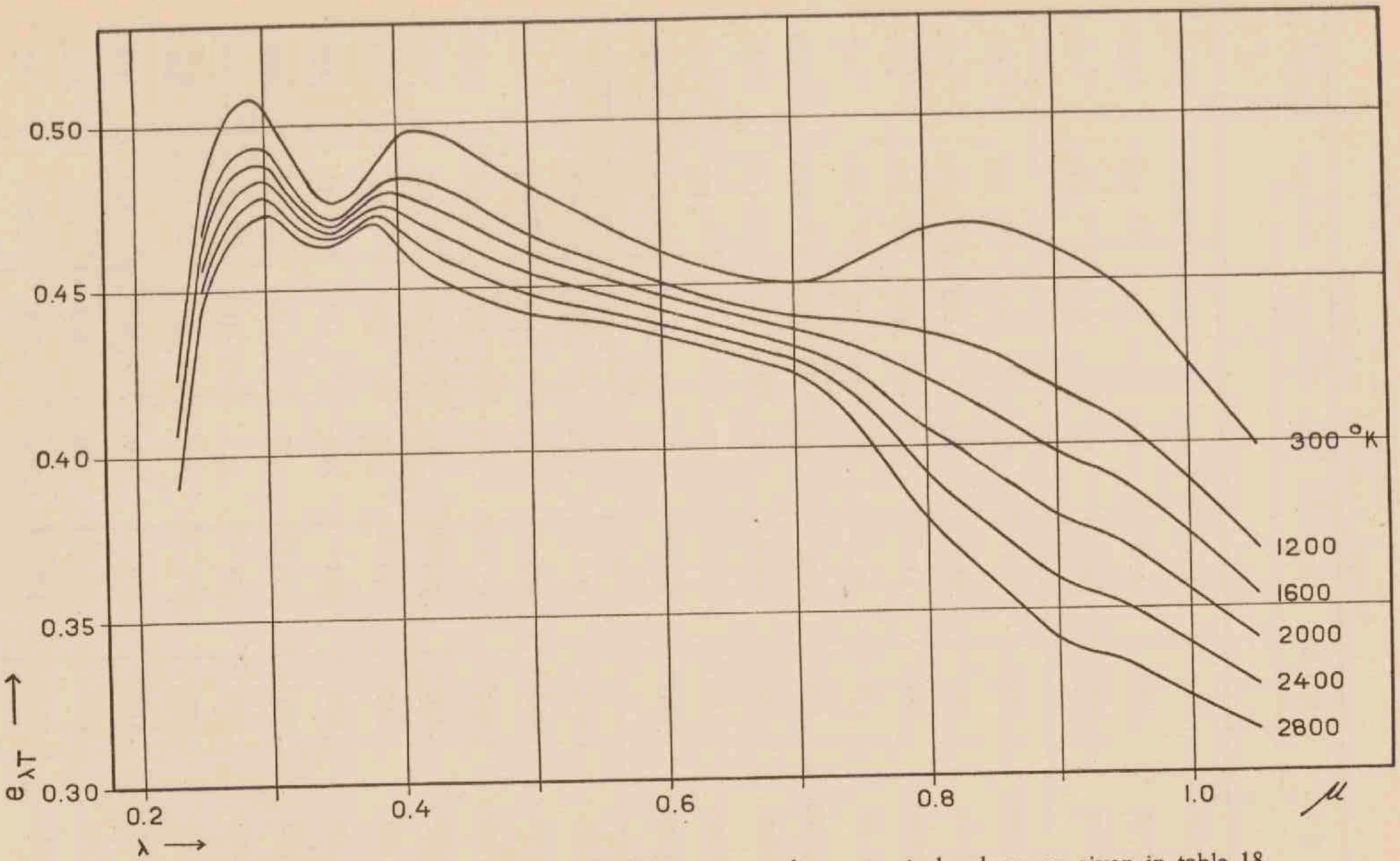

FIG. 21. Spectral emissivities of tungsten. The corresponding numerical values are given in table 18.

TABLE 18.

Spectral emissivities of tungsten.

$T=$	1000	1200	1400	1600	1800	2000° K
λ in μ						
0.23	0.414	0.411	0.408	0.406	0.403	0.400
0.24	.440	.438	.435	.432	.429	.427
0.25	.470	.467	.465	.462	.459	.456
0.275	.494	.491	.488	.485	.482	.478
0.30	.496	.493	.491	.488	.486	.483
0.325	.481	.479	.477	.476	.474	.472
0.35	.472	.471	.470	.469	.468	.467
0.375	.479	.478	.477	.476	.475	.473
0.40	.487	.484	.482	.479	.476	.474
0.425	.484	.481	.477	.473	.470	.467
0.45	.480	.477	.473	.470	.466	.463
0.50	.468	.465	.462	.459	.456	.453
0.55	.460	.458	.456	.453	.451	.448
0.60	.453	.451	.449	.447	.445	.443
0.65	.446	.444	.442	.440	.438	.436
0.70	.442	.440	.438	.436	.434	.431
0.75	.443	.438	.434	.430	.426	.422
0.80	.441	.434	.427	.420	.412	.405
0.85	.436	.428	.419	.410	.401	.393
0.90	.426	.416	.407	.397	.388	.378
0.95	.415	.406	.397	.388	.379	.370
1.00	.396	.388	.380	.372	.364	.355
1.05	.375	.368	.361	.354	.348	.341

TABLE 18 (continued).

$T=$	2200	2400	2600	2800	3000° K
λ in μ					
0.23	0.398	0.395	0.392	0.390	0.387
0.24	.424	.421	.418	.416	.413
0.25	.453	.450	.448	.445	.442
0.275	.475	.472	.469	.466	.463
0.30	.481	.478	.475	.473	.470
0.325	.470	.469	.467	.465	.463
0.35	.466	.465	.464	.463	.462
0.375	.472	.471	.470	.469	.468
0.40	.471	.468	.466	.463	.460
0.425	.463	.460	.457	.453	.450
0.45	.459	.456	.452	.449	.445
0.50	.450	.447	.444	.441	.438
0.55	.446	.443	.441	.439	.436
0.60	.441	.438	.436	.434	.432
0.65	.434	.432	.430	.428	.426
0.70	.429	.427	.425	.423	.420
0.75	.418	.414	.410	.405	.401
0.80	.398	.391	.384	.377	.370
0.85	.384	.375	.366	.358	.349
0.90	.369	.359	.350	.340	.331
0.95	.361	.352	.343	.334	.325
1.00	.347	.339	.331	.323	.315
1.05	.334	.327	.320	.313	.306

TABLE 19.

Relative mean square errors of the spectral emissivities of table 18.

$T \rightarrow$ in °K	1200	1600	2000	2400	2800
λ in μ	Errors in $^0/_0$				
0.23 to 0.65	0.6	0.6	0.7	0.8	0.9
0.7	0.7	0.9	1.0	1.3	1.6
0.8	1.2	1.8	2.3	2.7	3.5
0.9	1.5	2.1	3.0	3.8	5.0
1.0	1.3	1.9	2.6	3.5	4.5

quite exact. The principal error is here caused by inaccurate adjustment of the relative data of table 16 and this error will have the same relative value from 0.75 μ to 1.05 μ for all wave lengths. Consequently the total errors in this region are not entirely independent of each other and by applying the law of squares we certainly get too large values for the error in the ratio of two emissivities.

In figure 21 the emissivity has been plotted against wave-length for some temperatures. As might be expected from figure 19 the curve gradually changes with increase of temperature.

§ 3. *Comparison with other observations.*

In figure 22 WORTHING's emissivities (4) are compared with the values that I found.

WORTHING made his measurements by means of an optical pyrometer with red and blue filter and with a black body made of tungsten. Up to now his values are in general use and they are included in the International Critical Tables. As we see the deviations are only small; at 0.467 μ WORTHING's values are higher by a constant amount of 0.01; at 0.665 μ the lines cross each other. It is not altogether impossible that there may be systematic errors in WORTHING's measurements, but it is equally possible to ascribe the deviations to a difference in the composition of the tungsten used. The material I used contained 1 % thorium oxide.

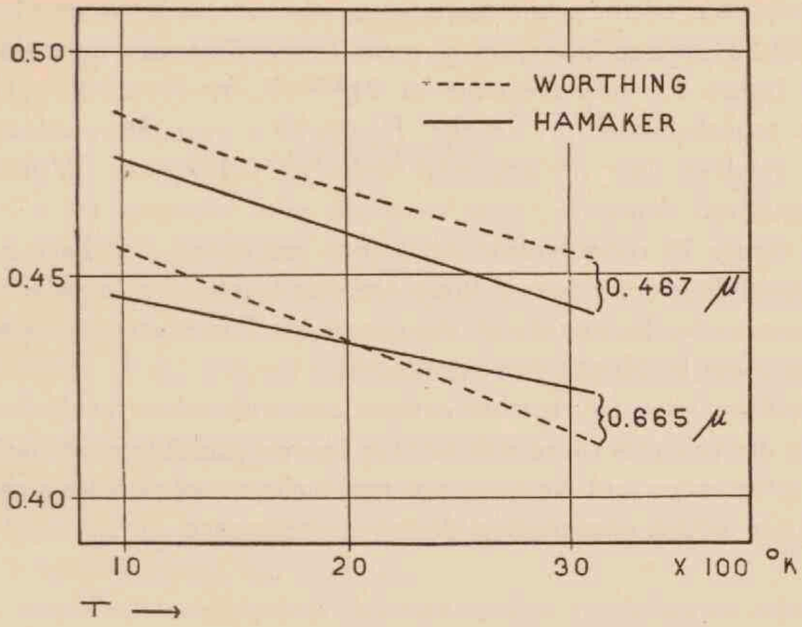

FIG. 22.
Comparison of the spectral emissivities of table 18 with Worthing's values.

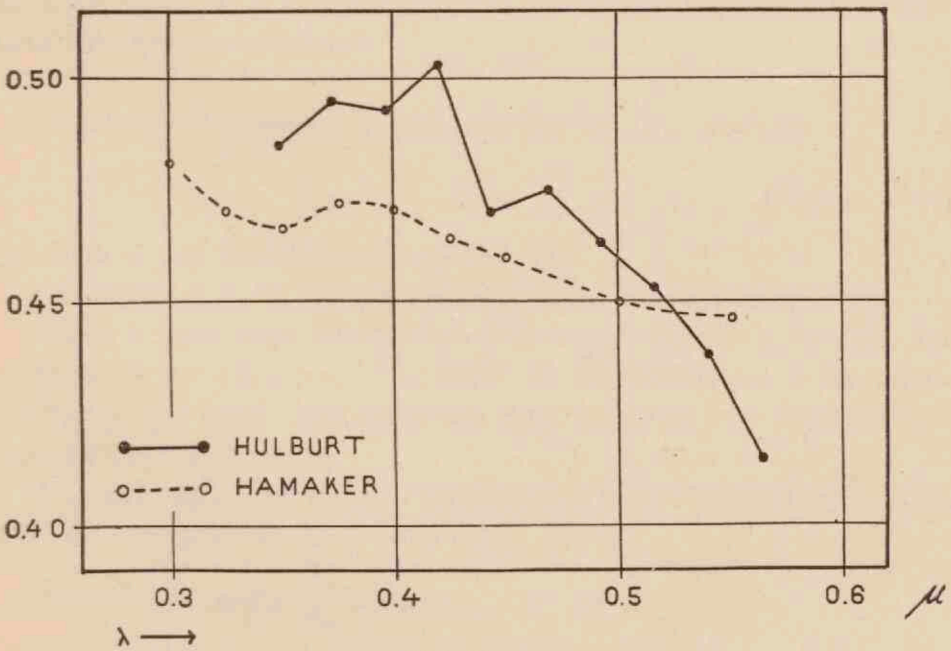

FIG. 23.
Comparison of the spectral emissivities of table 18 with observations by Hulbert.

Deviations of the same order were also ofund between COBLENZ and EMERSON's reflectivities at room temperature and mine (fig. 14).

In figure 23 measurements at 2143° K by HULBURT (17) are given together with my results. HULBURT's own observations were only relative, but he adjusted them at 0.467 μ to WORTHING's emissivities.

In figure 23 considerable deviations are found, but here too it is not possible to ascertain if these are due to observation errors or to differences in the quality of the tungsten. The maximum at 0.4 μ is also present in HULBURT's results.

I will not give any further comparison with other results here. As in the above cases there will always be two possible explanations of any differences and we have not much chance of deciding with any certainty which explanation deserves preference.

CHAPTER 6.

§ 1. *On the accuracy of a standard lamp calibrated by means of an optical pyrometer.*

The calibration of a standard lamp consists of a determination of the temperature of the strip by means of an optical pyrometer and a calculation of the energy distribution from PLANCK's law and the emissivities.

Important errors may arise

1. from errors in the emissivities;
2. from errors in the temperature determination;
3. from absorbtion by the bulb.

The mean square errors in the emissivities are given in table 19. I will now shortly discuss the sources of error mentioned under 2 and 3; from a mutual comparison of the possible errors we shall have to decide whether the emissivities are sufficiently accurate and by what methods a further increase of the accuracy of a standard lamp can best be obtained.

Denoting the emission of the tungsten by $E_{\gamma T}$ we have

$$E_{\lambda T} = e_{\lambda T} \cdot c_1 \cdot e^{-\frac{c_2}{\lambda T}} \cdot \lambda^{-5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

in which $e_{\lambda T}$ is the emissivity. $c_2 = 1.433$.

The value of c_1 is, for the present, of no consequence.

Above I have used WIEN's law instead of PLANCK's formula but as long as $\lambda < 1.0\,\mu$ and $T < 3000°$ K the difference is less than 0.7 %. In the error discussion we may therefore use WIEN's law without any restrictions.

When dealing with general properties of the emission the notation $E_{\lambda T}$ will often be replaced by E or E_1 and E_2.

Differentiating $log\,\frac{E_1}{E_2}$ we get

$$d\,\frac{E_1}{E_2} : \frac{E_1}{E_2} = \frac{dE_1}{E_1} - \frac{dE_2}{E_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

from which we conclude that

the percentage error in the relative intensity of two wave-lengths is equal to the difference of the percentage errors in the absolute intensities of both separately.

Thus if we calculate the latter as a function of λ and T, the former can be computed in a very simple way.

Differentiating the logarithm of formula 1 with respect to T we get

$$\frac{dE_{\lambda T}}{E_{\lambda T}} = \frac{c_2 \, . \, dT}{\lambda \, T^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

We may consider an observation with the pyrometer either as a determination of the temperature T or as a determination of the emission E; the errors dT and dE will then be connected by equation 3.

The calibration of a pyrometer is made

1. by a relative calibration;
2. by adapting the relative calibration to an absolute standard, usually given by a black body at the melting temperature of gold.

Errors in the temperature determination by means of a pyrometer may arise from the following sources.

1. An error in the melting temperature of gold.
2. An error in the black body, the emissivity not being exactly 1.00.
3. An error in the temperature of the black body.
4. An observation error made in observing the black body with the pyrometer.
5. Errors in the relative calibration of the pyrometer.
6. An observation error made in observing the tungsten strip.
7. An error due to an erroneous correction for the absorbtion and reflection in the bulb.
8. An error caused by an inaccuracy in the emissivity.

All errors, except the one mentioned under N⁰. 5, are such that they will cause a relative error dE/E in the emission independent of the temperature. Let us for the present assume that the same also applies to the error N⁰. 5, it then follows from formula 3 that the errors dT must increase with the square of the temperature T, so that

$$dT = C \, . \, T^2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

In reality this will not be exactly so, but since we are aiming at a rough estimation of the errors only we need not be concerned with such details.

Mr. VERMEULEN of the Utrecht Institute, who has much experience in the use of a pyrometer, informs me that the mean square error of a temperature determination with a HOLBORN-KURLBAUM pyrometer is about 10° K at 2500° K. I will here use this value; if it is either too large or too small all errors computed below must be reduced by a corresponding factor.

We may now calculate the constant C in formula 4 and we find

$$C = \frac{10}{2500^2} = 1.6 \times 10^{-6}$$

Inserting this value in equation 3 and expressing λ in microns (10^{-4} cm) we get

$$100 \frac{dE_{\lambda T}}{E_{\lambda T}} = \frac{2.3}{\lambda} \quad (\lambda \text{ in } \mu) \quad \ldots\ldots \quad (5)$$

from which, according to equation 2, we find

$$100 \left(d\frac{E_1}{E_2} : \frac{E_1}{E_2} \right) = 2.3 \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) \quad \ldots\ldots \quad (6)$$

By this simple formula the percentage mean square error in the relative intensities is expressed independent of temperature.

In table 20 I have compared the errors calculated from equation 6 with the errors caused by inaccuracies in the emissivity. Te latter were computed as indicated on page 62; they are not independent of temperature so that the table is valid for 2000° K only.

In most cases the errors e_2 are considerably greater than the errors e_1 and the differences increase with the interval between the two wave-lengths. From this we may draw the conclusion that *to increase the accuracy of a standard lamp, we have in the first place to effect more accurate determinations of temperature.*

If the two wave-lengths lie close together the errors due to the temperature determination will become very small, but since the values of table 18 were derived from smooth curves the same will

TABLE 20.

Percentage mean square errors in the relative intensities of two wave-lengths. ($T = 2000\ ^\circ K$).

e_1 = error due to errors in the emissivities.

e_2 = error due to errors in the temperature determination.

$\lambda_1 \downarrow$	$\lambda_2 \rightarrow$	0.30	0.40	0.50	0.60	0.80	1.00
0.25	$e_1 =$	1.0	1.0	1.0	1.0	2.4	2.7
	$e_2 =$	1.6	3.5	4.6	5.5	6.3	6.9
0.30	$e_1 =$		1.0	1.0	1.0	2.4	2.7
	$e_2 =$		1.8	3.0	3.9	4.6	5.3
0.40	$e_1 =$			1.0	1.0	2.4	2.7
	$e_2 =$			1.2	2.1	2.9	3.5
0.50	$e_1 =$				1.0	2.4	2.7
	$e_2 =$				0.9	1.7	2.3
0.60	$e_1 =$					2.4	2.7
	$e_2 =$					0.9	1.4
0.80	$e_1 =$						3.5
	$e_2 =$						0.6

λ_1 and λ_2 in μ.

be the case with the errors in the relative emissivities. It is impossible to decide which error will be most important in these cases.

On page 69 the absorbtion by the bulb was also mentioned as a source of serious errors. How the absorbtion can be determined has been explained in § 5 of Chapter 1 and in figure 6 some absorbtion curves have been given.

With the magnesium oxide cylinder and mercury arc lamp measurements of the absorbtion were also made in the ultra-violet part of the spectrum. In figure 24 the results for two quartz bulbs are given.

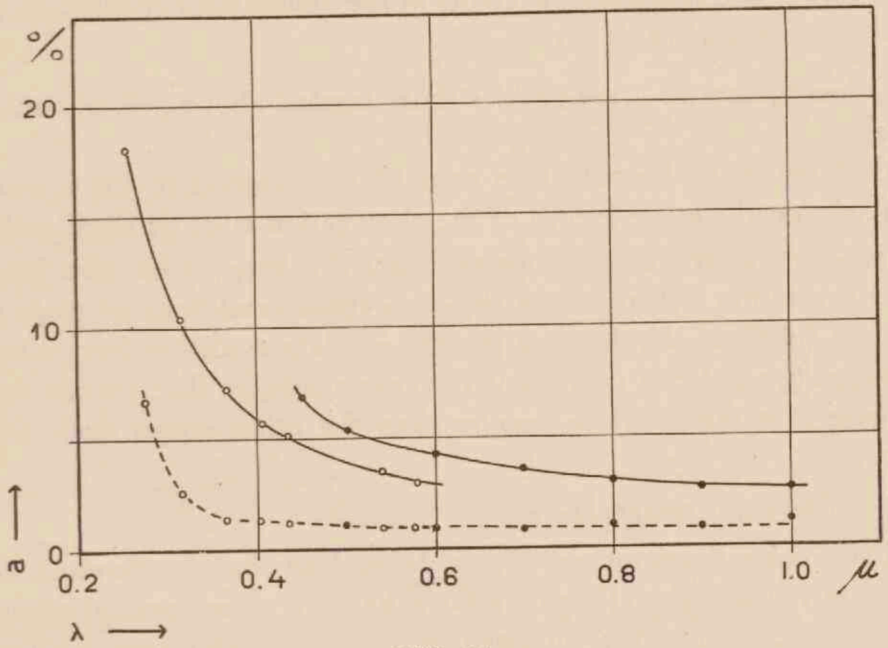

FIG. 24.

Absorbtion curves for two different quartz bulbs

One of the bulbs was not quite clean and shows a strong absorbtion. In the region from 0.4 μ to 0.6 μ the observations made with a mercury arc lamp lie 1½ % lower than the values obtained with the arrangement of figure 2. This indicates that the absorbtion was not the same everywhere on the bulb. In the ultra-violet the absorbtion is seen to increase to no less than 18 %.

The other bulb was quite clear and the absorbtion is almost constant, about 1 %, from 1.0 μ to 0.4 μ; here too, however, the absorbtion increases in the ultra-violet, in this case to 7.0 %.

That serious errors may arise does not need any further demonstration; especially in the ultra-violet they may be many times as large as the values of table 20.

In virtue of the above we may conclude that to increase the accuracy of a standard lamp we must

1. *Increase the accuracy of the temperature determinations.*
2. *Either avoid or determine the absorbtion of the bulb.*

3. *Carry out more accurate observations of the increase of reflectivity with temperature in the region from 0.7 μ to 1.0 μ. In this part of the spectrum the errors in the emissivities of table 18 are still quite considerable.*

Other sources of error besides those treated above will, of course, exist.

Differentiating formula 1 on page 69 with respect to λ it is easily proved that the wave-length of the observation must be fixed within 10 or 20 Å. Every worker, however, has this error in his own hand.

Other errors will occur as the theoretical conditions of page 60 are not fulfilled. We have at present no data available from which to judge the order of magnitude of these errors, so that we can only surmise that they will be smaller than the other errors dealt with above.

§ 2. *The color temperature.*

The "color temperature" T_c is defined as that temperature for which a black body (PLANCK's law) has the same spectral energy distribution, as has the tungsten at its true temperature T. True temperature T and color temperature T_c will thus be connected by the equation

$$e_{\lambda T} \cdot e^{-\frac{c_2}{\lambda T}} = A_T \cdot e^{-\frac{c_2}{\lambda T_c}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which the factor A_T is independent of wave-length. $e_{\lambda T}$ is the emissivity.

The conception of a color temperature has been introduced to describe the energy distribution of incandescent tungsten by a simple formula. We see, however, from equation 7 that by the color temperature the relative emissivities are entirely fixed as a function of wave-length and a T_c will therefore only be valid within a limited region of the spectrum.

Taking the logarithm of formula 7 we get

$$\log e_{\lambda T} = \log A_T - \frac{c_2}{\lambda}\left(\frac{1}{T_c} - \frac{1}{T}\right) \quad . \quad . \quad . \quad . \quad (8)$$

or

$$\log e_{\lambda T} = a_T + \frac{b_T}{\lambda}$$

from which it is seen that, when a color temperature exists, the logarithm of the emissivity must be a linear function of the inverse of wave-length.

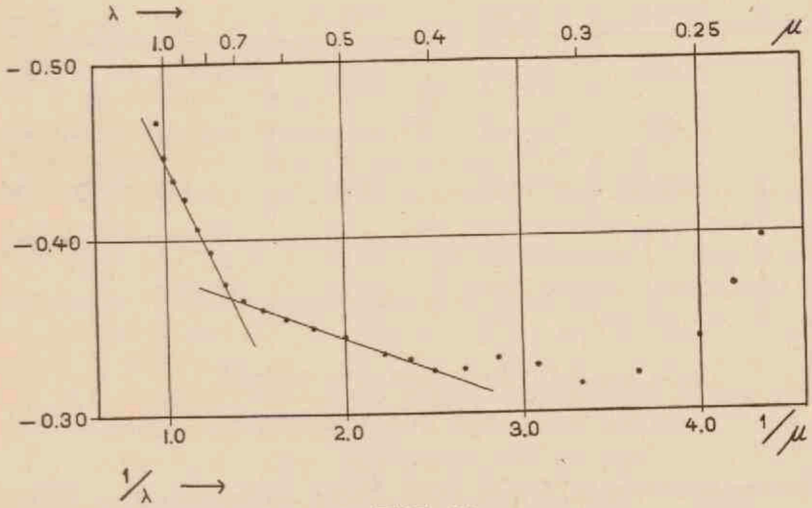

FIG. 25.

$^{10}Log.\ e_{\lambda T}$ plotted against $1/\lambda$ ($T = 2000$ °K).

In figure 25 I have plotted *log.* $e_{\lambda 2000}$ against $1/\lambda$. From this figure we at once infer that to tungsten a color temperature can be assigned in the region from $0.4\ \mu$ to $0.7\ \mu$, but outside this region the conception of a T_c does not hold at all. From the slope of the straight line T_c was computed to 2023° K (See formula 8).

Conversely we may calculate the emissivities from the color temperature by reading *log.* $e_{\lambda T}$ from the straight line in figure 25. In table 21 the emissivities thus found have been compared with their true values from table 18.

In the region from $0.4\ \mu$ to $0.7\ \mu$ the agreement is very close, the differences not exceeding 0.5 %, but outside this region the discrepancies vary from 4 to 20 %.

By the graphic method of figure 25 the color temperatures were computed for different values of the true temperature. The results of these computations are given in table 22.

Especially for higher temperatures the differences between the new determinations and the old values (19) are rather large.

It is seen from figure 25 that we may also assign a color tempera-

TABLE 21.

Comparison of the emissivities calculated from the color temperature with their true values from table 18.

λ in μ	$e_{\lambda\, 2000}$	$e'_{\lambda\, 2000}$	λ in μ	$e_{\lambda\, 2000}$	$e'_{\lambda\, 2000}$
0.25	0.456	0.534	0.6	0.443	0.441
0.3	0.483	0.505	0.7	0.431	0.433
0.35	0.467	0.486	0.8	0.405	0.426
0.4	0.474	0.472	0.9	0.378	0.422
0.5	0.453	0.453	1.0	0.357	0.418

$e_{\lambda\, 2000}$ is the true value from table 18.

$e'_{\lambda\, 2000}$ is the value calculated from T_c.

ture in the region from 0.75 μ to 1.0 μ. Here we find for $T = 2000°$ K, $T_c = 2150°$ K, a considerable difference with the value of 2023° K valid in the visible region.

TABLE 22.

Color temperatures for the visible region.

T	T_c New	T'_c Old
1200	1210	1210
1600	1616	1619
2000	2023	2033
2400	2432	2452
2800	2844	2878

It should finally be remarked that from a purely scientific point of view the conception of a color temperature has no meaning at all. It has however great practical value, the more so since the color temperature is valid over the whole visible range of the spectrum.

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STELLINGEN

1.

Het is gewenscht, dat in leerboeken over en bij het onderwijs in de waarschijnlijkheidsrekening ook aandacht wordt besteed aan de methoden en opvattingen der mathematische statistiek.

2.

De foutentheorie van GAUSZ is gecompliceerd en gaat van te veel premissen uit; zij wordt daardoor voor den praktischen natuurkundige weinig toegankelijk.

Een eenvoudiger en doeltreffender opzet eener foutentheorie is zeer goed mogelijk.

3.

F. MÖLLER, Integration der Bewegungsgleichungen im gekrümmten Isobarenfelde.

Ann. der Hydrographie und maritime Meteorologie
1930, H 4.

Het in dit artikel door MÖLLER behandeld probleem behoort tot de puntmechanica; daarbij te spreken van een isobarenveld is onjuist.

4.

De z.g. diffusievergelijking van FICK in den vorm

$$\frac{D\chi}{Dt} = \frac{\partial}{\partial x}\eta_x\frac{\partial\chi}{\partial x} + \frac{\partial}{\partial y}\eta_y\frac{\partial\chi}{\partial y} + \frac{\partial}{\partial z}\eta_z\frac{\partial\chi}{\partial z}$$

is onvolledig. De grootheden η_x, η_y en η_z zijn in werkelijkheid drie componenten van een tensor, waarvan de andere zes componenten niet zonder meer mogen worden verwaarloosd.

De correspondeerende algemeene vergelijking kan ook langs zuiver mathematischen weg worden afgeleid.

TH. HESSELBERG, Ann. der Hydrographie und maritime Meteorologie 1929, H 10, p. 319.
O. G. SUTTON, Proc. Roy. Soc., 1932, A 135, p. 143.
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5.

In een één- of meerzijdig begrensden oceaan met stabiel gelaagde watermassa's kunnen geen stationnaire driftstroomen bestaan, wanneer de wind ergens een component heeft loodrecht op de kustrichting.

6.

Voor de mathematische beschrijving van verschillende problemen in oceanographie en meteorologie is het gewenscht, dat wordt gezocht naar een algemeen stel vergelijkingen, waarin hydrodynamica en diffusieverschijnselen tezamen als één probleem worden behandeld.

7.

Het is niet juist, dat de vergelijking van RICHARDSON voor de thermische electronen-emissie strikt genomen geldt voor een veldsterkte nul.

S. DUSHMAN, Reviews of modern physics, 1930, p. 392

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