An accurate method for the relative measuring of spectral intensities : with an application to the determination of Planck's constant C2

AN ACCURATE METHOD FOR THE RELATIVE MEASURING OF SPECTRAL INTENSITIES

WITH AN APPLICATION TO THE DETERMINATION OF PLANCK'S CONSTANT C_2

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Diss Utrecht 1935

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PROEFSCHRIFT

TER VERKRIJGING VAN DEN GRAAD VAN DOCTOR IN DE WIS- EN NATUURKUNDE AAN DE RIJKSUNIVERSITEIT TE UTRECHT, OP GEZAG VAN DEN RECTOR MAGNIFICUS DR. H. BOLKESTEIN, HOOGLEERAAR IN DE FACULTEIT DER LETTEREN EN WIJSBEGEERTE, VOLGENS BESLUIT VAN DEN SENAAT DER UNIVERSITEIT TEGEN DE BEDENKINGEN VAN DE FACULTEIT DER WIS- EN NATUURKUNDE TE VERDEDIGEN OP DINSDAG 9 JULI 1935, DES NAMIDDAGS TE 3 UUR

DOOR

DIRK VERMEULEN

GEBOREN TE KAPEL-AVEZAATH

AMSTERDAM — 1935

N.V. NOORD-HOLLANDSCHE UITGEVERSMAATSCHAPPIJ

Aan de nagedachtenis van mijn Vader.
Aan mijn Moeder.
Aan mijn Vrouw.

Bij het beëindigen van dit proefschrift betuig ik mijn oprechte dank, aan allen, die tot mijn wetenschappelijke vorming hebben bijgedragen.

In het bijzonder is het mij een behoefte U, Hooggeleerde ORNSTEIN, Hooggeachte Promotor, dank te brengen voor de leiding en levendige belangstelling, die ik zoowel bij de onderzoekingen voor dit proefschrift, als bij andere onderzoekingen van U heb mogen ondervinden. Dat ik onder Uw leiding in aanraking ben gekomen met problemen behoorend tot het gebied der toegepaste wetenschappen, acht ik een groot voorrecht.

U zult wel willen gelooven, dat de belangstelling, die ik naast mijn laboratoriumwerk van U mocht ondervinden, voor mij van groote beteekenis is.

TRUUS EYMERS, ik ben je dankbaar voor onze langdurige samenwerking, waarin wij aan vele problemen werken mochten. Dat ik als onderwerp voor een proefschrift één onzer problemen kiezen mocht, waarbij ik mijn neiging tot precisie metingen volgen kon, dank ik aan jouw vriendschappelijke welwillendheid.

Ook aan jou, ZAALBERG, mijn dank voor je prettige samenwerking.

CONTENTS

INTRODUCTION.

In a report, issued by the Dutch Committee for photometry for the I. C. I. (1931) a method was described for measuring spectral intensities, on which the definition of the light unit can be based (1). For reasons of principle, defining the light unit with the aid of a method, by which the spectral energy distribution can be determined in an absolute measure, is to be preferred to fixing it by means of a standardized light source as e.g. the black body. For, as a general rule, one must, in fixing units, define the unit in question, if at all possible, by means of already established standard values. In the case of the light unit, this can be done by choosing as a unit the flow of energy characterised by a definite number of erg/sec, and a spectral distribution, both of which are to be agreed upon by general consent.

During the discussions at Cambridge in 1931 concerning the light unit stress was laid on the condition that only when the measuring method for objective spectral photometry should have reached an accuracy of 0,1 %, the spectral energy distribution could serve as a basis for the definition of that unit.

On account of this condition, investigations have been carried out in the Utrecht laboratory, with a view to increasing the accuracy.

The problem involved, has been treated in two parts, firstly the relative measuring in the spectrum has been worked out to the highest possible degree of accuracy, and secondly the precision of an absolute energy measurement has been investigated.

The present thesis deals with the former part, for the latter part, the reader is referred to the thesis written by J. WOUDA.

In the first chapter the measuring method is described in detail. In the second chapter the method is applied to the determination of the relative energy distribution in black radiation at the melting temperature of gold, from which as a further result the value of PLANCK's constant c_2 is deduced.

CHAPTER I.

Determination of the relative spectral distribution of energy according to the monochromator method.

§ 1. PRINCIPLE OF THE METHOD.

In a paper on the measurement of intensity ratios between spectral lines at great distances from one another, L. S. ORNSTEIN has shown the principle of the method of measuring which we shall explain by means of a schematic figure (fig. 1) of the arrangement (2, 3).

With an auxiliary lamp Q, a tungsten band lamp, and a monochromator *M*, images of a definite colour are obtained of the back-

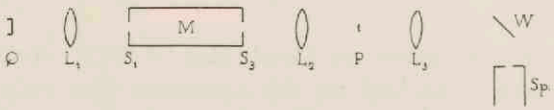

Fig. 1.

slit of the latter instrument in *P*. The proportions of the energy in the coloured images in *P* are measured by a non-selective radiation-receiver. Once these proportions known, the images play the part of a primary standard source of light with which a lamp to be standardized is compared for various wave-lengths by means of a spectral apparatus *Sp*. To this end the proportions of the intensity in the image *P* and the source of light to be standardized placed also at *P* are measured for various wave-lengths.

This must be carried out in such a way that the percentage of energy lost by the light on its way from *P* to the receiver behind the spectral apparatus is the same for the coloured images and for the lamp to be standardized for each wave-length, since in that case only, the measured intensity ratio will indeed be the true intensity ratio. In order to meet these requirements the following conditions must be borne in mind.

1. Those parts of the spectral apparatus actually traversed by

the light when it is illuminated by the images *P* and by the lamp to be standardized must be identical.

2. The light emerging from the monochromator is partly polarized. The ordinary light from the lamp to be standardized is thus compared with the polarized light of the images *P*, which may give rise to errors, owing to the fact that the power of transmission of the spectral apparatus for various wave-lengths is dependent on the polarization of the light falling upon it.

The following contrivance suggested by W. J. D. VAN DYCK enables us to satisfy both these conditions at the same time. Images are formed of the images at *P* and of the lamp to be standardized on a diffusely reflecting white surface *W* (magnesium oxide) which, as a secondary light source, radiates on to the spectral apparatus. The polarized light is thus depolarized by the diffuse reflection; the light therefore, which the images at *P* as well as the lamp cast on to the apparatus is ordinary light which, moreover, traverses the instrument in both cases along the same paths.

We want to measure the relative spectral distribution of the lamp to be standardized, that is, the intensity, measured in an arbitrary unit, radiated per spectral region of 1 Å, as a function of the wave-length. It is, therefore, necessary to find out the energy of the images per Å. The relative energy per Å of the images *P* is found from the energies measured and from the knowledge of the wave-length regions let through.

From the above it is seen that a determination of the spectral distribution of energy in light sources requires the three following measurements:

a. The measurement of energy of the coloured images.

b. ,, ,, ,, the wave-length regions.

c. ,, ,, ,, ,, ratio of intensity in the images *P* and the lamp to be standardized, at various wave-lengths.

In the following paragraphs we give first a brief description of the arrangement used, followed by a detailed description of the methods of measuring and the apparatus employed, in which we discuss as well the various sources of errors. The accuracy with which a spectral energy-measurement can be carried out is discussed in the last paragraph.

§ 2. SHORT DESCRIPTION OF THE ARRANGEMENT.

A tungsten bandlamp provided with a vertical band, serves as the auxiliary lamp in fig. 1. A compensating arrangement is used for the accurate testing whether the strength of the electric current is constant. It is further necessary to ascertain whether the radiation is constant for constant current. The measurements carried out to this purpose are described in § 7.

For these experiments a double monochromator, after VAN CITTERT (4) has been used. A bolometer has been selected as a radiation receiver. An image of the back slit of the monochromator is formed, by means of a combination of two Zeiss Tessar lenses L_2 on to a horizontal bolometer strip of the vacuum bolometer placed at P. In this way we measured the energy with small slit-height. This offers advantages as regards the curvature of the images of the front-slit ; reference will be made to this point in further detail in the description of the double monochromator (§ 7).

For measuring the wave-length regions (§ 4) an image of the back slit is formed by a rectangular prism and a microscope objective *aa* on the slit of a Hilger spectroscope placed perpendicularly to the path of light behind the monochromator. As light sources, of which the spectral distribution of energy is measured, we chose tungsten band lamps. In order to get the band exactly in point P (the place of the illuminated bolometer-strip) we used a telescope with cross-wires, placed as perpendicularly as possible to the direction of the light-path behind the monochromator. The light reflected by the bolometer-strip is just sufficient to focus the telescope accurately on P. The band-lamp to be standardized is mounted on a stand which allows of small displacements in three mutually perpendicular directions. By lens L_3 an image of the image P, respectively of the band-lamp, is formed on the white surface.

Since the comparison between the spectral distribution of the energy of the lamp, to be standardized, and of the images P, is made via a white surface, it is necessary to measure weak intensities. The photographic, photo-electric and the spectral-pyrometric methods, can, therefore, be applied. In our measurements we made use of the last mentioned method.

§ 3. MEASURING OF THE ENERGY OF THE COLOURED IMAGES.

In our measurements we must concern ourselves with the total energy-current in the direction of the width of the images. For the energy per surface-unit in different parts of the image of the back-slit is not constant but diminishes towards the smaller wave-lengths. The radiation receiver, therefore, must collect the radiation of the whole image in its full breadth, and be of a homogeneous sensitivity in that direction. A bolometer, of which the strip is placed perpendicular to the images, meets with this requirement. We made use of a vacuum-bolometer provided with two bolometer-strips. (See description of bolometer in § 7).

Two kinds of disturbances occur in the use of the bolometer, those, namely, due to changeable fields and to sudden extraneous temperature changes. The same disturbances occur with thermo-elements. In this respect the vacuum-bolometer with two strips has a decided advantage over the thermo-element, for both kinds of disturbances can be compensated by its construction and by its electrical connecting system.

The changes in the resistance, caused by the radiation on to one of the strips, is measured in a Wheatstone bridge. The electrical arrangement is shown in fig. 2. The heating-current is supplied by an accumulator A_1, which is regulated by a resistance-controller W_1. The constancy of the current is checked by a compensating arrangement. C_1 and C_2 are constant resistances of about 60 Ohm; they are two arms of the connecting-system of the bridge, while

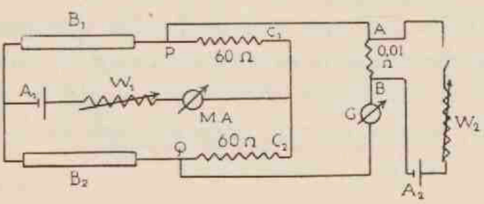

Fig. 2.

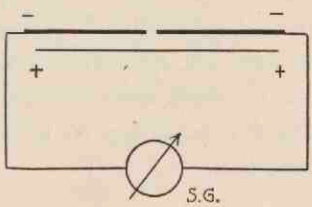

Fig. 3.

the bolometer-strips B_1 and B_2 are the two others. The relay-galvanometer of MOLL and BURGER is used as reading instrument (5).

A rectifier cell of which the sensitive layer is divided into two parts by a scratch, and which is arranged as indicated in fig. 3

acts as relay. The deviation of the primary galvanometer is increased to such an extent that the disturbances due to Brownian movement amount to about 0.2 mm, which corresponds to an E. M. F. in the primary galvanometer circuit of 5×10^{-10} volt.

The changes in the resistance dW arising from the exposure of one of the strips to the radiation causes a difference of potential idW between the points P and Q. This difference is compensated by applying a E. M. F. between A and B. In order to take into account a slow change of zero we proceeded as follows:

The resistance W_2 (precision rheostat) is chosen in such a way that the potential difference between P and Q is almost compensated and is subsequently changed by successive steps until a deflection of the opposite sign is obtained. The deflections are registered while after every 3 or 4 deflections zero is registered. A registrogram is thus obtained which is schematically shown in fig. 4. It will be clear that a slight change in zero can be brought into account by tracing a dotted line through the registered zeros. In order to increase the accuracy of our measurings we have photographed on the registrogram, with the drum at rest, equidistant lines of reference, which indicate the direction of the deflection. When measuring the deflections the measuring-rule is held parallel to the line of reference. Particularly in the case of considerable changes in zero the accuracy of the measuring is greatly increased by the use of these lines of reference, as without them there would be a decided tendency to put the measuring-rule perpendicular to the zero-line.

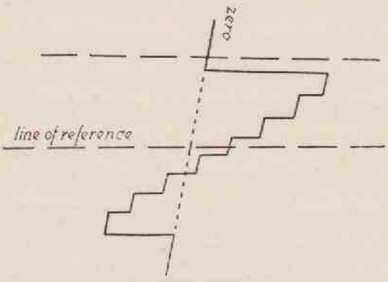

Fig 4.

Accuracy in the measuring.

In order to eliminate the effects of fluctuations in the auxiliary-lamp Q (fig. 1) during our investigation of the properties and the accuracy of our arrangement, we have included a constant and reproducible E. M. F. in the circuit $PABQ$ of fig. 2 (cp. fig. 5).

The resistance is varied in steps in such a way that we obtain a deflection to the right and to the left of zero. The relay is adjusted in such a way that during dead current the secondary galvanometer, too, reads zero. We found that for deflections up to 4 cm to the right and left of zero the reading is linear. Furthermore, we have investigated whether the sensitivity is dependent on the changes of zero. To this end the sensitivity was measured for the case that with the contactors S_1 and S_2 open, the secondary galvanometer too is deflected relatively to its own zero. (This means, therefore, that the image reflected by the mirror of the primary galvanometer falls asymmetrical with relation to the scratch on the rectifier-cell). It then appeared that the sensitivity is not dependent on the asymmetrical position of the image relatively to the scratch, so long as the degree of asymmetry corresponds to deflections of the secondary galvanometer smaller than 1.5 cm [1]).

In order to investigate the accuracy of measuring we made our registrations with a constant resistance of 10500 Ohm at *a* and a

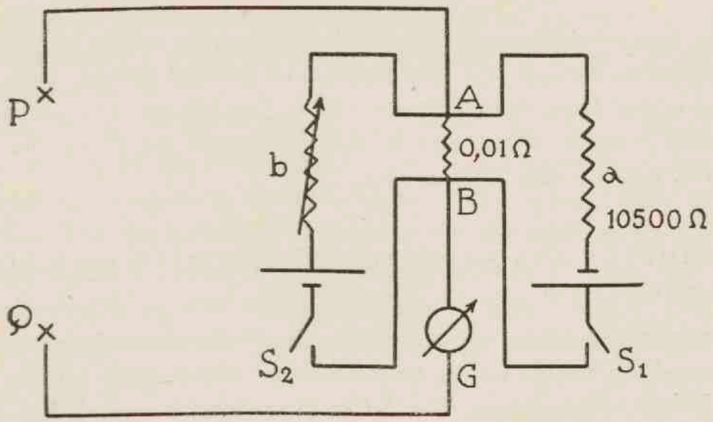

Fig. 5.

resistance in *b*, which starting from 11000 Ohm, was diminished in steps of 100 Ohm down to 10400 Ohm, while after every 3 or 4

1) The amount of this deflection counts only for the arrangement as used by us.

deflections zero was registered. We repeated this registration 7 times. From each of them we obtained the E. M. F. which compensates the voltage supplied by AaB (fig. 5). From these values we calculated the mean error of a single observation $\sqrt{\frac{\Sigma \Delta^2}{n-1}}$ for which we found 0,4 %.

§ 4. Determination of spectral regions.

In order to deduce the energy per Å from the measured energy of an image P, it is necessary to know the spectral region let through, that has contributed to it: more precisely, the transmission curve for the wave-length adjustment of the monochromator associated with that region. We understand by "transmission" for a spectral line with a given adjustment of the monochromator, the ratio between the energy let through with that adjustment, and the energy let through when the adjustment of the monochromator corresponds exactly to the wave-length of the line.

The transmission-curve can be determined by means of a direct experimental method, viz. the method of crossed prisms, or, it can be calculated from the dispersion-curve. For various reasons which we shall mention at the end of this paragraph we chose the first method, using a line light source.

Method of the crossed prisms with line light source.

An enlarged image of the back-slit of the monochromator M is formed by means of an *aa*-microscope object-glass on the slit, of a Hilger spectroscope. The latter instrument is mounted in such a way that the axis of the eye-piece is vertical. By this arrangement the slit of the spectroscope and the image of the back-slit are mutually perpendicular. The images of the back-slit jaws run parallel to the direction of dispersion, in the field of the spectroscope. The latter is provided with a micrometer eye-piece.

Let us now observe what is shown in the spectroscope when the front-slit of the monochromator is illuminated by a line light source. A number of monochromatic images of this front-slit is formed in the plane of the back-slit. We adjusted the monochromator, in such a way, that a monochromatic image of the front-slit coincides exactly with the back-slit. In this case one observes in the spectroscope a small rectangle of which the edges, parallel to the direction of dispersion, are the images of the right and left jaw of the back-slit of the monochromator (See fig. 6 type *b*). The width of the rectangle is determined by the slit-width of the spectroscope. The wave-length adjustment of the monochromator may be such, however, that the back-slit is not quite filled and part of the monochromatic image is cut off by the left jaw. In the spectroscope one sees then a rectangle, type *a*, as in fig. 6, whereas monochromatic images cut off by the right jaw are rectangles of type *c*. When the front-slit of the monochromator is illuminated by a light-source in which many lines lie close together the spectroscope shows all three types described at the same time.

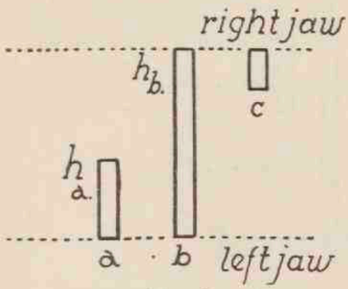

Fig. 6.

From our definition of transmission it will be clear that with a given reading of the wave-length index of the monochromator, the transmission is equal to the ratio between the actual height of the rectangle and the maximum height that can be obtained by moving the wave-length index. When the adjustment is such that the spectroscope shows two small rectangles of type *a* and *b*, (fig. 6), the transmission for the spectral line *a* for that adjustment will be $\frac{h_a}{h_b}$.

Now, the transmission-curve is found from the heights, measured with the eye-piece micrometer of the various monochromatic rectangles arising from the illumination with a line light-source.

The measuring of the heights of the various spectrum-lines is, however, tedious and takes a long time; by the following device it can be considerably shortened. An object micrometer with divisions of 0,1 mm is attached to the slit of the spectroscope.

In the latter instrument one sees then, dark lines corresponding to the scale divisions at right angles to the spectrum-lines (fig. 7).

Adjustment for longer wave-lengths will lengthen the spectrum-

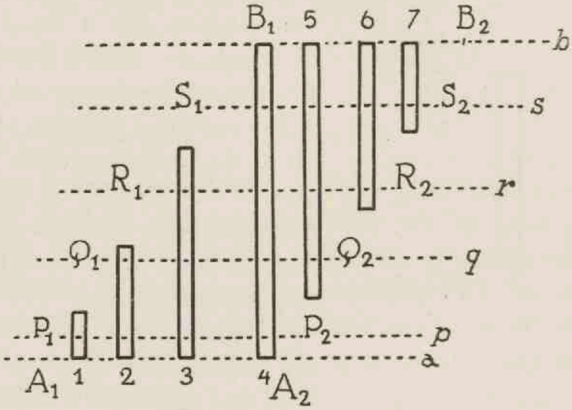

Fig. 7.

lines 5, 6 and 7 and shorten the lines 1, 2, 3 and 4; now, one can adjust to a high degree of precision the wave-length index so that either the upper or the lower edge of a spectrum-line coincides exactly with one of the scale divisions of the object micrometer, or that a line is on the point of appearing at A_1 or of disappearing at B_2, or that the end of a line just touches b in B or begins to shorten at A_2. We can find the connection between the position of the wave-length index and the true wave-length when the various spectrum-lines touch in $A_1P_1Q_1R_1S_1B_1$ and in $A_2P_2Q_2R_2S_2B_2$, taking care to turn the screw in the same direction all the time to avoid backlash. Curves, showing this connection are given in fig. 8. The distances between the divisions p and s and the edges of the images of the back-slit jaws, i.e. the distances *bs* and *ap* in fig. 7 are measured with the eye-piece micrometer.

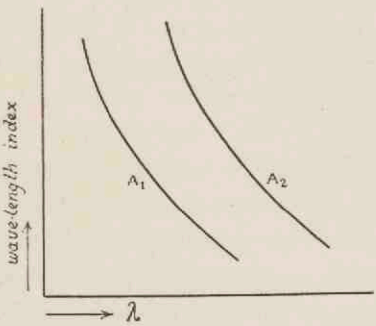

Fig. 8.

For a given reading of the wave-length index the transmission as a function of the wave-length is then found from the curves A_1P_1 etc. of fig. 8, by simply reading from them the value for λ associated with this reading. For, the transmission for that wave-length of the curve Q_1 is proportional to 0.5 mm + ap.

When the image of the front-slit is of the same size as the back-slit, one can represent the transmission D_λ approximately as a function of the wave-length by two sides of a triangle. This would be rigidly true if the second derivate of the dispersion-curve were zero; as it is, one finds for D_λ an asymmetrical figure relatively to λ_0, when λ_0 denotes the wave-length of maximum transmission (see fig. 9*a*). When the size of the front-slit image is not the same as that of the back-slit, the transmission as a function of the wave-length can be represented approximately by three sides of a trapezium (fig. 9*b*).

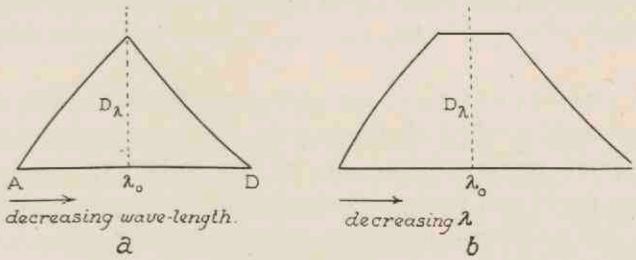

Fig. 9.

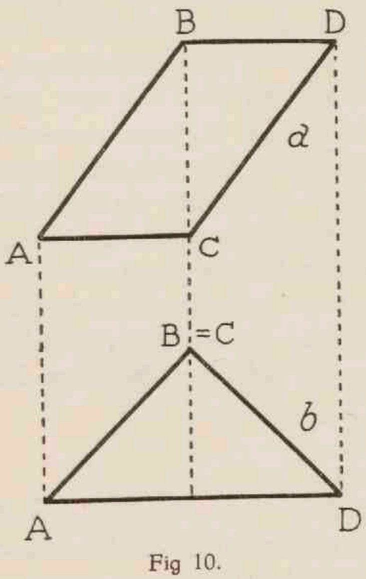

Fig 10.

Method of crossed prisms and continuous light source.

VAN ALPHEN (3) applied the method of crossed prisms for measuring the wave-length region while working with a continuous light-source. In this case the spectroscope shows a parallelogram (fig. 10a). It will be clear that the wave-lengths A, B, C and D in that figure are the same as those denoted by the same letters in fig. 10.

The measuring of the wave-length at the vertices of the parallelogram must be done by means of reading the drumhead of the Hilger-spectroscope; the highest accuracy obtainable by these readings is 1 Å. This means that for a region of 100 Å the precision is at most 1 % to 2 %. One could perhaps obtain a higher precision photographically.

From the dispersion-curve and the slit-width.

A very usual method of determining the transmission-curves for various wave-length adjustments of the instrument is from the dispersion-curve and the known slit-width. This is done as follows:

For a certain wave-length adjustment x_0 of the monochromator, the monochromatic image of the front-slit coincides exactly with the back-slit for the wave-length λ_0. For this wave-length D_{λ_0} is put arbitrarily equal to 1. The points A and D of the transmission-curves (see fig. 9) are formed by reading from the dispersion-curve the wave-lengths at distances b to the right and left of x_0, equal to the slit-width; the image of the front-slit and the back-slit are here supposed to be of the same size. The shape of the transmission-curve is then found by reading the wave-lengths to the right and left of x_0 at various distances y, where D_λ is proportional to $b-y$ $(y \leqq b)$. The precision attainable in the determination of D_λ is in this case dependent on the precision with which the width of the back-slit and the width of the front-slit image can be measured. By means of a comparator it is possible to measure a width of 0,5 mm with an error of 0,02 mm; now, assuming the error in measuring the front-slit image to have the same value, the result is that it is impossible to determine the wave-length region let through with an error smaller than 1 or 2 %.

Comparison of the methods.

From a comparison of the three methods here described it will be apparent that the method of crossed prisms with line light source illumination offers various advantages, namely:

1. The shape of the transmission-curve as a function of the wave-length can be determined wholly by experiment.

2. The wave-lengths of the lines used can be looked up with sufficient accuracy in the international tables.

3. The slit-width of the spectroscope cannot be a source of errors.

4. Index errors of the monochromator play no part. (By index errors we understand, in the case of our double monochromator, the differences between the readings of the wave-length index and the true displacements of the intermediate-slit.)

§ 5. Energy of the images per Å.

For the spectral comparison of the intensities of the lamp to be standardized with the images P (§ 6) it is necessary to know the brightness per Å of the images. To this end the energy of the images is measured and also the transmission as a function of the wave-length at the corresponding index readings. From these measurements the brightness per Å (in an arbitrary measure) is obtained in the following way:

The energy incident on the bolometer-strip is determined by the integral:

$$\int_{\lambda'}^{\lambda''} E_\lambda D_\lambda \, b \, s \, d\lambda$$

here λ' and λ'' denote the limits of the wave-length region let through for a definite adjustment of the monochromator, D_λ the transmission as defined above, E_λ the energy pro unit of surface and pro Å of the image at P, b the width of the image and s the width of the bolometer-strip.

When the size of the images P is independent of the wave-length the indication of the bolometer is proportional to:

$$\int_{\lambda'}^{\lambda''} E_\lambda D_\lambda \, d\lambda.$$

The integral is computed easily as follows:

For a given adjustment the shape of the transmission-curve can be represented with very good approximation by a trapezium (fig. 4) i.e. expressed mathematically:

$D_\lambda = 1 + \frac{x}{a}$ for the wave-length region $\lambda_2 - \lambda_1$, where $x = \lambda - \lambda_2$ and $a = \lambda_2 - \lambda_1$

$D_\lambda = 1$ from λ_2 to λ_3

$D_\lambda = 1 - \frac{y}{b}$ from λ_3 to λ_4 , where $y = \lambda - \lambda_3$ and $b = \lambda_4 - \lambda_3$.

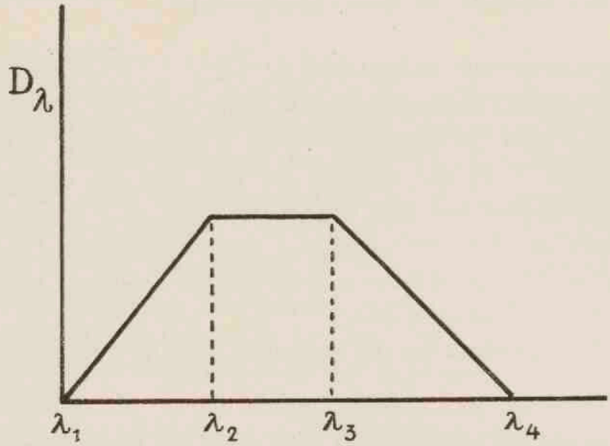

Fig. 11.

In the case of continuous light source illumination the energy of the image E_b is given by

$$E_b = \alpha \int_{-a}^{0} E_\lambda \left(1 + \frac{x}{a}\right) dx + \alpha \int_{\lambda_2}^{\lambda_3} E_\lambda \, d\lambda + \alpha \int_{0}^{b} E_\lambda \left(1 - \frac{y}{b}\right) dy \quad . \quad (1)$$

where the reducing factor α is supposed to be independent of the wave-length over the region λ_4—λ_1.

If we have equi-energy distribution in our continuous light-source, we can write:

$$E_b = \alpha E_\lambda \left(\tfrac{1}{2} a + \lambda_3 - \lambda_2 + \tfrac{1}{2} b\right).$$

In this case the energy per Å in the images is equal to the measured energy divided by the wave-length region $\frac{1}{2}(a + b) + \lambda_3 - \lambda_2$. This is what we shall call in future the "effective" wave-length region. For our energy measuring we used a continuous light-source for illuminating the front-slit, namely, a tungsten band lamp of which the distribution of energy can be described by a colour temperature of 2800° K.

On developing E_λ we can write for the energy in the image:

$$E_b = \alpha \left[\int_{-a}^{0} E_{\lambda_2} (1 + e^1_{\lambda_2} x + e^2_{\lambda_2} x^2) \left(1 + \frac{x}{a} \right) dx + \right.$$

$$+ \int_{0}^{b} E_{\lambda_3} (1 + e^1_{\lambda_3} y + e^2_{\lambda_3} y^2) \left(1 - \frac{y}{b} \right) dy +$$

$$\left. + \int_{0}^{\lambda_3 - \lambda_2} E_{\lambda_2} (1 + e^1_{\lambda_2} x + e^2_{\lambda_2} x^2)\, dx \right].$$

where:

$$e^1_\lambda = \frac{1}{E_\lambda} \frac{\partial E_\lambda}{\partial \lambda} = -\frac{5}{\lambda} + \frac{c_2}{\lambda^2 T}.$$

$$e^2_\lambda = \frac{1}{2 E_\lambda} \frac{\partial^2 E_\lambda}{\partial \lambda^2} = \frac{15}{\lambda^2} - \frac{6 c_2}{\lambda^3 T} + \frac{c_2}{2 \lambda^4 T^2}.$$

$$E_b = \alpha \left\{ \left[E_{\lambda_2} a \left(\frac{1}{2} - \frac{e^1_{\lambda_2}}{6} a + \frac{e^2_{\lambda_2}}{12} a^2 \right) + E_{\lambda_3} b \left(\frac{1}{2} + \frac{e^1_{\lambda_3}}{6} b + \frac{e^2_{\lambda_3}}{12} b^2 \right) + E_{\lambda_2} (\lambda_3 - \lambda_2) \left\{ 1 + \frac{e^1_{\lambda_2}}{2} (\lambda_3 - \lambda_2) + \frac{e^2_{\lambda_2}}{3} (\lambda_3 - \lambda_2)^2 \right\} \right] \right\}. \quad (2)$$

For small wave-length regions we have approximately:

$$E_b = \alpha E_{\lambda_2} [\tfrac{1}{2} (a + b) + (\lambda_3 - \lambda_2)] \quad . \quad . \quad . \quad . \quad . \quad (3)$$

We have used formula (2) for computing the correction to be applied when the energy per Å is determined as the quotient of the energy measured and the effective wave-length region. For the computation of these corrections the values of the quantities e^1_λ and e^2_λ introduced above, must be known for 2800° K. These values computed with $c_2 = 1.432 \times 10^8$ (Å degrees) are collected in table 1; where the wave-lenght is given in Å, the temperature in degrees Kelvin.

TABLE 1.

Wave-length	Colour temperature 2800° K	
	e_λ^1	e_λ^2
4000 Å	20.5×10^{-4}	1.27×10^{-6}
4500 „	14.2×10^{-4}	0.56×10^{-6}
5000 „	10.5×10^{-4}	0.23×10^{-6}
5500 „	7.8×10^{-4}	0.08×10^{-6}
6000 „	5.9×10^{-4}	0.007×10^{-6}
6500 „	4.4×10^{-4}	-0.03×10^{-6}
7000 „	3.2×10^{-4}	-0.046×10^{-6}

A few remarks on the use of the double monochromator.

We must mention here a complication, occurring when a double monochromator is used. In this instrument an image of the front-slit is formed on the intermediate slit and again an image of the latter on the back-slit (§ 7). Now the widths of these three slits may be combined either in such a way that the width of the monochromatic images at the back-slit is larger or in such a way that it is smaller than the width of the back-slit itself. We must inquire therefore, how the widths of the various slits must be combined, in order, to make an accurate determination of the brightness per Å of the images possible. Suppose the width of the back-slit to be larger than that of the monochromatic images, the energy, incident on the bolometer-strip is then given by:

$$\int_{\lambda'}^{\lambda''} E_\lambda \, D_\lambda \, b_\lambda \, s \, d\lambda,$$

where b_λ denotes the width of the images. With the aid of the method of crossed prisms (§ 4), b_λ must then be measured as a function of the wave-length, so that in computing the brightness per Å, one can take into account the change of b_λ with the wave-length.

That this complication is not a superfluous refinement of the computation, is proved by the fact that for our double monochromator b_λ appeared to be indeed dependent on the wave-length.

It will be clear, however, that for the sake of accuracy, this complication must be avoided.

It is advisable, therefore, to choose the widths of the three slits in such a way that the width of the back-slit is smaller than the monochromatic images in the plane of the latter.

§ 6. SPECTRAL COMPARISON OF A LAMP TO BE STANDARDIZED WITH THE COLOURED IMAGES.

For the spectral comparison of a lamp to be standardized with the images as explained in § 1 a white surface is essential. Owing to diffuse reflection due to that surface we are limited to weak intensities; the following three methods are, therefore, indicated:

a. the photographic,
b. the spectral pyrometric,
c. the photo-electric.

In the region 4800 Å—6800 Å the intensity comparisons were carried out with the aid of a spectral pyrometer. This involves the visual adjustment on equal brightness, it can, therefore, not be called an objective-method. In visual monochromatic photometry the mean error of one measurement amounts to 1 %; using the photographic method, taking every precaution, it amounts to 1 %. To form an opinion of the precision of photo-electric intensity measurements in the case of weak intensities (with which we are here concerned) we have compared the sensitivity of the photo-electric arrangement as described in the thesis of H. C. HAMAKER (6), with that of the spectral pyrometer. From this comparison it appeared that by the photo-electric method one can attain in our case a precision of 0.1 %.

What induced us to choose the spectral pyrometric method rather than the photographic and the photo-electric methods, was its great simplicity.

If, however, one wishes to either work objectively or to cover a greater spectral region, or to attain a higher precision, the photoelectric method is, without doubt, the most suitable. In carrying out the energy comparison of the lamp to be standardized with the images, the following must be taken into account. The energy per Å at various wave-lengths of the coloured image on the diffuse white surface is proportional to $E_{\lambda Q}\, D_{\lambda M}$ if $E_{\lambda Q}$ denotes the energy per Å of the auxiliary lamp Q in fig 1, and $D_{\lambda M}$ the spectral transmission associated with a given adjustment of the monochromator. Now, when the brightness is measured by means of the spectral pyrometer and the latter is adjusted to the same wave-length λ_0 for which the transmission of the monochromator is a maximum, we have, for equal adjustment:

$$a_{\lambda_0} \int_{\lambda_0 - \triangle\lambda}^{\lambda_0 + \triangle\lambda} E_{\lambda Q}\, D_{\lambda M}\, D_{\lambda S}\, O_\lambda\, d\lambda \,(:)\, \int_{\lambda_0 - \triangle\lambda}^{\lambda_0 + \triangle\lambda} E_{\lambda P}\, D_{\lambda S}\, O_\lambda\, d\lambda$$

where a_{λ_0} denotes the reducing coefficient of the monochromator, $E_{\lambda P}$ the energy per Å of the pyrometerlamp, $D_{\lambda S}$ the transmission of the spectral apparatus of the pyrometer and O_λ the sensitivity-factor of the eye.

For small wave-length regions we have:

$$a_{\lambda_0}\, E_{\lambda_0 Q} \int_{\lambda_0 - \triangle\lambda}^{\lambda_0 + \triangle\lambda} D_{\lambda M}\, D_{\lambda S}\, O_\lambda\, d\lambda \,(:)\, E_{\lambda_0 P} \int_{\lambda_0 - \triangle\lambda}^{\lambda_0 + \triangle\lambda} D_{\lambda S}\, O_\lambda\, d\lambda$$

where $a_{\lambda_0}\, E_{\lambda_0 Q}$ for which we shall write $E_{\lambda_0 B}$ denotes the energy per Å of the coloured image.

With the lamp to be standardized, radiating on to the white surface, we have, on adjusting on equal brightness, the relation:

$$E_{\lambda_0 X} \int_{\lambda_0 - \triangle\lambda}^{\lambda_0 + \triangle\lambda} D_{\lambda S}\, O_\lambda\, d\lambda \,(:)\, E'_{\lambda_0 P} \int_{\lambda_0 - \triangle\lambda}^{\lambda_0 + \triangle\lambda} D_{\lambda S}\, O_\lambda\, d\lambda.$$

$E_{\lambda_0 X}$ denoting the energy per Å of the lamp to be standardized. Now, it follows from the method of standardizing of the spectral-pyrometer that:

$$E'_{\lambda_0 P} = a_{\lambda_0}\, E_{\lambda_0 P}$$

$$E_{\lambda_0 X} \int_{\lambda_0 - \triangle\lambda}^{\lambda_0 + \triangle\lambda} D_{\lambda S}\, O_\lambda\, d\lambda \,(:)\, d_{\lambda_0}\, E_{\lambda_0 B} \int_{\lambda_0 - \triangle\lambda}^{\lambda_0 + \triangle\lambda} D_{\lambda M}\, D_{\lambda S}\, O_\lambda\, d\lambda \quad . \quad . \quad (1)$$

For the computation of $E_{\lambda_0 X}$ from this formula it is necessary to know the transmission of the monochromator $D_{\lambda M}$ as well as the transmission $D_{\lambda S}$ of the spectral apparatus of the pyrometer at the wave-length adjustments concerned. One can, however, avoid the transmission-measurings of the latter instrument by making the front-slit of the monochromator so wide [1]) that the transmission is the same for all wave-lengths in the region let through by the spectral pyrometer. In this case the transmission curve of the monochromator can be represented by three sides of a trapezium, fig. 11, in which $\lambda_2 - \lambda_1$ must at least be equal to the region let through (fig. 12) by the spectral pyrometer. In formula 1 we have then $D_{\lambda M} = 1$ and one finds for the various wave-lengths:

$$E_{\lambda_0 X} \, (:) \, a \lambda_0 \, E_{\lambda_0 B}.$$

When a double monochromator after VAN CITTERT is used we can avail ourselves of the fact that by removing the intermediate slit the emergent light is white, so that here the above-mentioned

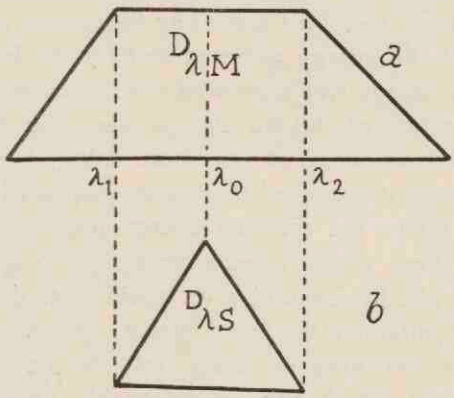

Fig. 12.

condition which the wave-length regions, let through by the monochromator and the spectral apparatus, must satisfy, no longer holds good. Similar considerations apply to the photographic and the photo-electric methods, only, the sensitivity-factor of the eye must

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[1]) A wider back-slit would serve no purpose here, for one must remember that the spectral-pyrometer measures surface brightness and that, therefore, the monochromatic images on the white surface must coincide for the various wave-lengths, for which $D_{\lambda M} = 1$. This occurs, in fact, when the front-slit is wider than the back-slit.

then be replaced by that of the plate and of the photo-cell respectively.

§ 7. DESCRIPTION OF THE INSTRUMENTS.

1. *The light sources.*

Annealed tungsten band lamps with vertical band served as light-sources. The light of the auxiliary lamp Q (fig. 1) should remain constant during the measurings; the light of the lamp to be standardized should be constant and reproducible. The sources actually employed by us were selected as follows. In the first place the behaviour of the current was checked by means of a compensation-connection. From this test it appeared that the electric-current of various lamps was subject to irregular fluctuations. The second test consisted in lightly tapping the bulbs of the lamps in order to see if this had any effect. Those lamps whose electric current showed irregularities and which were at all effected by the tapping were excluded from the third test. The latter was concerned with the constancy of the energy radiated at 6200 Å. In order to find this the radiated energy was measured every quarter of an hour for three hours at a stretch, taking care to keep the strength of the current accurately adjusted at the same value throughout. From these measurements it appeared that in the course of a three hours' running of the lamps the radiated energy had not changed. The mean error of these measurements amounted to 0.5 $^0/_{00}$. Finally we investigated whether switching on again after dead current the radiated energy attained the same value, measuring the energy 15 minutes after switching on. Here, too, the deviations did not exceed the amount which can be reasonably expected from the measuring errors.

2. *The monochromator.*

We used the double monochromator after VAN CITTERT, which consists of two similar monochromators M_1 and M_2 placed symmetrically relatively to the place of the slit S_2 (fig. 13). By moving S_2 in the plane of the image of S_1 one can make the various colours emerge from the instrument. The movable slit S_2 must be considered as the front slit of the second monochromator which forms an image of S_2 in S_3. Owing to scattering and reflection in the first monochromator there will pass through S_2 not only light of the spectral region, determined by the slit width and the dispersion, but also light of different wave-lengths, the so-called stray light. This stray light, however, forms a spectrum in the plane of S_3 and can, therefore not emerge from the back-slit.

The jaws of the intermediate slit are curved; their curvature is the same as that of the monochromatic image of S_1 in green. The image which the second monochromator forms of the curved slit S_2 on the slit S_3 is again straight and rectangular. By this device, a greater light intensity is obtained when a line light source is used, and when a continuous source is used it has this advantage that the wave-length region let through remains the same along the height of the back-slit. We checked this for that part of the slit, which is focussed on the bolometer strip.

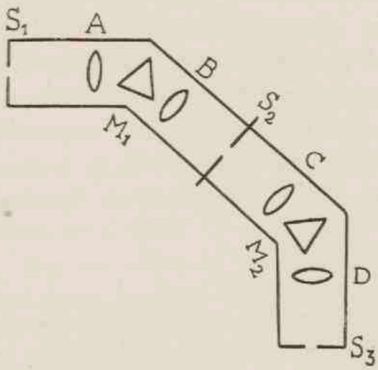

Fig. 13.

The lenses A, B, C and D can be adjusted separately. By means of a telescope focussed on infinity, A and C have been adjusted once and for all at distances equal to their focal lenghts for yellow from the slits S_1 and S_2 respectively. The focussing of S_1 on S_2 and of S_2 on S_3 for the various wave-lengths is done by adjusting the lenses B and D.

To examine the qualities of the monochromator we used the same arrangement as for measuring the wave-length regions. The spectroscope shows the back slit magnified about a hundred times. The adjusting of the monochromator is accomplished as follows:

The front slit is illuminated by a line light source, for instance a *He*-vacuum tube and an image of the back-slit is thrown on to the slit of the spectroscope by means of the microscope object glass; whereupon the intermediate slit is made narrower than the third one. By adjusting lens D, the slit S_2 is focussed on S_3 for the various wave-lengths. From these adjustments it appeared that the lenses of the monochromator were badly corrected both for spherical as well as for chromatic aberration. The wavelength region let through can only be accurately measured according to the method described, when the images in the field of the spectroscope are sharply defined. The VAN CITTERT monochromator has lenses on both sides of the intermediate slit. Since better images of the slits, however, are obtained by removing these lenses, we carried out our measurements without them, but the use of a diafragm remained necessary. The size of the aperture of the diaphragm should be such that sharp images are formed in the spectroscope and that at the same time the light intensity is still sufficient for an accurate energy measurement. A diaphragm of 1.5 cm diameter appeared to meet both requirements. Finally, when an image of S_2 is formed on S_3 then, by adjusting B, an image of S_1 is formed on S_2 after the latter has been opened wide [1]).

Scattered light.

A careful examination of the monochromatic images of the back-slit in the spectroscope will reveal the existence of weak wings as indicated in fig. 14 which must be ascribed to scattered light of the double monochromator. These wings show particularly clearly whenever the wave-length adjustment is such that a monochromatic image of the back-slit is on the point of appearing or of disappearing. Since the energy per Å is found as the quotient of the measured energy and the effective wave-length region, we investigated to what extent our determination of the energy per Å is affected by neglecting the

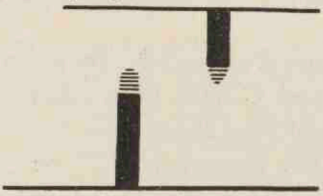

Fig. 14.

[1]) Owing to the chromatic aberration of the lenses, one must focus for each spectral region separately. When high precision is required, one can therefore not make use of the fact, that, on removing the intermediate slit, a white image is formed in P, of which the spectral energy distribution is known. This property of the double monochromator was applied by VAN ALPHEN. It has great advantages when the lamp to be standardized is photographically compared with the images. When, however, the photo-electric or the spectral-pyrometric method is used, there is not the least objection to focussing for each spectral region separately.

wing energy. To that end, we measured the ratio between the brightness of the wing and of the rectangular image as follows: an image of the back-slit of the double monochromator was formed on the filament of a small standardized lamp (of which the intensity as a function of the current strength is known) and of this filament an image was formed on the slit of the HILGER spectroscope. The latter showed then the continuous spectrum of the filament, at right angles to the monochromatic images of the back-slit. The ratio between the brightnesses of the wings and of the rectangles was then measured by us for the red and the yellow *He*-line, taking care to measure the brightness of the wing as closely as possible to the rectangle. In this way we found for the intensity ratio $1 : 10^3$. In the green and blue regions this ratio is more favorable.

For an effective wave-length region of *a* Å, the energy of the radiation incident on the bolometer-strip, when a continuous light source is used for the illumination, is proportional to aE_λ, if the energy contribution of the wing is neglected; E_λ denoting the energy per Å of the auxiliary lamp.

Assuming the wing to cover one tenth of the back-slit width with the same intensity as measured at the edge of the rectangular image in the spectroscope, the energy contribution of the wing will be proportional to $a \times 0.1 \times 0.001\ E_\lambda$. Owing to the wing, therefore, the result of the energy measurement will be 0.1 $^0/_{00}$ too high so that our determination of the energy per Å of the images, will also give a value 0.1 $^0/_{00}$ too high.

3. *The vacuum bolometer.*

After a few preliminary investigations we chose platinum as the metal for the bolometer-strips. The blackening of the strips, when the electrolytic method of LUMMER and KURLBAUM is employed, is largely dependent on the strength of the current applied. We obtained a satisfactory blackening by using a current of 12 milliampères during 4 minutes.

The energy-measurements were carried out with a bolometer, provided with 2 Pt. strips, 1 μ thick and 0,5 mm wide.

The shape of the bulb must be such, that, once the instrument is ready for use, the degree of blackness can be measured as often as may prove advisable. To this end it should be possible, when the incident light is perpendicular to the strips, to measure the reflection coefficient at various angles. This condition led to the construction shown schematically in fig. 15.

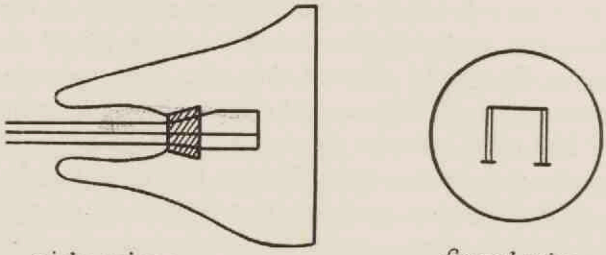

Fig. 15.

Measurement of blackness.

To determine the degree of blackness of the bolometer-strips, we compared the reflected energy with the incident energy by means of the arrangement shown in fig. 16 [1]).

An image of a tungstenbandlamp *B. L.* is formed, by means of

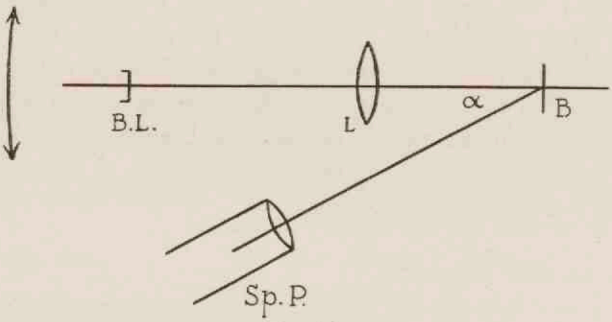

Fig. 16.

lens *L*, on the bolometer-strip *B*, which is placed perpendicularly to the incident beam of light.

The lamp, lens and bolometer can revolve together round the centre of the strip. To measure the incident energy we put in *B* a white surface (*MgO*), of which the intensity $I(\lambda, \alpha)$, expressed

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[1]) Owing to the construction of our bolometer, we could not employ the method of measuring the power of reflection as worked out by HAMAKER. (6).

in an arbitrary measure, was determined at various wave-lengths as a function of the angle. The influence of the glass window of the bolometer was eliminated by placing a glass plate between L and B in such a way, that the light incident on the white surface, passes through the glass but that the intensity measurements are made on reflected light which has not passed through it. Furthermore, we measured the ratio between the brightness $f(\lambda, \alpha)$ of the strip and the white surface as a function of α at various wave-lengths; this time the influence of the glass window was eliminated by placing a glass plate immediately in front of the white surface. The total amount of energy of a given wave-length, reflected by the white surface per unit of area, is (fig. 17):

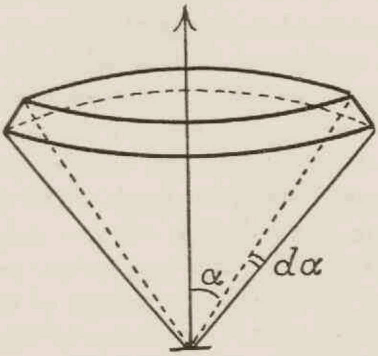

Fig. 17.

$$D_{W.\lambda} = \int_0^{\frac{\pi}{2}} 2\pi I_W(\lambda, \alpha) \sin \alpha \cos \alpha \, d\alpha = \pi \int_0^{\frac{\pi}{2}} I_W(\lambda, \alpha) \sin 2\alpha \, d\alpha.$$

For the bolometer-strip this amount is therefore given by:

$$D_{B.\lambda} = \pi \int_0^{\frac{\pi}{2}} f(\lambda, \alpha) I_W(\lambda, \alpha) \sin 2\alpha \, d\alpha.$$

Assuming that the white surface does not absorb any energy at all the power of reflection of the bolometer-strip is given by:

$$R_\lambda = \frac{D_{B.\lambda}}{D_{W.\lambda}}.$$

From measurements in the wave-length region from 5000 Å to 6800 Å it appeared that $f(\lambda, \alpha)$ does not depend on λ. The construction of the bolometer and the way it was mounted permitted us to carry out measurements over a range of α from 15° to 70°.

To investigate the behaviour of $f(\alpha)$ for α smaller than 15° we have measured its value after placing the bolometer-strip at an angle of 80° to the incident light. Its behaviour proved the same as with perpendicular incidence. For small angles too, $f(\alpha)$ is independent of λ.

As regards $I_W(\lambda, \alpha)$, its values appeared to change at the various wave-lengths in a way similar to α. Column 3 of table 2 shows the behaviour of $I_W(\lambda, \alpha)$; the value of $I_W(\lambda, 10°)$ is put arbitrarily equal to 100.

With the aid of the data given in table 2 we can compute the total power of reflection of the bolometer-strip; the result is 2,51 %. Its power of absorption is therefore 97.49 %. The white surface, we assume here, does not absorb at all; this assumption can influence only the absolute value of the power of reflection of the bolometer-strip. The selectivity, of the power of absorption of MgO in the spectral range considered, is not more than 1 %. This gives for the selectivity of the power of absorption of the bolometer-strip an error of 0.25 ‰. The values given for $f(\alpha)$ and $I_W(\alpha)$ are averages of six measurements, the mean error is smaller than 1 %.

From these measurements we can therefore conclude, with a precision of at least 0.5 ‰ that the power of absorption is independent of the wave-length in the range from 5000 Å to 6800 Å.

The method described above of measuring the reflection of the bolometer-strip can be applied to wave-lengths ranging from 4600 Å to 6800 Å. For smaller wave-lengths one can use the photo-electric method.

TABLE 2.

Angle	$f(\alpha)$	$I_W(\alpha)$
10°	0.0530	100
20	0.0377	100
30	0.0270	99.4
40	0.0225	97.2
50	0.0195	93.2
60	0.0165	86.8
70	0.0145	74.3

Sensitivity.

The sensitivity of the bolometer depends on the temperature of the strips. To determine the sensitivity as a function of the heating-current we used the arrangement shown in fig. 2, reading the deflection of the galvanometer at various current strengths, while one of the strips was exposed to constant radiation. From these measurements (fig. 18) it appears that the sensitivity is a maximum when the current strength is 100 mA. But on investigating the

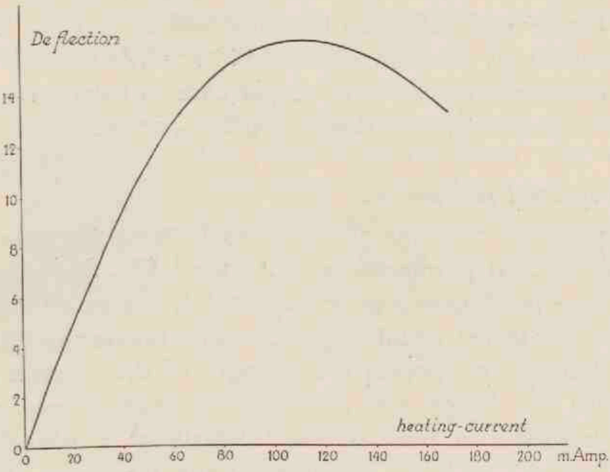

Fig. 18.

behaviour of zero in the arrangement shown in fig. 2 this turned out to be decidedly better when a current of 50 mA was used than one of 100 mA. This must be ascribed to the fact that an accumulator will stand the drain of a current of 50 mA with great constancy for quite a long time, but not the drain of a current of 100 mA. Moreover, the precision of the measurements with a current of 50 mA is higher than with one of 100 mA, notwithstanding the smaller sensitivity. This was our reason for always using a current strength of 50 mA for our energy-measurements.

Linear Dependence.

We checked the proportionality between the galvanometer deflections and the incident energy by varying the amount of energy over a range from 1 to 100. To this end, images were formed on the bolometer-strip, of a white surface of which the brightness was altered in known ratios by variation of distance. The precision of each measurement in this experiment did not exceed 0.5 %. The deviations from direct proportionality are, therefore, of the same order as those which may be expected from measuring errors. Higher precision might be obtained by varying the distance of the white surface more carefully. In the meantime it has appeared from a mathematical treatment (not yet published) by WOUDA of the type vacuum bolometer mentioned, that the linear connection between incident energy and deflection is guaranteed up to 1 ‰ so long as the incident energy does not exceed 5×10^{-4} Watt/cm^2. In carrying out our measurements we took proper care to keep the deviations from direct proportionality down to 0.2 ‰ at the most.

4. *The spectral pyrometer.*

The pyrometer principle of HOLBORN-KURLBAUM is the basis on which the spectral pyrometer is constructed (7). An image of the object, whose brightness is to be measured, is thrown on the filament of a pyrometer lamp; equal brightness of the image and the filament is then obtained by varying the strength of the current. In order to compare the image and the filament monochromatically the filament is placed in front of the front-slit of a monochromator. Looking into the back slit the filament and the image are seen in monochromatic light. The apparatus is shown schematically in

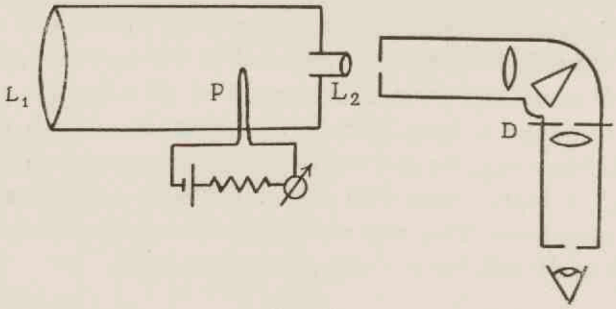

Fig. 19.

fig. 19. As spectral apparatus we used a Hilger spectroscope, provided with two slits. The slit-height was chosen in such a way that the troublesome diffraction and reflection phenomena which

occur just whenever the brightness of the filament is almost equal to that of the background and which, therefore considerably interfere with the precision of the measurements are entirely absent (8, 9).

Scattered light.

To investigate the influence of scattered light we measured the brightness of a white surface of constant brightness, of which we varied the extent of the reflecting area by means of a diaphragm. From these measurements we found between the results for a small and a large area, differences of 1 % to 2 %, which must be ascribed to scattering. The amount of scattered light on the spot where the image of the pyrometer filament is formed will be nearly proportional to the size of the object. It is, therefore, very necessary when using a spectral pyrometer to employ for brightness measurements objects of the same size as for standardizing. A second effect of scattering consists in the fact that the light emerging from the back-slit contains also wave-lengths differing from those determined by slit-width and dispersion; this is a source of errors whenever the spectral distribution of the pyrometer lamp and of the object, to be tested by the pyrometer, differ largely. The amount of this stray light is nearly proportional to the illuminated area of the back lens of the spectroscope. To eliminate its influence we placed a rectangular diaphragm *D*, of the same size as the image of the white surface *W* in fig. 1.

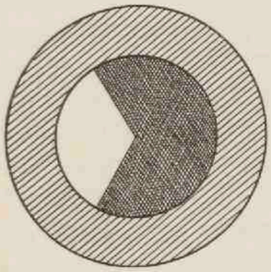

Fig. 20.

Standardizing.

In order to measure the power of reflection of the bolometer-strip the standardizing was accomplished with the aid of a band lamp of which at various wave-lengths the energy is known as a function of the current strength. The bandlamp radiated on to a white surface of which the brightness was altered in known ratios by varying the current strength. We measured the strength of the electric current of the pyrometer lamp, which made the filament disappear at known brightness of the white surface. This method of standardizing is, however, too inaccurate for the comparison of the spectral energy of the band lamp to be standardized, with the images of which the energy is known; we proceeded, therefore, as follows:

A sector diaphragm was placed in front of lens L_3 with its centre on the optical axis of the lens. This contrivance for reducing the light has the advantage, that the reduction is the same for every wave-length and that, moreover, the amount of the reduction can be very precisely measured in the region of the longer wave-lengths. The error in the reduction over a range from 80 % to 10 % is smaller than 0.5 %. To eliminate errors which might possibly arise from differences between the optical behaviour of the various sectors of the lens, the diaphragm was made to revolve rapidly.

The spectral pyrometer was standardized with the aid of sector diaphragms; as light source we used a tungsten band lamp of which the light corresponds to a colour temperature of 2800° K and which was placed in P (fig. 1). The measurements were carried out for various wave-lengths lying between 6800 Å and 4800 Å and with intensities varying from 1 to 10. In this way a set of curves with the wave-length as parameter was found which represent the intensity as a function of the pyrometer current. The largest deviation of the points actually observed from the smooth curve drawn through them is 1 %. Each measurement of brightness is the mean of 3 observations. We measured the strength of the pyrometer current in compensation connection.

Influence of slit-width and effective wave-length.

When the brightness of the filament of the pyrometer lamp is adjusted so as to be equal to that of the background we get the equation:

$$\frac{\int_{\lambda_0 - a}^{\lambda_0 + a} E_{L\lambda}\, O_\lambda\, D_\lambda\, d\lambda}{\int_{\lambda_0 - a}^{\lambda_0 + a} E_{P\lambda}\, O_\lambda\, D_\lambda\, d\lambda} = 1$$

where:

$E_{L\lambda}$ denotes the brightness of the background,
$E_{P\lambda}$ „ „ „ „ „ filament,
O_λ „ „ sensitivity factor of the eye,
D_λ „ „ transmission of the spectral pyrometer,
λ_0 „ „ central wave-length of the region let through,
$2a$ „ „ wave-length region let through.[1])

When $E_{L\lambda}$ changes in a similar way as $E_{P\lambda}$ with the wave-length in the region from $(\lambda_0 - a)$ to $(\lambda_0 + a)$, one can infer from this equation that $E_{L\lambda_0} = E_{P\lambda_0}$. When, however, the energy distribution $E_{L\lambda}$ differs from $E_{P\lambda}$ over the range $2a$ one might compute the correction to be applied to the equation $E_{L\lambda_0} = E_{P\lambda_0}$. Usually, however, one computes in the case of pyrometric measurements the so-called effective wave-length, λ_e. The simplest way of defining the latter is by the mathematical equation:

$$\frac{\int_{\lambda_0 - a}^{\lambda_0 + a} E_{L\lambda}\, O_\lambda\, D_\lambda\, d\lambda}{\int_{\lambda_0 - a}^{\lambda_0 + a} E_{P\lambda}\, O_\lambda\, D_\lambda\, d\lambda} = \frac{E_{L\lambda_e}}{E_{P\lambda_e}} = 1. \quad \ldots \ldots \quad (1)$$

When the energy distributions of the object to be investigated by the pyrometer, and of the filament, can be described with the temperatures T_L and T_P one can write for (1), as an elementary but rather lengthy computation would show:

$$\lambda_e - \lambda_0 = \frac{a^2}{6} \lambda_0^2 \frac{e^2_{\lambda L} - e^2_{\lambda P} + o'_\lambda (e'_{\lambda L} - e'_{\lambda P})}{c_2 \left(\frac{1}{T_L} - \frac{1}{T_P} \right)}$$

where the e's are the coefficients in the development of:

$$E_{L\lambda} = E_{L\lambda_0} (1 + e'_{\lambda L} x + e^2_{\lambda L} x^2).$$
$$E_{P\lambda} = E_{P\lambda_0} (1 + e'_{\lambda P} x + e^2_{\lambda P} x^2).$$
$$O_\lambda = O_{\lambda_0} (1 + o'_\lambda x + o^2_\lambda x^2).$$

---

[1]) For those wave-length regions under consideration (max. 100 Å) the transmission as function of the wave-length may be taken to be represented by the two sides of an isosceles triangle.

We computed $\lambda_e - \lambda_0$ for those cases in which, when adjusted for equal brightness, the object to be measured and the filament had the colour temperatures 2800° K and 1200° K, 2800° K and 1400° K, 2000° K and 1200° K and 2000° K and 1400° K respectively, while the wave-length region let through was 200 Å. With the aid of the table given below we can then easily compute $\lambda_e - \lambda_0$ for the various other cases.

TABLE 3.

λ_0	$\lambda_e - \lambda_0$			
	2800° K-1200° K	2800° K-1400° K	2000° K-1200° K	2000° K-1400° K
6800 Å	−10.2 Å	10.4 Å	− 9.8 Å	− 9.9 Å
6600	− 7.3	− 7.5		
6400	− 4.6	− 5.2	− 4.4	− 4.6
6200	− 3	− 3.2		
6000	− 1.2	− 1.5	− 0.6	− 0.9
5800	+ 2	+ 3		
5600	+ 2.4	+ 1.9	+ 2.9	+ 2.4
5400	+ 4.3	+ 4.3		
5200	+ 7.4	+ 7.4	+ 8.2	+ 7.4
5000	+10.2	+10.2		
4800	+11.5	+11.5	+11.5	+11.5

§ 8. The precision of the relative measurements.

We shall now discuss, in a short survey, the various factors on which the precision of a relative energy measurement depends. Errors in the results of such measurements of a tungsten band lamp arise from two sources:

1. Errors in the energy per Å of the coloured images.
2. Errors in the comparison of the intensities of the lamp to be standardized with those of the images.

1. The energy per Å is computed from the measured transmitted energy and the wave-length region let through (See § 5).

Now, systematical errors can arise in energy measurements from a variety of causes, as, for instance, because the bolometer reading is not proportional to the incident energy. The energies actually measured lie within an intensity range for which the maximum deviation from direct proportionality amounts to 0.2 $^0/_{00}$. The energy of the auxiliary lamp Q of fig. 1 is sufficient to enable one to measure the energy of the images in the region from 6800 Å to 4800 Å with a mean error in a single observation of 0.5 $^0/_{00}$. In this connection we may remark that the relative energy measuring is reduced to the measuring of an electromotive force supplied by a voltage source and a precision rheostat (see fig. 2) [1]).

In addition to the error just mentioned, of 0.5 $^0/_{00}$, a systematic error, for which a safe estimate is 0.1 $^0/_{00}$ can arise from errors in the resistances of the rheostat.

To form an opinion concerning the precision of measuring for smaller wavelengths, we registered the deflections of the relay galvanometer while the monochromator was adjusted on 4200 Å and the effective wave-length region was 100 Å. This deflection was 3 cm with a mean error of one single observation of 5 $^0/_{00}$. The selectivity of the absorption of the bolometer is determined (see § 7 sub. 3) in the region from 6800 Å to 4800 Å with a precisio of 0.5 $^0/_{00}$. By the photo-electric method this determination can be extended to 4000 Å with the same precision.

The measuring of the transmission curve belonging to a given monochromator adjustment was accomplished to the method explained in § 4. The precision is influenced by the occurrence of a weak wing on the monochromatic rectangular images in the spectroscope (§ 7 sub. 2). To investigate whether the wing gives rise to systematic errors in the adjustment, the transmission curves belonging to various wave-length adjustments were measured by two observers. It appeared then that systematic errors did not occur; the highest difference between the measurements of the

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[1]) As a source for the E. M. F. we used an accumulator, checked as to its constancy by means of a normal element. All precision rheostats are manufactured by O. WOLFF.

two observers in the effective wave-length region (200 Å) amounted to 2 ‰. The amount found for the energy when influenced by the wing is at the most 0.1 ‰ too high (see § 7 sub. 3).

The resulting error in the determination of the energy per Å of the images is, therefore, composed of the following factors:

a.	energy measurement	0.5 ‰.
b.	effective wave-length region	1 ‰.
c.	scattered light	0.1 ‰.
d.	selectivity of the bolometer	0.5 ‰.
e.	direct proportionality of the bolometer	0.2 ‰.
f.	precision rheostats	0.1 ‰.

The errors under *a.* and *b.* are of a fortuitous character; it is, therefore, possible to attain a higher precision by increasing the number of observations. The errors sub *c. d. e.* and *f.* are systematic; their combined influence is smaller than 1 ‰.

2. The spectral intensity comparison of the lamp to be standardized with the image of which the energy is known was carried out with a spectral pyrometer in the region from 6800 Å to 4800 Å. The precision of the mean of three measurements amounts in this case to 5 ‰. As described in § 6 the photo-electric method can be applied in the region from 6800 Å to 4000 Å; the precision is then 1 ‰. For the intensity comparison we have thus an objective and sufficiently precise method at our disposal.

When comparing the energy of the lamp to be standardized with the images for various wave-lengths, the degree of weakening suffered by the radiation of the images and by that of the lamp to be standardized, on the way from P to the receiver behind the spectral apparatus, must be the same. The contrivance of the white surface eliminates the influence of difference in the filling of the spectral apparatus and that of the degree of polarisation of the light from the images P (see § 1). The tungsten band lamp to be standardized is put in its right place by the aid of a telescope directed slantingly on to the light path (see § 2). The error in the position of the band is at most 0.05 mm, and this influences the brightness of the white surface. A computation showed us that an error of 0.05 mm in the position of the band gives rise to an error in the brightness at the centre of the image on the white plane, amounting to 0.5 $^0/_{00}$; its influence on the changes in the brightnesses as function of the wave-length is smaller.

We have, we believe, shown by the above that a relative energy measurement of the tungsten band lamp with a precision of 1 $^0/_{00}$ is possible. For a detailed discussion of the measuring results we refer the reader to the next chapter, where we give the measurement of the spectral energy distribution of a black body at the melting temperature of gold.

CHAPTER II.

The measuring of Planck's constant c_2.

§ 1. A FEW REMARKS ON THE VARIOUS DETERMINATIONS OF c_2.

The investigation, carried out with a view to testing the law of radiation and to determining the constants, occurring in that law, are very numerous. On a close examination of the work of many investigators in this field, one can not help doubting, whether the values, actually obtained, are entirely free from systematic errors. Indeed, considering the variety of such errors to which this kind of work is liable, the agreement between the values, found by the various writers, is a matter of some surprise!

In table 4 we give a list of values of c_2 together with the names of those, who obtained them [1]). From a comparison of these values with the one, adopted by the 7th „Conférence générale des Poids et Mesures" ($c_2 = 1.432$ cm degree) the impression will most probably result that the choice of the latter value is more or less an arbitrary one.

The investigators, mentioned in table 4, obtained their values for c_2 from intensity measurements on the radiation of a black body, of which the temperature was determined by means of the gas thermometer. c_2 was then computed, either from the energy ratio of the radiation, for one and the same wave-length at two different temperatures, (the isochromatic method) or the wave-length λ_m, for which, at one and the same temperature, the radiated energy was a maximum, was computed from the change of the energy per Å (the isothermal method) according to the first method c_2 is given by the formula:

$$c_2 = \lambda \frac{T_2 . T_1}{T_2 - T_1} \log \frac{E_2}{E_1};$$

the second method gives for c_2 the expression:

$$c_2 = 4{,}9651\ \lambda_m\ T.$$

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[1]) Reprinted from an article by COBLENTZ. (10).

TABLE 4. Observed value and the probable value of the constant c_2.

Observer	Date	$\lambda_m I$ observed	$\lambda_m T$ corrected	c_2	c_2, probable value in micron deg.
PASCHEN	1899	2891	2891		14360
		2907	2907		
LUMMER and	1900	2921	2894		
PRINGSHEIM	1900	2879	2879	14290	
		2876	2876		
		2940	2882	14310	14300
WARBURG and	1911			14200—14600	
COLLABORATORS	1912			14300—14400	
	1912			14360	
	1913			14370	
	1915			14250	
	1915			14300—14400	14300
COBLENTZ	1913	2911		14456	
	1916	2894		14369	
	1920			14311—14318	14318
Value adopted by the 7th. „Conférence générale des Poids et Mesures"					14320

We confine ourselves here to the simple statement of the principles, underlying the methods which were used, it is not our aim to criticise the values, so obtained, or to discuss once more, the many sources of error, which, doubtlessly, have influenced the results of these investigations. For such a discussion we refer the reader to the original publications, to the handbooks and to a very comprehensive article by Coblentz (10).

The new value of c_2, computed by us, is obtained from the measured energy-distribution in the spectral region from 0.68 μ to 0.50 μ, of the radiation of a black body at the melting temperature of gold. For the quantitative description of this energy distribution, we used, instead of PLANCK's law, WIEN's well-known formula:

$$E_{\lambda T} = \frac{c_1}{\lambda^5 e^{\frac{c_2}{\lambda T}}}.$$

That WIEN's expression yields in our case a very good approximation, is clear from the following: denoting the energy, at the wave-length λ, radiated according to PLANCK's- and to WIEN's formula by E_p and E_W, respectively, their ratio is given by:

$$\frac{E_p}{E_W} = \frac{e^{\frac{c_2}{\lambda T}}}{e^{\frac{c_2}{\lambda T}} - 1},$$

which depends solely on λT.

Putting $\lambda T = 1400$, we find $\frac{E_p}{E_W} = 1{,}00004$. Now in our actual measurements, we have $\lambda T < 1400$, which means that, in our case, $\frac{E_p}{E_W}$ is even smaller still.

We can re-write WIEN's formula in the form:

$$log\,(E_{\lambda T}\lambda^5) = log\,c_1 - \frac{c_2}{\lambda T},$$

plotting, therefore, the lefthand member against $\frac{1}{\lambda}$, we obtain straight lines which, by their inclination, give a value for $\frac{c_2}{T}$; since, moreover, the melting point of gold is known from measurements with the gasthermometer, we find in this way the value of c_2.

However, a new determination of c_2 will only be of any use if the method used, enables one to reach a higher accuracy than the methods hitherto used.

Now, in order to form an opinion of the accuracy attainable, let us compute c_2 from the ratio between the energies, radiated at λ_1 and λ_2, which is given by:

$$V = \frac{E_{\lambda_1}}{E_{\lambda_2}} = \left(\frac{\lambda_2}{\lambda_1}\right)^5 e^{\frac{c_2}{T}\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)}.$$

From this formula it follows that

$$d\,c_2 = \frac{d\,V}{V} \cdot \frac{T}{\frac{1}{\lambda_2} - \frac{1}{\lambda_1}}.$$

Putting $\lambda_2 = 5000$ Å and $\lambda_1 = 6800$ Å, we get:

$$d\,c_2 = \frac{d\,V}{V} \,.\, 3{,}5 \times 10^8 \text{ (Å, degree)}.$$

This means, that an error of 1 % in the determination of the energy ratio, will cause an error of 0,003 (Å, degrees) in the value for c_2.

It will be clear from these considerations, that thanks to the method of measuring energy ratios, explained in Chapter I, we are indeed in a position to determine the value of c_2 to a very high degree of precision.

§ 2. THE METHOD OF MEASURING.

The energy distribution in the radiation of a black body at the melting point of gold was measured with the aid of a primarily standardized light source, viz. the images in P (fig. 21). Now, the obvious way to proceed in measuring the black body radiation in

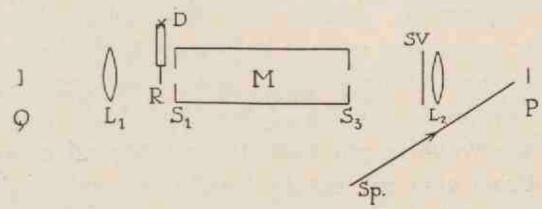

Fig. 21.

question would seem to place the body in P, as was done also in standardizing the tungsten band lamp, and then to carry out the comparison of the energy distribution by means of a spectral apparatus Sp (fig. 21). In the present case, however, this procedure would lead us into unsurmountable difficulties. Indeed, the brightnesses of the light sources to be compared, are now so widely different (in yellow e.q. they differ by a factor of about 1000), that the brightness of the images in P can no longer be put in line with the brightness of the black body at the melting-point of gold, by simply weakening sufficiently the strength of the current in the auxiliary lamp Q (fig. 21), for in this way it would no longer be possible to measure the energy of the images at P with the required precision.

We succeeded, however, in reducing the brightness of the images in P to a sufficient extent, to make it comparable with the brightness of the blackbody at the melting point of gold, and we were at the same time able accurately to take into account the change of the spectral energy distribution in the images, due to the method of reduction, by the following contrivances. In the first place, a diffusely reflecting white surface, with a known reflective power was placed in P. The images on this surface serve as a primarily standardized light source; in the red region of the spectrum the brightness of the images is now about the same as that of the black body. It was, in the second place necessary as a further contrivance, to use in the green region a smoked glass reducer, with a transmission power of about 20 %, which was applied in front of the monochromator. The brightness of the images on the white surface was measured for various wave-lengths, with the aid of a spectral pyrometer by altering the brightness in known ratios by means of standardized sector reducers. In this way the connection between the pyrometer current and the relative brightness was obtained. The pyrometer current belonging to the brightness of the black body at the melting point of gold, was then also measured, so that the relative brightness of the black body was found as a function of the wave-length from the established connection, just mentioned. From this we obtain finally the value of c_2, in the manner, described in § 1.

Apart from serving as a reducer, the white surface is an essential feature in our arrangement in another respect as well; for its diffuse reflection and depolarising action bring about, that the light, radiated on to the spectral apparatus either by the images or by the black body is ordinary light, which, moreover, travels, in both cases, along identical paths through the apparatus (c.f. Chapter I § 1).

The spectral pyrometer is mounted vertically so that the image on the white surface and the wire are observed in the instrument as is indicated in fig. 22*a*, whereas they show as in fig. 22*b*, when the pyrometer is in a horizontal position[1]).

a b
Fig. 22.

It follows from the above, that apart from the measuring of the spectral energy distribution in the images in *P*, it is necessary to carry out the following measurings as well:

1. of the pyrometer current, on adjusting for a brightness, equal to that of the black body at the melting temperature of gold.
2. of the reflective power of the white surface, and of the transmission curve of the smoked glass.
3. of the transmission of the sector reducers.

The method of determining the spectral energy distribution in the images was already explained in detail in Chapter I, as was also the spectral pyrometer. We shall, therefore, now proceed to give particulars of the black body used by us, and the complete series of measurements, which have served to obtain the value for c_2.

§ 3. THE BLACK BODY.

Our black body consisted of nickel and was of the LUMMER and KURLBAUM type. It was heated in an electric Heraeus furnace. (length 30 cm, aperture 6,5 cm).

In order to obtain an even temperature throughout the furnace, the heating platinum band was wound closer together in the centre than at the ends. The way, in which the black body is mounted in the furnace is shown schematically in fig. 23.

For the determination of the temperature of the black body, at the melting point of gold we used the so-called „wire method", according to this method a thermo-element (*Pt, Pt-Rh*), fitted with

1) Since, for the sake of accuracy, the pyrometer wire and the slit of the spectral instrument must be at right angles to each other; a horizontal position of the spectral pyrometer (i.e. slits of the monochromator vertical and pyrometer wire vertical) is in our case excluded. Of these two positions the former allows of the better adjustment.

a gold[1]) wire between the *Pt.* and *Pt-Rh* wires (see *Au* in fig 23), is inserted in the interior of the black body. There is still another thermoelement, inserted in it, which, however, is not shown in the figure.

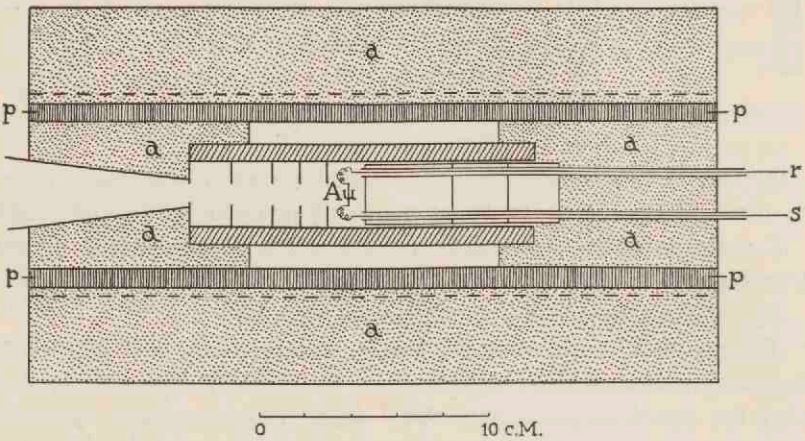

Fig. 23.

With the aid of an electrical connecting-system, as shown in fig. 24, the E. M. F., of the two elements are measured, with the temperature slowly increasing, as a function of the time. One notices the moment, at which the temperature in the furnace has

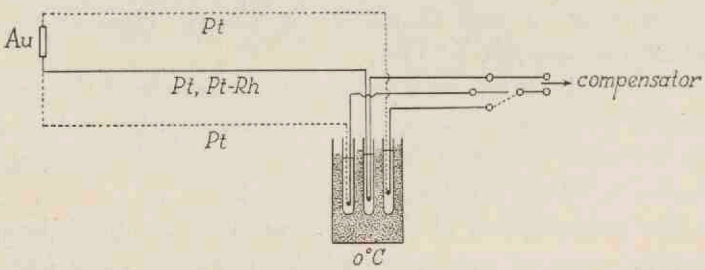

Fig. 24.

reached the melting point of gold, by the fact that, from then onwards, the E. M. F. of the gold-wire thermoelement remains

[1]) The gold, used in our experiments, was supplied by the Royal Dutch Mint, at Utrecht. Our sincere thanks are due to the mint-master Dr. V. HETEREN for his courteous help in this matter.

constant for a few minutes, where upon the circuit is suddenly interrupted by the melting of that wire [1]) whereas in the meantime the E. M. F. of the second element continues to increase slowly (see fig. 25).

This element, once standardized by the above procedure, for

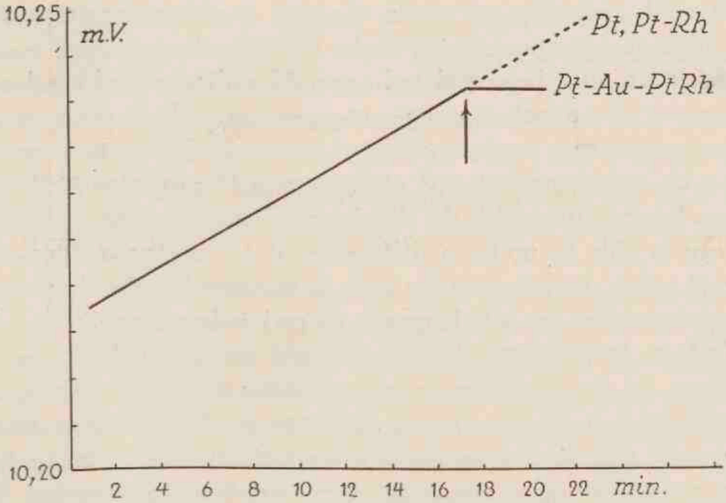

Fig. 25.

the melting temperature of gold, serves for heating the black body to that temperature again later on.

The temperature and the temperature-differences in the black body.

In order to make the actual radiation comply with the definition of black radiation, the radiating body must satisfy two conditions, viz., its coefficient of absorption must be equal to 1, and the walls of the enclosure must possess the same temperature throughout. As regards the first condition, the observation may suffice here, that part of the radiation, incident in the enclosure, may eventually find its way out of it, in which case the absorption coefficient would

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1) The actual interruption of the circuit takes place at a higher temperature of the furnace. The latter is at the melting point of gold, at the moment. the constancy of the E. M. F. sets in; this means that the temperature of the black-body, yielded by the method of melting a gold wire is too low.

differ from 1. As for the second condition, there are two causes, from which an uneven distribution of the temperature over the walls can arise. There is, first, a wrong way of heating, and, consequently a temperature gradient in the length direction of the black body, and there is, secondly, the fact, that the part of the black wall, radiating outwards through the diaphragms, will be at a lower temperature, owing to the fact that the radiation, emitted by that part, must be completed again by radiation and conduction from parts of the wall at a higher temperature. To form an opinion of the temperature gradient along the black body, we may remark, that with a heating velocity of 0,1° to 0,2° pro minute, the eye can no longer distinguish the diaphragms from the rear wall. Now, the eye is able to notice a difference of 1°, at the melting-point of gold; it is, therefore, quite safe to assume that the drop in the temperature over the whole length (6 cm) is less than 1°.

Various investigators have computed an upper limit for the deviation of the radiation in an enclosure from the theoretically defined black radiation, so e.g. FLEURY in his "Etalons Photométriques" and RIBAUD in his "Pyrométrie Optique". On applying these considerations to our case, it follows that the deviation from the theoretically black radiation in the visible region of the spectrum, amounts at the most to 0,1 %. The deviations in our spectral energy distribution, are therefore, certainly less than 0.1 %.

§ 4. THE MEASURINGS CONCERNING THE WHITE SURFACE, THE SMOKED GLASS REDUCER, AND THE SECTOR REDUCER.

The white surface.

As explained already in § 2, we must know the spectral energy distribution in the images in P. Now, in our experiments, this surface (magnesium oxide) is placed in P at right angles to the light path, and the brightness of the images is measured by means of a spectral pyrometer, of which the optical axis makes an angle of about 20°, with the light path behind the monochromator. It is therefore, necessary to measure the selectivity of the reflective power of the surface under the same conditions i.e. for light, incident at right angles and reflected at an angle of 20°. This was done in two steps. First, the reflective power was measured, according to the method of HAMAKER (6) as a function of the wave-length, in the case of perpendicular incidence. This gives the ratio between the total amount of the reflected light and the incident light.

Secondly, we measured, for various wave-lengths the brightness of the white surface as a function of the angle of reflection, from which it appeared that, in the visible part of the spectrum at least the connection between the brightness and the angle of reflection, was not influenced by the wave-length [1]).

These twofold measurements teach us, that, with perpendicular incidence, the reflective power, in the case of an angle of reflection of 20°, is proportional to the numbers measured, according to the method of HAMAKER.

In § 2, we mentioned already the essential part, played by our white surface owing to its depolarising action. Now, in our arrangement, the vibrations of the polarised light, which has travelled through the double monochromator, are at right angles to the vibrations of the light, which has passed through the monochromator of the spectral pyrometer. One may expect, therefore, that, if the degree of polarisation of the former vibrations should depend on the wave-length, a non complete depolarisation by reflection on the white surface, would be a source of errors. In order to make this out, we measured, with the aid of the arrangement, shown in fig. 26, the brightness, for various positions of the nicol N, of the image of a tungsten band lamp Q, on the white surface $W.vl$.

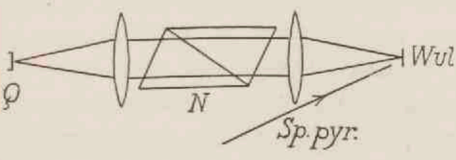

Fig. 26.

The brightness appeared, for the various wave-lengths, to be independent of the position of the nicol, from which we can infer,

[1]) One can not dispense with the measuring of the brightness as a function of the angle, because it is by no means self-evident, that the relation between the reflective power and the angle of reflection, is independent of the wave-length. If a dependence should exist, the functional relation between the reflective power and the wave-length in the case of perpendicular incidence and reflection at an angle of 20°, would be different from that relation in the case of perpendicular incidence and the total amount of reflected light.

that a non complete depolarisation caused by the reflection on the white surface does not occur.

The smoked glass reducer.

In measuring the transmission curve of this reducer, the complication arises, that the arrangement used in measuring the energy of the images in P, in which the bolometer-strip is perpendicular to the length-direction of the images (fig. 27*a*) is not sensitive enough to secure accurate results. The bolometer-strip was, therefore, placed parallel to that direction (fig. 27*b*).

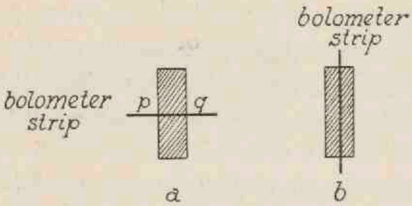

Fig. 27.

As mentioned already, the smoked glass reducer serves to weaken the brightness, in the green region of the spectrum, of the images on the white surface, sufficiently to make a direct comparison possible with the brightness of the black body, at the melting point of gold. Now, when the reducer is inserted in the lightpath, the brightness of the images is computed from the measured energy, and the transmission of the smoked glass. In doing so, however, one must not overlook the fact, that in measuring the energy, the relative position of bolometer-strip and image, is as shown in fig. 27*a*, whereas, when the transmission is measured, it is as shown in fig. 27*b*. Now, it will be clear, that in our case, we must know in particular the transmission of the smoked glass, corresponding to that part (viz. pq) of the image, which radiates on to the bolometer-strip in the orientation of fig. 27*a*, and one must admit the possibility, that the transmission of the smoked glass corresponding to the various parts of the image, is different from the transmission, corresponding to its central part (p, q), for the paths of the light-rays, belonging to the various points of the image, are different. In order to make sure about this, we measured the transmission for various heights of the slit, using red light, since the accuracy of measuring attainable, with this part of the spectrum, is amply sufficient. It appeared, as a result from these measurings, that a systematic change of the transmission with the height of the slit, was absent.

As an instrumental detail, we mention here, that the reducer is fitted with a revolving and arresting arrangement, so that it can be inserted in the lightrays, in front of the double monochromator, in a position, which is reproducible at will.

The sector reducers.

These reducers, placed in front of lens L_2 fig. 22, serve to vary the brightness of the images in P over a range, which includes the brightness of the black body at the melting point of gold. The use of these sector reducers has the important advantage, that the transmission is independent of the wave-length. It is, therefore, possible to measure the transmission with red light, which allows of a high precision. During the measuring, bolometer-strip and image, are at right angles to each other. It appeared to be necessary to mount the stand and holder of the reducers in such a way that the mechanical vibrations, arising from the rapid rotation, can not have a disturbing influence on the bolometer-strip. When for example, the stand, which carried the reducers, was fastened on to the same table, on which the bolometer was placed, the zero of the relay galvanometer showed rapid unsystematic fluctuations of about 3 mm. These fluctuations, which made an accurate determination of the transmission impossible, must be ascribed to the induced vibrations of the bolometer-strip.

The construction of the stand and of the revolving holder was such as to prevent an arrangement, which would make the position of the sector reducers in the path of the light reproducible at will. It was, therefore, necessary, contrary to the adjustment of the smoked glass reducer, to measure the transmission of the sector reducers anew, each time they were inserted in the light path.

§ 5. ACCOUNT OF THE COURSE OF THE MEASURING.

From the above it will be clear that the determination of c_2 requires:

1⁰. The measurings of the current running the pyrometerlamp, when the latter is adjusted for a brightness, equal to that of the black body at the melting temperature of gold.

2⁰. The standardizing of the spectral pyrometer with the aid of the images in P (fig. 21) i.e. the relative standardizing of the brightness.

1°. In order to determine the pyrometer current corresponding to the brightness of the black body, at the melting temperature of gold, we proceeded as follows:

While the temperature was made to increase at the rate of 0,1° per minute, the pyrometer current was determined, for the various wave-lengths, as a function of the heating time, as were also the E. M. F.'s of the two thermoelements. Now, at the moment, the constancy of the E. M. F. of the gold wire element sets in, the black body has reached the melting temperature of gold; the pyrometer current in question, is therefore found for the various wave-lengths from the known connection pyrometer current-time by interpolating to that moment.

Table 5 gives for the various wave-lengths the pyrometer currents corresponding to the brightnesses of the black body [1]).

TABLE 5.

λ in Å	6605.0	6499.5	6406.5	6004.5	5805.0	5610.5	5404.5	5301.5	5198.5
Pyrom. current in m. amp.	261.00	260.98	260.72	259.82	259.40	258.95	258.45	258.25	258.00

As regards sub. 2, the condition must be satisfied that the brightness of the auxiliary lamp Q remains constant during the relative standardizing of the spectral pyrometer. In order to obtain a constant energy radiation. the current was, therefore, adjusted for unvarying strength bij means of a compensating arrangement, which made it possible to reach an accuracy of 0.1 ‰ in the constancy [2]). However, since the constancy of the current is not a sufficient quarantee for the constancy of the radiated energy, it was still necessary to check the latter during the relative standardizing of the spectral pyrometer. As already mentioned in § 4, the sector reducers could not be inserted in the lightpath in a reproducible position, and their transmission must, therefore, be measured each time a new.

[1]) We observed, in a few cases that the E. M. F. did not remain constant during the melting, but showed a slight decrease.

[2]) A fluctuation of 0,1 ‰ in the current means a change of 0,3 ‰ in the radiated energy for the type of lamp, used in our case.

It follows from the above, that the bolometer-strip and the white surface must alternately be put in *P* (fig. 21) in a reproducible way. In order to make this possible the bolometer and the white surface are each mounted on a stand, of which the legs end in sharp points, resting on three metal blocks. Of the latter, one is plane, one has a groove in it, and the remaining one has a hole in it. The three blocks are fastened on to the same iron plate, which carries the whole of the arrangement.

The adjustment of the white surface and of the bolometer is then effected as follows:

The white surface is placed in *P*, in such a way, that the images on it, are sharp, and its position fixed by means of a telescope provided with cross threads, which is put, as nearly as possible, at right angles to the lightpath behind the monochromator. The bolometer-strip is then, in its turn, adjusted with the aid of the telescope. The stands, which carry them, make small displacements in three mutually perpendicular directions possible. Each time, the bolometer-strip and the white surface were inserted in the light path, their positions were checked by the telescope, and the experiments proved that this way of proceeding was indeed successful for obtaining reproducible adjustments.

In order to eliminate errors, arising from slight changes in the arrangement which preliminary experiments had shown actually to occur, we tried to obtain the values for the required quantities from observations, which were, as nearly as possible, simultaneous.

We shall now give the complete set of measurings, to be carried out, which is involved in our method of proceeding:

1°. Of the selectivity of the reflective power of the white surface, § 4, and of the bolometer-strip, Ch. I, § 7.

2°. of the transmission of the smoked glass reducers, § 4, and of the bolometer window (bolometer-strip parallel to the length direction of the images).

3°. of the wave-length regions, let through by the spectral pyrometer, § 4, and besides, measurings for the determination of the connection, wave-length-reading — true wave-length.

4°. of the regions, let through by the double monochromator, Ch. I, § 4.

5°. of the energy of the images, Ch. I, § 3; measurings from which to deduce the connection brightness-pyrometer current, § 3, and the measuring of the transmission of the sector reducers, § 4. (bolometer-strip at right angles to the length-direction of the images).
6°. the checking of the measurings sub. 4, by repeating them.
7°. the checking in the same way of no. 3.
8°. the checking in the same way of no. 2.
9°. the checking in the same way of no. 1.

A few points ask for more elucidation. For the determination of the transmission of the bolometer window, a glass plate was used, cut from the same plate as the window itself.

The measuring of the wave-length-regions, let through by the spectral pyrometer, is required for the determination of the effective wave-length; a high accuracy is not necessary in this case.

As regards the check measurings sub. 6 and 7 preliminary experiments showed them to be absolutely necessary; it has actually occurred, for example, that the adjustment of the slits had for some unknown reason, suddenly changed by which, of course, the regions, let through, as well as the energy of the images, were also altered.

The measurings sub. 5 require an alternate determination of the energy and the brightness of the images in P; the order of the necessary manipulations is here chosen in such a way as to make their number a minimum.

In table 6 we give the measurements referring to the measuring programme mentioned sub. 5. The measuring ot the energy of the images is carried out for wave-lengths, lying close to those for which the brightness was measured.

The transmission of the sector reducers was measured in the red region, the reading of the wave-length adjustment being 14.70.

By the arrangement of the measuring programme as given above, we are in a position to check the constancy of the radiated energy, during the relative standardizing of the spectral pyrometer, and to measure the transmission of the sector reducers, when inserted in the light parth.

TABLE 6.

Energy measuring of the images	
Wave-length reading of the double monochromator	Resistance in W_2
— — —	— — — — — — — — — —
— — —	— — — — — — — — — —
19.90	38000 Ω — 36000 Ω (steps 500 Ω)
20.00	38000 Ω — 36000 Ω (steps 500 Ω)
14.70	7500 Ω — 7300 Ω (steps 50 Ω)
Sector reducers in light path, measuring of the transmission	
14.70	8600 Ω — 8400 Ω (steps 50 Ω)

Brightness measuring, with sector 1 in light path	
Wave-length reading sp. pyr.	Pyrometer current in m. Amp.
6060	2663^5
6140	2627^5
6230	2605
Smoked glass reducer in light path	
5380	2596^5
5200	2629^5
5110	2655
Sector 1, replaced by 5	
5110	2582^5
5020	2613^5
5200	2557^5

TABLE 6. (*Continued*).

Smoked glass reducer out of light path	
5720	2688^5
5890	2627^5
6060	2571^5
Energy measuring, sector 5 in the light path	
Wave-length reading double mon.	Resistance
14.70	13400 Ω — 12800 Ω (steps 200 Ω)
Sector reducer out of light path	
14.70	7500 Ω — 7300 Ω (steps 50 Ω)
Energy measuring of the images	
1520	8600 Ω — 8400 Ω (steps 50 Ω)
— — —	— — — — — — — — — —
— — —	— — — — — — — — — —

§ 6. RESULTS OF THE MEASURING.

The results obtained from a series of measurings which served to deduce the value of c_2 are collected in table 7. The measurements, from which the relative brightness of the images per Å was computed, are given in the last column.

The relative brightness per Å of the images is determined as the quotient of the energy obtained and the effective wave-length-region. As already explained, however, in Ch. I, § 5, a correction must be applied to the value, so computed, owing to the change of the energy with the wave-length in the auxiliary lamp. Besides, there is still another correction required, due to the fact, that the shape of the area, covered by the transmission as an ordinate over the wave-length-region, let through, differs slightly from a trapezium. These corrections were computed from the approximate values

TABLE 7.

Wave-length reading double monochromator	λ_0	Effect. wave-length-region	Energy image	Rel. brightn. pro Å
14.70	6590 Å	330.2 Å	13452	203.5
14.90	6511.5	318.3	12722	199.8
15.—	6474.0	312.6	12405	198.5
15.15	6418.0	304.1	11865	195.1
15.25	6381.5	298.5	11483	192.4
15.70	6227.0	273.9	9990	182.4
15.80	6194.5	268.7	9738	181.1
16.35	6023.0	241.9	8172	168.9
16.45	5993.5	237.5	7928	166.9
17.00	5842.5	215.7	6713	155.6
17.10	5816.0	212.8	6496	153.2
17.85	5628.5	187.1	5209	139.2
17.95	5606.0	184.3	5044	136.8
18.80	5424.0	162.5	3855	118.5
18.90	5403.5	160.3	3745	116.8
19.35	5316.0	150.7	3249	107.7
19.45	5297.5	148.4	3135	105.3
19.90	5216.5	140.2	2732	97.4
20.00	5199.5	139.4	2653	95.8

of the relation energy pro Å, given in column 5 of table 7. The latter was computed according to the formula:

$$A = E_{\lambda_0} \triangle \lambda,$$

where A denotes the measured energy of the image, $\triangle\lambda$ the corresponding effective wave-length-region and E_{λ_0} the energy per Å, belonging to λ_0 —, i.e. the wave-length corresponding to the apex of the triangle (point B. fig. 10) when the transmission area has a triangular shape, and to the centre of the shorter parallel side, when it has the shape of a trapezium.

The correction factor f was determined for the various wave-lengths by means of a graphical solution of the equation:

$$f E_{\lambda_0} \triangle_{\lambda} = \int_{\lambda'}^{\lambda''} E_{\lambda} D_{\lambda} d\lambda,$$

where E_{λ} denotes the approximate value for the energy per Å, given in Column 5 of table 7.

In table 8 the values for f, obtained in this way for the various wave-lengths.

TABLE 8.

λ in Å	6800	6200	5600	5000
f_{λ}	1.0006	1.0007	1.0008	1.001

Since we have to deal, in our case, with relative energy measurements, the maximum error, which can arise from neglecting this correction, amounts to 0,4 $^0/_{00}$.

Each time a sector reducer was inserted in the lightpath, the energy was measured anew, for which the red part of the spectrum was used (with the wave-length reading of the double mono-chromator at 14.70). The values, so obtained, which give an account of the behaviour of the energy radiated during the standardizing of the spectral pyrometer, are collected in table 9.

TABLE 9.

No. measuring	1	2	3	4	5	6	7	8
Energy	13462	13452	13464	13420	13454	13425	13450	13469

Table 10 gives the values, found for the transmission of the various sector reducers.

TABLE 10.

No. sector	1	2	3	4	5	6	7
Transmission in %	86.8	70.8	60.5	57.2	42.0	41.1	29.3

For the comparison with the brightness of the black body, at the melting point of gold, the brightness of the images was measured over a range, which, was reduced so as to include the former brightness, by the use of the smoked glass- and sector reducers. In table 11 are given the pyrometer currents, measured for this comparison.

From the connection pyrometer current-transmission of sector reducers, found in measuring the brightness of the images, we obtained, by interpolating to the pyrometer current, belonging to the brightness of the black body at the melting point of gold, the reduction factor r. (last column of table 11), which must be applied in order to make the brightness of the images on the white surface equal to that of the black body.

In this way, therefore, we arrive at a brightness of the images, as observed in the spectral pyrometer, which is equal to that of the black body at the melting point of gold, likewise observed in the spectral pyrometer. This equality is expressed mathematically by:

$$\int_{\lambda'}^{\lambda''} f_\lambda E_{B\lambda} D_\lambda O_\lambda \, d\lambda = \int_{\lambda'}^{\lambda''} E_{Z\lambda} D_\lambda O_\lambda \, d\lambda.$$

Here, λ' and λ'' denote the limits of the spectral region, let through by the spectral pyrometer,

$E_{B\lambda}$ the brightness of the images on the white surface,

$E_{Z\lambda}$ the brightness of the black body at the melting point of gold,

f_λ the reduction factor,

D_λ the transmission of the spectral pyrometer,

O_λ the sensitivity-factor of the eye.

TABLE 11.

Adjustment Sp. Pyr.	λ_e	Pyrometer currents in m amp. occurring with the reducers									
		No	Smoked glass	1	2	3	4	5	6	7	r in %
5020	5198.5					2624*	2613.5*	2571*	2573*		43.7*
5110	5301.5			2655*	2621.5*	2593.5*	2582.5*	2539*			56.0*
5200	5404.5			2629.5*	2596.5*	2571.5*	2557.5*				68.2*
5380	5610.5		2618	2596.5*	2557*						85.0*
5550	5805		2545.5						2683.5	2618	25.7
5720	6004.5						2688.5		2623.5	2563	35.6
5890	6206.5					2638	2627.5	2574	2567.5	2511	50.0
6060	6406.5			2663.5	2616.5	2584.5	2571.5				68.8
6140	6499.5	2661.5		2627.5	2593.5						78.1
6230	6605	2632		2605	2562						90.4

*) For these measurements, the smoked glass and the sector reducers were together inserted, at the same time in the light path.

In Ch. I, § 7, we explained already how from this equation the brightness per Å of the black body is computed at the effective wave-length; the latter being defined as that particular wave-length, for which the energies per Å of the images and of the black body are equal.

We obtained the effective wave-length from the data of table 3 and the measured spectral region, let through by the spectral pyrometer. The results are given in table 12.

TABLE 12.

Wave-length adjustment Sp. pyr.	Wave-length region	λ_0	Effective wave-length λ_e
6230	244 Å	6615.5 Å	6605.0 Å
6140		6507.5	6499.5
6060	218	6412.5	6406.5
5890		6209.5	6206.5
5720	180	6005.5	6004.5
5550		5805.0	5805.0
5380	148	5609.0	5610.5
5200		5402.5	5404.5
5110	120	5299.0	5301.5
5020		5196.0	5198.5

For the relative brightness per Å of the black body at the melting point of gold, we have the expression:

$$E_{Z\lambda} = E_\lambda \, r \, d_\lambda \, w_\lambda \, \frac{1}{b_\lambda},$$

where $E_{Z\lambda}$ denotes the brightness per Å of the black body,
E_λ denotes the brightness per Å of the images,
r the reduction factor given in table 11,
d_λ the transmission of the smoked glass,
w_λ the reflection coefficient of the white surface,
b_λ the transmission of the bolometer window.

The values for these quantities are collected in table 13, where one finds besides in column 7 and 8 the values for $\frac{1}{\lambda_e}$ and for $log.\, E_{\lambda_e}\, \lambda_e^5$ respectively.

TABLE 13.

λ_e	E_{λ_e}	r	d_λ	n_λ	b_λ	$1/\lambda_e$	$^{10}log\, E_\lambda\, \lambda^5$
5198.5 Å	95.70	43.7%	18.30%	91.80%	92.00%	1.9236×10^{-4}	2.471
5301.5	105.5	56.0	17.84	91.76	91.94	1.8862	2.644
5404.5	116.8	68.0	18.12	91.71	91.87	1.8502	2.821
5610.5	138.5	85.0	20.52	91.59	91.70	1.7832	3.128
5805.0	152.2	25.7	—	91.46	91.42	1.7227	3.412
6004.5	167.6	35.6	—	91.29	91.18	1.6653	3.669
6206.5	181.6	50.0	–	91.09	90.84	1.6112	3.923
6406.5	194.2	68.8	—	90.88	90.48	1.5609	4.161
6499.5	199.3	78.1	—	90.73	90.30	1.5385	4.259
6605.0	208.0	90.4	—	90.56	90.10	1.5140	4.376

To the computation of the value for c_2 which is obtained from the inclination of the line $\left(\frac{1}{\lambda}, log E_\lambda \lambda^5\right)$, the method of least squares was applied. The result was

$$c_2 = 1{,}4277 \times 10^8 \ (\mathring{A},\ degree).$$

From a second series of measurements, we obtained the value

$$c_2 = 1{,}4336 \times 10^8 \ (\mathring{A},\ degree).$$

As a final result from our measurements, we can therefore take c_2 to be the mean of these two values, i.e.

$$c_2 = 1{,}4306 \times 10^8 \ (\mathring{A},\ degree).$$

Some remarks on the accuracy attained.

As already explained in § 1, an error of 0,003 in the value for c_2 from an error of 1 % in the value for the measured energy ratio, and in connection with what was said of the high accuracy of the relative energy measuring, one would probably expect a closer agreement between our two separate results for c_2. The explanation of this discrepancy, however, is, that one must distinguish, in our case, between the method as such, and the apparatus, at our disposal. As regards the latter, it is more in particular the reliability of the double monochromator, in its present construction, which falls decidedly short of what is required of it.

LIST OF LITERATURE CITED.

1. L. S. ORNSTEIN and D. VERMEULEN: Proc. Int. Ill. Con. **115**, 1931.
2. L. S. ORNSTEIN: Scripta Univ. atque Bibl. Hierosolym. 1923.
3. H. C. v. ALPHEN: Thesis Utrecht, 1927.
4. P. H. v. CITTERT: Revue d'Opt. 2, 57, 1923.
5. L. S. ORNSTEIN, W. J. H. MOLL and H. C. BURGER: Objektive spektralphotometrie.
6. H. C. HAMAKER: Thesis Utrecht, 1934.
7. L. S. ORNSTEIN, Miss J. G. EIJMERS and D. VERMEULEN: Ztschr. f. Phys. **75**, 575, 1932.
8. FAIRCHILD and HOOVER: Journ. Opt. Soc. Amer. **7**, 543, 1923.
9. G. RIBAUD: Traité de Pyrométrie optique.
10. W. W. COBLENTZ: Bur. Stand. Bull. **8**, 1920.

STELLINGEN

I.

RUSSELL vervangt in zijn berekeningen de werkelijke steratmosfeer door het model van SCHUSTER-SCHWARZSCHILD, daarbij de „effectieve diepte" van de fotosfeer constant aannemend. Hierdoor kunnen aanzienlijke fouten ontstaan.

H. N. RUSSELL, Ap. J. 78, 239, 1933.

II.

Een betere en meer economische wegverlichting is te verkrijgen, indien de bouw van de lantaarns aan de reflectie-eigenschappen van het wegdek aangepast wordt.

III.

Alvorens de monochromatische wegverlichting algemeen toe te passen, behoort o.a. onderzocht te worden, of de mérites van genoemde wegverlichting ook voor kleurenblinden gelden.

IV.

Voor de vraagstukken van dag- en avondverlichting is een systematische studie der lichtverstrooiende en lichtrichtende materialen noodzakelijk.

V.

De gevoeligheid van spectraal fotometrische meetmethodes volgens het beginsel der gelijktijdige waarneming, is kleiner dan van de methodes volgens het beginsel van successieve waarneming.

VI.

Het is gebruikelijk de temperatuurschaal aan de smeltpunten van verschillende stoffen vast te leggen; als zoodanige stof is Palladium niet geschikt.

VII.

Er is verband tusschen electronen emissie en licht emissie.