The description and use of the trianguler-quadrant: being a particular and general instrument, useful at land or sea; both for observation and operation : more universally useful, portable and convenient, than any other yet discovered

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THE Deſcription and Uſe OF THE TRIANGULER-QUADRANT:

BEING

A Particular and General *Inſtrument*, uſeful at Land or Sea; both for Obſervation and Operation.

More Univerſally uſeful, Portable and Convenient, than any other yet diſcovered.

With its Uſes in

Arithmetick.
Geometry, Superficial and Solid.
Aſtronomy.
Dyalling, Three wayes.
Gaging.
Navigation.

In a Method not before uſed.

By *John Brown*, Philomath.

London, Printed by *John Darby*, for *John Wingfield*, and are to be ſold at his houſe in *Crutched-Fryers*; and by *John Brown* at the *Sphear* and *Sun Dial* in the *Minories*; and by *John Sellers* at the *Hermitage-ſtairs* in *Wapping*. 1671.

To the Reader.

FRiendly Reader, Thou haſt once more preſented to thy view, a further Improvement and uſe of the *Sector*, under the name of the *Trianguler Quadrant*, ſo called from the ſhape thereof.

In the *year* 1660, it was my lot, firſt, to apply and improve this former Contrivance of Mr. *Samuel Foſter* on a *Quadrant*, to a joynt Rule or Sector; and did, in 1661, publiſh my preſent Thoughts thereof, in a ſmall Diſcourſe, under the name of the *Joynt Rule*.

Since then, through my perſwaſions, and aſſiſtance, another Piece was publiſhed 1667, by *I. T.* under the name of the *Semi-Circle on a Sector*: But neither of theſe, that is to ſay, neither my own nor his, ſpoke what

I would have it ſpeak ; neither have I hopes ever to produce a Diſcourſe either for method or matter, worthy or becoming ſo excellent, univerſal, and uſeful an Inſtrument, for the moſt Mathematical Occaſions, being for acurateneſs, conveniency, cheapneſs, and univerſality, before all others. For,

1. If it is made of Wood, if the Wood keep but ſtreight, it is as true to be made uſe of as of Metal.

2. It may be made of any Radius or bigneſs, and yet in little Room in compariſon of other Quadrants.

3. More convenient to uſe when large, than other Quadrants.

4. As to the Projection for Hour and Azimuth, particularly uſing only two Lines of Natural Sines, the Thred and Compaſſes for thoſe two difficult (and many more eaſie) Propoſitions.

5. The neat Conveniency of a greater and a leſs Radius, double, treble,

treble, or quadruple one to another.

6. The convenient Contrivance that happens to it, of three Inſtruments in one, *viz.* A *Sector*, *Quadrant*, and *Gunter*'s *Rule*; all three conveniently in one.

The conſideration of theſe things, and the love and willingneſs I alwayes had, to the communicating of them to others, hath put me on this hard task of writing this Collection of the uſe thereof.

Wherein I do moſt heartily beg thy Pardon and Acceptance, to accept in good part the willing endeavours of my poor Ability, which I doubt not but to have from moſt that know me; For, firſt, my inſufficiency in the Tongues, Arts, and Sciences: Secondly, my Meanneſs and Poverty in the World, for theſe Imployments, which take up ſo much of a mans time, and ability, to perform them to purpoſe, may plead my excuſe; for firſt, here is the Product of more than Two years Improvement of more than vacant Hours; with the great disadvantage of taking three Weeks at times, to do that which three Dayes together might have as well, if not much better, performed; And at last, to call the Assistance of two others, to undertake the Charge thereof, to midwife it into the World.

Thus, as Widows Mites are accepted, which are offered in sincerity; so I hope will mine, though attended with much disorder, as to Method; more uncouthness, as to Stile and Matter; What it is, it is as at first Composing, for I could never get Time nor Liberty, from my daily Trade and Calling, to transcribe it twice.

Yet was it not done at any time carelesly, but with good will and a free intent of plainness and usefulness for the publick good of others, as well as my own recreation and delight,

The

The *Gunters* Rule, the Quadrant, and Sector, I need not commend, they are ſo well known already; but this I will add, a better Contrivance and more general hath not yet to my knowledge been produced; nor a Diſcourſe where the uſe of all the Three together hath ſo been handled, nor many more *Examples*, though Mr. *Windgate* and Mr. *Patridge* have done ſufficiently for the *Gunters*-Lines, and Mr. *Gunter* for the Sector, and Mr. *Collins* with the Quadrant, and all of them diſtinctly far beyond this; yet this Diſcourſe of all the Three together, may give content to ſome others, as well as to me.

The Diſcourſe of *Dialling*, is gathered from Mr. *Wells*; and yet thoſe that ſhall read Mr. *Wells*, and this, may often-times think otherwiſe; for I aſſure you, I ſaw not one leaf of his Book all the while it was doing; but, I hope, it may pleaſe in in moderate ſort, an ordinary capacity, both for plainneſs, convenience, and variety.

The cutting of the Regular Bodies, I learned from Mr. *John Leake*, and the way is ready, convenient, and exact, and worthy of remembrance.

The Theorems, from Mr. *Thomas Diggs*, as in its due place, is obſerved.

The way of Meaſuring Superficies and Solids, from Mr. *Gunter*: and my conſtant Experience in thoſe Imployments; and the Learner may here be ſupplied with what is often complained on, *viz.* the Interpretation of Hard-words, as much as I could call to mind, or think to be convenient for that purpoſe.

In the 15th Chapter, I have gathered many Cannons from Mr. *Collins* his Workes, and applied them to the *Trianguler Quadrant*; and been more large than needs in ſome places, yet I hope to the content of ſome inquiring Perſons.

The buſineſs of Navigation, I fear, may prove moſt defective; for my part, I never yet ſaw *Graveſ-end*, much leſs the Streights of *Gibralter*; but for Obſervation and Operation, the Inſtrument will do as well as any, if well made and applied.

So for the preſent, I reſt and remain, ready to ſerve you in, and ſupply defects by well making of theſe Inſtruments, at the *Sphear* and *Sun-Dial* in the *Great Minories*.

John Browne.

The

The *Argument* of the *Book*, and the *Authors* Apologie.

AT length my pains hath brought to the things I long intended, (pass
And doubt not but in every place,
hereafter 't may be mended.
To me it hath been of great use,
to others more likewise;
Therefore let no man it abuse,
before he doth advise.
One Part thereof hath had renown,
with Artists far and near:
The other Part I strive to crown,
with use and plainness here.
Although my Parts and Time be small,
to hold forth Arts aright;
Yet have I plainly set forth all,
seemed useful in my sight.
And though I have not seen so far,
as some perhaps might see;
I doubt not but that some there are,
will pleased with it be.
For first the Tyroes *young may find,*
some terms to be explained; *Which*

Which when well fixed in his mind,
time quickly will be gained.
In the next place Mechanicks mean,
that have ſmall time to ſpare;
But yet may have a Love extream,
to Mathematicks fair.
And others that of wordly Means,
have little to afford,
For various Mathematick Theams,
this having, they are ſtor'd;
As firſt with Gunters Sector, *and,*
his Quadrant *eke alſo;*
By Foſter *altred after, and,*
with Gunters Rule *and* Bow.
The Traviſs Quadrant *and* Croſs-
the Davis Quadrant *too;* (ſtaves,
Their uſes all to more than halfs,
this Inſtrument *will do:*
With this advantage more beſide,
of lying in leſs room,
A fault that Saylors *muſt abide,*
when they on Ship-board come.
In the next place, the Rudiments
of Geometry *exact*
The right Sines & *their* complements,
and how they lie compact, *With-*

The Argument of the Book,

Within a Circle, *and the reſt,*
the Chords *and* verſed Sines
About a Circle *are expreſt,*
the Tangents, Secants, Lines.
And how their uſe and place is ſeen,
in Round *and* Plain Triangles ;
Which ſerve to deck Urania *Queen,*
as Jewels, Beads, and Spangles.
In the next place Arithmetick,
by Numbers *and by* Lines ;
In wayes that won't be far to ſeek,
by them that uſe their times ;
Becauſe the Precepts are explain'd,
by things of frequent uſe,
That for the moſt part are contain'd,
in City, Town, or Houſe ;
As Land and Timber, Boards & Stones,
Roofs, Chimneys, Walls and Floor,
Computed and reduc'd at once,
in Thickneſs, Leſs or More.
The cutting Platoe's *Bodies five,*
which are not yet made ſix ;
And them the beſt way to contrive,
and Dials *on them fix :*
Their Meaſure and their Magnitude,
in Circle *circumſcribed ;* *Whoſe*

Whose Properties by old Euclide,
and Diggs, *have been described.*
Then also in Astronomy,
are many Propositions,
Which fitly to th' Rule *I apply,*
avoiding repetitions.
And after, in the pleasant Art,
of Shadows, *I do wander,*
To draw Hour-lines *in every part,*
both upright, over, *and* under:
And all the usual Ornaments,
that on Sun-Dials *be,*
Which are describ'd to the intent,
Sol's *travels for to see;*
As first, his Place *and* Altitude,
his Azimuth *likewise;*
His Right Ascention, Amplitude,
and how soon he doth Rise.
The same also to Moon *and* Stars,
is moderately appli'd;
Whereby the time of Night *appears,*
the Moons Age, *and the* Tide.
Then Heights *and* Distances *to take,*
at one, or at two Stations,
Performed by those wayes that make,
the fewest Operations.

And

And also ready Rules *to use,*
the Logarithmal Table ;
Which may prove ready Hints to these,
that are in those most able :
And many other useful Thing,
is scattered here and there,
Which formerly by Me hath been,
accounted very rare.
And lastly, for the Saylors *sake,*
I have spent many an Hour,
Th' Trianguler-Quadrant *for to make,*
more useful than all other :
Sea-Instruments *that they do use,*
at Sea for Observation ;
And sure I am, it won't abuse
them in their Operation ;
As in the following Discourse,
to them that willing be,
It will appear with easie force,
if they have eyes to see :
The Method and the Manner us'd,
as neer as I was able,
To follow the old Wayes still us'd,
and counted warrantable.
And in this, having done my best,
a[illegible] [illegible] up my male ;

Ascri-

Ascribing to my self the least,
would have the Truth prevail;
And give the honour and the praise,
to him that hath us made,
Of willing minds his Fame to raise,
by his assisting aid.
To whom be honour now and eke,
henceforth for evermore,
Ascribed by all them that seek
the Truth for to adore.

J. B.

ERRATA.

ERRATA.

PAge 28. line 8. for *Rombords*, read *Remboides*. P. 73. l. last. f. 337, r. 247. p. 75. l. 1. f. 7. r. 8. p 87. l. 14. r. *multiplied by*. p. 89. l. 14. f. 537 1616. r. 538-1616. & l. 21. f. 537, r. 538. p. 90. l. 4. f. 537, r. 538. & l. 5. add, *being better done with a parallel answer*. p. 100. l. 2. add, *the Thred*. p. 128. l. 2. dele 10 *min*. p. 133. l. 6. f. 60, r. 16. p. 143. l. 10, 11. f. *from* 12 *to* 7, r. *from* 7 *to* 12. p. 146. l. 22. f. 12 *Section*, r. 13 *Section*. p. 158. l. last, dele *and*. p. 160. l. 11. f. 72, r. 720, also in line 15 & 23. p. 164. l. 19. f. *Diameter*, r. *Area*. p. 165. l. last, add, *to* 707. p. 184. l. 10 f. *foot*, r. *brick*. & l. 20. f. $\frac{1}{2}$, r. 1 $\frac{1}{2}$. p. 187. l. 17. f. *Ceiling*, r. *Tileing* p. 201. l. 11. f. 52 *Links* r. 55 *Links*. & l. 12. f. 48 *Acres*, r. 4 *Acres*, 3 *Roods*, *&* 8000 *Links*. p. 102. l. 5. f. 21 *Acres* 42 *Links*, r. 2 *Acres*, 0 *Roods*, *but* 14760 *Links*; read so likewise in l. 11. of the same page. p. 204. l. 1. f. 16 $\frac{1}{2}$ r. 18 $\frac{1}{2}$. p. 205. l. 8. f. 55, r. 50. & r. 50 f. 55 in l. 21 & 22. p. 206. l. 19. f 4-50, r. 4-50000. & l. 21. f. 1 *Chain* 25, r. 11 *Chains* 23. p. 229. l. 16. f. 8-10*th*, r. 8-100. p. 231. l. 15. f *of*, r. *at*. p. 234. l. 22. f. 1 *of a foot*, r. 1 10*th of a foot*. p. 236, the 3 lines over 134-5, are to come in after 134-5. Also, the two lines over 3-545, should come in after 3-545. p. 257. l. 13. f. 2496, r. 249-6. p. 370. l. 3. f. *sine* r. *Co-sine*. p. 383. l. 22. add, *by the general Scale*. p. 384. l. 14. f. = S. ☉. r. = *Co-sine*. p. 414. l. 11. f. *or* r. *on*. p. 420. l. 22. f. 71 r. 31. p. 429. l. 15. f. *Declination*, r. *Suns Right Ascention*.

The

The Description, and some Uses of the Triangular Quadrant, *or the Sector made a Quadrant ; being an excellent Instrument for Observations and Operations at Land or Sea, performing all the Uses of the Fore-staff,* Davis-*Quadrant*, Gunter's-*Bow*, Gunter's-*Cross-staff*, Gunter's-*Quadrant and Sector, with far more conveniency and as much exactness as any, or all of them will do.*

The Description thereof.

1. FIrst, it is a joynted Rule (or Sector) made to what Length or Radius you please, (as to 6, 9, 12, 18, 24, 30, or 36 inches Length, when it is folded or shut together ; the shorter of which Lengths is big enough for Land uses, or Paper draughts ; the four last for Sea uses, or Observations.) To which is

added, a third Piece of the ſame length of the Sector, with a Tennon at each end, to fit into two Mortice-holes at the two ends of the inſide of the Sector, to make it an Æquilateral Triangle; from which ſhape, and its uſe, it is properly called a *Triangular Quadrant*.

2. Secondly, as to the Lines graduated thereon, they may be more or leſs, as your uſe of them, and as the coſt you will beſtow, ſhall pleaſe to command: But to make it compleat for the promiſed Premiſes, theſe that follow are neceſſary to be inſcribed thereon, as in the Figure thereof.

And firſt you are in order hereunto to conſider, The outer-edges of the (Sector or) Inſtrument, the inner-edges, the Quadrantal-ſide, the Sector-ſide, and the third or looſe-piece, alſo the fixed or Head-leg, the moving-leg, the head, and the end of each leg, alſo the head and leg center; of which more in its proper place.

1. And firſt, on the outer-edge is placed the Lines of Artificial Numbers, Tangents, Sines, and verſed Sines, to as large a Radius as the Inſtrument will bear.

2. Secondly, on the in-ſide or edge, on ſhort Rules is placed inches, foot meaſure, the line of 112, or ſuch-like. But on larger Inſtruments, a Meridian line to one inch,

or

or half an inch (more or leſs) for one degree of the Æquinoctial, for the drawing of Charts, according to *Mercator*, or any other more uſeful Line you ſhall appoint for your particular purpoſe.

3. Thirdly, in deſcribing the Lines on the two ſides; firſt I ſhall ſpeak to the Sector-ſide, where the middle Lines all meet at the Center at the head where the Joynt is: the order of which (when the head or joynted end lyeth toward your left hand, the Sector being ſhut, and the Sector-ſide uppermoſt) is thus:

1. The firſt pair of Lines, and lying next to you, is the Line of Sines, and Line of Lines, noted at the end with S, and L: for Sines and Lines, the middle Line between them that runs up to the Center, and wherein the Braſs center pricks be, is common both to the Sines and Lines in all Parallel uſes, or entrances.

2. The Line next theſe, and counting from you, is, the Line of Secants beginning at the middle of the Rule, and proceeding to 60 at the end, and noted alſo with *Se* for Secants, one of which marginal Lines continued, would run to the center as the other did.

3. The next Lines forward, and next the inner-edge on the moving-leg are the Lines of

Tangents; the firſt of which, and next to you, is the Tangent of 45, being the largeſt Radius, (as to the length of the Rule:) the other is another Tangent to one fourth part of the length of the other, and proceeds to 76 degrees, a little beyond the other 45: the middle Line of theſe alſo is common to both, in which the Center pricks muſt be. At the end of theſe Lines is uſually ſet T. T. for Tangents.

4. On the other Leg of the Sector, are the ſame Lines again, in the ſame order counting from you; wherein you may note, That as the Lines of Sines and Lines on one Leg, are next the outward-edge; on the other Leg, they are next the inward-edge: ſo that at every, or any Angle whatſoever the Sector ſtands at, you have Lines, Sines, and Tangents to the ſame Radius: and the Secants to juſt half the Radius, and conſequently to the ſame Radius by turning the Compaſſes twice; Alſo any Tangent to the greater Radius above 45, and under 76, by turning the Compaſſes four times, as afterwards will more appear: Which contrivance is of excellent convenience to avoid trouble, and ſave time; and happily made uſe of, in this contrary manner to the former wayes of ordering them.

5. Fifthly, without or beyond, yet next

to

to the greater Line of Tangents on the head-leg, is placed the firſt 45 degrees of the leſſer Tangents, which begin from the Center at 45 degrees, becauſe of the ſtraitneſs of the room next the Center, where they meet in a Point: yet this is almoſt of as good uſe, as if it had gone quite to the Center, by taking any parallel Tangent from the middle or common Line on the great Tangents, right againſt the requiſite Number counted on the ſmall Tangent under 45.

6. Sixthly, next to this will not be amiſs to adde a Line of Sines, to the ſame Radius of the ſmall Tangent laſt mentioned, and figured both wayes for Sine and co-Sine, or ſometimes verſed Sines.

7. Seventhly, next to this a Line of Equal Parts, and Chords, and the Secants in a pricked line beyond the little Tangent of 45, all to one Radius: To which (if you pleaſe) may be added, Mr. *Foſters* Line Soll, and his Line of Latitudes; but theſe at pleaſure.

8. Eighthly, on the outermoſt-part of both Legs next the out-ſide, in Rules of half an inch thick and under, is ſet the Line of Artificial verſed Sines, laid next to the Line of Artificial Sines, on the outer-edge; but if the Rule be thick enough to bear four

Lines, then in this place may be ſet the Meridian Line, according to Mr. *Gunter*, counting the Line of Lines as a Scale of Equal Parts. Thus much as for the Sector-ſide of the Inſtrument.

4. Fourthly, The laſt ſide to be deſcribed is the Quadrantal-ſide of the Inſtrument, wherein it chiefly is new. Therefore I ſhall be as plain as I can herein.

To that purpoſe I ſhall in the deſcription thereof imagine the looſe piece, (or third piece) to be put into the two Mortiſe-holes, which poſition makes it in form of an Æquilateral Triangle, according to the Figure annexed, noted with ABCD; where in AB is for brevity and plainneſs ſake called the Moveable-leg, DB the Head or Fixed-leg, DA the looſe-piece, B the Head, A and D the ends, C the Leg-center, at the beginning of the general Scale; the center at B the head-center, uſed only in large Inſtruments, and when you pleaſe on any other.

For the Lines graduated on this ſide.

Firſt, On the outer-edge of the moveable-Leg, and looſe-piece, is graduated, the 180 degrees of a ſemi-circle, C being the center thereof.

And

And these degrees are numbred from 0|60 on the loose-piece toward both ends, with 10, 20, 30, 40, *&c.* and about on the moveable-leg, with 20, 30, 40, 50, 60, 70, 80, and 90 at the head: Also it is numbred from 60|0 on the moveable-leg, with 10, 20, toward the head; and the other way, with 10, 20, 30, 40, 50, 60 on the loose-piece; and sometimes also from the Head along the Moveable-leg, with 10, 20, 30, *&c.* to 90 on the loose-piece; and the like also from the end of the Head-leg, and sometimes from 60 on the loose-piece both wayes, as your use and occasion shall require.

Secondly, On the Quadrantal-side of the loose-piece, but next the inward-edge is graduated 60 degrees, or the Tangent of twice 30 degrees, whose center, is the center-hole or Pin at B, on the Head or Joynt of the Sector.

Which degrees are numbred three wayes, *viz.* First from D to A for forward Observations; and from the middle at 30 to A the end of the Moving-leg, with 10, 20, 30; and again, from D the end of the Head-leg to A, with 40, 50, 60, 70, 80, 90, for Observations with Thred and Plummer.

Thirdly, Next to these degrees on the Moving-leg, is the Line of the *Suns* right Ascention, numbred from 60|0 on the degrees, with 1, 2, 3, 4, 5, 6, toward the Head, and then back again with 7, 8, 9, 10, 11, 12, &c. 1, 2, 3, 4, 5, on the other side of the Line, as the Figure annexed sheweth: The divisions on this Line is (for the most part) whole degrees, or every four minutes of time.

Fourthly, Next above this is the Line of the *Suns* place in the Zodiack, noted with ♈ ♉ ♊ ♋ toward the Head; then back again with ♌ ♍ ♎ over 60|0 in the degrees, and 12 and 24 in the Line of the *Suns* right Ascentions: then toward the end, with ♏ ♐ ♑; then back again with ♒ and ♓, being the Characters of the 12 Signes of the Zodiack, wherein you have exprest every whole degree, as the number of them do shew, there being 30 degrees in one Sign.

Fiftly, Next above this is a Kalender of Months and Dayes; every single Day being exprest, and three or more Letters, of the name of every Month being set in the Month, and also at the beginning of each Month, and every 10th day noted with a Prick on the top of the Line representing it, as is usual in such work.

Sixtly,

Sixtly, Next over the Months, is the Line to find the Hour and Azimuth in a particular Latitude. Put alwayes on ſmaller Inſtruments (and very rarely on large Triangular Quadrants for Sea Obſervations) the loweſt Margent whereof, and next the Months, is numbred from the end toward the Head, with 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, near the Head Center. For the Semi-diurnal Ark of the *Suns* Azimuth, and in the Margent next above this, with 4, 5, 6, 7, 8, 9, 10, 11, 12, near the end, for the Morning hours; then the other way, *viz.* toward the Head on the other-ſide the Hour Line, with 1, 2, 3, 4, 5, 6, 7, 8, for the Afternoon hours.

Seventhly, On the ſame Quadrantal-ſide, and Moveable-leg on the ſpare places, beyond the Months toward the end, is ſet an Almanack; and the Names of 12 or more Stars, to find the hour of the Night; which 12 Stars are noted with 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. among the degrees in ſmall Figures; as in the Figure.

Eightly, Next of all to the in-ſide, is the Line of Natural verſed Sines drawn to the Center, with his correſpondent Line on the other, or Head-leg. Expreſt ſometimes in a pricked Line, for want of room.

Ninthly, On the Head-leg, and next to the

the versed Sines last mentioned, is first the Line of Equal Parts, or Line of Lines: and on the same common Line wherein is the Center, is the Line of Natural Sines, whose length is equal to the measure from the center at C to 60|0 on the moveable-leg; so that the Line of degrees is a Tangent, and the measure from C to any Tangent, a Secant, to the same Radius of the Natural Lines of Sines, and Lines: Also beyond the Center C on the same common middle Line is another smaller Line of Natural Sines, whose length is equal to the measure from C to 60 on the loose-piece; then if you count from the Center pin at 60, on the loose-piece, toward the end of the movable-leg, they shall be Tangents to the same Radius, and the measure from the Center C to those Tangents, shall be Secants to the same Radius, which may be well to be ordered, to a third, or fourth part of the former, from the Center downwards: These twoLines of Sines are best figur'd with their Sines; and Cosines, the other way with a smaller figure, and the Line of Lines from the Center downward from 1 to 10 where 90 is, which Lines of Sines may be called a general Scale for all Latitudes.

Tenthly, Next to this toward the outer-edge is another Line of Natural Sines, fitted to the particular Line of Hour and Azimuths, for one particular Latitude, noted, *Pert. Scale of Altitudes ; or Sines.*

Eleventhly, Next to this is the Line of $29\frac{1}{2}$, for so many dayes of the Moon's age, in short Rules of the whole length, but in longer not; being easily known by the single strokes, and Figures annexed to those strokes.

Twelfthly, Next the outer-edge is a Line of 24 hours, 360 degrees, or 12 Signs, or in most Rules inches also, used together with the former Line of $29\frac{1}{2}$, and as a Theory of the Sun and Moon, and ready way of finding the Hour by the Moon or fixed Stars.

Thirteenthly, To this Instrument also belongs a Thred and Plummet, and Sights, as to other Quadrants; and a pair of Compasses as to other Sectors; a Staff and Ball socket also, if you will be curious and accurate.

And for large Instruments for Sea, a Square and an Index, which makes it a perfect sinical Quadrant, and two sliding sights also, which makes it a fore and back-staff, and bow, as will appear more at large afterward.

Some

Some Uſes of the *Trianguler Quadrant*, for Land and Sea Obſervations and Operations.

CHAP. I.

Numeration on the Lines graduated on the Inſtrument.

IN the firſt place it will not be amiſs to hint a few words, as to the reading the Lines, or (more properly) Numeration on the Lines; wherein take notice, That all Lines of Equal Parts, or Lines applicable to Arithmetick, as the Line of Lines, the Line of Numbers, the Line of Foot-meaſure, and the like; wherein Fractions of Numbers are requiſite: they are moſt commonly accounted in a Decimal way, and as much as may be, the ſmall diviſions are numbred, and counted accordingly.

But

But in the Lines of Sines, Tangents, Secants, and Chords; being Lines belonging properly to a Circle: in regard that the Sexagenary Fraction is ſtill in uſe, the intermediate Diviſions are, as much as may be, fitted to that way of account, *viz.* by whole degrees, where they come cloſe together, (or the Line of no great uſe.) And if more room is, to half degrees or 30 minuts, and ſometimes to quarters of degrees or 15 minuts; but toward the beginning of the Line of Natural Sines, or the end of the Natural Tangents and Secants: where the degrees are largeſt, they are divided to every 10th minute in all large Rules, as by conſidering and accounting you may plainly perceive.

Take two or three *Examples* of each kind.

1. Firſt, On the Line of Lines, to find the Point that repreſents 15. In the doing of this, or any the like, you muſt conſider your whole Scale, Radius, or length of the Line, may be accounted as 1, as 10, as 100, as 1000, or as 10000; and no further can be applicable to any ordinary Inſtrument.

Wherein obſerve, That if the whole Line be one, then the long ſtroke by every Figure doth repreſent one tenth of that Integer: and the next ſhorter without Figures, are hundredth parts of that one Integer; and a 1000th part is eſtimated in ſmaller Inſtruments, and ſometimes expreſt in larger: But the hundredth thouſand part is alwayes to be eſtimated by the eye in all Inſtruments whatſoever.

2. But if the whole Line of Lines ſhall repreſent 10, as it uſually doth, and as it is figured, then the long ſtroke at every Figure is 1, and the next longer are tenths, and the ſhorteſt are hundred parts, and the thouſand parts as near as can be eſtimated.

3. But if the whole Line repreſents a hundred, as here in our preſent Example, then the long ſtroke by every Figure repreſents 10, and every ſhorter ſtroke is one, and the ſhorteſt ſtrokes are tenths, and the hundredth parts as much as can be eſtimated.

4. But if the whole Line ſhall repreſent a 1000, then the long ſtroke by the Figure ſhall repreſent a hundred, and every ſhorter 10, and every of the ſhorteſt ſtrokes is one Integer, and a 10th part as near as can be eſtimated.

5. But laſtly, if the whole Line repreſent 10000, then every long ſtroke is 100, and every ſhorteſt cut is ten, and every ſingle Integer is as near as can be eſtimated by any ordinary Inſtrument.

Now

Now our present Example will properly come under the third Rule, by conceiving the whole Line to represent 100; then the first long stroke by 1 is 10, then the next shorter is for 11, the next 12, &c. to 15; which is cut up a little above the Line, for the more ready reckoning without telling the parts: which 15 is the Point required to be found.

Example the second, to find out 1550 *on the Line.*

This will come under the Notion of the 5th Rule, wherein the whole Line is conceived to represent 10000; then the first 1 is for the 1 thousand, then the fifth longer stroke next is for the 500; and lastly, the middle between the 500 stroke and the 6000 stroke is for the 50, being a little beyond the Point for 15 in the first Example.

A third Example of 5025.

This third Example may suffice for this work, being so plain after a little due consideration: For first, the whole Line is conceived to represent 10000, then the long stroke by 5 is for 5000, then there is no hundreds, therefore the Point required must be short of the next longer stroke, which signifies hundreds, and being it is just 25, which is $\frac{1}{4}$ of an hundred, the true Point readily sheweth it self: If you require

quire a more plainer and larger wording of this matter, I refer you to the third Chapter of Mr. *Windgates* Rule of Proportion; or the firſt Chapter of the *Carpenters Rule*, by *J. Brown*.

Laſtly, In nameing of any Point found out on the Line, great care and reſpect muſt be had as to the true value of the Number, according to the rate of the queſtion propounded: for the ſame Point that repreſents 15, doth repreſent 150; and alſo 1500, or 15000, (increaſing above the bounds before mentioned) alſo it ſignifies one and a half, or 15 of one hundred, which is uſually expreſt thus in a Decimal Fraction $\frac{15}{100}$, or more readily, 0. 15.

Alſo if it ſhould be a Number with a digit, two ciphers and another digit, as 2,005, this Number would be found cloſe to the long ſtroke, by the figure 2: and may repreſent either two thouſand, and 5 of 1000 more; or 20 and 5 of a hundred; or 2 hundred and 5 of another 10 more, or plainly as it is ſet down, two thouſand, no hundred, but five: Thus you ſee the manner of expreſſing whole Numbers, or whole Numbers and Decimal Fractions, which on the Lines is one and the ſame thing; and thus all Decimal Scales are to be accounted, and in the ſame manner is the Line of Numbers to be reac, as you may ſee more at large in the two Books before mentioned.

SECT. II.

But for Numerateon on all Circular Lines, it is much eaſier : For firſt, very few Inſtruments, unleſs at one part of the Line, can expreſs nearer than minutes of a degree.

Secondly, The whole Radius or Line of Lines is but 90 degrees, or but 45 of the Tangents, or 60 of the Chords, or Secants: So that in Inſtruments of 12 or 18 inches Radius, you may expreſs very well every tenth minute, to 60 on the Line of Sines : and every half degree to 75, and whole degrees to 90. And on the Tangents or Chords, every 10th minute quite through: and the Secants as the Sines.

So that any degree or minute being named, to find the ſame on the reſpective Line, count thus;

Firſt, every 10th degree is noted with a long ſtroke, and figures ſet thereunto. Secondly, every whole degree is cut between two, or three Lines, and ſometimes with a Point or Mark on the end of the ſtroke; and every 5th degree cut up higher than the reſt, and ſometimes with three Points, on the end of the Line, or ſome other convenient di-ſtinction, for readineſs ſake: and every 10th, 15th, or 30th minute, is cut only be-tween two Lines and no more; as will ap-pear very plain with a little practice.

Example, to find the Sine of the Latitude being at London, 51 degrees, 32 minutes.

1. Firſt, look on any Line of Sines, on the Quadrantal, or Sector ſide, according as you have occaſion, till you ſee 50, which is 50 degrees; then one degree forward, to-ward 60 is 51 degrees, then count three 10ths of minutes more for 30 minutes, and then for the odde two minutes, eſtimate one fifth part of the next 10 minutes forwarder, and that is the preciſe Point for the Sine of 51 degrees 32 minutes, the latitude of *Lon-don*, where ſometimes is ſet a Braſs Center-Pin.

Example the ſecond.

2. To find the Coſine of the Latitude, there are two wayes to count the Comple-ment of any Ark or Angle.

Firſt, by ſubſtracting the Ark or Angle out of 90 by the Pen, and count the reſidue from the beginning of the Line of Sines, and that ſhall be the Sine Complement of the Latitude required.

Example.

51 32 taken from 90, the remainder is 38 28, now if you count ſo much from the beginning of the Line of Sines, according to the laſt Rule, that ſhall be the Point for the Sine of 38 28, the Complement of 51 32, or the Sine Complement of the Latitude.

Or Secondly, If you count 51 32 from 90, calling 80, 10; and 70, 20; and 60, 30; 50, 40; 40, 50, &c. whereſoever the Number whoſe Complement you would have ſhall end, that is the Sine Complement required, which will be at 38 28, from the Center or beginning, for the Coſine of 51 32; The like work ſerves for any other Number, or on any other Line, as on the Degrees, Tangents, or Secants, Natural or Artificial, as by practice will more plainly appear, to the willing Practitioner.

SECT. III.

To find the verſed Sine of an Ark or Angle, or the Sine of an Ark or Angle above 90 degrees, or the Chord above 180 degrees, obſerve theſe Rules.

1. First, a right Sine, is the measure on the Line of Sines, from the center or beginning of that Line, to the Point that doth represent the Ark or Angle required.

2. The right Sine of an Ark or Angle above 90 degrees, is equal, to the right Sine of the Complement thereof to 180 degrees, being readily accounted, thus; Count the excess above 90 backwards, from 90 toward the Center; then the measure or distance from the end of the account to the Center, is the Sine of the Ark above 90 required: *Example.* Let the Sine of 130 be required, first, if you take 130 from 180, the remainder is 50; then I say that the right Sine of 50, is also the right Sine of 130; for if you count backwards from 90, calling 80, 100; and 70, 110; and 60, 120; and 50, 130; the measure from thence to 00, or the Center, is the right Sine of 130 degrees.

3. The versed Sine of an Ark or Angle, is the measure on the Line of Sines from 90 toward the Center, counted backwards, as the small figures for Complements shew, counting 90 for 00, and the Center for 90, (as the Azimuth Line is figured) opening the Line of Sines to a strait Line, and then counting beyond 90 for the versed Sines above 90, as on the versed Sines is plainly

seen

seen in the figure of the Rule.

4. For Chords of any Ark or Angle, do thus:

Halve the Ark or Angle required, and take the right Sine thereof, and that shall be the Chord thereof.

Example.

I would have the Chord of 40, the half of 40 is 20; then I say the right Sine of 20 is the Chord of 40, to that Radius that is equal to the right Sine of 30 degrees, at the Radius the Rule stands at.

5. To find a Chord to an Ark or Angle above 180 degrees, you must count as you did the right Sines; for note, the Chord of 180 is equal to the right Sine of 90 doubled, which is the full Diameter of a Circle: and a longer right Line than the Diameter cannot be taken in a Circle; therefore it must needs follow that Chords of above 180, are shorter than the Diameter which is the biggest Chord; therefore the Chord of 260, is equal to the Chord of 100 degrees, or right Sine of 50, the Sine of 30 being Radius.

6. In using the Artificial Sines and Tangents, or Secants; if you are to use a Sine above 90, then count 80 for 100, 70 for 110, 60 for 120, &c. But for Secants,

then count after the manner of versed Sines: Thus the Secant of 60 is as far beyond 90, as it is from 30 to 90; so that when you have occasion to use an Artificial Secant, which is not often, Then set the end of the Rule against a Table, and counting backwards from 90 to the number of the Secant required, turn that distance beyond 90 on the Board or Table, and that shall be your Secant required, as will be afterward hinted, as they come in use.

CHAP. II.

A brief Description of the Lines of a Circle, and the Explanation of some termes used in the following Discourse.

FOR the better understanding of the following discourse, it is needful to understand these Elements or Principles, as the Letters are necessary to be known before reading.

1. A Circle is a figure enclosed in one circular Line, called the Circumference; in the middle whereof is a Point called the Center:

ter: From which Point all right Lines drawn to the Circumference are equal one to another; as in the Circle A B C D, E is the Center, A B C D the Circumference, the Lines E A, E B, E C, E D are equal.

2. Any right Line crossing the Circumference, and passing through the Center of a Circle, is called the Diameter; and it divides the whole Circle into two equal parts, called Semi-circles (or half-Circles.) And the half of that Line is called the Semi-diameter or Radius to that Circle. As the Line A C is the Diameter, and E C the half-Diameter or Radius.

3. Any other Right-line crossing the Circumference is called a Chord, or Subtence, as the Line F G, which divides the Circle into two unequal parts: And note, that this Subtence belongs both to the lesser, and also to the greater part of the Circumference; that is to say, the Chord of 90 deg. is also the Chord of 270 deg. so that F G is Chord to the Ark F B G 90 deg. and also to the Ark F D G being 270 deg. much more than half the Circle.

4. Half the Chord of any Ark, is the right Sine of half that Ark: thus the right Line H G, the half of F G, is the right Sine of the Ark B G the half of F B G.

5. The Sine Complement or Cosine of

any Ark is the neareſt diſtance from the Circumference to the Diameter: Perpendiculer to that Diameter from whence you counted the degrees and minutes of the Ark or Angle. As thus, G I is the Coſine of the Ark B G, and the Right Sine of any Ark is the neareſt diſtance from the Circumference to the Diameter you counted the degrees from, as G H is the Right Sine of B G.

6. The verſed Sine of any Ark or Angle, is the Segment of the Diameter between the right Sine of the ſame Ark and the Circumference.

Thus H B is the verſed Sine of the Ark BG, and H D the verſed Sine of G D. So alſo is GH the right Sine of the Ark GCD or the Angle G E D 45 degrees above 90, *viz.* 135 degrees.

7. A *Tangent* is a right Line drawn perpendiculer to the Diameter, beginning at one extreme of the given-Ark, and terminated by a right Line drawn from the Center to the other extreme, of the given-Ark in the Circumference, till it inter-ſect the perpendiculer; Thus C K is the Tangent of the Ark C G, or the Angle C E G, 45 degrees.

8. A *Secant* is a right Line drawn from the Center thorow one extreme of the given-

Ark,

Ark, till it meet with the Tangent rais'd perpendicularly from the Diameter, drawn to the other Extreme of the ſaid Ark; Thus the Line EK is the Secant of the Ark CG, or the Angle GEC.

9. Note, as in a (Natural) Sine, the neareſt diſtance from the Ark to one Diameter, from whence you counted the degrees of the Ark or Angle, was the Right Sine; and the neareſt diſtance from the ſame Point to the other perpendiculer Diameter, is the Coſine of that Ark or Angle.

So likewiſe the neareſt diſtance from the Point where the Tangent and Secant meets, to one of the Diameters aforeſaid, is the Tangent of the Ark or Angle; ſo the neareſt diſtance from the meeting Point of the ſame Secant-line is the other Tangent-line to the other Diameter aboveſaid, is the Co-Tangent of the Ark or Angle aboveſaid.

Thus the Right-line KC is the Tangent of 45, and the Right Line KB the Co-Tangent of 45; Alſo the Line LC is the Tangent of 53, 30; and the Line MB is the co-Tangent thereof, *viz.* the Tangent of 36, 30.

Alſo the neareſt diſtance from L to EB, is the Tangent of 36, 30, to the Radius LC.

10. Every *Circle* is ſuppoſed to be divided

ded into 360 degrees; the Semi-circle into 180, the Quadrant or Quarter into 90.

11. Every *Degree* is ſuppoſed to be divided into 60 minutes, and every minute into 60 Seconds, and every Second into 60 Thirds, &c.

12. A *Radius*, or *Semidiameter*, is in our Inſtrumental Practice, ſuppoſed to be divided into 10000 parts, and every Chord, Sine, Tangent, or Secant, is to be divided by the Parts of the ſame Radius, or Radius and Parts more.

13. An *Angle* is the meeting of two Right Lines, as G E, and E C, meeting at E, do conſtitute the Angle G E C, called a Right-lined Angle; or when two Circles croſs one another, it is called a Spherical Angle, the Anguler Point being noted alwayes by the middle Letter of three that ſhew the Triangle.

14. A *Plain Triangle* is the meeting of three Right Lines croſſing one another; and a *Spherical Triangle* is conſtituted by the croſſing of three Circles, as in the two Figures noted II and III, you may plainly ſee.

15. All *Angles*, Plain and Spherical, are either Acute, Right, or Obtuce.

16. An *Acute Angle* hath a ſharp Point containing an Angle leſs than 90 degrees,

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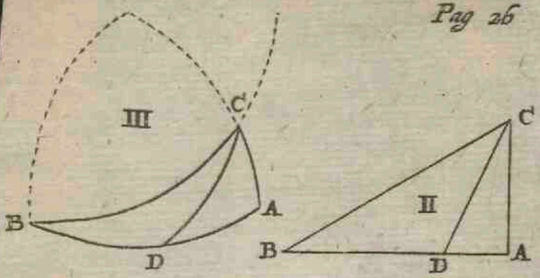
III
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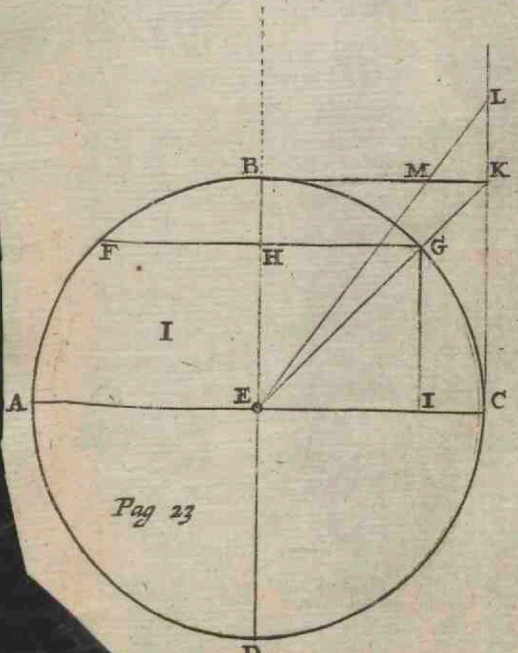
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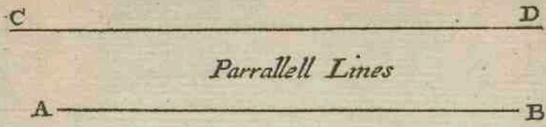
D


C
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Parrallell Lines
A
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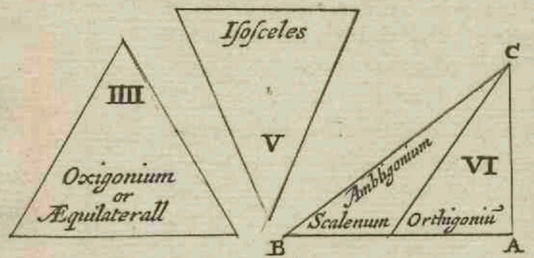
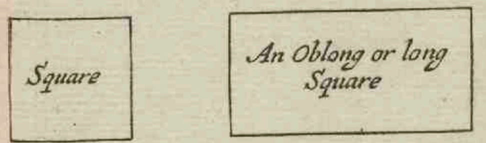
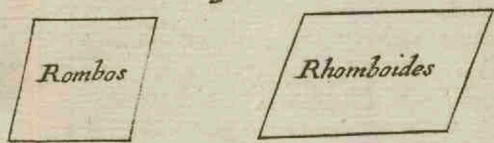
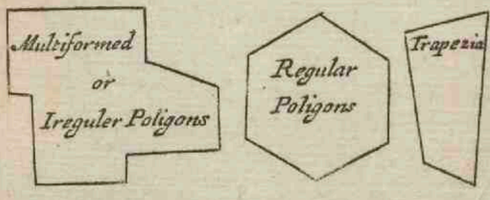

Isosceles
IIII
V
C
Oxigonium
or
Æquilaterall
Ambligonium
VI
Scalenum
Orthigoniū
B
A


Square
An Oblong or long Square

Pag 26


Rombos
Rhomboides


Multiformed
or
Ireguler Poligons
Regular
Poligons
Trapezia

grees, as the Angle C B A, ſheweth in Figure II.

17. A *Right Angle* is alwayes juſt 90 degrees, as you may ſee in the Figures II, and III, by the Angles at A in both of them.

18. An *Obtuce Angle* is alwayes more than 90 degrees, as the Angles at D in both Figures ſhew.

19. A *Parallel Line*, is any Line drawn by another Line in ſuch a way, that though it were infinitely produced, yet they would never meet or croſs one another, as the Lines A B, C D.

20. A *Perpendiculer Line*, is when one Line ſo falleth on another Line, that the Angles on each ſide are equal, as C A falls on the Line B A, Figure VI.

21. All *Triangles* are either with three equal ſides, as Figure IIII, or two equal ſides, as Figure V, or all unequal ſides, as Figure VI; the firſt of which is called *Equilateral*, the ſecond *Iſoſceles*, the third *Scalenum*.

22. Again, they may be ſometimes named from their Angles; thus: *Orthigonium*, with one Right Angle, and two Acute Angles. *Ambligonium*, with one Obtuce Angle, and two Acute Angles. *Oxigonium*, with three Acute Angles only.

23. The

23. The three Angles of every *Plain Triangle*, are equal to two Right Angles.

24. All *Four-ſided Figures* are either Squares, with four ſides, and four right Angles all equal; or long Squares (or Oblongs) with the two oppoſite ſides equal, or the ſame cruſhed together, or not Right-Angled, as the *Rombus*, and *Romboides* or elſe with four unequal Sides, called *Trapeziaes*.

25. Laſtly, many ſided-Figures, are ſome Regular, having every ſide alike, as 5, 6, 7, 8, 9, 10, &c. Or elſe unlike, as Fields, and Woods, and Meadows, which being infinite, cannot be comprehended under any Regular Order or Rule.

26. *Multiplicator*, is a term uſed in Multiplication, by which any Number is to be multiplied, as in ſaying 5 *times* 6, 5 is the Multiplicator of 6.

27. *Multiplicand*, is the Number to be multiplied, as 6 by 5, as above named.

28. The *Product*, is the Iſſue or Reſult of two Numbers multiplied one by the other, as 30 is the Product of 6 multiplied by 5; for 6 times 5 is 30.

29. *Diviſor*, is a term uſed in Diviſion, and is the Number by which another Number is to be divided; as to ſay, How many times 5 in 30? 5 here is the Diviſor.

30. *Dividend*,

30. *Dividend*, is the Number to be divided, as 30 abovesaid.

31. *Quotient*, is the Answer to the How many times (as in the abovesaid) 5 is in 30? 6 times: 6 then is the Quotient.

32. *Square*, is the Product of two Numbers multiplied together, as the Square of 6 multiplied by 6, is 36.

33. *Square-root* of any Number, is that number, which being multiplied by it self, shall have a Product or Square equal to the given Number; thus the Square-root of 36 is 6; for 6 multiplied by 6, is 36, equal to the first given Number.

But if it be a Number that cannot be squared, as 72, the content of half a Foot of Board; whose near Square-root is 8: 4852811 of 10000000, then is the Square-root to be exprest as near as you may (or care for) as here the Square-root of 72, which is called a Surd Number, that will not be squared.

34. *Cube*, is a second Product, or power of two Numbers increasing or multiplied together, as thus; the Square of 6 is 36, the first power: and the Cube of 6 is 216, that is to say 6 times 36, the second power.

35. *Cube-root* of any Number, is a second Quotient decreasing between the Number given to be cubed and 1, as thus; the

Cube-root

Cube-root of 216, is to be found out by the Line of Numbers, the third part of the diſtance between 216 and 1, is at 2 repetitions found to ſtay at 6; for if you ſhould have 216 Cubes or Dies, which is a proper Cube, you ſhall find that 36 laid together one by another, making a Square repeated 6 times, will uſe or take up 216, the juſt number, and make one great ſquare Die; and no other Number whatſoever, except 6, will do the like; therefore 6 is the Cube-root of 216.

In Mr. *Windgate's* Book of *Arithmetick*, is the way of doing it by Numbers or Figures, being one of the hardeſt Leſſons in Arithmetick.

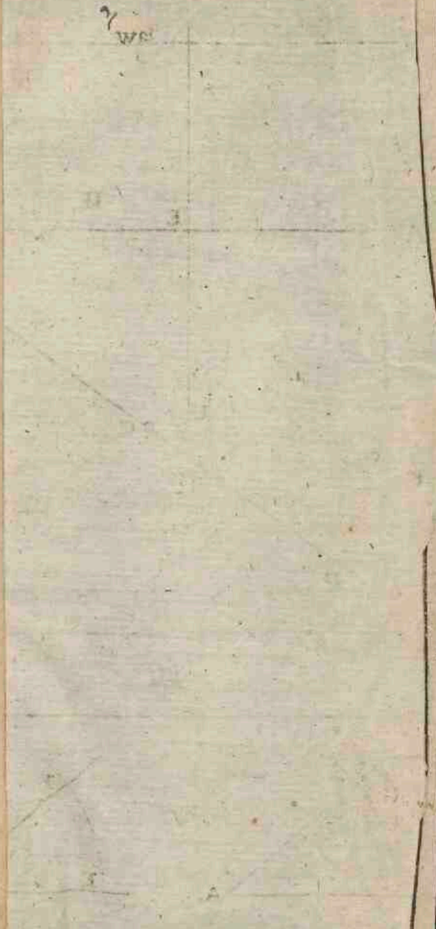
CHAP. III.

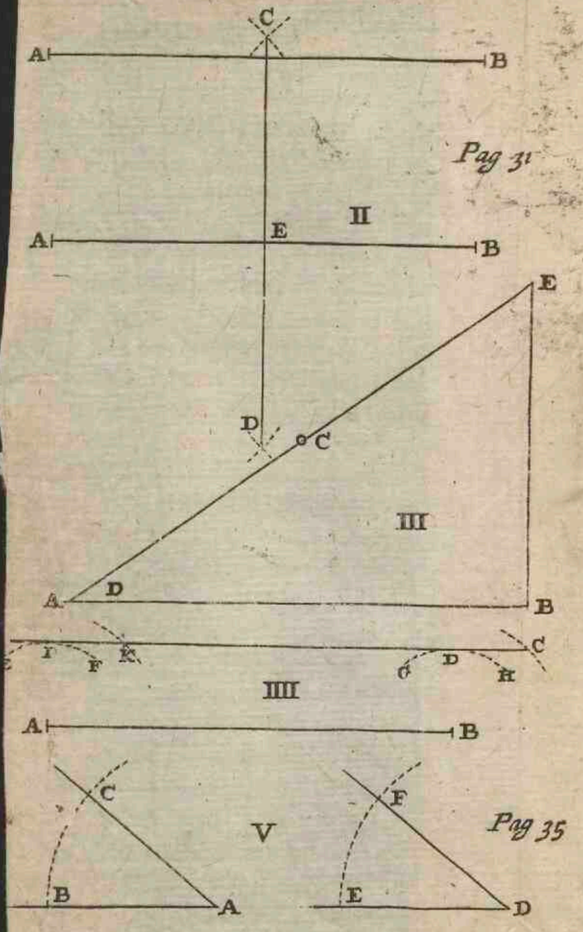
Certain Geometrical Propositions, fit to be known as Preparatory Rudiments for the following Work.

1. *To draw a Right Line between two Points.*

EXtend a Thred or Hair, from one Point to the other, and that ſhall be the Line required. But if you uſe a Rule (being the fitteſt Inſtrument) to try your Rule, do thus; apply one end to one Point as to A, and the other end to the other Point at B, and cloſe to the edge draw the Line required: then turn the Rule, and lay the firſt end to the laſt Point (yet keeping the ſame ſide of the Rule toward the Paper) and draw the Line again, and if the two Lines appear as one, the Rule is ſtreight, or elſe not. Note the Figure I.

2 *To*





2. *To draw a Line Perpendiculer to another, on the middle of a Line.*

On the Point E on the Line AB, I would raiſe the Perpendiculer Line C E, ſet one point of the Compaſſes in E, and open them to any diſtance, as E B and E A, and note the Points A and B, then open the Compaſſes wider, and ſetting one Point in A, make the part of the Arch by C upwards; and if you have room do the like downwards, near D: Then the Compaſſes not ſtirring, ſet one Point in B, and with the other, croſs the former Arks, near C & D: a Rule laid, and a Line drawn, by thoſe two croſſings, ſhall cut the Line A B perpendicuIerly juſt in the Point E, which was required.

3. *To let fall a Perpendiculer from a Point to a Line.*

But if the Point C had been given from whence to let fall the Perpendiculer to the Line A B, do thus: Firſt, ſet one Point of Compaſſes in C; open the other to any diſtance, as ſuppoſe to A and B; and then (if you have room upon A and B, ſtrike both the Arks by D, which finds the Point D, if not)

not) the middle between A and B gives the Point E; by which to draw C E D, the Perpendiculer from C desired. Note Figure 2.

Note, That if you can come to find the Point D, by the crossing, it doth readily and exactly divide the Line A B in two equal parts, by the Point E.

4. *To raise a Perpendiculer on the end of a Line.*

On the end of the Line A B, at B, I would raise a Perpendiculer: First, set one Point of the Compasses in B; open them to any distance, as suppose to C; and set the other Point any where about the middle, between D and E, as suppose at C, then keep that Point fixed there; turn the other till it cut the Line, as at D, and keep both Points fixed there, and lay a streight Rule close to both Points, and there keep it; then keep the middle-Point still fixed at C, and turn the other neatly close to the other end and edge of the Rule, to find the Point E; then a Rule laid to the Points E and B, shall draw the Perpendiculer required.

Or else, when you have set the Compasses in the Point C, prick the Point D in the Line, and make the touch of an Ark near to E; then a Rule laid to DC cuts the Ark laſt made, at or by E, in the Point E: There are other wayes, but none better than this. Note the Figure 3.

5. *From a Point given, to let fall a Perpendiculer to the end of a Line, being the converſe of the former.*

Firſt, from the Point E, draw the Line ED, of which Line find the middle between E and D, *viz.* the Point C: then the extent CE, or CD, keeping one Point in C, ſhall croſs the Ground-line in the Point B, by which, and E, you may draw the perpendiculer Line EB, which is but the converſe of the former.

6. *To draw a Line Parallel to another, at any diſtance.*

To the Line AB, I would have another Parallel thereto; to the diſtance of AI, take AI between your Compaſſes, and ſetting one Point in one end of the Line, as at A, ſweep the Ark EIF; then ſet the Compaſſes in the other end, as at B, and ſweep the Ark GDH; then juſt by the Round-ſide of thoſe Arks, draw a Line,
which

which ſhall be the parallel-Line required.

Or thus,

Take B C, the meaſure from the Point that is to cut the Parallel-line, and one end of the given-Line, *viz.* B; with this diſtance, ſet one foot in A, at the other end of the given-Line, and draw the Arch at K; then take all A B, the given-Line, and ſetting one Point in C, croſs the Ark at K, then C and K ſhall be Points to draw the Parallel-line by. Note the Figure 4.

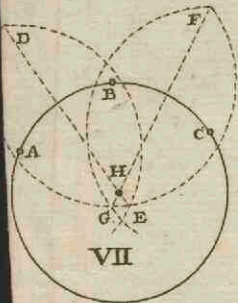
7. To make one Angle equal to another.

The Angle B A C, being given, and I would have another Angle equal unto it; ſet one point of the Compaſſes in A, and draw the Arch C B; then on the Line D E from the Point D, draw the like Ark E F; then in that Ark make E F equal to C B, then draw the Line D F, it ſhall make the Angle E D F, equal to the Angle B A C, which was required.

8. To divide a Line into any Number of parts.

Let A B repreſent a Line to be divided

 into



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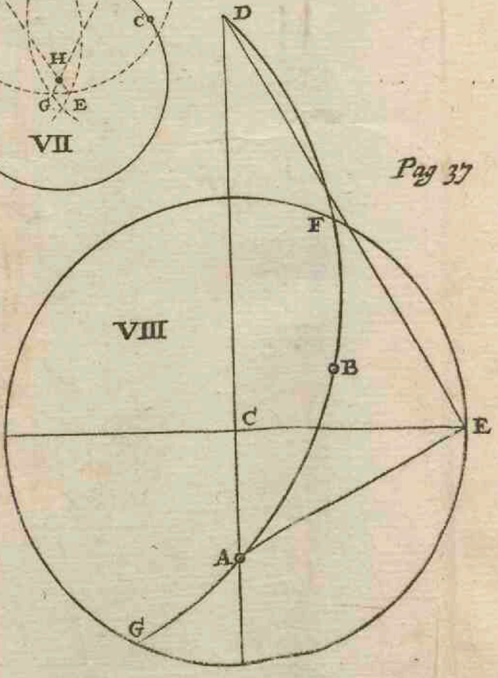


into Eight parts: On one end, *viz.* A, draw a Line, as A D, to any Angle; and from the other end B, draw another Line Parallel to A D, as B E, then open the Compasses to any convenient distance, and from A and B, divide the Lines A D, and B E, into eight parts; then Lines drawn by a Ruler, laid to every division, in the Lines A D, and B E, shall divide the Line A B in the parts required. Note the Figure marked VI.

This Proposition is much easier wrought by the Line of Lines on the Sector, thus; Take A B between your Compasses, and fit it over parrally in 8, and 8 of the Line of Lines; then the Parallel distance between 1 and 1, shall divide A B into 8 parts required.

9. *Any three Points given, to bring them into a Circle.*

Let A B C be three Points to be brought into a Circle; first set one Point on A, and open the other above half-way to C, and sweep the part of a Circle above and below the Point A, as the two Arches at D and E; not moving the Compasses, do the like on C, as the Arks F and G; then set the Compass-point in B, and cross those Arks

Arks in D E F and G; then a Rule laid from D to E, and from F to G, and Lines drawn do inter-ſect at H, the true Center, to bring A B C into a Circle.

10. *Any two Points given in a Circle, to draw part of a Circle, which ſhall cut them, and the Circumference firſt given into two equal parts.*

Let A and B be two Points in a Circle, by which two Points, I would draw an Arch, which ſhall cut the whole Circumference into two equal parts. Firſt, draw a Line from A, the Point remoteſt from the Center, through the Center, and beyond the Circumference, as A D; then draw another Line from A, to a Point in the Circumference, perpendiculer to A D, (and cutting the Center C) as the Line A E: Then on the Point E, draw another Line perpendiculer to the Line A E, till it inter-ſect A D at D; then theſe three Points A B D brought into a Circle, or Arch, by the laſt Rule, ſhall divide the Circumference into two equal parts. Note the Figure 8, where the firſt Circle is cut into two equal parts at F and G, by part of a Circle paſſing through the Points A and B.

 11. *Any*

11. *Any Segment of a Circle given, to find the Diameter and Center of the Circle belonging to it.*

Let A B C be the Segment of a Circle, to which I would find a Center; any where about theниddeſt of the Segment, ſet one point of the Compaſſes at pleaſure, as at B; on the point B (at any meet diſtance) deſcribe a Circle, and note where the Circle doth croſs the Segment, as at D and E, then (not ſtirring the Compaſſes) ſet one point in D, and croſs the Circle twice, as at F and I; and again, ſet one point in E, and croſs the Circle twice in G and H: Laſtly, by the Points G H, and F I, draw two Lines, which will meet in the point O, the center required.

12. *Or elſe to find the Diameter, thus.*

Multiply the Chord (or flat-ſide) of the half-Segment, *viz.* AK, 12 by it ſelf (which is called Squaring) which makes 144; then divide that Product 144 by 8, the Line KB, called a Sine, the Quotient which comes out will be found to be 18; then if you adde 8 the Sine, and 18 the Quotient together, it ſhall make 26 for the

the Diameter required to be found.

13. *Any Segment of a Circle given, to find the Length of the Arch of the Segment.*

Lay the Chord of the whole Segment, and twice the Chord of half the Segment, from one Point ſeverally ; and to the greateſt extent, adde one third part of the difference between the Extents, and that ſum of Extents ſhall be equal to the Arch.

Example.

Let A B C repreſent the Segment of a Circle; the length of whoſe Arch I would know, or have a Line equal thereunto: Take all the Chord A C, and lay it on any Line, as from D to E; alſo take the Chord of half the Arch, as A B, or B C, and lay it twice from D to F; then E and F are the two Extents, whoſe third part F G is to be added to D F the greateſt Extent to make up D G, a Line equal to the Arch A B C, which was required to be done. Which operation, may very well be performed on a Line of Lines, or inches on your Rule, or by Numbers in Figures, thus; Suppoſe A C be 35 inches and 6 tenths; and twice

AB be 42 inches, 7 tenths: The difference between them you may count, on the Rule, to be 7 inches, and 1 tenth; a third part of which is 2 inches 4 tenths; which added to 42, 7, makes 45, 2. the meaſure of the Arch ABC, which was required.

14. To draw a Helical Line from any Three Points, to ſeveral Radiuſſes without much gibbioſity; uſeful for Architects, Shipwrights, and others.

Let ABCDE be five Points, to be brought into a Helical-Line, ſmoothly, and even without gibbioſity or bunches, as the under-ſide of an Arch, or the bending of a Ship, or the like.

Firſt, between the two remote Points of 3, as A and C, draw the Line AC, then let fall a Perpendiculer from B, to cut the Line AC at Right Angles, and produce it to F: draw the like perpendiculer-Line from the point D, to cut the Line CE at Right-Angles produced to F. I ſay, the Center both for the Arches AB the leſſer, and BC the greater, will be found to be in the Line BF; the like on the other-ſide for DE and CD, the Helical-Circle, or Arch required.

But

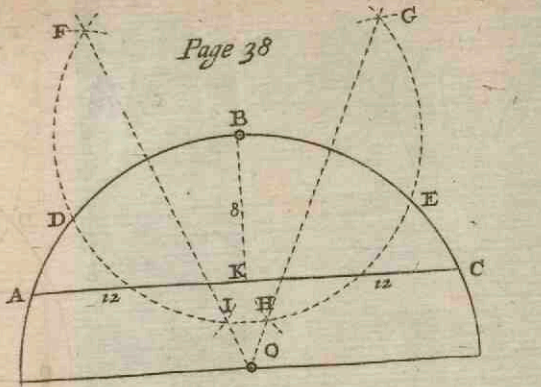
But if you divide the Arch ABCDE into 24 or more parts, the ſeveral Centers of the ſplay-Lines are thus found; Take the meaſure A G, and lay it from B, or D, or C, on the Line G F; and thoſe Points on G F, ſhall be the ſeveral Points to draw the ſplay-Lines of the Arch, and Key-ſtone by.

CHAP. IV.

Of the Explanation of certain Terms uſed in this following Book.

1. R*Adius*, or Sine of 90, or Tangent of 45, or Secant of 00, are all one and the ſame thing, yet taken reſpectively in their proper places, and is the whole Line of Sines, or Tangents, to 45; or more particulary that point at the end of the Natural-Sines, on the Sector-ſide, and at 90 and 45 on the edge of the Rule for the Artificial Sines and Tangents, or 10 on the Line of Numbers, and 10 and 90 on the Line of Lines, and Sines, on the Quadrantal-ſide of the Inſtrument.

2. A



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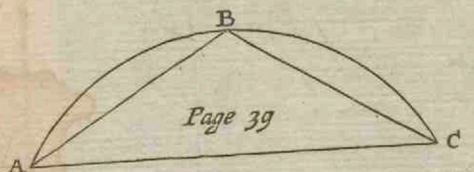

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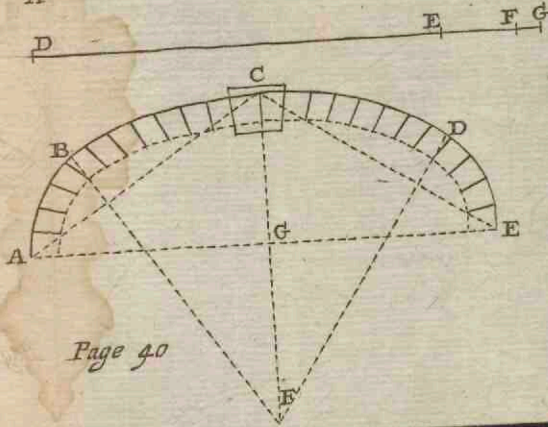

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Right-Sine. 2. A *Right Sine* of any Ark, or Angle, is the measure from the beginning of the Line of Natural-Sines, to that Point on that Line of Sines, which represents the degrees and minutes contained in that Ark or Angle required. But on the Artificial-Sines we respect not any measure but the Point only.

Tangent. 3. The same account is used both for the Right-Tangent, and Secant also; the Natural-Tangent taken from the beginning to the degree and minute required; the Artificial respecting the Point only.

Secant, Chord. 4. In the same manner count for the Secants, and Chords, Lines, or versed Sines.

Cosine. 5. A *Cosine*, or *Sine Complement* of any Ark or Angle, is the measure from the Point representing the Ark or Angle, counted from 90, to the beginning of the Line of Sines, being in effect the Right-Sine of the Cosine of the Ark or Angle required: As for *Example*; I would take out the Cosine of the Latitude of *London*, which is 51, 32; Count 51 32 from 90 toward the beginning, and you shall find your account to end at the Right-Sine of 38 28, which is the Complement of 51 32; for both put together, makes 90, the whole Sine or Radius.

But on the Artificial-Lines count back-

ward

ward to the Point required, without minding any diſtance or meaſure, till you come to Proportion.

6. A *Lateral Sine*, Tangent, or Secant, or Scale of Equal Parts, is any Sine Tangent, or Secant, taken along the length of any Line, from the beginning onwards, being a term uſed only in operation with a Sector, or one Line and a Thred, and opposed to a parallel-Sine, Tangent, or Secant, the thing next to be explained. *Lateral Sine.*

7. A *Parallel* Sine, Tangent, or Secant, is any Sine, Tangent, or Secant, taken acroſs from one Leg to the other of a Sector; or from any degree and minute on one Line to a Thred drawn ſtreight with the other hand, or any other fixed Line whatſoever, at the neareſt diſtance. *Parallel.*

8. The *Neareſt Diſtance* to any Line, is thus taken; When one Point of the Compaſſes ſtands in any one Point, and the Line being laid, I open or cloſe my Compaſſes till the other moveable-Foot, being turned about, will but juſt touch or cleave the Thred. But if you are to lay the Thred to the neareſt diſtance, then one Point of the Compaſſes being ſet faſt, the other is to be turned about, and the Thred alſo ſlipped to and fro, till the Compaſs-point ſhall juſt cleave the Thred in the middeſt. *Neareſt Diſtance.*

9. To

Addition on Lines. 9. To *adde* one Sine or Tangent, to a Sine or Tangent, is to take the Right-Sine, or Tangent of any Ark or Angle between your Compaſſes, and ſetting one Point of the Compaſſes in the Point of the other Number, and then to ſee how far the other Point will extend Laterally. *Example.* To adde the Sine of 20, to the Sine of 30, take the Sine of 20 between your Compaſſes, and then putting one Point in 30, the other ſhall reach to the Sine of 51 21; therefore the diſtance from the beginning to 51 21, is the ſum of the Sine of 30 and 20 added together. The like way is to add Tangents.

Subſtraction on Lines. 10. To *Subſtract* a Sine from a Sine, or a Tangent from a Tangent, is but to take the Lateral leaſt Right-Sine or Tangent between your Compaſſes, and ſetting one Point in the term of the greateſt turn, the other toward the beginning, and note the degree and minute that the other Point ſtayes in, for that is the difference or remainder.

Example.

Suppoſe I would take the Sine of 10 degrees from 25; Take the diſtance 10 between your Compaſſes, and ſetting one foot in 25, and the other turned toward the

the beginning, ſhall reach to 14 23, the reſidue or difference required.

Or, you may ſometimes take the diſtance between the greater and the leſs, and lay this from the beginning, ſhall give the remainder in diſtance on the Sines as before.

11. The *Rectifying-Point*, is a Point or Hole on the Head of the *Trianguler Quadrant* in the inter-ſecting of the hour and Azimuth-line, and the common Line to the Lines and Sines on the Head-leg; in which Point you are, when the Rule is open, to ſtick a ſmall Pin to look to the object whoſe Altitude above the Horizon you would have in degrees and minutes. *Rectifying Point.*

Of Terms uſed in DIALLING.

PLain, is that Board, Glaſs, or flat Superficies you intend to draw the Dial upon, either ſingle of it ſelf, or joyned to ſome other. *Plain.*

Pole of the Plain, is an imaginary Point in the Horizon (for all upright Dials) directly *Pole of the Plain.*

rectly opposite to the Plain, or in all Plains, a Point every way 90 degrees from the Plain.

Declination. *Declination of a Plain*, is only the number of degrees and minutes, that the Pole-point of the Plain is distant from the North and South-points of the Horizon.

Perpendiculer-line on the Plain. The *Perdendiculer-Line on the Plain*, is a Line Square to a Horizontal-line, being part of a Circle passing through the Zenith, and Nadir, and Pole-point of the Plain.

Horizontal-line. The *Horizontal-line*, is a Line drawn on any Plain, exactly parallel to the true Horizon of the place you dwell in.

Reclination and Inclination. *Reclination*, is when a Plain beholdeth the Zenith-point over our heads: But *Inclination*, is, when a Plain beholdeth the Nadier; as in a Roof of a House, the Tiled-part reclines, and the Celid-part inclines.

Meridian-line. The *Meridian-line*, on all Plains is the Hour-line of 12; but the Meridian of the Plain, is the great Circle of Azimuth perpendiculer to the Plain, being the same with the Perpendiculer-line on the Plain, passing through the Points of Declination.

Substile. The *Substile-line* on all Dials, is that Line wherein the Stile, Gnomon, or Cock of the Dial doth stand, usually counted from 12, the Meridian-line, or from the Perpendiculer-line, which in all erect Dials is 12.

The *Stile* of a Dial. is the Angle, between the common Axis of the World and the Plain, upon the Subſtile-line on the Plain, on all Dials. *Stile.*

The *Angle between* 12 *and* 6, is onely the number of degrees and minutes contained between the Hour-line of 12, and the Hour-line of 6 a clock, on any kind of Plain; eſpecially thoſe having Centers. *Angle between 12 and 6.*

The *Inclination of Meridians*, is the number of degrees and minutes, counted on the Æquinoctial, between the Meridian or Hour-line of 12: and the Subſtile being the diſtance, between the Meridian of the place, *viz.* 12 a clock, and the Meridian of the Plain, but counted on the Æquinoctial; and doth ſerve to make the Table of Hour-Arks at the Pole, and to prove your work. *Inclination of Meridians.*

The *Lines Parallel* to 12, are two Lines peculiar to this way of Dialling by the Sector, and are only two Lines drawn equidiſtant from, and parallel to the Hour-line of 12. *Parallels.*

The *Contingent* or Touch-line in this way of Dialling with Centers, is a Line drawn parallel to the Hour-line of 6; but in thoſe without Centers, it is drawn alwayes perpendiculer to the Subſtile, and ſo may it be alſo, if you pleaſe, in thoſe with Centers alſo. *Contingent.*

The

Vertical-line. The *Vertical Line* on the Plain, is the ſame with the Perpendiculer-line on the Plain, being perpendiculer to the Horizontal-line.

Nodus or Apex. By the word *Nodus*, is meant a Knot or Ball, on the Axis or Stile of the Dial, to make a black-ſhaddow on the Dial, to trace out the *Suns* motion in the Heavens; or ſometimes an open or hollow-place in the Stile, to leave a light-place to do the ſame office.

But by *Apex* is meant the ſame thing, when the Top-end, or Point of an upright Stile ſhall ſhew the Hour and *Suns* place, as the Spot doth in Celing-Dials, where the Hours and Quarters are all of one length, and diſtinguiſhed by their cullours or greatneſs only.

Perpendiculer height of the Stile. The *Perpendiculer height of the Stile*, is nothing elſe but the neareſt diſtance from the *Nodus* or *Apex* to the Plain.

Foot of the Stile. The *Foot of the Stile* is properly right under the *Nodus* or *Apex* at the neareſt diſtance.

Virtical-point. The *Vertical-Point*, is a Point only uſed in Recliners and Incliners, being a Point right over, or under the *Apex*; and yet in the Meridian, being let fall from the Zenith, by or through the *Apex* or *Nodus*, to the Plain in the Meridian-line.

The

The *Axis of the Horizon*, is only the measure from the Apex to the Vertical-point last spoken to, being the Secant of the complement of the Reclination to the Radius of the Perpendiculer height of the Stile. *Axis of the Horizon.*

Erect, is when Plains are upright, as all Walls are intended to be. *Erect.*

Direct, is when the Dial-plain beholdeth one of the Four Cardinal Points of the Horizon, as South or North, East or West, that is to say, when the Pole of the Plain, being 90 degrees every way from the Plain, doth lie precisely in one of those Four Cardinal Azimuths: Which in an *Erect* and *Direct-Plain* will be in the Horizon. *Direct.*

Declining, and Reclining, or Inclining-Plaines, are as the upper or under-side of Roofs at any Oblique Scituation from the Cardinal Points of the Horizon. *Declining Reclining or Inclining-plains*

Oblique, is only a wry, slanting, crooked; contrary to direct, right, plain, flat, or perpendiculer; and applied variously, as to the Sphear, to Triangles, to Dial-plains, to Discourse and Conversation. *Oblique.*

Circles of Position, or rather Semi-circles making 12 Houses, are Circles, whose Pole or Meeting-point is in the Meridian and Horizon of every Country, dividing the Æquinoctial into 12 equal parts, being then called Houses, when used in *Astrologie*, and *Circles of Position.*

 some-

ſome times drawn on Sun-Dials.

But when they are uſed in *Aſtronomy*, they require a more near account, as to degrees and minutes.

Of certain Terms in Aſtronomy, and Spherical Definitions of Points and Lines in the Sphear.

NOt to be curious in this matter, a Sphear may be underſtood to be a united Spherical Superficies, or round Body, contained under one Surface; in the middle whereof is a Point or Center, from whence all Lines drawn to the Circumference are equal: Or you may conceive a Sphear to be an Inſtrument, conſiſting of ſeveral Rings or Circles, whereby, the ſenſible motion of the Heavenly Bodies are conveniently repreſented.

For the better Explanation whereof, Aſtronomers have contrived thereon, *viz.* on the Sphear, ten imaginary Points, and ten Circles, which are uſually drawn on Globes and Sphears; beſides others not uſually drawn, but apprehended in the fancy, for

De-

Demonstrations sake, in Spherical Conclusions.

The ten Points are, the two Poles of the World, the two Poles of the Zodiack, the two Æquinoctial Points, the two Solstitial Points, the Zenith, and Nadir.

The ten Circles are, The Horizon, the Meridian, the Æquinoctial, the Zodiack; the two Colures, *viz.* that of the Equinoxes, and that of the Solstices; the Tropick of *Cancer*, and the Tropick of *Capricorn*; and the two Polar-Circles, *viz.* The Artick or North, the Antartick or South, Polar-Circle.

The *first six*, are great Circles, cutting the Sphear into two equal parts: And the *four last* are lesser Circles, dividing the Sphear unequally.

All which Points and Circles shall be represented by the Figure of the Analemma, from whence the *Trianguler Quadrant* is derived, as a general Instrument, and also by the Horizontal projection of the Sphear fitted for *London*, being better for the fancy to apprehend the Mystery of Dialling, one thing mainly intended in this Discourse.

Of the 10 Points in the Sphear.

Poles. THe two Poles of the World, are the
1. two Points P and P in the Analem-
2. ma, being directly oppoſite one to another; about which two Points, the whole frame of the Heavens moveth from Eaſt to Weſt; *one* of which Poles may alwayes be ſeen by us, called the Artick or North-Pole; repreſented in the particular Scheam by the Point P. The *other* being not ſeen, is not repreſented in the particular Scheam; but the Line P E P, in the general Scheam, drawn from Pole to Pole, is called the Axis, or Axeltree of the World, becauſe the whole Sphear appears to move round about it.

Poles of the Zodiack. The Poles of the Zodiack are two Points diametrically oppoſite alſo, upon which Points the Heavens move ſlowly from Weſt
3. to Eaſt, repreſented by the two Points, I
4. and K, 23 degrees and 31 minutes diſtant from the two former Poles, in the Analemma, and by the Point P Z in the Horizontal projection; but the other Pole of the Zodiack cannot be repreſented in that particular Scheam.

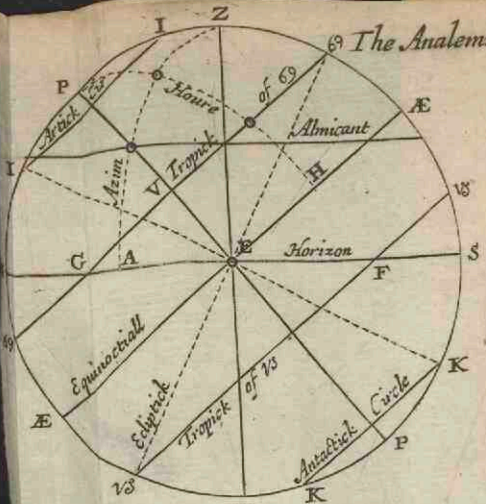
Equinocti-al-Points. The Equinoctial Points, are the Points of *Aries* and *Libra*; to which two Points,
5. when
6.

when the *Sun* cometh along the Eclipitck, it maketh the Dayes and Nights equal in all places; at *Aries* March 10th or 11th; to *Libra* about the 13th of *September*, where the Spring, and Autumn begins; being represented in the Analemma by the Point E, and in the particular Scheam by the Points E, and W.

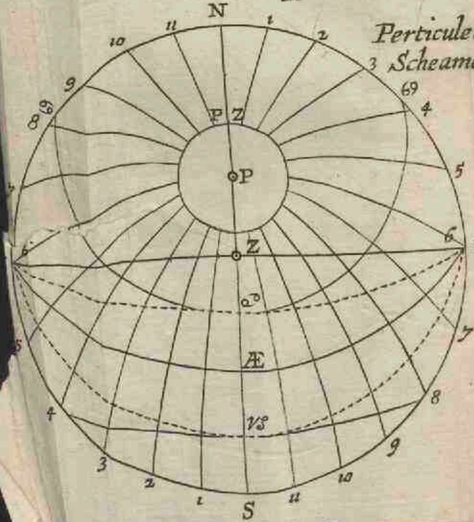
Solsticial Points. The two Solsticial Points, are represented one by the Point ♋, and the other by the Point ♑, in both Scheams; to which 7.
Points when the *Sun* cometh along the E- 8.
cliptick, it makes the Dayes in *Cancer* ♋, longest; in *Capricorn* ♑, shortest; ♋ being about the 11th of *June*; and ♑ about the 11th of *December*.

Zenith. The Zenith is an imaginary Point right over our heads, being every way 90 degrees 9.
distant form the Horizon; in which Point all Azimuth Lines do meet, represented by the Points Z, in both Scheams.

Nadir. The Nadir is an imaginary Point under our feet, directly opposite to the Zenith, 10.
represented by the Point N in the Analemma, but not in the particular Scheam, because it is not seen at any time.



The Analemma



Perticuler Scheame

Of the Circles of the Sphear.

THe *Horizon* is twofold, *viz. Rational*, and *Sensible* : The Rational Horizon, is an imaginary great Circle of the Sphear, every where 90 degrees distant from the Zenith, and Nadir; Points cutting, or dividing the whole Sphear into two equal parts, the *one* called, The upper or visible Hemisphear; the *other* the lower or invisible Hemisphear.

This Rational Horizon, is distinguished also into Right, Oblique, and Parallel-Horizon.

1. The Right Horizon is when the two Poles of the World lie in the Horizon, and the Equinoctial at Right Angles to it; which Horizon is peculiar to those that live under the Equinoctial, who have their Dayes and Nights alwayes equal, and all the Stars to Rise and Set, and the Sun to pass twice in a year by their Zenith-point, thereby making two Winters, and two Summers; Their Winters being in *June* and *December*, and their Summers, in *March* and *September*.

2. The Oblique Horizon is when one Pole-point is visible, and (the other not) having Elevation above, and depression below

low the North or South part of the Horizon, according to the Latitude of the place: in which Horizon when the Sun cometh to the Equinoctial, the Dayes and Nights are only then equal; and the nearer the Sun comes to the viſible Pole, the Dayes are the longer, and the contrary; alſo ſome Stars never ſet, and ſome never riſe in that Horizon: And all Horizons but two, are in a ſtrict ſenſe Oblique Horizons, *viz.* The Right Horizon already ſpoken to: And

The Parallel Horizon, is that Horizon which hath the Equinoctial for its Horizon, and one of the Pole-points for its Zenith; peculiar only to thoſe Inhabitants under the Pole, (if any be there.) In which Horizon, one half of the Sphear doth only alwayes appear, and the other half alwayes is hid; and the Sun, for one half year, doth go round about like a Skrew, making it continual Day, and the other half year is continual Night, and cold enough; which Circle in the Analemma is repreſented by the Line H E S, but in the particular Scheam by the Circle N E S W.

The viſible or ſenſible Horizon, is that Circle where the Heavens and the Earth ſeem to touch, where the ſight of the Sun and Moon doth ſeem to begin, or ceaſe to appear in our ſights, being not much differing in Observation from the true Horizon: and from thence hath been called by *Blagrave*, and others, The *Finitor*, or ender of our sight of the Heavenly Bodies.

Meridian. 2. The *Meridian* is a great Circle which passeth through the two Pole-points, the Zenith and Nadir, and the North and South-points of the Horizon, and is called Meridian, because when the Sun (or Stars) cometh to that Circle, it maketh Mid-day, or Mid-night, which is twice in every 24 hours: Also all places, North and South, have the same Meridian; but places that lie Eastward, or Westwards, have several Meridians. Also, when the Sun or Stars come to the South, or North-part of the Meridian, their Altitudes are then highest, and lowest. And the difference of Meridians is the difference of Longitudes of Places, noted by the Circle Z H N S in the Analemma; and N Z ♋ S in Horizontal-projection.

Equinoctial. 3. The *Equinoctial* is a great Circle, every where 90 degrees distant from the two Poles of the World, dividing the Sphear into two halfs, called the North and South Hemisphear; and is called also the Æquator, because when the Sun passeth by it twice a year, it makes the day and nights equal in all places; noted by W Æ E, and Æ E Æ in both.

The

The *Zodiack*, or Signifer, is another great Circle that divides the Sphear and Equinoctial into two equal parts, whose Poles are the Poles of the Zodiack, being 90 degrees from it; and it inter-sects the Equinoctial in the two Points of *Aries* and *Libra*; and one part of it doth decline Northward, and the other Southward, 23 degrees 31 minutes, as the Poles of the Zodiack decline from the North and South-Poles of the World: The breadth of this Zodiack, or Girdle, is counted 14 or 16 degrees, to allow for the wandring of *Luna*, *Mars*, and *Venus*; the middle of which breadth is the Ecliptick-Line, because all Eclipses are in, or very near in this Line. And this Circle is divided into 12 Signs, and each Sign into 30 degrees, according to their Names and Characters, ♈ *Aries*, ♉ *Taurus*, ♊ *Gemini*, ♋ *Cancer*, ♌ *Leo*, ♍ *Virgo*, ♎ *Libra*, ♏ *Scorpio*, ♐ *Sagittarius*, ♑ *Capricornius*, ♒ *Aquarius*, ♓ *Pisces*. 6 being Northern, and the 6 latter Southern. *Zodiack.* 4.

The two *Colures* are only two Meridians, or great Circles, crossing one another at Right Angles; the one Colure passing through the Poles of the World, and the Points of *Aries* and *Libra*, there cutting the Equinoctial and Ecliptick: And the other Colure *Colures.* 5. & 6.

Colure passeth by the Poles of the World also, and cuts the Ecliptick in ♋, and ♑, making the Four Seasons of the year; that is, the equal Dayes, called the Equinoctial-Colure; and the unequal Dayes, in *June* and *December*, called the Solsticial-Colures, represented in the Analemma by Z P ♋ N S, and P E P; and in the particular Scheam by W P E, and N P S, the Solsticial-Colure.

Tropicks. The lesser Circles are the *Tropicks* of ♋,
7. and ♑; being the Lines of the Suns moti-
& on in the longest and shortest dayes, noted
8. in the Scheams by ♋, ☉, ♋, and 6 ♋ E, and ♑, ♑; and W ♑ 6; to which two Circles when the Sun cometh, it is on the 11th of *June*, and the 11th of *December*, making the Summer and Winter Solstice.

Polar-Circles. The *Polar Circles*, are two Circles drawn about the Poles of the World, as far off as
9. the Poles of the Zodiack are, *viz.* 23 de-
& grees, 31 minutes; That about the North-
10. Pole is called the *Artick*, and that about the South the *Antartick*, being opposite thereunto, shewed in the Analemma by II, and KK; and by the small Circle about P in the particular Scheam.

Of

Of the other Circle imagined, but not described on Spheres or Globes.

1. HOurs are great Circles, passing through the two Poles, and cutting the Equinoctial in 24 equal parts, as the Lines P1,P2,P3,&c. in the Particular; and P ☉ H in the Analemma; such also are degrees of Longitude, and Meridians; the Meridian being the hour of 12. *Hours:*

2. *Azimuths* are great Circles, passing through, or meeting in the Zenith and Nadir-points, numbred and counted on the Horizon, from the Four Cardinal Points of North and South, East and West, according to Four 90ties, or 180 degrees, or according to the 32 Rombs or Points of the Compass, as Z ☉ A, and Z E, the Azimuth of East and West, being called the prime Virtical; *viz.* S E, W Z. *Azimuths*

3. *Almicanters*, or Circles of Altitude, are lesser Circles, all parallel to the Horizon, counted on any Azimuth from the Horizon to the Zenith, to measure the Altitude of the Sun, Moon, or Stars above the Horizon, being the portion of some Azimuth, between the Center of the Sun, or Star; and the Hori- *Almicanters.*

Horizon, commonly called its Altitude above the Horizon, ſhowed by A ☉ in the Analemma, and S Æ in the particular Scheam.

Declination. 4. Parallels of *Declination*, are parallels to the Equinoctial, as the Almicanters were parallel to the Horizon, as ♋ ☉ ♋, the greateſt Declination or Circle of ♋: Theſe parallels have the 2 Poles of the World for their Centers, and in reſpect of the Sun or Stars, are called degrees of Declination; but in reſpect of the Earth, degrees of Latitude; being the Arch on the Meridian of any place, between the Pole and Horizon, as 4 ♋ 4 in the Particular, and HP in the Analemma.

Latitude. 5. Parallels of *Latitude*, in reſpect of the Stars, are Lines drawn parallel to the Ecliptick, as the Almicanters were parallel to the Horizon; ſo that the Latitude of a Star is counted from the Ecliptick toward the Poles of the Zodiack; but the Sun being alwayes in the Ecliptick, is ſaid to have no Latitude.

Longitude. 6. Degrees of *Longitude*, in reſpect of the Heavens, are meaſured by the degrees on the Ecliptick, from the firſt point of *Aries* forward, according to the ſucceſſion of the 12 Signs of the Zodiack.

But

But *Longitude* on the Earth, is counted on the Equinoctial Eastwards, from some principal Meridian on the Earth, as the Isles of *Azores*, or the Peak of *Tenneriff*, or the like.

7. *Right Ascention* is an Arch of the Equinoctial (counted from the first Point of *Aries*) that cometh to the Meridian with the Sun, Moon, or Stars, at any day, or time of the year, being much used in the following discourse, noted in the Analemma by EH, or the like; but counted as afterward is shewed. *Right Ascention.*

8. *Oblique Ascention* is an Arch of the Equinoctial, between the beginning of *Aries*, and that part of the Equinoctial that riseth with the Center of a Star, or any portion of the Ecliptick in an Oblique-Sphear. *Oblique-Ascention.*

9. *Ascentional Difference*, is the difference between the Right and Oblique Ascention, to find the Sun or Stars rising before or after 6. *Ascentional Difference.*

10. *Amplitude* is an Arch of the Horizon, between the Center of the Sun and the true East-point, at the very moment of Rising, represented by ♋ F, in the particular Scheam, and G E, and F E in the Analemma: useful at Sea. *Amplitude.*

11. A

Circles and Angles of Position. 11. A *Circle of Position* is one of the 12 Houses in Astronomy or Astrology.

12. An *Angle of Position*, is the Angle made in the Center of the Sun, between his Meridian, or Hour, and some Azimuth, as the Prime, Vertical, or the Meridian, or any other Azimuth, being useful in Astronomy, and sometimes in Calculation, represented bp P ☉ Z in the Analemma.

Thus much for Astronomical terms.

CHAP. V.

Some Uses of the Trianguler Quadrant.

Use I.

And first to rectifie the Rule, or make it a Trianguler Quadrant.

First open the Rule, and put in the loose piece into the two Mortice-holes, (which putting together makes it a *Trianguler Quadrant*) but if you do not use the loose-piece, then open it to an Angle of 60 degrees, which is thus exactly done: Measure from the Rectifying-point, to any Number

ber on the Sines or Lines; then keeping the Point of the Compasses still fixed in the Rectifying-point, turn the other to the Common-Line of the Hour and Azimuth-Line, that cuts the Rectifying-point, and there keep it; then removing the Point of the Compasses from the Rectifying-point, open or close the Rule till the other Point shall touch the distance first measured in the Line of Sines or Lines, then shall you see the Lines on the Head, and Moveable-leg, to meet; and also see quite through the Rectifying-point, to thrust a Pin quite through; and thus is it set to an Angle of 60 degrees, without the help of the loose-piece, or to an Angle of 45, or whatsoever else the Rule is made for.

Use II.

To observe the Sun or a Stars Altitude above the Horizon.

Put a Pin in the Center-hole on the Head-Leg, and another in the Rectifying-point, and a third (if you please) in the end of the Hour-line on the Moving-leg. Then on the Pin in the Leg-center, hang a Thred and Plummet; then if the object be low, *viz.* under 25 degrees high; Look along by the two Pins in the Rectifying-

point,

point, and the Moving-leg, and ſee that the Plummet playeth evenly and ſteady, then the degrees cut by the Thred, ſhall be the Altitude required, counting from 60|0 toward the Head, as the ſmaller Figures ſhew.

But if the Object be above 25 degrees high, then look by the Pin in the Rectifying-point, and that on which the Plummet hangeth; and obſerve as before, and the Thred ſhall ſhew the Altitude required, as the Figures before the Line ſheweth; If you have Sights, uſe them inſtead of Pins; and by Practice learn to be accurate in this Work, the ground and foundation in every Obſervation; and according to your exactneſs herein, is the following Work alſo.

Note alſo, that this looking up toward the Sun, is only then when the Sun is in a cloud, and may be ſeen in the Abiſs, but will not give a clear ſhadow: Or elſe you muſt uſe a piece of Red, or a Blue, or Green Glaſs, to darken the luſter that it offend not the eyes.

But if the Sun be clear and bright, then you need not look up toward it, but hold the *Triangular Quadrant* ſo, that the ſhadow of the Pin in the Center may fall juſt on the ſhadow of the Pin in the Rectifying-point, and both thoſe ſhadows on your finger

finger beyond them, and the Plummet being somewhat heavy, and the Thred ſmall and playing evenly by the Rule, then is the Obſervation ſo made, likely to be near the very truth.

Note alſo further, That the ſhaking of the hand, you ſhall find will hinder exactneſs; therefore, when you may, find ſome place to lean your Body, or Arm, or the Inſtrument againſt, that you may be the more ſteady.

But the ſureſt and beſt way is with a Ball-ſocket, and a Three-leg-ſtaff, ſuch as Land Surveighers uſe to ſupport their Inſtruments withal, then you will be at liberty to move and remove it, to and fro, till the Sights or Pins, and Plummet and Thred play to exactneſs; without which care and exactneſs, you cannot certainly and knowingly attain the Sun's or a Star's Altitude to a minute, either by this nor any other Inſtrument whatſoever, though they be never ſo truly made: Yet I dare affirm to do it, or it may be done as well by this, as by any other graduated Inſtrument whatſoever: The Line of degrees on this, being only two thirty degrees of a Tangent laid together; of which, that on the in-ſide of the looſe-piece is the largeſt, and conſequently the beſt; to diſtinguiſh the minuts of a degree withal.

Use III.
To try if any thing be Level, or Upright.

Set the Moving-leg of the *Trianguler Quadrant* on the thing you would have to be Level; then if the Thred play just on 60 degrees, or the stroke by 60|0, then is it Level, or else not.

But to try if a thing be upright or not; apply the Head-leg to the Wall or Post, and if it be upright, the Thred will play just on the common Line between the Lines and Sines on the Head-leg, and cut the stroke by 90 on the Head of the Instrument, or else not.

Use IV.
To find readily what Angle the Sector stands at, at any opening.

First, on the Sector side, about the Head, is 180 degrees, or twice 90 graduated to every two degrees; so that opening the Rule to any Angle, the in-side of the Moving-leg, passing about the semi-circle of the Head, sheweth the Angle of opening to one degree. But to do it more exactly, do thus:

The two Lines of Sines that iſſue from the Center in Rules of a Foot, ſhut, are drawn uſually juſt 5 degrees aſſunder; or rather the two innermoſt Lines, on each Leg, are always juſt one degree from the in-ſide, ſo that if you put a Center-pin in the Line of Tangents, juſt againſt the Sine of 30, it makes the two innermoſt Lines that come from the Center, juſt 2 degrees aſ-ſunder, which is eaſie to remember either in adding or ſubſtracting as followeth, two wayes.

1. Take the Latteral Sine of 30, *viz.* the meaſure from the Center to 30: the Compaſſes ſo ſet, ſet one Point in the Center-pin in the Tangents juſt againſt 30; and turn the other till it cut the common Line, in the Line of Sines on the other Leg, and there it ſhall ſhew what Angle the two innermoſt-Lines make, counting from the end toward the Head, and two degrees leſs is the Angle the Sector ſtands at, both on the in-ſide and out-ſide, the Legs being parallel; which Number muſt nearly agree with what the in-ſide of the Leg cuts on the Head-ſemicircle, or there is a miſtake.

As thus for Example.

Suppose I open the Rule at all adventures; and taking the Latteral Sine of 30 from the Sines on the Sector-side, and putting one Point of the Compass in the Center on the Tangents, right against the Sine of 30 on the other Leg (or the beginning of the Secants on the same Leg) and turning the other Point to the Line of Sines on the other Leg, it cuts the Sine of 60 on the innermost Line that comes from the Center; then I say, that the Lines of Sines and Tangents are just 30 degrees assunder, and the in-side or out-side of the Legs but 28, *viz.* two degrees less, as a glance of your eye to the Head will plainly shew.

2. This way will serve very well for all Angles above 20, and under 80: But for all under 20, and above 80, to 120, this is a better way;

Open the Rule to any Angle at pleasure, and take the distance parallelly (that is, across from one Leg to the other) between the Center-pin at 30 in the Sines, and that in the Tangents right against it, and measure it latterally from the Center, and it shall shew the Sine of half the Angle the Sines and Tangents stand at; and one degree less is the Sine of half the Angle the Sector ſtands at.

Example.

Suppoſe that opening the Sector at adventures, or to the Level of any thing, I would know the Angle it ſtands at: I take the parallel Diſtance between the two Centers; and meaſuring it latterally from the Center, I find it gives me the Sine of 51 degrees, *viz.* the half Angle the Lines ſtand at; or 50, the Angle the Rule ſtands at; which doubled, is 102 for the Lines, or 100 for the Legs of the Sector, as a glance of the eye preſently reſolves by the inner-edge of the Moving-leg, and the divided ſemi-circle.

3. On the contrary, Would you ſet the Legs or Lines to any Angle, take the half thereof latterally, or one degree leſs in the half for the Legs, and make it a Parallel in the two Centers, and the Sector is ſo ſet accordingly.

Example.

I would ſet the Legs to 90 degrees, or a juſt Square: take out the Latteral Sine of 44, one degree leſs than 45, the half of 90, and make it a Parallel in the two Centers aboveſaid, and you ſhall find the Legs ſet juſt to a Square, or Right-Angle, as by looking to the Head you may nearly ſee.

At the ſame time if you take Latteral 30, and lay it from the Center, according to the firſt Rule, you ſhall ſee a great deficiency therein, as above is hinted.

Uſe V.

The Day of the Month being given, to find the Suns Declination, true place in the Zodiack, Right Aſcention, Aſcentional Difference, or Riſing and Setting.

1. Lay the Thred to the Day of the Month (in the upper Line of Months, where the length of the Dayes are increaſing; or in the lower-Line, when the Dayes are decreaſing, according to the time of the year) then in the Line of degrees you have his Declination; wherein note, that if the Thred lie on the right hand of 60]0, then the Suns Declination is Northwards; the contrary-way is Southwards: Alſo on the Line of the Sun's Right Aſcention, you have his Right Aſcention, in degrees and hours, (counting one Hour for 15 degrees) as the Months proceed from *March* the 10th, or Equinoctial, the Right Aſcention being then 00, and ſo forward to 24 hours, or 360 degrees, as the Months and Dayes proceed.

Again,

Again, on the Line of the Sun's true place, you have the sign and degree of his place in the Ecliptick; *Aries*, or the Equinoctial-point being the place to begin, and then proceeding forward as the Months and Dayes go.

Lastly, on the Hour-line you have the Ascentional-difference, in degrees and minutes, counting from 6; or the Suns Rising, counting as the morning hours proceed; or his Setting, counting as the afternoon hours proceed.

Of all which, take two or three Examples.

1. For *March* the 12th, lay the Thred to the Day, and extend it streight; then on the Line of degrees, it sheweth near 1 degree, or 54 minutes Northward.

2. The *Suns Right Ascention*, is in time 8 minutes and better, or in degrees, 2 deg. 5 minutes.

3. The *Suns Place*, is 2 degrees and 16 minutes in *Aries*, ♈.

4. The *Ascentional Difference*, is 1 degree and 10 minnutes; or the Sun riseth 4 minutes before, and sets 4 minutes after 6.

Again, for *May* the 10th, the Thred laid thereon, cuts in the degrees, 20 deg. 9 min. for Northern Declination; and 57 deg. 24 min. or 3 hours 52 min. Right Aſcention; and 29 37 in ♉ *Taurus* for his true place; and 27 12 for difference of Aſcentions, or riſeth 11 minutes after 4, and ſets 49 minutes after 7.

Again, on the *laſt of October*, or the 21 of *January*, near the Declination, is 17 22 Southwards, the Right Aſcention for *October* 31, is 225, 53, for *January* 21, 314 21: The true place for *October* 31, is ♏ *Scorpio*, 18 deg. 22 min; but for *January* 21, ♒ *Aquarius* 11, 52; according as the Months go to the end at ♑, and then back again; but the Aſcentional difference, and Riſing and Setting, is very near the ſame at both times, *viz.* 23 10, and Riſeth 32 minutes, and more, after 7; and Sets 28 minutes leſs after 4.

Uſe VI.

The Declination of the Sun, or a Star, given, to find his Amplitude.

Take the *Declination*, being counted on the particular Scale of Altitudes, between your Compaſſes; and with this diſtance, ſet one foot in 90 on the Azimuth-Line, the other

other Point applied to the ſame **Line**, ſhall give the Amplitude, counting from 90.

Example.

The *Declination* being 12 North, the Amplitude is 19 deg. 15 min. Northwards. Or the *Declination* being 20 South, the Amplitude is 34 deg. 10 min. Southwards.

Uſe VII.

The Right Aſcention and Aſcentional-difference being given, to find his Oblique-Aſcention.

When the *Declination* is North, then the difference between the Right Aſcention, and the Aſcentional-difference, is the Oblique-Aſcention.

But in *Southern declinations*, the ſum of the Right Aſcention, and difference of Aſcentions, is the Oblique Aſcention.

Example.

On or between the 25 and 26 of *July*, the Oblique-Aſcention is by Subſtraction 112, 15: On the 30th of *October*, the Oblique-Aſcention is ~~137~~, 45 by Addition.

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Use VIII.

The Day of the Month, or Sun's Declination and Altitude being given, to find the Hour of the Day.

Take the Suns Altitude, from the particular Scale of Altitudes, ſetting one Point of the Compaſſes in the Center, at the beginning of that Line; and opening the other to the degree and minute of the Sun's Altitude, counted on that Line; then lay the Thred on the Day of the Month (or Declination) and there keep it: Then carry the Compaſſes (ſet at the former diſtance) along the Line of Hours, perpendiculer to the Thred, till the other Point, being turned about, will but juſt touch the Thred; the Compaſſes ſtanding between the Thred and the Hour 12, then the fixed Point in the Hour-Line ſhall ſhew the hour and minute required; but whether it be the Fore or Afternoon, your judgment, or a ſecond obſervation muſt determine.

Example.

On the *firſt of Auguſt* in the morning, at 20 degrees of Altitude, you ſhall find it to be juſt 52 minutes paſt 6; but at the ſame

Altitude

Altitude in the afternoon, it is 8 minutes past 5 at night, in the Latitude of 51 32 for *London*.

Use IX.

The Suns Declination and Altitude given, to find the Suns Azimuth from the South-part of the Horizon.

First, by the 4th Use, find the Suns Declination, count the same on the particular Scale, and take the distance between your Compasses; then lay the Thred to the Suns Altitude, counted the same way as the Southern-Declination is from 60|0, toward the loose-piece; and when need requires on the loose-piece, then carry the Compasses along the Azimuth-line, on the right-side of the Thred, that is, between the Thred and the Head, when the Declination is Northward; and on the left-side of the Thred, that is, between the Thred and the End, when the Declination is Southward. So as the Compasses set to the Declination, as before, and one Point staying on the Azimuth-line, and the other turned about, shall but just touch the Thred at the nearest distance; then, I say, the fixed-Point shall, in the Azimuth-line, shew the Suns-Azimuth required.

Example

Example 1.

The Sun being in the Equinoctial, and having no Declination, you have nothing to take with your Compasses, but only lay the Thred to the Altitude counted from 60|0 toward the loose-piece, and in the Azimuth-line it cuts the Azimuth required.

Example. At 25 degrees high, you shall find the Suns Azimuth to be 54, 10; at 32 degrees high, you shall find 38, 20, the Azimuth.

Again, At 20 degrees of Declination, take 20 from the particular Scale, and at 10 degrees of Altitude, lay the Thred to 10 counted as before; then if you carry the Compasses on the right-side for North-Declination, you shall find 109, 30, from South; but if you carry them on the left-side for South-Declination, you shall find 38, 30, from South.

The rest of the *Uses* you shall have more amply afterwards.

CHAP.

CHAP. VI.

The use of the Line of Numbers on the Edge, and the Line of Lines on the Quadrantal-side, or on the Sector-side, being all as one.

HAving shewed the way of Numeration on the Lines, as in *Chapter* the first. Also to add or substract one Line or Number to or from one another, as in *Chapter* 4th, Explanation the 9th. I come now to work the Rules of Multiplication and Division, and the Rule of Three, direct and reverse, both by the Artificial and Natural-Lines; and first by the Artificial, being the most easie; and then by the Natural-lines both on the Sector and Trianguler Quadrant, being alike: and I work them together; First, because I would avoid tautology: Secondly, because thereby is better seen the harmony between them, and which is best and speediest. Thirdly, because it is a way not yet, as I know of, gone by by any other. And laſt of all, becauſe one may explain the other; the Geometrical Figure being the ſame with the Inſtrumental-work by the Natural way.

Sect. I.

To multiply one Number by another.

1. By the Line of Numbers on the Edge Artificially, thus:

Extend the Compaſſes from 1 to the *Multiplicator*; the ſame extent applied the ſame way from the *Multiplicand*, will cauſe the other Point to fall on the Product required.

Example.

Let 8 be given to be multiplied by 6; If you ſet one Point of the Compaſſes in 1, (either at the beginning, or at the middle, or at the end, it matters not which; yet the middle 1 on the Head-leg, is for the moſt part the moſt convenient) and open the other to 6, (or 8, it matters not which, for 6 times 8, and 8 times 6, are alike; (but yet you may mind the Precept if you will) the ſame Extent, laid the ſame way from 8, ſhall reach to 48, the Product required; which.

which, without these Parenthesis, is thus ;

The Extent from 1 to 6, shall reach the same way from 8 to 48. Or, The Extent from 1 to 8, shall reach the same way from 6 to 48.	the Product required.

By the Natural-Lines on the Sector-side, or Triangular Quadrant with a Thred and Compasses, the work is thus ;

1. For the most part it is wrought by changing the terms from the Artificial way, as thus ;

The former way was, as 1 to 6, so is 8 to 48 ; or as 1 to 8, so is 6 to 48 ; but by the Sector it is thus : As the Latteral 6 taken from the Center toward the end, is to the Parallel 10 & 10, set over from 10 to 10, at the end counted as 1 ; so is the Parallel-distance between 8 & 8, on the Line of Lines taken a-cross from one Leg to the other, to the Latteral-distance from the Center to 48, the Product required.

Or shorter thus.

As the Latteral 8, to the Parallel 10 ;
So is the Parallel 6, to the Latteral 48.

See Figure I.

2. Another way may you work without altering the terms from the Artificial way, as thus, by a double Radius; Take the Latteral-Extent from the Center to 1, (or from 10 to 9, if the beginning be defective) make this a Parallel in 6 & 6, then the Latteral-Extent from the Center to 8 of the 10 parts between Figure and Figure, ſhall reach acroſs from 48 to 48, as before. *See Fig.*II.

The ſame work as was done by the Sector, is done by the Line of Lines, and Thred on the Quadrant-ſide, that if your Sector be put together as a Trianguler Quadrant, you may work any thing by it, as well as by the Sector, in this manner; (or by the Scale and Compaſs, as in the Figure I.) and firſt, as above, Sector-wiſe.

Take the Extent from the Center to 6 latterally, between your Compaſſes; ſet one Point in 10, and with the other lay the Thred in the neareſt diſtance, turning the Compaſs-Point about, till it will but juſt touch the Thred, then there keep it; then ſet one Point of the Compaſſes in 8, and take the neareſt diſtance to the Thred; this diſtance laid latterally from the Center, ſhall reach to 48 the Product.

Or as Latteral 6, to Parallel 10; ſo is the Parallel 8, to the Latteral 48, the Product required.

Or the laſt way by a double Radius, or a greater and a ſmaller Scale, as the Latteral Extent from the Center to 1, is to the Parallel 6, laying the Thred to the neareſt diſtance ; ſo is the latteral Extent from the Center to 8 parts, leſs than the 1 before taken, carried parallelly along the common-line, till the other Point will but juſt touch the Thred, it ſhall on thoſe conditions ſtay only at 48, the Product required. Obſerve and note the Figure, by protraction, with Scale and Compaſs.

3. But if you have an Index and a Square, as is uſed in the Demonſtrative Work of *Plain Sayling*, as you ſhall have afterwards, then the repreſentation of this Natural-way will moſt evidently appear, as thus :

Set the edge of the Square to 1 on the Line of Lines, counted from the Center downwards, where the Figure 1 is, then move the Index till the edge cut 8 of the ſmall parts, counted on the Square, from the Line of Lines toward the end of the Square, and then there keep the Index ; then remove the Square downward to 6, on the Line of Lines ; then there holding it ſquare, you ſhall ſee the Index to cut 48 on the Square, counting after the ſame rate that the 8 parts was accounted, Note the

Figure III. for illuſtration ſake.

From hence you may obſerve, That the firſt and third Numbers muſt alwayes be accounted alike, and on like Scales; and the ſecond and the fourth in like manner on like Scales and counting; and the Latteral-firſt Number, muſt alwayes be leſs than the Parallel-ſecond, in length or quantity, or you cannot work it; which you muſt make ſo, either by changing the terms, or uſing a leſs Scale, to begin and end upon.

Here you muſt except a Decimal gradation, as thus; ſometimes the ſame place which is called 10 in the firſt, may be counted 100 in the third, and the contrary; or more or leſs differing in a Decimal Account.

But if you would ſee a Figure of the Sector-way of operation, then it is thus; Let the Line C 6, repreſent one Leg of the Sector; and the other Line C 6, repreſent the other Leg of the Sector; then take 1 out of any Scale, as 1 inch, or one tenth part of a foot, or what you pleaſe: or the diſtance from the Center to 1, or 2, on the Line of Lines between your Compaſſes: put this diſtance over in 6 & 6, of the Line of Lines. Then is the Sector ſet to its due *Iſoſcles* Angle.

Then take 8 parts, or rather 8 tenth

parts,

parts of the former 1, from the Scale from whence you took the first Latteral distance, and carry it parallel between the Line of Lines till it stay in like parts, which you shall find to be at 48, the Product required.

Or to get the Answer in a Latteral-line, is generally most convenient, by changing the terms; work thus: Take the Latteral-distance from the Center to 8, on the Line of Lines, make it a Parallel in 10 & 10; then the Sector being so set, take the Parallel-distance between 6 & 6, and lay it latterally from the Center, and it shall reach to 48, the Product required. *See Fig.* IIII.

Thus you see that the way of the Sector-side, and of the Quadrant-side, is in a manner all one; and the laying of the Thred, or Index to the nearest distance, is the same with setting the Legs of the Sector to their Angle; and the taking the nearest distance from any Point or Number to the Thred, is the same with taking parallelly from Point to Point, or from Number to Number: So that having thus fully explained the Latteral and Parallel-Extent, and laying of the Thred, and setting of the Sector, the following Propositions will be more easie, and ready; and to that purpose, these brief Marks for Latteral, Parallel, and Nearest-diſtance, will frequently be uſed; as thus, for Latteral, thus –; for Parallel, thus =; for Neareſt-diſtance, thus |, or N D, thus: for the Sine of 90, or Radius, or Tangent of 45, thus R; &c.

In all which wayes you may ſee, that for the want of ſeveral Radiuſſes, which do properly expreſs the unites, tens, hundreds, and thouſands, and ten thouſands of Numbers, there muſt a due and rational account, or conſideration, go along with this Inſtrumental manner of work, elſe you may give an erroneous anſwer to the queſtion propounded; to prevent which, obſerve, that in Multiplication there muſt be for the moſt part, as many figures in the Product, as is in the Multiplicator and Multiplicand, put together; except when the firſt figures of the Product, be greater than any of the firſt figures of the Multiplicator, or the Multiplicand, and then there is one leſs;

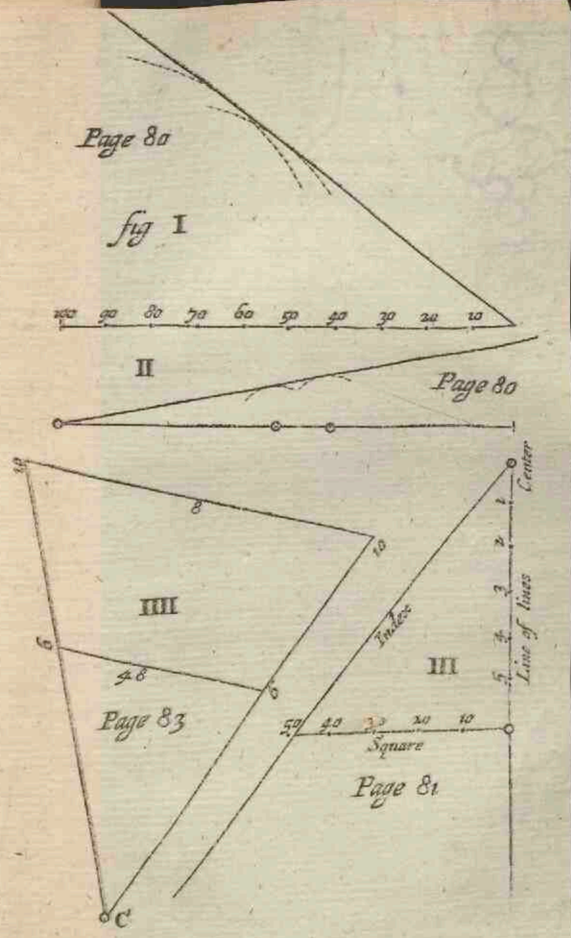
As for Example.

2 times 2 makes 4, being only one figure, becauſe 4 is greater than 2; but 2 times 5 is 10, being two figures; wherein 1, the firſt figure, is leſs than 5. Again, in a bigger ſum; 52 multiplied by 23, makes 1300, conſiſting of four figures, as many as

is in the Multiplier and Multiplicand put together; but if you multiply 42 by 22, it makes but 924, which is but three figures; because the first figure 9 is greater than 2 or 4, as in the former, the first figure 1 was less than 5 or 2: And this Rule is general, as to the number of places, or figures, in any Multiplication whatsoever; but note, that no Instrument extant, and in ordinary use, is capable to express above 5 or 6 places: Yet with this help you may come true to 5 places, with a good Line of Numbers.

As thus;

Suppose I was by a Line of Numbers to multiply 168 by 249; the extent from 1 to 168, will reach the same way from 249, to 41832: Now by the Line of Numbers, you can only see but the 418, and estimate at the 3; but the last figure 2, I cannot see by any Line usually put on two foot Rules, therefore the 168, and 249 being before you, say (according to the vulgar Rules of Multiplication) 9 times 8 is 72; therefore 2 must needs be the last figure; and if you can see the former 4, you have the Product infallibly true: if not, multiply a figure more; But by this help you shall be sure



to come right alwayes to 4 figures, or places, in any Multiplication whatſoever.

4. Alſo, by theſe operations, you may plainly ſee, That the Line of Numbers, or *Gunters*-Line, as it is uſually called, is the eaſieſt and exacteſt for Arithmetical operations, being performed with an Extent of the Compaſſes only, without any opening or ſhutting of the Rule, or laying a Thred or Index ; But in Queſtions of *Geometry*, where a lively draught or repreſentation is required, as to the reaſon of the work, there the Natural-Lines are more demonſtrative.

In which Natural Work, Note, That the Parallel-diſtance muſt alwayes be the greateſt, or you cannot work it, unleſs you make uſe of a greater, and a leſſer Scale ; to which purpoſe, this Inſtrument is well furniſhed, with three or four Radiuſſes, bigger and leſs, both of Sines, Tangents, and Secants, and Equal-Parts, as in their due places ſhall be obſerved ; and taken notice of, in the *Aſtronomical*-Work.

And note alſo, That if the Line of Lines were repeated to 4 Radiuſſes, or 400 inſtead of 10, you might work right-on to 4 figures ; but then the Radiusses must be very small, or the Instrument very large.

Therefore this of 10, being the most usual, I shall make use of, and work every Question the most convenient way; that by a frequent Practice, the young Beginners, for whom only I write, may see the Reason and Nature of the Work, and the sooner understand it.

For a Conclusion of this Rule of Multiplication, take three or four Examples more, both by the Line of Numbers, and Equal-Parts also;

1. First 15 foot, 8 tenths, multiplied 9 foot and 7 tenths, by the Line of Numbers; the Extent of the Compasses from 1 to 9 foot, 7 tenths, shall reach the same way from 15--8, to 153, 26.

By the Line of Lines, or Equal Parts.

As the Latteral — 15--8, to the Parallel = 10, at the End counted as 1;
So is the Parallel = 9--7, to the Latteral — 153--26.

Where you may observe, that the first and fourth, are measured on like Scales; and the second and third, also on like Scales.

But note, that as you diminished the account in the third = work, counting 9--7 less than 10 reckoned as 1:

So likewise in the — fourth, you count 153--26, which is less in Extent than the 15--8, first taken Latterally; yet is to be read as before, *viz.* 153--26, because 9 times 15, must needs be above 100.

2. As 1, to 9 foot 10 inches; So is 10 foot 9 inches, to 105 foot 6 inches $\frac{1}{2}$.

To work this properly by the Line of Numbers, you are first by the inches and foot-measure on the in-side of the Rule, to reduce the inches into decimals of a foot; as thus: Right against 10 inches, in the Line of Foot-measure, you shall find 83 $\frac{1}{3}$. Also, right against 9 inches, on the Foot-measure, you shall find 75; this being done, which is with a glance of your eye only, on those two Lines, then the work is thus:

As 1, to 10 75; so is 9 83, to 105 $\frac{1}{2}$, or 105 foot, 6 inches: Now for the odd 6 square inches, you cannot see them on the Rule, but must find them by the help before mentioned, as thus, Having set down the 9--10, & the 10--9, as in the Margent, say by vulgar Arithmetick thus, Ten times

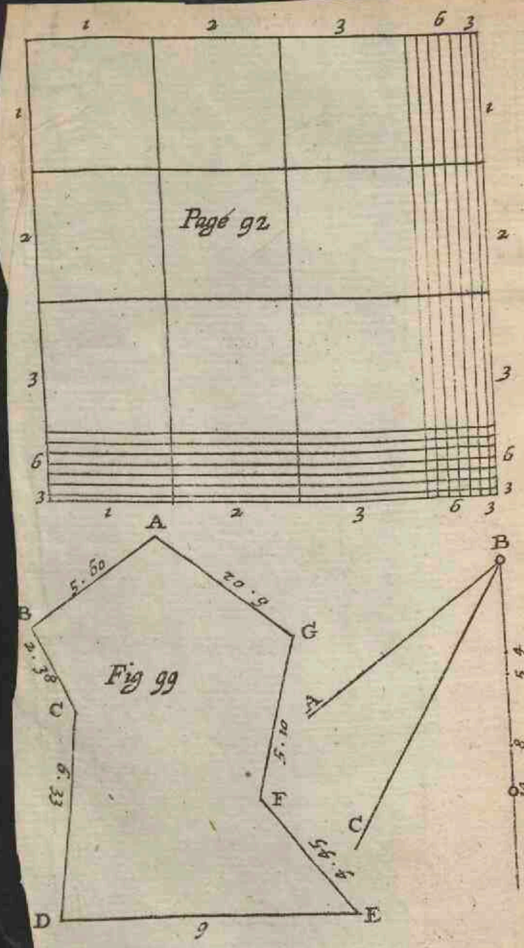
9	10
10	9
7	6

9 is

9 is 90; for which you muſt ſet down 7 foot and 6 inches, which is the 6 inches you could not ſee on the Line of Numbers, and there muſt needs be 105 foot, and not 10 foot and an half, and better, which is as to the right Number of Figures.

But by the Line of Lines as the — 10--75, from any Scale, as the Line of Lines doubled, or foot-meaſure, or the like, is to the = 10, ſo is the = 9--83, to the Latteral 105--6 $\frac{1}{2}$, as before, though not ſo quick or plain, as by the Line of Numbers.

3. As 1, to 1528, ſo is 3522, to 5381616, the true anſwer; which indeed is to more places than poſſible the Rule can come to without the help laſt mentioned: But if the Queſtion had been thus, with the ſame Figures, 15 foot $\frac{28}{100}$ parts, by 35 foot $\frac{22}{100}$ parts, as ſo many feet, or yards, and hundred parts: Then the anſwer would be as before, 538 foot; and cutting off four figures for the four figures of Fractions, both in the Multiplicator and Multiplicand, *viz.* the 1616, which makes near 2 inches of a foot more, or $\frac{16}{100}$ parts of a yard more, which in ordinary meaſuring is not conſiderable.



By the Line of Lines.

4. As — 15--28, to = 1, next the Center:
So is = 35--22, to 537 -- $\frac{1616}{10000}$.

To multiply 3 pound, 6 shillings, and 3 pence, by it self; the Product is, 10.*l.*—19.*s.*—5.*d.*—1.*f.*— $\frac{7}{10}$: For the Extent from 1, to 3--3125, the Decimal number for 3.*l.*—6.*s.*—3.*d.* shall reach from thence to 10—9726, which reduced again, is as before, 10—19—5—2; as followeth.

Note, That in this way of Multiplication by the Pen, works thus; You must first multiply Pounds by Pounds, one over the other, as 3 by 3: Then the Shillings by the Pounds cross-wise both-wayes, as the black-line sheweth. Then Pounds by Pence, as the long Prick-lines sheweth both-wayes also. Then Shillings by Shillings, as the 6 by 6. Then Shillings by Pence, both-wayes, as the short Prick-lines sheweth. Then lastly, the Pence by the Pence, as 3 by 3; whose true power, or denomination, is somewhat hard to conceive; which is thus:

First,

First, 3 times 3 (next the left-hand) is 9 Pounds.

Secondly, 3 times 6, is 18 Shillings.

Thirdly, 3 times 6, is 18 shillings again.

Fourthly, 3 times 3, is 9 Pence, as the long Prick-line sheweth.

Fiftly, 3 times 3, is 9 Pence more.

Sixtly, 6 times 6 is 36, every 20 whereof is 1 Shilling; and every 5 thereof is 3 Pence; and every 1 is 2 Farthings and $\frac{4}{10}$ths of a Farthing: So that 36 make 1 Shilling, 9 Pence, 2 Farthings, $\frac{4}{10}$ths of a Farthing.

Seventhly 6 times 3 is 18; every 5 whereof is a Farthing, and every 1 is two tenths of a Farthing, as the short Prick-line sheweth.

Eightly, 6 times 3 is 18, or 3 Farthings and 6 tenths, as before.

Ninthly and lastly; to the right-hand, 3 times 3 is 9; where note, that there goes 60 to make 1 Farthing; therefore 6 makes one tenth of a Farthing: So that here is 1 tenth and $\frac{1}{2}$: Consider the Scheam and the Decimal-work, to prove it exactly to the hundreds of millions of a Pound, and you will find it to be very near.

Example.

Example.

l.	s.	d.	f.	10.

9	18	9	
	18	9	
	1	9	2. 4
			3. 6
			3. 6
			1 $\frac{1}{2}$

Product is 10. 19. 5. 1. 7 $\frac{1}{2}$.

The same Decimally.

```
    3.3125
    3.3125
   -------
    165625
    66250
   33125
  99375
 99375
----------
10.97265625
```

Which

Which Sum, being brought to Shillings, Pence, and Farthings, and tenths of Farthings, is just as aforesaid, *viz.*

$10 - 19 - 5 - 1 \frac{7}{10}$.

Or else find the Square of the least Denomination in 20 *s.* and divide the Product of the Sums being brought to that least Denomination thereby, and the Quotient shall be the Answer required.

Example.

960, the Farthings in 20 *s.* squared, is 921600. The sum of 3 *l.* -- 6 *s.* -- 3 *d.* in Farthings, is 3180; multiplied by it self is 10112400: This Product divided by 921600, the square of the Farthings in 20 *s.* makes 10 *l.* $\frac{896400}{921600}$ in the Quotient, which reduced, is 10 *l.*—19 *s.*—5 *d.*—1 $\frac{7}{10}$.

To find this Decimal Fraction is very easie thus, by the Line of Numbers; for if 20 shillings be 1000, what shall 6 shillings and 3 pence be? Set one Point in 2, representing 20; and the other in 1, representing 1000: then the same Extent laid the same way from 6 and $\frac{1}{4}$, shall reach to 3.125, the decimal fraction for 6 shillings and 3 pence,

Or

Or by inches and foot-measure; for if you account every 8th of an inch a farthing, then every inch is 2 *d.* and 6 inches is 12 *d.* right against which, in the Foot-measure, is the Decimal Fraction required: So that right against 12 farthings, or 1 inch and ½ on the Line of Foot-measure, is 125, the Decimal Fraction sought for.

Or if one Pound be the Integer, or whole Number, then every 10th part is 2 shillings; and every 5th is 1 shilling: and the inter-mediate pence and farthings is very near the 5th part; for if you conceive a 5th part, or 50 of an hundred to contain one shilling, or 48 farthings; then one of 50 is very near one farthing, for 12 and ½ is just 3 *d.* and 25 is just 6 *d.* 37½ is just 9 *d.* and 50 just 12 *d.* So that to set the Compass-point to 3 *l.* 6 *s.* and 3 *d.* is to set the Point on 3.3125, as before; which a little practice will make easie.

By the Line of Lines on the *Triangular-Quadrant*, or Sector.

As the Latteral 3.3125 is to the Parallel 10, So is the Parallel 3.3125 to the Latteral 10 *l.* —19 *s.*— 5 *d.*—2 farthings *ferè*, or 10.973.

Sect. II.

Sect. II.

To divide one Number by another.

First, by the Line of Numbers the Rule is,

Extend the Compasses from the Divisor to 1, then the same extent of the Compasses, applied the same way from the Dividend, shall reach the Quotient required.

Or the Extent from the Divisor to the Dividend, shall reach the same way from 1, to the quotient required.

Example the first.

Let 40 be a Number given, to be divided by 5; here 40 is the Dividend, and 5 the Divisor; and the answer to how many, *viz.* 8 is the Quotient.

Extend the Compasses from 5 to 1, the same Extent shall reach the same way from 40 to 8 the Quotient required; or the Extent from 5 to 40, shall reach the same way from 1 to 8 the quotient required.

F *But by the Line of Lines, the work is thus;*

As the — Latteral 40, to the = Parallel 5; So is the = 10 counted as 1 to the — 8.

Or, so is the = 1, to the — 8 of the smaller part.

Observe the Figure with the Line A B.

Or, as the — 5, to the = 10;

So is the — 40, to the = 8; As the Line CB in the Figure doth demonstrate, being the manner of working by the *Trianguler-Quadrant*, the way of the *Sector* being the same.

A second Example.

Let 1668, be divided by 19.

As 19 to 1, so is 1668 to 87 $\frac{78}{100}$;

Or, 15 of 19: Or,

As 19 to 1668, so is 1 to 87 $\frac{78}{100}$;

Or, 15 of 19, as before.

For the Extent from 19 to 1668, shall reach the same way from 1 to 87--$\frac{78}{100}$; the work by the Lines is as before.

In this work of Division, for most ordinary questions, where there is not above four figures in the quotient, you may come very near with a good Line of Numbers, as that on Serpentine-lines, and the like; but

but the difficulty is, to know the Number of Figures, which is thus most certainly done: Write down the Dividend, and set the Divisor under it, as in the vulgar way of Division; and there must alwayes be as many Figures as the Dividend hath more than the Divisor; and one more also, when the first figure of the Dividend is greater than the first figure of the Divisor; as if 152178 were to be divided by 365, then there would be 3 figures in the Quotient; for the Divisor would be written 3 times under the Dividend, in the usual way of Division; and those figures be 417 almost: But if 9172318, is divided by 8231, you will have 4 figures, *viz.* 1115, being one figure more than 3, the difference of places. In this Rule also you may see the excellency of the Artificial-Lines of Numbers, before and above the Natural-Lines.

Sect. III.

To two Lines or Numbers given, to find a third in continual Proportion Geometrical.

By the Line of Numbers work thus :

The Extent from one Number to the other, shall reach the same way from that second to a third, &c.

Example.

As 5 to 7, so is 7 to 9--82;
So is 9--82 to 13--76, &c. *ad infinitum.*

By the Trianguler-Quadrant, or Sector.

As the Lateral first Number to the Parallel second, laying the thred to the nearest distance, there keep it :

Then so is the Latteral second, to the Parallel third.

Sect. IV.

Any one side of a Geometrical Figure being given, to find all the rest, or to find a Proportion between two or more Right Lines.

This Proposition is most proper to the Line of Lines, and not to the Line of Numbers; and done thus:

Take the Line given, and make it a Parallel in its respective Numbers; the Thred so laid to the nearest distance, or the Sector so set, there keep it: then take out all the rest severally, and carry the Compasses parallelly till they stay in like parts, which shall be the Numbers required. Note the Figure.

Note also, That the Line of Sines, and the Thred will readily lie on all the Angles, and be removed from Radius to Radius more nimbly than any Sector whatsoever, only by drawing the Thred streight, and observing on what degree and part it cuts being so laid.

Let A B C D E F G be the Plot of a Field, whose side E D is only given to be

9 Chains; and I would know all the reſt: Take E D, make it a = in 9; lay the Thred to N D, or ſet the Sector to that gage, and there keep it: then meaſure every ſide ſeverally, and you ſhall find what every one is in the ſame proportional parts, by carrying the Compaſſes parallelly, till it ſtay in like parts by the Sector, or N D, by the Quadrant.

Sect. V.

To lay down any Number of parts in a Line, to any Scale leſs than the the Line of Lines.

Take 10, or any other Number, out of your given Scale, or deſign any diſtance to be ſo much as you pleaſe, and ſet one Point in the ſame Number, on the Line of Lines; and with the other, lay the Thred to the neareſt diſtance, and there keep it, by noting the degree cut by; then take out any other Number that you would have, ſetting one Point in that Number on the Line of Lines: and opening the Compaſſes, till the other Point will but juſt touch the middle of the Thred, at N D, and that ſhall be the other part required; or the length of so much, according to the first Scale given.

Example. Figure I.

Let A B represent a Line which is 100 parts, and I would lay down 65, 30, 42, 83 parts of that 100.

First, take all A B between your Compasses, and set one Point in 100 at 10 with the other, lay the Thred to N D, then take out 65, 30, 42, 83, &c. parallelly, and lay them down for the parts required, as here you see.

The like work is by the Sector, making A B a = in 100 & 100; then take out = 30, 42, 65, 83, or any thing else for the parts required.

But note, If the Line be too large for your Scale, or Line of Lines, then take half, or one third, or fourth part of the given Line; then if you take half, you must at last turn the Compasses two times: If you take one third, then turn the Compasses three times; which may prove a very convenient help in many cases, in Surveying and Dialling.

Note, That by this Rule you may add to, or take from, any given Line, or Number, any number of Parts or Lines required;

which is called the increasing, or diminishing a Line, to any Proportion required.

Sect. VI.

To divide a Line into any Number of Parts.

Take the whole length of the Line between your Compasses, and setting one Point in the Number of Parts, you would have the Line divided into; with the other, lay the Thred to N D, and there keep it; then take the N D from 1 to the Thred, and that shall divide the Line into the parts required.

Example.

Let A B be to be divided into 7 parts: Take A B, make it a Parallel in 7, laying the Thred to the N D, there keep it; then the = 1 shall divide the Line into 7 parts. But if the Line were to be divided into many parts, as suppose 73: Then first, fit the whole Line in = 73; then take out the = 72, 71, & 70, for the odd 3; then the = 10 *s*, for every 10th division, then the = 1 for the smaller parts; or else you shall find it almost an impossible thing, to take at once

once any diſtance, which, being turned above 50 times over, ſhall not at laſt happen to be more or leſs than the deſired Number required.

Note, That if the given Number happen to be ſuch, that the Part will fall too near the Center; as ſuppoſe 11, 12, or any Number under 30; then you may double, treble, or quadruple the Number, and then count 2, 3, or 4, for one of the Numbers required.

As for Example.

Suppoſe I would divide a Line into 15 parts; multiply 15 by 6, and it makes 90: Now in regard you have multiplied 15 by 6, you muſt take the = 6, inſtead of the = 1, to divide the Line into 15 parts, between your Compaſſes, becauſe the whole Line is ſet in = 90, inſtead of = 15; which is 6 times as much as 15.

Note alſo, if the Line be too big for your Scale, then take half, or a third, and make it a = in the given Line; then take out the = 1, and turn two or three times, to divide the Line according to your mind, when it is too large for your Scale.

Theſe two laſt are not to be done by the Line of Numbers, but proper for the Line of Sines only; unleſs you turn your Lines to be divided into Numbers, and then work by Proportion, as thus ;

As the whole Number of Parts, is to the whole Line, in any other parts ;
So is 1, to as many of thoſe Parts as belongs to 1.

Sect. VII.

To find a mean Proportion, between two Lines, or Numbers given.

A mean Proportion between two Lines or Numbers, is that Number, which being multiplied by it ſelf, ſhall produce a Number equal to the Product of the two Numbers given, when they are multiplied the one by the other.

Example.

Let 4 and 9 be two Numbers, between which a Geometrical mean is required.

4 and 9, multiplied together, make 36 ;

So alſo 6, multiplied by it ſelf, is 36 : Therefore 6 is a mean Proportional between 4 and 9. To find this by Arithmetick, is by finding the Square-root of 36.

But by the Line of Numbers, thus;

Divide the diſtance between 4 and 9 into two equal parts, and the middle-point will be found to be 6, the Geometrical mean proportional required.

But to do it by the Line of Lines, do thus;

Firſt, joyn the Lines, or Numbers, together, to get the ſum of them, and alſo the half ſum; and ſubſtract one from the other to get the difference, and half the difference; then count the half difference from the Center down-wards; and note where it ends: then taking the half ſum between your Compaſſes, lay your Thred to 00 on the looſe-piece; then, ſetting one Point in the half-difference, on the Line of Lines: See where, on the looſe-piece, the other Point ſhall touch the Thred; and mark the place, with a Bead on the Thred, or a ſpeck of Ink, or otherwiſe: for the meaſure from thence to the Center is the mean Proportional required.

Or elſe uſe this moſt excellent way by Geometry.

Draw the Line A B, and from any Scale of of Equal Parts, take off 4 and 9, and lay them from C, to A and B; then find out the true middle between A and B, as at E; and draw the half Circle ADB; then on C erect a perpendiculer Line, as CD; then if you take CD between the Compasses, and measure it on the same Scale that you took 4 and 9 from, and you shall find it to be 6, the true mean proportional required: being only the way by the Line of Lines, as by considering the Triangle CDE will appear.

To do this by the Sector, open the Line of Lines to a Right-Angle (by 3, 4, & 5, or 6, 8, & 10. *thus:* Take 10 Latterally between your Compasses, make it a Parallel in 6 and 8, then is the Line of Lines opened to a Right-Angle; or if your Rule be large, and your Compasses small, then take Latteral 5, the half of 10, and make it a Parallel in 3 and 4, the half of 6 and 8, and it is rectangle also:) Then set half the difference on one Leg from the Center, then having half the sum between your Compasses, set one Point in the half-difference last counted, and turn the other Point to the other Leg, and there it shall shew the mean proportional Number required.

To

1. *To make a Square, equal to an Oblong.*

Find a mean proportion between the length and the breadth of the Oblong, and that ſhall be the ſide of a Square equal to the Oblong.

Example.

Let the breadth of the Oblong be 4, and the length 9, the mean proportion will be found to be 6; Therefore a Square, whoſe ſide is 6, is equal to an Obſong, whoſe breadth is 4, and length 9, of the ſame parts.

2. *To make a Square, equal to a Triangle.*

Find a mean proportion between the half Baſe, and the whole Perpendiculer; and that ſhall be the ſide of a Square equal to the Triangle.

Example.

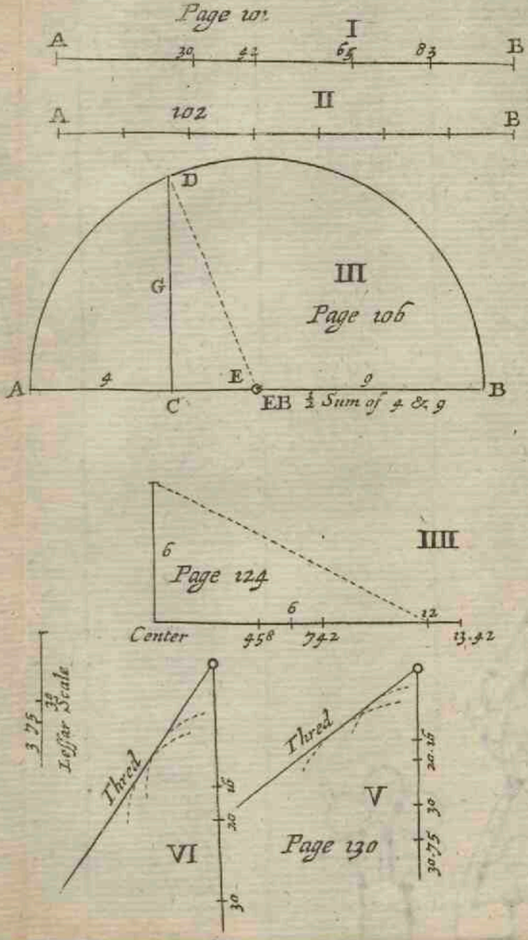
If the half-Baſe of a Gable-end be 10, and the whole Perpendiculer 11-18; the mean proportion between 10 and 11-18, is 10-575; the ſide of the Square equal to that Triangle, or Gable-end required.

3. *To find a Proportion between the Superfecies, though unlike to one another.*

Firſt, to every Superfecies, find the ſide of his equal Square, whether it be Circle, Oblong, Romboides, or Triangle; then the proportion between the ſides of thoſe Squares, ſhall be the Proportion one to another.

Example.

Suppoſe I have a Triangle, and a Circle, and the ſide of the Square, equal to the Circle, is 10 inches; and the ſide of the Square, equal to the Triangle, is 15 inches: The Proportion between theſe two Squares, as they are Lines, is as 10 to 15; but as Superfecies, as 100 to 45; being thus found out, Take the Extent between 15 and 10, on the Line of Numbers, and repeat it two times the ſame way from 100, and it ſhall reach to 45, the Proportion as Superfecies, between that Circle and Triangle, whoſe Squares equal were 15 and 10.



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4. *To make one Superfecies, equal to another Superfecies, of another shape: but like to the first Superfecies given.*

First find a mean proportion between the unequal sides of the given Superfecies, that you are to make one like; and find the mean proportion also between the unequal sides of the Figure that you are to make one equal to.

As thus for Example.

I have a Romboides, whose base is 5, and perpendiculer is 3, (and side is 3-55) the mean proportion between is 3-866: Also, I have a Triangle, whose half-base is 8, and the perpendiculer 4, the mean proportional is 5-6552; and I would make another Romboides as big as the Triangle given, whose Area is 32: Then by the Line of Numbers, say, As 3-866, the one mean proportion, is to 5-6552, the other mean proportion; so is the Sides of the Romboides, whose like I am to make, to the sides and perpendiculer of the Romboides required, to make a Romboides equal to a Triangle given, and like to another Romboides first given.

As

As thus for Example.

As 3-866 is to 5-6552; so is 5, the base of the Romboides given, to 7-30, the base of the Romboides required. And so is 3, the given perpendiculer of the Romboides, to 4-38, the perpendiculer of the Romboides required: So also is 3-55, the side of the Romboides given, to 5-19, the side of the Romboides required: for, if you multiply 7-30, the base thus found, by 4-38, the perpendiculer now found, it will make a Romboides, whose Area is equal to 32, the Area of the Triangle, that I was to make the Romboides equal to; and making the side to be 3-55, it will be like the first Romboides propounded.

If it had been a *Trapesia*, or other formed Figure, it might have been resolved into Triangles, and then brought into Squares, as before: Then all them Squares added into one sum, whose Square-root is the mean proportional or side of a Square, equal to that many-sided Figure, whose like or equal is desired to be made and produced.

5. One

5. *One Diameter and Content of a Circle given, to find the Content of another Circle, by having the Diameter thereof only given.*

The Extent from one Diameter to the other, being twice repeated the right-way from the given Area, ſhall reach to the Area required.

If the Area's of two Circles be given, and the Diameter required; then the half-diſtance on the Numbers, between the two Area's, ſhall reach from the one Diameter to the other.

Sect. VIII.

To find the Square-root of a Number.

To do this by the Line of Numbers, you muſt firſt conſider, whether the Figures, whereby the Number, whoſe Root you would have, is expreſſed, be even or odd figures; that is, conſiſt of 2, 4, 6, 8, or 10; or 1, 3, 5, 7, or 9 figures.

For if it be of even figures, then you muſt count the 10 at the end for the unite; and the Root and Square are backwards toward 1.

But

But if it consist of odd Figures, then the 1, in the middle of the Line, is the unite; and the Root and Square is forwards towards 10: for the Square-root of any Number, is alwayes the mean proportional, or middle space between 1, and the Number propounded; counting the unite according to the Rule abovesaid: So that the Square-Root of 1728, consisting of four figures, it is at 41 and $\frac{6}{10}$, counting 10 for the unite; for the Number 42 & $\frac{6}{10}$, is just in the middest between 1728 and 10.

And to find the Square-root of 144, consisting of three figures; divide the space between the middle 1 and 144, counted forwards, into two equal parts, and the Point shall rest at 12, the Square-root required.

To do this by the Line of Lines, or Sector.

First, find out a Number, that may part the Number given evenly, or as even as may be; then the Divisor shall be one extream, and the Quotient another extream; the mean proportional between which two, shall be the Square-root required, working by the last Rule.

Example

Example.

To find the Square-root of 144. If you divide 144 by 9, you ſhall find 16 in the Quotient: Now a mean proportion between 9 the Diviſor, and 16 the Quotient, is 12 the Root, found by the laſt Rule, *viz.* the 7th.

Sect. IX.

To find the Cubick-Root of a Number.

The Cubick-root of a Number, is alwayes the firſt of two mean proportionals between 1, and the Number given; counting the unite with the following cautions: Set the Number down, and put a Point under the 1ſt, the 4th, the 7th, and the 10th figure; and look how many Points you have, ſo many figures ſhall you have in the Root.

Then if the laſt Point fall on the laſt Figure, then the middle 1 muſt be the unite, and the Root, the Square, and Cube will fall forwards toward 10.

But if the laſt Point fall on the laſt but one, then the unite may be placed at either end, *viz.* at 1 at the beginning, or at 10 at the end; and then the Cube will be one Radius beyond the unite, either forwards or backwards.

But if it fall on the laſt but two, then 10 at the end of the Line muſt be the unite; and the Root, the Square, and Cube will alwayes be in the ſame Radius, that is between 10 at the end, and the middle 1.

So that by theſe Rules, the Cubick-root of 8 is 2; for putting a Point under 8, being but one figure it hath but one Point, therefore but one figure in the Root: Secondly, the Point being under the laſt figure, the middle 1 is the unite; then dividing the ſpace between 1 and 8, into three equal parts; the firſt part ends at 2, the Root required.

So likewiſe in 1331, there is two Points, therefore two figures in the Root; and the laſt Point being under the laſt Figure, the middle 1 is the unite; and the ſpace between 1 and 1331, being divided into three equal parts, the firſt part doth end at 11, the Cubick-root of 1331.

Again, for 64 there is one Point, and it falls on the laſt figure but one; therefore the Root contains but one figure, and 1 at the beginning, or 10 at the end, which you pleaſe, may be the unite.

But yet with this Caution, That the Cube muſt be in the next Radius beyond that which belongs to the unite; ſo that dividing the ſpace between 10 and 64, beyond the middle 1; towards the beginning, into three equal parts; the firſt part falls on 4, the Cubick-root required: Or, if you divide the ſpace between 1 and 64, near the 10, into three equal parts, the firſt part falls on 4 alſo.

Again, for 729, there is but one Point; therefore but one figure: Again, it falls on the laſt but 2; therefore 10 at the end is the unite; and between 10 and the middle 1 backwards, you ſhall have both Root, Square, and Cube, for the Number required, which will be at 9; For if you divide the ſpace between 10, and 729, into three equal parts, the firſt part will ſtay at 9, the Cubick-root required.

Note, if it be a ſurd Number that cannot be cubed exactly; yet the Number of figures to be accounted as Integers is as before; and the reſidue diſcoverable by the Line, is a Decimal Fraction.

Example.

For 1750 the Root reſulting, is $12\frac{048}{1000}$, or 12, and near 5 of a 100.

Thus you have a very good and ready way for this hard queſtion in Arithmetick, and will come near enough for moſt uſes.

But to perform this by the Natural-lines, at the beſt it is very troubleſome, and cannot come to no ſuch exactneſs, as by the Line of

 Num-

Numbers; and therefore I ſhall omit it as inconvenient.

For Application or Uſe of this laſt Rule of finding the Cube-Root, obſerve with me as followeth:

1. *Between two Numbers, or Lines given, to find two mean proportional Numbers, or Lines required.*

Divide the ſpace on the Line of Numbers, between the two Numbers given, into three equal parts; and the Numbers where the Points of the Compaſſes ſtay at each repetition, (or turning) ſhall be the two mean proportional Numbers required.

Example.

Let 4 and 32, be two extream Numbers (or the meaſure of two extream Lines) between which I would have, two mean proportional Numbers (or Lines) required.

In dividing the ſpace on the Line of Numbers, between 4 and 32, into three equal parts, you ſhall find the Compaſſes to ſtay firſt at 8, ſecondly at 16; the two mean proportionals, between 8 and 32, the two extream Numbers firſt given.

For the Square or Product of 4 and 32, the two extreams, 128, is equal to the Square or Product of 8 and 16, the two means multiplied together, being 128 alſo.

2. To apply it then thus for *Example:* If I have the ſolid content of a Cube to be 1728 Cubick inches, and the ſide thereof be 12 inches; I would know what ſhall the ſide of the Cube be, whoſe ſolid content is 3456, the double of 1728? Divide the ſpace between 1728, and 3456, into three equal parts; then, lay the ſame diſtance the Compaſſes ſtand at from 12, the ſide of the Cube given, and it ſhall reach to 15-12, the ſide of the Cube required, whoſe ſolid content is 3456 inches.

Alſo, If I have a Shot of Iron, whoſe weight is 3 pound, and the diameter thereof 2 inches, and 780 parts of an inch in a 1000; what ſhall the diameter of a Shot be, whoſe weight is 71 pound? One third part of the diſtance, on the Line of Numbers, between 3 pound, and 71 pound, ſhall reach from 2 inches, 780 parts, the given diameter, to 8 inches, the true diameter of a caſt Iron Bullet, whoſe weight is 71 pound.

2 Secondly, *on the contrary, if the Diameter and Content of one Globe, or Cube be given, and the Diameter of another Globe or Cube, to find the content thereof.*

As the diameter of the Globe, whoſe content is alſo given, is to the diameter of the Globe whoſe content is required ; ſo is the content given, to the content required ; by repeating the Extent the ſame way three times. *Example.* Suppoſe the capaſſity or content of a Globe, whoſe diameter is 10 inches, be 523 inches ſolid, and 80 parts ; what ſhall the content of that Globe be whoſe diameter is 20 inches ? the Extent from 10 to 20, being turned three times from 523-8, the content of a Globe of 10 inches diameter, ſhall reach to 4190 $\frac{10}{1}$, the Cubick inches contained in a Globe of 20 inches diameter, being 8 times as much as the former.

3. *The Proportion between the weights and magnitudes of ſeveral Metals, are as followeth, according to* Marinus Ghetaldi.

If 7 pieces of the 7 Metals, are all of one ſhape, and bigneſs, either Spheares, or Cubes, or Cilenders, or Parallelepipedons ; then their weights are in proportion as followeth, according to *Marinus Ghetaldi.*

The

The Shape and Magnitudes equal,
The Weights are in proportion, as,

♃ Tinn——	1554
♂ Iron ——	1680
♀ Copper——	1890
☽ Silver ——	2030
♄ Lead ——	2415
☿ Quickſilver—	2850
☉ Gold ——	3990

So that if a Cillender of Tinn, whoſe ſide is one inch, weigh 1824 grains; What ſhall a Cillender of Gold weigh, the height and diameter being juſt one inch, *viz.* 4682.

For as 1554, is to 3990; So is 1824, to 4682, the grains in one inch of Gold.

The Shapes and Weights of the pieces of the ſeven ſeveral Metals being equal, then the Magnitudes of the ſides are as followeth, according to Mr. Gunter.

☉ Gold———	3895
☿ Quickſilver—	5433
♄ Lead ———	6435
☽ Silver ———	7161
♀ Copper———	8222
♂ Iron ———	9250
♃ Tinn———	10000

So that if I have a Sphear of Iron, weigheth 9 pound, whoſe Diameter is 4 inches; What muſt the Diameter of a Leaden Sphear, or Bullet, be of the ſame weight?

Say thus;

One third part of the ſpace between 9250, and 6435, ſhall reach from 4 the Diameter of the Iron Bullet, to 3 inches 54 parts, the diameter of the Leaden Bullet, that weighs 9 pound.

4. *So*

4. So that if I have the weight and magnitude, of a body of one kind of Metal, and would know the magnitude of a body of another Metal, having the ſame weight: work thus;

The firſt of two mean proportionals, between the two Points on the Line of Numbers, repreſenting the Numbers in the laſt Table, for the two Metals, ſhall reach the right way, from the Magnitude given, to the Magnitude required.

As in the Example before, and illuſtrated by another thus;

Suppoſe a Cube of Gold, whoſe ſide is 2 inches, weigh 29000 grains; What ſhall the ſide of a Cube of Tinn be, having the ſame weight? Divide the ſpace on the Line of Numbers, between 3895, the Point on the Numbers for Gold; and 10000, the Point for Tinn; and this extent parted into 3 equal parts, and that diſtance laid from 2, the ſide of the Cube of Gold, ſhall reach to 2-74; the ſide of the Cube of Tinn required.

5. The

5. *The Magnitudes of two bodies of ſeveral Metals being given, and the weight of the one, to find the weight of the other.*

Take the Extent between their Points, on the Line of Numbers, according to the laſt Table, for each ſeveral Metal; and this Extent laid from the given weight, ſhall reach to the enquired weight, of the other Metal propounded.

Example.

If a Bullet of Iron, of 4 inches Diameter, will weigh 9 pound; a Bullet of Lead, of the ſame Diameter, will weigh 13 pound.

6. *A body of one Metal being given, to make another body like unto it of another Metal, and any other weight, to find the Diameters and Magnitudes thereof.*

First, by the 4th laſt paſt, find the Magnitude of the ſide, or Diameter of the Sphear, having equal weight; and note that down, or keep it.

Then find out two mean proportions, between the weights given; and ſetting this diſtance, on the Line of Numbers, the right way, either increaſing, or diminiſhing from the

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To find this by Arithmetick, do thus;

First, square the given Number (that is, multiply it by it self) then multiply the Product by 5, and divide this Product by 4; then find the Square-root of the Quotient, and from it take half the given Number, the residue is the greater portion, then the greater part taken from the whole, leaves the lesser-part.

By the Sector, work thus;

Open the Sector to a Right Angle, in the Line of Lines (making Latteral 10, a Parallel in 8 and 6) or else make the Latteral 90, a Parallel Sine of 45 (or the Latteral Sine of 45, a Parallel Sine of 30) then upon both Legs count the given Number; then take the Parallel Extent from the whole Number on one Leg, to the half on the other Leg, and lay this from the Center Latterally; and whatsoever the Point reacheth beyond the whole Number, must be added to the half Number, to make up the greater Number; or taken from the half to make the lesser.

Example.

Let the Number given be 12, which may be represented at 6 on the Line of Lines; then

then the Sector ſtanding at Right Angles, take the Parallel-diſtance from 3, the half of 6 (counted as 12) on one Leg, to 6 on the other Leg ; and you ſhall find it reach to 6-71, which doubled is 13-42 ; from which if you take 6, the half ſum, reſt 7-42 for the greater part : and if you take 7-42 from 12, there remains 4-58, the leſſer-part.

But by the Line of Numbers work thus : following the Arithmetical way.

Extend the Compaſſes from 1 to 12, and that extent ſhall reach from 12 to 144 ; then next the extent from 1 to 144, ſhall reach from 5 to 720 ; then the extent from 4 to 1, ſhall reach from 720 to 180 : then to find the Square-root of 180 ; the half-diſtance between 180 and 1, you will find to be at 13-42, as before ; which uſed as abovesaid, gives the extream, and the mean proportional parts of 12 required.

Another Example of 26.

Extend the Compaſſes from 1 to 26, and repeat the ſame again forward from 26, ſhall reach to 676.

Again, the Extent from 1 to 676, ſhall reach from 5 to 3380.

Laſtly,

Laſtly, the Extent from 4 to 1, ſhall reach from 3380, to 845; and the Square-root of 845, is 29-07: from which Number or Root, if you take half the given Number, *viz.* 13; then there will remain 16-07, one extream: then 16-07 taken from 26-0, reſt 9-93, the other extream required.

For the Extent from 26 to 16-07, will reach from 16-07, to 9-93.

Another way by the Line of Sines, Geometrically.

The beſt and quickeſt way is by the Line of Sines, thus; Make the given Line a Parallel-Sine of 90; then take out the parallel-Sine of 38 degrees, 10 minutes, and that ſhall be the greater part.

Alſo, take out the Parallel-Sine of 22 degrees, 27 minutes, and that ſhall be the leſſer extream required: Or, according to Mr. *Gunter*, uſe 54 for the whole Line, 30 for the greater part, and 18 for the leſs.

Alſo by conſequence, having the mean, or greater part, make it a parallel-Sine of 38 degrees, 10 minutes; then Parallel 90 ſhall be the whole Line, and Parallel 22 degrees, 27 minutes, ſhall be the leſſer part.

And laſtly, having the leaſt part, make

it a Parallel in 22 degrees, 27 minutes; then Parallel 90 deg. 10 min. shall be the whole Line; and Parallel 38-10, the greater part.

The Use whereof, you shall have afterwards in the 11th *Chapter*, about the cutting off the *Platonical Bodies*.

Sect. XI.

Three Lines or Numbers given, to find a Fourth, in Geometrical Proportion; or, the Rule of Three direct.

1. In all Questions of the Rule of *Three*, there be three terms propounded, *viz.* two of Supposition, and one of Demand.

2. Also note, that two of the terms propounded, are of one denomination, (or at least to be reduced to one denomination) and one of another denomination.

3. Of the three termes propounded, (in direct proportion) that of Demand is alwayes the third term, and one of the terms of Supposition, *viz.* that of the same Denomination, with the term of Demand, is alwayes the first; then the other of Supposition left, must needs be the second term in the Question..

In

4. In direct proportion *Alwayes*; As the first term is to the second, so is the third to the fourth term required.

5. Having discovered which be the first, second, and third terms; If the first and third term be of divers Denominations, they must be reduced to one Denomination, if it cannot be done on the Line in the operation, as many times it may; *As thus for instance:*

If one pound cost two shillings, what shall 30 ounces cost?

Here you see that the term of Demand, 30 ounces, *viz*. the third term, is not directly of the same Denomination with one pound, the first term; but is thus to be reduced to ounces: *Saying*;

If 16 ounces cost 2 shillings, what shall 30 ounces cost? 3 *s* — 9 *d*.

Thus the first and third terms, are brought to one Denomination: Also you see that the Demand or Question, *viz*. What shall 30 ounces cost? is joyned to the third term; and also that 16 ounces the first term, is of the same Denomination; therefore the 3 *s*. must needs be the second term, and the Answer to the Question is the fourth.

6. Having thus discovered, which are the first, second, and third terms, and re-

duced the first and third to the like Denominations; then the work by the Line of Numbers is alwayes thus;

As the first, to the second; so is the third, to the fourth.

Or the Extent of the Compasses upon the Line of Numbers, from the first, to the second; shall reach the same way, from the third, to the fourth required.

As 16 ounces is to 2 *s*; so is 30 ounces to 2 *s* $\frac{1}{2}$, or 9 pence. *Or*,
As 16 ounces is to 24 *d*; so is 30 ounces to 45 *d*; which is 3 *s*. — 9 *d*. as before.

7. But by the Line of Lines or Sector, if you will work on one Scale only, you must consider which term of the first or second is biggest; for you must alwayes order it so, that the Parallel work must be the largest, (or at least so as it may be wrought) and as much as may be, that the fourth term may be a Latteral Extent, as the first alwayes is; for then it is wrought the soonest, and also the exactest.

Yet by this Instrument, you need not much care for these Cautions, having several Scales of Equal Parts, to begin and end the work on, you are freed from that trouble.

As

As thus for Example.

When the ſecond term is greater then the firſt, then the Work is well performed thus, two wayes.

As the Latteral firſt	16		As Latteral third	30	from a leſſer Scale.
To the Parallel ſecond	02	or	To Parallel firſt	16	
So is the Latteral third	30	elſe	So Parallel ſecond	2	
To the Parallel fourth	$3\frac{75}{100}$		To Latteral fourth	$3\frac{75}{100}$	by the ſame Scale.

Or, As —ſecond, to = firſt ; So = third, to — fourth; by a leſs Scale alſo, if need be.

But when the ſecond term is leſs than the firſt, then the work is performed thus:

If 50 Foot of Timber coſt 40 *s.* what ſhall 20 Foot coſt?

As the Latteral ſecond	40		As —3d	20 foot
To the Parallel firſt	50	Or as	To = 1ſt	50 foot.
So is the Parallel third	20	before;	So = 2d	40 *ſhill.*
To the Latteral fourth	16		To —4th	16 *ſhill.*

8. Thus you ſee ſeveral wayes of working: but for Beginners, I would adviſe thus, briefly.

Firſt, either to obſerve this Rule of changing the terms, from the firſt to the ſecond, *viz.* To take the ſecond Latterally, and make it a Parallel in the firſt; then the Parallel third gives you a Latteral Anſwer. Or elſe to work directly, as the firſt to the ſecond, and ſo be content with a Parallel Anſwer, which you may alwayes do with the help of a ſmaller Scale, when need requires it.

Note the Figures of Operation, by the *Trianguler Quadrant.*

Sect. XII.

The Rule of Three inverſed.

1. The Rule of Three inverſed, or the back-Rule of Three, is, when the term required, or fourth term, ought to proceed from the ſecond term, according to the ſame proportion, that the firſt term proceeds from the third.

As thus for Example.

Car.	*Hour.*	*Car.*	*Hour.*
20.	16.	10.	32.
1.	2.	3.	4.

If 20 Carts carry 60 Square yards of Earth in 60 hours, how many Square yards of Earth ſhall 10 Carts carry in 16 hours?

Here it is apparent that fewer Carts muſt have a longer time to carry the like quantity; therefore to the ſame time muſt leſs work be allotted, as in the work doth follow.

Car.	*Hour.*	*Car.*	*Hour.*
20.	16.	10.	32.
1.	2.	3.	4.

For if you extend the Compaſſes from 10 to 20, terms of like Denomination, *viz.* that of Carts; the ſame extent applyed the contrary way, from 16, the time required, by 20 Carts, ſhall reach to 32, the time required by 10 Carts, to carry 60 Load.

For *Note*, as in the former Rule of Three direct;

Look how much the third term is greater

than the firſt ; ſo much the fourth is greater than the ſecond.

And contrarily,

Look how much the third term is leſs than the firſt, by ſo much is the fourth term leſs than the ſecond.

As thus in Numbers.

As 2 is to 4, ſo is 6 to 12; for as 6 the third term, is thrice as much as 2, the firſt term; ſo is 12, the fourth term, thrice as much as 4, the ſecond.

And contrarily decreaſing.

As 12 is to 6, ſo is 4 to 2; For as 4 is one third part of 12, ſo is 2 one third part of 6.

2. But now in this Rule of Three inverſed, or the back-Rule of three; it is contrarily ordered, *as thus*;

Look how much the third term is greater (or leſſer) than the firſt, by ſo much is the fourth term leſſer (or greater) than the ſecond.

As thus in Numbers.

As 5 is to 60, ſo is 30 to $2\frac{1}{2}$; that is, If 5 *s*, is 60 *d*. How many ſhillings is 30 pence?

pence? The Answer is, 2 *s.* ½. For as 30 is greater than 5; so is 2½ less than 60.

Again, as 2½ is to 30; so is 60 to 5, in the like manner.

Pion.	*Dayes.*	*Pion.*	*Dayes.*
18.	40.	15.	48.
1.	2.	3.	4.

If 18 Pioneers make a Trench in 40 days, how many Pioneers is needful to perform the same in 15 dayes?

As 40 to 15, so is 18 to 48: Here, as the third is lesser than the first; so is the fourth greater than the second.

Hors.	*Dayes.*	*Hors.*	*Dayes.*
12.	30.	24.	15.
1.	2.	3.	4.

Again, if 12 Horses eat 20 bushels of Provender, in 30 dayes; how soon will 24 Horses eat up the like quantity of Provender? The Answer is in 15 dayes.

3. The manner of working this Rule on the Line of Numbers, *is thus.*

Extend the Compasses from one term to the other of like Denomination; the same extent laid the contrary way from the other term, ſhall reach to the Anſwer required.

As in the *laſt Example* ; the extent from 12 to 24, the terms under the denomination of Horſes, ſhall reach the contrary way from 30 to 15, the number of dayes required.

4. *Note*, That by due conſideration, this back-Rule may be wrought by the Precepts, for the direct-Rule,

Thus :

In all Queſtions of this nature, there be three terms given to find a fourth ; of which three terms, two are of one Denomination, and one of a different Denomination ; of which, the fourth muſt alwayes be ; which in the firſt Rule of the tenth Section before going, are called two termes of Suppoſition, and one of Demand. Now here you are to conſider, That

5. When the fourth term required, ought to be greater than that of Demand ; which by reaſon you may certainly know ;

Then ſay,

As the leſſer term of Suppoſition is to the greater ;

So is the term of Demand, to his Anſwer, the fourth.

Example.

Example.

Men.	*Dayes.*	*Men.*	*Dayes.*
80.	12.	40.	24.
1.	2.	3.	4.

If 80 Men do a Work in 12 dayes, how ſoon may 40 Men do the like Work? Here Reaſon tells me, that fewer Men muſt have longer time; therefore the fourth term required muſt be greater. Therefore,

As 40 to 80, *viz.* As the leſſer term of Suppoſition 40, to the greater 80;
So is 12, the term of Demand, to 24, the Anſwer required.

6. But if the required term ought to be leſſer, which Reaſon will diſcover in like manner; *Then thus:*

As the greater term of Suppoſition, is to the leſſer; ſo is the term of Demand to the fourth term required.

As 80 to 40, ſo is 24 to 12; extending the Compaſſes the ſame way from the third to the fourth, as from the firſt to the ſecond.

But *Note* here, That you are not tyed to obſerve which is the firſt, ſecond, or third term; but to conſider only the nature of the

the Queſtion, that you may Anſwer accordingly; and indeed this way will, generally, take in the direct Rule alſo. For alwayes in Direct Proportion, you may as well ſay, As the third term is to the firſt, ſo is the ſecond to the fourth; as to ſay, As the firſt to the ſecond, ſo is the third to the fourth.

Alſo backwards, or inverſly; As the third to the firſt, ſo is the ſecond to the fourth; extending the Compaſſes the contrary way.

As 80 to 40; So is 12 to 24.

8. To perform this by the Sector, or general Scale and Thred, on the Quadrantal-ſide, you may generally obſerve this Rule; Enter the ſecond term taken Latterally, Parallelly in the firſt, keeping the Sector, or Thred, at that Angle; then the Parallel-third, ſhall give the Latteral-fourth, LATTERALLY.

9. Or elſe, As the Latteral-firſt, to the Parallel-ſecond; ſo is the Latteral-third, to the Parallel-fourth, PARALLELLY.

And if the ſecond be leſs than the firſt, make uſe of a ſmaller Scale; or change the terms, as is ſhewed before.

Sect.

Sect. XIII.

The Double or Compound Rule of Three, Direct and Reverse.

Having premised the way to bring the back (or inversed) Rule of Three, to be performed by the Rules for the Direct; and considering that the Double and Compound Rules of Three are alike by the Line of Numbers; I have therefore joyned them together in one Section.

1. The Compound, or Double (Golden) Rule of Three; is, when more than three terms are propounded, or given.

2. The Double Rule of Three, is when five terms are propounded, and a fixt term proportional unto them is demanded.

As thus;

If 6 Men spend 18 *l.* in three months; How much will serve 12 Men for 6 months?

Or, again.

If two Barrels of Beer serve 12 Men for 14 dayes; How many dayes will 4 Barrels serve 24 Men?

3. The five terms given consist of two parts,

parts, *viz.* a Supposition, and Demand; as in the Rule of Three direct.

The Supposition lies in these three Numbers first propounded, *viz.* If 6 Men spend 18 *l.* in 3 months; and the Demand lies in the two remaining, *viz.* How much will serve 12 Men 6 months?

Or in the other Example, *viz.* If 2 Barrels of Beer serve 12 Men 14 dayes, are the terms of Supposition; and, how many dayes will 4 Barrels serve 24 Men, are the terms of Demand?

4. The next work is to rank the three terms of Supposition, and the two of Demand, in their due and proper order, for convenience of Operation; which may be thus:

Of the three terms of Supposition, that which hath the same Denomination with the term required, place in the second place; and the other two, one above another in the first place:

Thus;

6 —— 18 —— 12
3   6

And then place the two terms of Demand one above another in the third place, only observing to keep the Numbers of like Denominomination in the ſame ranks ; as 6 Men, and 12 Men in the upper rank ; and 3 Months, and 6 Months in the lower rank ; as in the Work is expreſt.

5. When Queſtions of this nature are reſolved by two ſingle Rules, then the Analogy, or Proportion, is *thus* ;

Operation I.

As the firſt term, in the upper Rank, is to the ſecond ;
So is the third, in the ſame Rank, to a fourth. Again,

Operation II.

As the firſt term in the lower Rank, is to the fourth laſt found ;
So is the other term in the lower Rank, to the term required.

As in the firſt Example ; As 6 to 18 ; ſo is 12 to 36 a fourth.
Again, as 3 to 36 ; ſo is 6 to 72, the term required.

Which by the Line of Numbers, is thus wrought ;

Extend the Compaſſes from 6 to 18 ; the ſame extent applyed the ſame way from 12, ſhall reach to 36. Then again, extend the Compaſſes from 3 to 36, the ſame extent applied the ſame way from 6, ſhall reach to 72, the term required.

By the Trianguler Quadrant, or Sector, thus ;

6. As — 18 to = 6 ; so is = 12 to — 36. Again,

As — 36 to = 3 ; so is = 6 to — 72, the term required.

Or else work it Parallelly, observing the same order, as by the Line of Numbers, thus ;

As — 6 to = 18 ; so is — 12 to = 36, the fourth term. Again,

As — 3 to = 36 ; so is — 6, to = 72, the sixt term required.

The Double Rule of Three inversed.

7. In the other Example, is comprehended the double Rule of Three inverse ; which runs thus ;

If two Barrels of Beer, serve 12 Men 14 dayes ; How many dayes will 4 Barrels serve 24 Men ?

If you Rank the terms, according to the former Precept, they will stand thus :

2–14–4 or thus, 12–14–24
12 24 2 4

8. Which

8. Which if you work according to the back-Rule, the way is thus;

Operation I.

Extend the Compasses from 2 to 4, terms of like Denomination, *viz.* of Barrels; the same Extent applied the contrary way from 14, shall reach to 7, for a fourth Proportional.

Operation II.

Again, Extend the Compasses from 12 to 7, the fourth last found; the same Extent shall reach the contrary way, from 24 to 14, the number of dayes required.

9. But if you would reduce this, to be wrought by two single direct Rules; you must consider the Precept Rule, the 5th and 6th, of the Eleventh Section; and the terms of Supposition and Demand; and the increasing, or decreasing of the fourth term, which is required.

As thus;

First, I part this into two single Rules, thus:

Operation I.

If 12 Men drink 2 Barrels in 14 dayes, then 24 Men may drink 2 Barrels in 7 dayes.

Opera-

Operation II.

Again, If 2 Barrels last 24 Men 7 dayes, 4 Barrels will last them 14 dayes; the Answer to the Question required.

Here by the 6th Rule, where the Number sought is to be less; As 24, the greater term of Men, is to 12 the less of the same Denonomination; So is 14 to 7, the fourth.

Again.

As 2 the lesser term, is to 4 the greater of the same Denomination; so is 7 to 14, the Answer required, by the 5th Rule of the 11th Section.

Or else thus;

As 2 to 7, so is 4 to 14; that is, the Extent from 2 to 7, shall reach the same way from 4 to 14, the term required.

To work this by the Trianguler Quadrant, or Sector, the general Rule in this Section, Rule 6 and 7, giveth sufficient direction.

10. The Rule of Three, compounded of five Numbers, is no other than the double Rule of Three; and is, or for the most part, may be wrought by one Operation, having prepared the Numbers by Multiplication, for that purpose: Which two Multiplications by the Line of Numbers, though they

are

are presently wrought, yet the two Rules of Three are done as soon; so that the Compound Rule, is here of no advantage at all, therefore I might wave it; yet because the only difficulty lies in the ordering the Question, I shall propound it, for the addition sake of another *Example*, which is this;

If the Carriage of 2 hundred weight, 30 miles, cost 4 *s.* What will the Carriage of 5 hundred weight cost for 100 miles? The Numbers Ranked, according to the first Precept, will stand thus, as followeth.

C.		S.		C.		
2	——	4	——	5	——	10
30				100		

II. Then for the Operation, multiply the two first Numbers one by the other; as 2 times 30 is 60, which is the first term; and let the middle Number be the second term; and the Product of the two last (multiplyed together) for the third term: Then the Numbers being so prepared, *say*,

As 60, the Product of the two first Numbers, is to 4, the middle Number;

So is 500, the Product of the two last, to 33 $\frac{1}{3}$, the Answer required.

By the Line of Numbers, the Extent from 60 to 4, will reach the same way from 500 to 33 $\frac{1}{3}$. or, thirty three shillings and four pence, the price of 5 hundred weight, carried 100 miles.

Note, This Rule serves when it is performed by the Compound Rule of Three direct.

12. But if the inverse, or backer Rule of Three, be used in the work; then Operate thus:

As in this following Example, is manifest.

A Merchant hath received 10 *l.* 10 *s.* for the Interest of a certain sum of Money for six Months; and he received after the rate of 6 *l.* for the use of an hundred pound in a year; the Question is, how much Money was Principal to 10 *l.*—10 *s.* for 6 Months?

First, I range the Numbers, according to the order first propounded, in the 4th Rule of the 12th Section, as followeth.

Mon.	lib.	Mon.
12 ——	100 ——	6
6		10 *l.* 10 *s.*
—		——
36		126

Then

Then I observe diligently, whether the inverse Proportion be in the first or second Operation, or Line, as thus in this Question it is in the lower Line; therefore after the Cross Multiplication, it is to be wrought by the single inversed Rule of Three; but when the inverse Proportion is in the upper Line, it is wrought by the single Rule direct.

		l.	*s.*
6	100	10	10
12		6	
——		——	
126		36	

Then I multiply the double terms across; that is, the lowest on the right-hand by the uppermost on the left; and the uppermost on the right, by the lowest on the left; *As thus:*

6 by 6, which makes 36, to be set under 6; and 12 by 10-5, or 10*l.* (which is 126, and 10 *s.*) and set it under 10: then say by the inversed Rule, thus; *

As 126 to 36, so is 100 to 350, the Answer demanded; So that 350*l.* as Principal will yield 10 *l.* — 10 *s.* in 6 Months; Or, the Extent from 126 to 36, shall reach the contrary way from 100 to 350, the Principal Money required.

* *In both these the inversed Proportion is in the lower line*

Which you may more readily prove by reasoning thus:

13. If 3*l.* be the Interest of 100*l.* in 6 Months, to how much Money shall 10 *l.* 10 *s.* be interest in 6 Months? *work thus*;

The Extent of the Compasses from 3 to 100, shall reach the same way from 10 *l.* 10 *s.* to 350, the Principal Money answering to 10 *l.* — 10 *s.* the Answer required.

By the Line of Lines, work thus;

As — 3 to = 10, counted 100;
So is — 10 $\frac{1}{2}$ at the first 1 next the Center, to = 350.

Or,

As — 100, to = 3; so is = 10$\frac{1}{2}$ to — 350.

Sect. XIV.

The Rule of Fellowship.

1. Rules of Plural Proportion are those, by which those Questions are resolved, which require more Golden Rules than one; and yet cannot be Resolved by the double (Rule of Three, or) Golden Rule, which was last mentioned.

2. Of

2. Of theſe Rules there be divers kinds and varieties, according to the nature of the Queſtion propounded; for here the terms given, are ſometimes four, five, or ſix, or more; and the terms required alſo more than one, two, or three.

3. The Rule of Fellowſhip, is to diſcover the Gain or Loſs of every Partner in the Stock, by their ſeveral Stocks, and the whole gain or loſs of the whole Stock.

Alſo obſerve, That the Rule of Fellowſhip may be either *ſingle* or *double*; of both which in order.

4. The *ſingle* Rule of Fellowſhip is, when the Stocks propounded are ſingle Numbers.

As thus for Example.

A B C and D, repreſenting the Names of 4 Men, put into one common Stock 100 *l.* to trade withal: A puts in 10 *l.* B puts in 20 *l.* C 30 *l.* and D 40 *l*; and with this Stock, in a certain time, they gained 10 *l.* or 200 *s*; Now the Queſtion is, what ought each man to have of the 200 *s.* that may be proportionable to his particular Stock?

5. The Rule of Operation is, firſt, by Addition find the total of all the particular Stocks, for the firſt term; the whole gain (or loſs), for the ſecond term; and each

particular Stock for a third term ; and repeating the Rule of Three as often as there be particular Stocks in the Question, you shall bring forth, or find out, as many fourths for the particular gains (or losses) of each particular Man required.

As thus for Instance.

The sum of the four Stocks are 100 *l.*
The whole gain is 10 *l.* or 200 *s.*

Then,

	l.				*s.*		
As 100 *l.* to 200 *s.* so is	10	the stock of	A	to	20	the gain of	A
	20		B		40		B
	30		C		60		C
	40		D		80		D

For the Extent from 100 to 200, shall reach from 10 to 20, and from 20 to 40, and from 30 to 60, and from 40 to 80; the particular gains due to A B C D, which was required.

6. For proof whereof, if you add 20, 40, 60, and 80 together, they make up 200 *s*, or 10 *l*; the whole gain of the whole Stock.

7. The

7. The *double* Rule of Fellowship is; when the Stocks propounded are double Numbers.

As thus for Example.

A B and C, holds a Field in common, for which they pay 50 *l.* a year; and in this Field, A had 25 Oxen went 30 dayes; B had 15 Oxen there 40 dayes; and C had 20 Oxen went there 40 dayes: What ought each man to pay for his part of the Rent, *viz.* 50 *l*? Here you see the Stocks propounded are double Numbers, as of Oxen, and their dayes, or time of feeding; as 25 & 30, 15 & 40, 20 & 40, being double Numbers.

8. The Rule of Operation is thus, in the double Rule of Fellowship:

Multiply the double Numbers, severally one by the other, one after another, and take the sum of their several Products, for the first term; and the whole gain or loss, for the second term; and the particular Products of every double Number, for the third term, one after another: This done, repeating the Rule of Three, as often as there be double Numbers, the 4th term produced from those Operations, shall be Answers to the Questions required, *viz.* the quantity of each mans gain or loss.

Example.

25 & 30, A's Oxen and time of feeding, multiplied, is ——	750
15 & 40, B's Oxen and time of feeding, multiplied, is ——	600
20 & 40, C's Oxen and dayes of feeding, multiplied, is ——	800
The Sum ——	2150

As 2150, to 50; so is	750 A's	Stock to	*l. s.* 17-9 A's	Rent.
	600 B's		13-19 B's	
	800 C's		18-12 C's	

9. To work by the Line of Numbers, the Extent of the Compasses from 1 to 25, shall reach the same way from 30 to 750, the first Product of a A's double Number, or Stock.

And as 1 to 15, so is 40 to 600, the Product of B's double Number, and Stock.

And as 1 to 20, so is 40 to 800, the Product of C's double Number, and Stock.

Which three Products added, make 2150, the first term; and 50 is the second term; and 750, 600, and 800, the three Products severally, the third term. *Then,*

The Extent from 2150 to 50, shall reach the same way from 750 to 17-45, or 17*l.* 9*s.* And from 600 to 13-95, or 13*l.*—19*s.*

And

And from 800 to 18-60, or 18 *l.* — 12 *s.* the ſeveral Anſwers required; which being added together, make up 50 *l.* the whole Rent to be paid among them.

There be other Rules of Arithmetick, as the Rule called *Allegation*, *Medial*, and *Alternate*, and the Rule of *Poſition* or *Falſe-hood*; in the working of which, are ſo many Cautions in ordering the Numbers, before you come to the proportional work, that it would make the Book more bulky than uſeful; therefore I ſhall wave it, and refer you to the particular Books of Arithmetick, as that of Mr. *Record*, *Dee*, and *Mellis*; or that of Mr. *Wingate* Natural and Artificial, having in it plenty of *Examples*; and others alſo, as *Johnſons*, *Jaggers*, or *Moores* Arithmetick, any of which exceed the bounds I intend for this whole diſcourſe; I ſhall therefore paſs on to the Rules of Practice, in ſeveral kinds, as meaſuring Superfecies, and Solids, and Rules of double and treble Proportion and Queſtions of Intereſt, which are tedious by the Pen, without the help of particular Tables, and very eaſie by the Line of Numbers, as will fully appear in the next Chapters.

CHAP.

CHAP. VII.

The use of the Line of Numbers in measuring of any kind of Superficial Measure.

THe Measure that is commonly used in this Work, is a Foot-Rule, divided into 100 parts; or else into 12 inches, and those inches into halves, and quarters, or 8 parts; or inches and 10 parts: but in regard that the Numbers do most fitly agree to the 100 parts of a Foot, it will be convenient here to shew how to reduce them, or any other Fraction, from 12 *s.* to 10 *s.* or any other whatsoever, from one Fraction to the other, which by the Line of Numbers is quickly done; as thus, from 12ˢ. to 10ˢ.

Reduction.

Extend the Compasses from one Denominator to the other, the same Extent shall reach the same way from one Numerator to the other.

Example.

Example.

As 12 to 10, ſo is 6 half of 12, to 5 half of 10.

Again.

As 120 to 100, ſo is 30 a 4th of 120, to 25 a 4th of 100.

Which two Lines of Inches, and Foot-Meaſure, are uſually ſet together on Rules, for the ready way of Reduction by Occular inſpection, only in this manner, *as in the Figure*; And the like may be for any thing whatſoever, as Mr. *Edmond Windgate* hath largely ſhewed in his Arithmetick. Which Line being next to the Line of Numbers on your Rule, will be very plain and ready in the uſe of the Line of Numbers for feet and inches, or ſhillings and pence; and the ſame Rule of Reduction, ſerves for all manner of Fractions: For as the Denominator of one Fraction is to the Denominator of the other, (which in the Decimal work is alwayes a unite, with one, two, or more Cyphers) ſo is the Numerator of one, to the Numerator of the other.

And Note, That the operation of Decimal Numbers, and their Fractions, is no other than whole Numbers, except only the cutting off ſo many Figures as there is Fracti-

ons

ons in the Multiplicator and Multiplicand, after any Multiplication; as in the following *Examples* will appear.

This being premised, I come next to the Work.

Problem I.

The breadth of an Oblong Superficies given in Foot Measure, to find how much in length makes one Foot.

The Extent of the Compasses from the breadth to 1, shall reach the same way from 1, to the length required.

Example at 7 10th broad.

As 7 to 1, so is 1 to 1 Foot and 43 parts.

The breadth given in inches, to find how much make a Foot.

As the breadth in inches to 12, so is 12 to the length of a Foot in inches, and 10 parts.

Example.

At 8 inches broad, you must have 18 inches to make a Foot; for the Extent from 8 to 12, shall reach the same way from 12 to 18.

To

To work these two by the Line of Lines.

By Inches.
As –1 to =7, so is =1 to –1-43, the length in Foot-measure ;
As – 12 to = 8, so is = 12 to – 18 ; *Or else,*
As – 8 to = 12, so is – 12 to =18, the length in inches.

Problem II.

Having the breadth of an Oblong Superficies given in Foot-measure, to find how much is in a Foot long.

This is soon wrought ; for in every Foot long there is just as much as the breadth is, either in Foot-measure or inches ; for a piece of Board half a Foot broad, and a Foot long, is just half a Foot.

Problem III.

Having the length and breadth in Foot-Measure, to find the Content in Feet.

The Extent from 1 to the length, shall reach the same way from the length to the Content in Feet.

Example.

Example.

As 1 to 1 foot 50, the breadth; so is 11 foot, 10 parts, the length, to 16 foot and 65 parts, the Content required.

The breadth given in inches, and the length in feet, to find the Content in feet.

As 12 to the breadth in inches, so is the length in feet, to the Content in feet required.

Example at 9 inches broad, and 11 foot long.

The Extent from 12 to 9, shall reach the same way from 11 to 8 foot, 3 inches, or $\frac{1}{4}$.

By the Line of Lines.

As — 11, to = 12;
So is — 9, to — 8 $\frac{1}{4}$.

But Note, That in working this, and many such-like, it will be convenient to double your Scale in account, calling 10 at the end 20, and every single figure as much more, as to call 1 2, and 2 4, &c.

So

So that in this Operation, the work runs thus;

As — 11 taken from the Line of Lines, counting 1 for 10 as usually,
To = 6, the half of 12 reckoned double for 12:
So is = $4\frac{1}{2}$ counted for 9, to — $8\frac{1}{25}$ between the Center and 1.

Or else thus;

As — $5\frac{1}{2}$ counted for 11, is to = 6 counted for 12;
So is = 9, to — $8\frac{1}{4}$ near the end, and as large as may be.

Thus you may many times vary the manner of work to get the Answer latterally, and as large as may be on the Scale of Lines, by doubling or halfing the Numbers, or taking the whole Number of quarters, or using a less or a bigger Scale, as hath been hinted, and shall be more in places convenient, in the following Discourse, to attain exactness and ease, as much as may be, as time and practice will demonstrate to the willing Practitioner in these Operations.

Problem IV.

Problem IV.

Having the length and breadth given in Inches to find the Content in Superficial-Square Inches.

As 1 inch, to the breadth in inches; so is the length in inches, to the Content in Superficial inches.

Example, 20 inches broad, and 36 inches long.

The Extent of the Compasses from 1 on the Line of Numbers to 20, shall reach the same way from 36 to 72, the true Number of Superficial Square inches in that Oblong.

By the Line of Lines.

As —— 36 to = 5, counted as 1;
So is = 10 counted as 20, to —— 72, at the largest Extent.

For Note, The reason that the Latteral 72 and 36, are from the same Scale in account; and the Parallel 1 and 20 counted Decimally, are from the same Scale also, or else according to the Proportion by the Line of Numbers;

As — 1 to = 20, So is — 36 to = 72. Here also is the same advance Decimally from 1 to 20, as before.

Problem V.

Problem V.

Having the length and breadth given in Inches, to find the Content in Feet Superficial:

As 144 to the breadth in inches, so is the length in inches, to the Content in Feet Superficial.

Example at 40 broad, and 60 long.

For the Extent on the Line of Numbers from 144, the number of inches in one foot, to 40 the breadth, shall reach from 60 inches the length, to 16 foot $\frac{1}{2}$ and 26 inches. To count so many inches on the Line, observe with me this way of Reduction, the 16 foot and $\frac{1}{2}$ is very plainly seen. *And Note*, That there is 10 cuts in this place between 16 and 17; and 10 times 14 is 140, which is near 144, the inches in a foot; so the Point of the Compasses staying at near 2 10ths beyond the half-foot, I count almost twice 14, which is 26 inches for the Fraction above 16 foot and $\frac{1}{2}$.

By the Line of Lines.

As — 144 (found between 1 and 2 near the Center) is to = 40 (at the figure 4)

So is half — 60, or the meaſure from the Center to 3, to = 8-35, which is the half of 16 foot $\frac{1}{2}$, and 26 inches: For if you had taken all 60, it would have exceeded the whole Parallel Radius, where the Anſwer would have been right 16 $\frac{1}{2}$ and better, but taking the half, it gives the half alſo.

Or elſe work thus with a Latteral Anſwer.

As $\frac{1}{2}$ — 60 to = $\frac{1}{2}$ 144, So is all = 40 to — 16 $\frac{7}{10}$.

Or,

As all — 40, to $\frac{1}{2}$ = 144; So is all = 60 to — 33 and $\frac{4}{10}$ the double of 16 and $\frac{7}{10}$.

Note, That in both theſe two laſt workings, the 144 is at 72, which is the half of 144; to make the work the larger. By theſe the excellency of the Line of Numbers, over the Line of Lines, is evident in theſe kind of Proportions. And for diſcovering the Reaſon of theſe Proportions, read the beginning of the 6th Chapter, Section 3d.

Problem VI.

Problem VI.

The length and breadth of an Oblong Superficies being given, to find the side of a Square equal to it, by the Line of Numbers.

Divide the space between the length and breadth into two equal parts, and the middle Point shall be the side of the Square equal to the Oblong given, in quantity.

Example.

If a long Square, or Oblong, be 18 foot one way, and 12 foot the other way; the middle Point between 18 and 12 is 14 and $\frac{7}{10}$ *ferè*; for 18 multiplied by 12, makes 216, and 14-7 multiplied by 14-7, is near 216 also.

To do this by the Line of Lines is shewed at large in the 7th Section of the 6th Chapter.

Problem VII.

Having the Diameter, or Circumference of a Circle, to find the Circumference, or Diameter, or Squares equal, or Inscribed, and Content.

For this purpose there are certain Proportional Numbers found out, *As thus;*

If the Diameter of a Circle be 10, then

the *Perifera*, or Circumference, is 31 42; the ſide of the Square equal to the Circle, is 8-862, the ſide of the Square inſcribed is 7-071, and the Superficial Content is 78-54; ſo that any one of theſe being given, you may find out any of the reſt by the Line of Numbers.

Thus having the Diameter, to find the Circumference.

As 10 to 31 42, ſo is the given Diameter to the Circumference required.

Or,

As 10 to 8-862, ſo is the given Diameter to the Square equal.

As 10 to 7-071, ſo is the given Diameter to the Square inſcribed.

As 10 to 30, ſo is 78-54 to the Square of the Area of that Circle, whoſe Diameter is 30. *Or*, To the Diameter turning the Compaſſes twice.

As for Example.

Let the given Diameter of a Circle be 30,

The	Circumference	is	94 26	Diameter	10 00	
	qu are equal		26 58½	Circumference	31 42	
	Square within		21 21	Square equal	8 862	
	Content or Area		707 00	Square within	7 071	
	Diameter		030 00	Area or Content	78 54	

Also Note, If the Circumference be first given, then say, As 31 42 is to the Number 10 for the Diameter, or 8-862 for the Square equal; or to 7-071 for the inscribed Square; so is the given Circumference to the rest.

But to find the Area say, As the fixed Diameter 10, is to the given Diameter 30; so is the fixed Area for 10, *viz.* 78-54, to 707 — by turning the Compasses two times.

Or if the Circumference be given, and I would find the Area.

As 31-42, the fixed Circumference, is to 94-26 the given Circumference; so is 78-54, the fixed Area, answering the fixed Circumference 707, the Area

 required,

required, turning the Compaſſes two times the ſame way.

Thus by having five Centers at the five fixed Numbers; or four Centers anſwering to the four fixed Numbers; for a Circle whoſe Diameter is 10, having any one of thoſe 5 given, you may find any of the other required.

Thus you have eight Problems couched in one; therefore be the more diligent to underſtand it. To work theſe by the Line of Lines, obſerve the former directions, which for brevity ſake I now omit.

Problem VIII.

The Content of a Circle being given, to find the Diameter.

Divide the diſtance on the Line of Numbers, between the fixed Content, or the Point 78-54, and the given Content into two equal parts; that diſtance laid the ſame way, from the fixed Diameter, ſhall reach to the required Diameter.

Example. The Content being 707-00.

The half diſtance between 78-54, and 707, ſhall reach from 10 to 30, the Diameter required.

Problem IX.

Problem IX.

The Content of a Circle being given, to find the Circumference.

Divide the distance between the fixed and the given Contents or Area's into two equal parts, that distance laid from the fixed Circumference, shall reach to the required Circumference.

Example.

A Circle, whose Area is 707, shall be 94-26 about. For the half distance between 78-54, and 707-00, shall reach from 31-42 the fixed Circumference, to 94-26, the enquired Circumference.

And from 8-862, the fixed Square, equal to 26-58 $\frac{1}{2}$, the inquired Square Equal.

And from 7-071, the fixed Square inscribed, to 21-21, the inquired Square Inscribed.

Problem X.

Certain Rules to measure several Geometrical Figures Superficially.

For the Square, the long Square and Circle, hath been spoken to just before; All other Figures are to be reduced to a Square, or to a long Square; and then measured by Multiplication, as before.

Or thus.

Multiply the Diameter by it ſelf; and then that Product by 11: then laſtly, divide this laſt Product by 14, and the Quotient ſhall be the Area, or Content, of the Circle required.

Circle. For a *Circle* (otherwiſe) thus; Multiply half the Diameter, by half the Circumference, and the Product ſhall be the Content required.

Half-Circle. For a *Half Circle*; Multiply half the Diameter of the whole Circle, and a quarter of the whole Circumference together, and the Product ſhall be the Content.

Quadrant, or the quarter. For a *Quadrant, or a quarter* of a Circle; Multiply half the Arch, by the half Diameter, or Radius of the Circle, and the Product ſhall be the Superficial Content.

Leſſer-parts. The like Rule holds for any *leſſer portion* of a Circle, whoſe Point goeth to the Center, *viz.* to take half the Arch, and the whole Radius, and multiply them together, and the Product ſhall be the Content.

Any Segment of a Circle given, to find the true Diameter.

Square half the Chord, and divide the Product by the Sine, then add the Quotient and Sine together; the ſum is the Diameter.

Chord

Chord is 24 $\frac{1}{2}$ 12. Squared is 144; divided by 8, gives 18 in the Quotient, which added to 8, makes 26 for the Diameter.

For any other *Segment* of a Circle, find the true ſemi-Diameter, and meaſure it as before; then take out the Triangle, and the remainder is the true Content of the Segment. *See Chapter* 3. 11, 12. *Segments.*

Or elſe thus, by the Line of Segments joyned to a Line of Numbers, in this manner.

To the Segment given, find the true Diameter, by *Chap.* III. 11, 12. Then having the Diameter, find out the Area, or Content of the Circle, by any of the former Rules, then the Proportion or Analogy is thus;

As the whole Diamcer is to 100 on the Segments; So is the Altitude of the Segment, whoſe Area is required to a 4th Number on the Line of Segments, which you muſt keep.

Then Secondly,

As 1, to the whole Content of the whole Circle given; So is the 4th Number, kept, counted on the Numbers, to the Area of the Segment required.

If the Line of Segments is not on your Rule, then this Table annexed, will ſupply the defect, reaſonably well thus:

A Table to divide a Line of Segments, making the whole Circle 10000 *parts.*

A Table of Segments.

Seg.	*par*	*Seg.*	*par*	*Seg.*	*par*	*Seg.*	*Par.*
70	1	1127	2	3484		6682	
112	2	1151	4	3566		6786	
147	3	1177	6	3645		6850	
178	4	1201	8	3729		6935	
206	5	1224	7	3810	35	7020	75
233	6	1248	2	3892		7106	
258	7	1272	4	3971		7193	
282	8	1296	6	4050		7281	
307	9	1318	8	4121		7370	
329	1	1341	8	4211	40	7460	80
350	1	1365	2	4290		7550	
371	2	1388	4	4369		7642	
392	3	1411	6	4448		7735	
412	4	1433	8	4527		7829	
431	5	1455	9	4606	45	7924	85
451	6	1478	2	4686		8022	
469	7	1500	4	4766		8119	
487	8	1522	6	4844		8222	
507	9	1544	8	4922		8327	
524	2	1565	10	5000	50	8436	90
558	2	1673	11	5078		8552	
592	4	1778	12	5156		8669	
626	6	1881	13	5234		8788	
657	8	1978	14	5314		8908	
688	3	2076	15	5394	55	9029	95
718	2	2171	16	5473		9172	
749	4	2265	17	5552		9330	
779	6	2358	18	5631		9505	
808	8	2450	19	5710		9720	
0836	4	2540	20	5789	60	10000	100
0864	2	2630	21	5869			
0892	4	2719	22	5950			
0918	6	2807	23	6029			
0948	8	2894	24	6108			
0970	5	2980	25	6190	65		
1000	2	3065		6271			
1027	4	3150		6355			
1051	6	3214		6434			
1077	8	3318		6516			
1102	6	3402	30	6598	70		

The Diameter of the Circle, answering to the Segment given, being found out, Say,

As the whole Diameter to 100; so is the Altitude of the Segment to a 4th Number, which sought in the Table of Segments, or the nearest to it, gives in the parts the Number to be kept. Then again,

As the whole Content of the Circle fixed, *viz.* 100, is to the whole Content of the new Circle; so is the Number kept, being the Content, or Area, of the fixed Segment, to the Area of the Segment required.

Example.

Let the Segment of a Circle, whose whole Area is 314-2, and whose Diameter is 20, and let the Altitude of the Segment be 5, one 4th part of the whole Diameter.

Then say,

As 2-000, the whole Diameter given, is to 10000:

So is the Altitude of the Segment 5, to 2500, the 4th; which sought in the Table of Segments, in the Parts, gives 19-50 for a 5th Number to be kept.

Then again,

As 1, to 314-2, the whole Area; So is 19-50, to 61-30, the Area, or Content of the Segment required.

For

Triangles. For all manner of *Triangles*, multiply the longest side (being properly called the base) by half the perpendiculer, and the Product shall be the Content of the Triangle; or as 2 to the base, so is the perpendiculer to the Content.

Rhombus. For a *Rhombus*, being a Figure like a Quarry of Glass, containing 4 equal sides, and two pair of equal Angles: And any Figure having his opposite-sides Parallel one to another; then the length of one side and the nearest distance between the other two opposite-sides multplied together, shall be the true Area required.

Trapeziaes. For all other four-sided-figures, call'd *Trapeziaes*, being irregular Figures; draw a Line from one corner to the other, which makes it two Triangles; then multiply that Line, being the whole base of both the Triangles, by the half sum of both the Perpendiculers, and the Product shall be the Content required.

Or,

Regular-Polligons. For all *Regular Polligons*, or Figures, with equal sides, the measure from the Center to the middle of one side, and the half sum, of the measure of all the sides multiplied together, shall be the true Area, or Content thereof.

All

All other Figures whatsoever, of how many sides soever they be, may be reduced to Triangles, or to Trapeziaes, and measured as before; which kind of Figure, Surveyors and Builders oftentimes meet withal, in their Operations.

Problem XI.

For the measuring of an Oval, the best way is to reduce it to a Circle thus; Ovals.

Divide the distance on the Line of Numbers, between the length and the breadth of the Oval into two equal parts; and the middle Point where the Compass stayeth on, shall be the Diameter of a Circle equal in Area to the Oval given.

Example.

Suppose an Oval be 10 foot long (transverse) and 8 foot broad (conjugate); the mean proportion, between 10 and 8, is 8-95: I say, that a Circle whose Diameter is 8-95, is equal to an Oval of 8 broad, and 10 long; And how to measure the Circle, is shewed before.

Of these Figures.

If the Content be 100, then the sides of these Regular Figures are as followeth, and

also

alſo ſo in proportion, is any other quantity, or content required.

Perpendiculer-Triangle,	13.	123.
Trianguler-ſide,	15.	2.
Square, its Side,	10.	0.
Pantagon of five Sides,	7.	62.
Hexagon of ſix,	6.	02.
Heptagon of ſeven,	5.	26.
Octagon of eight,	4.	55.
Nonagon of nine,	4.	03.
Decagon of ten,	3.	06.
Half Diameter, or Radius,	5.	64.

Example as thus:

I would have a Triangle to contain 200, *What must the Sides be?*

The half diſtance on the Numbers between 100 and 200, ſhall reach from 15-2 to 21-5, the ſide required.

And from 13-123 the fixed perpendiculer for a Triangle, whoſe Area is 100, to 18-6, the perpendiculer of an equilatteral *Triangle,* whoſe Area is 200.

But

But if the Sides be given, and you would find the Area, work thus;

The Extent from the fixed-side, to the given-side, shall reach at two turnings, from the fixed Area, to the Area required.

The Extent from 15-2, to 21-5, shall reach, at twice repeating, from 100 to 200.

Problem XII.

To make an Oval equal to a Circle, having the Diameter of the Circle, and the length or breadth of the Oval given.

Set one Point of the Compasses in the Diameter of the Circle found out on the Line of Numbers, and the other Point to the Ovals length; then turn that distance the contrary way from the same Diameter-point, and it shall reach to the breadth of the Oval required.

Example.

Let the Diameter of a Circle be 10 foot, I would have an Oval to contain as much as the Circle, and be 12 foot long; the Query is, how broad must it be?

Set one Point in 10, and the other in 12, that Extent turned the other way from 10, shall reach to 8-34, the breadth of the Oval required.

If you pleaſe to alter the breadth or length, you ſhall ſoon find the length or breadth accordingly.

To work this by the Line of Lines, you muſt work by the Directions in the 7th Section of the 6th Chapter, as thus;

Firſt, To find the Content of the Oval, joyn the length and breadth in one ſum, to get the ſum, the half ſum, and difference, and half difference; then open the Sector, (or lay the Thred on 60|0) to a Right Angle; Then count half the difference from the Center downwards, and note the place; then take half the ſum between your Compaſſes, and ſetting one Point in the half-difference, and extending the other to the other Leg, (or perpendiculer Line) and it ſhall ſhew a Point, whoſe diſtance from the Center is the mean proportional required; which is the Diameter of a Circle, equal in Area, to the Oval, or *Elipſis* given to be meaſured; as before is ſhewed.

To make an Oval equal to a Circle.

Take the gueſſed half-ſum of the length and breadth of the Oval, and ſetting one Point in the Diameter of the Circle; and on the other Leg, ſet at a Right Angle, the other Point ſhall ſhew half the difference, between

between the length and breadth of the O-val; then if the mean proportional between them be equal to the Diameter, you have wrought right; if not, then reſolving upon the length or breadth of the Oval, take more or leſs, for the breadth or length accordingly: Herein alſo is ſeen the excellency of the Line of Numbers, in many operations.

Problem XIII.

The length and breadth of any Oblong Superficies given in Feet, to find the Content in Yards.

As 9 foot (the number of feet in one yard) to the length in feet and parts;
So is the breadth, in feet and parts, to the Content in yards.

Example at 13 Foot 6 Inches long, and 7 Foot 6 Inches broad.

The Extent of the Compaſſes from 9 to $13\frac{1}{2}$ the length, ſhall reach the ſame way from $7\frac{1}{2}$ the breadth, to 11 yards and a quarter, the Content.

Note, That if you meaſure by feet and hundred parts, you ſhall find this way exceeding ready; the Anſwer being given in Yards, and hundred parts of a yard. But if you have a yard divided into a 100 parts, to measure withal; *Then the Rule is thus:*

As 1 to the length or breadth, so is the breadth or length to the Content in yards.

Example at 3 yards, 72 parts broad, and 5 yards, 82 parts long.

The Extent of the Compasses on the Line of Numbers, from 1 to 3-72, shall reach the same way from 5-82, to 21 yards 65 parts, the Content in square yards, and 100 parts.

By the Line of Lines.

As —5-82, to = 1 at 10 the end;
So is = 3-72, to — 21-65 yards.

Or in the Example before.

As — 13-6, counting $6\frac{1}{2}$ for 13, is to = 9;
So is = $7\frac{1}{2}$ to — $11\frac{1}{4}$; as you counted at first.

Problem XIV.

The length and breadth of any Wall, being given in feet and 100 parts, to find how many Rods of Walling there shall be at a Brick and an half thick.

First you must Note, That 272 foot and a quarter, makes one Rod, (or so many feet is in a Rod).

Secondly, That let the Walls be half a Brick, one Brick, two Bricks, two and a half, or three Bricks thick; it is to be reduced to Brick and a half thick, as a standard thickness.

Thirdly Note, That this reducing to a Brick and half thick, may be at the measuring, or after the casting-up, as you please, as in the *Examples* following will plainly appear.

As thus for Instance;

A Front, or side-Wall of a House is to be measured, wherein the Celler-story Wall is 2 Bricks and a half thick; The Shop and first Chamber-story is two bricks thicks; the other Stories 1 Brick and a half thick; and the Gable-ends 1 Brick thick.

The nearest way to measure this Wall, I conceive is thus ;

1. The Cellar-ſtory is 10 foot high, but being 2 bricks and a half thick, I make it 16 foot 8 inches high, by adding two thirds of 10 foot, to the 10 foot high, which is 6 foot 8 inches, in all 16 foot 8 inches.

2. The other two Stories, are ſuppoſed 22 foot ; but in regard they are two bricks thick, I add one third part of 22 foot, which is 7 foot 4 inches, to 22 ; and it makes 29 foot and 4 inches, the height of the Shop and next Story above.

3. The other two Stories being a brick and half thick, need no alteration, which ſuppoſe may be 19 foot.

4. The Gable-end, or Garret-ſtory, if any be, being but one brick thick ; you muſt take away one third part to bring it to a brick and a half. Alſo if it be a Gable-end, *Note*, it is a Triangle, and you muſt meaſure but half the height, and the whole breadth, to find the Content ; which here may be 5 foot.

The

The Cellar Story,	16 — 8
Two next Stories,	29 — 4
Two next Stories,	19 — 0
The Garret,	5 — 0
	70 — 0

5. Add all these sums of feet high together, and they make 70; then measure the breadth, which is common to every Room, the out-side going upright, which in a double House may be 36 or 40 foot.

6. Then having gotten the Dimentions right by the Line of Numbers, *Say*,

As 272 $\frac{1}{4}$ (the feet in one Rod) is to 40 foot, the breadth of the House; so is 70 foot, the whole height of every several Story, (reduced) to 10 Rod and 29 parts; which 29 parts you may call a quarter of a Rod, and 10 foot and a half.

The reason whereof is apparent thus:

As 100 is to 272 $\frac{1}{4}$; so is 29 to near 79; of which 79-68 is a quarter of a Rod, or 25 of 100 is a quarter likewise, which by the Line of Numbers is apparently seen; then every 10th part is 2 foot, and 72 of a hundred, which is near two and three quarters; so that here 25 being a quarter of a

Rod, there is 4 hundred parts more in 29 : *Then thus*; the double of 4 is 8, or, twice 4 is 8, and four times three quarters is three foot more; of which you must abate somewhat (because 72 $\frac{1}{4}$ is not 75, which is just three quarters) and all put together, make ten rod, one quarter, ten foot and a half: for if you shall divide the Product of 40, multiplied by 70, which is 2800 by 272 $\frac{1}{2}$, you shall find the Quotient to be 10 rod, 78 $\frac{1}{2}$, which is, as before, 10 rod, 1 quarter and 10 foot and a half.

But note also by the way, That when you come to take out the deductions for the doors and windows, if any happen in a Wall of two Bricks and a half, or in two Bricks; you must add two thirds, or one third more to the length or bredth one ways; and then casting them up severally, when they be of several lengths or breadths, you shall do no wrong to the Work-master nor Work-man: For true *Arithmetick* and *Geometry* will lie for no man, or use any kind of partiality.

This I conceive is as near a way, as any such business can be performed. But if you will measure every Story severally, taking account of each Story severally in their thicknesses; then, after it is cast up, the best way, by the Rule, to reduce it, is *thus*;

As

As 3 half bricks, for a brick and a half, is to any other number of half bricks thick, over or under 3; So is the Content at that rate accordingly, to his Content, at a brick and a half required.

Example.

1269 foot at 5 half bricks thick is 2115, for two thirds of 1269, which is 846, added to 1269, makes 2115; For the Extent on the Line of Numbers, from 3 to 5, shall reach the same way from 1269 to 2115, the Number required to be found out.

Otherwise thus;

To bring any kind of thickness, to one brick and a half thick, at one operation, by the Line of Numbers.

For this purpose, you must use several Points, as so many gage Points, as in the short *Table* following doth appear,

For half a brick, use ——	3-00000
For 1 brick, —— ——	1-50000
For 1 brick & a half, use	1-0000
For 2 bricks, —— ——	0-7500
For 2 bricks & a half, use	0-6000
For 3 bricks, —— ——	0-5000
For 3 bricks & a half, ——	0-4285
For 4 bricks, —— ——	0-3750
For 4 bricks and a half, —	0-3333
For 5 bricks, —— ——	0-3000

For 5 bricks and a half, — 0-2727
For 6 bricks, ———— 0-2500
&c. *ad infinitum.*

Example at the 6 ordinary thicknesses.

Let a Wall be 30 foot long, and 10 foot high; and let it be supposed of any of these thicknesses following, from half a bricks length, to three bricks length in thickness; then thus in order, increasing, &c.

First, at half a Foot.

For ½ brick. As 3 to 30; so is 10 to 100 foot, at 1 ½.
For 1 brick. As 15 to 30; so is 10 to 200 foot, at 1 brick.
For 1 ½ thick. As 10 to 30; so is 10 to 300 foot, at 1 ½.
For 2 bricks. As 0-75 to 30; so is 10 to 400 foot, at 1 ½.
For 2 ½ thick. As 0-60 to 30; so is 10 to 500 foot, at ½.
For 3 bricks. As 0-50 to 30; so is 10 to 600 foot, at 1 ½.
For 3 ½ thick. As 0-4285 to 30; so is 10 to 700 foot, at 1 ½.
For 4 bricks. As 0-3750 to 30; so is 10 to 800 foot, at 1 ½.

And

And so for any other thickness, as far as you please; which Points are found *thus*;

The Extent, from the number of bricks, any Wall is thick to 15 (or 1 and ½) shall reach the same way from 10, or 1, to the Gage-Point required for that Wall, or Walls of that thickness.

Example.

As 2 to 1½; so is 10, to 0-750, for 2 bricks thick, &c.

Lastly, having the Number of Feet in the whole work, to find how many Rods there is. *Say*,

If 272¼, be one Rod; what shall any other Number of Feet make in Rods?

The Extent of the Compasses from 172¼, to 1, shall reach the same way, from the Number of Feet, to the Number of Rods, and hundred Parts, or Rods, and Quarters, and Feet; as by the 6th, last mentioned.

Example.

In 5269 Feet, how many Rods?

The Extent from 272¼, to 1, shall reach the same way, from 5269, to 19 Rod, and 36 parts of a 100; or, 19 Rod 1 quarter, and 29 foot, and a quarter of a foot. The 19 Rod, and a quarter, is seen plainly

plainly on the Rule; and 25 being a quarter, 36 is 11 parts more; for wch 11 parts more, I ſay, 2 times 11 is 22 foot, and 11, 3 quarters of a foot is near 8 foot, which put together, makes 29 foot, as before: Or, as the Compaſſes ſtand, turn them the contrary way, from the Decimal parts, above the even quarter, and it ſhall reach to the odd feet above the quarter required.

Example.

The Extent from 272 $\frac{1}{4}$, to 100; or 1, ſhall reach the contrary way from 10 $\frac{1}{2}$, to 29 foot, the feet above $\frac{1}{4}$ of a Rod.

8. Obſerve, That the Tyling, the Roof, the Floors, and Partitions, are meaſured by the Square; which is 10 foot Square every way, or 100 foot in Area. The Chimneys are uſually done by a certain rate for a Chimney; or if to be meaſured, thus are the height and breadths taken, &c.

If a Chimney ſtand ſingly and alone, not leaning againſt, or in a Wall, the uſual way is to girt it about; and if the Jaumes are but a brick thick, and wrought upright over the Mantle-tree to the Floor; then I ſay, girt it about for a length, and the height of the Story is the breadth, at a brick thick, becauſe of the gathering together, to make room for the next Hearth above in the next Story.

But

But if the Chimney-back be a Party-Wall, the Wall being first measured, then the brest and the depth of the two Jaumes is one side, and the height of the Story another side, to be multiplied together, at a brick and a half thick, or a brick thick, according as the Jaumes be, and nothing to be abated for the want between the Hearth and the Mantle-tree, because of the Withs and thickning for the next Hearth.

For measuring the Shafts of the Chimneys.

Girt with a Line, round about the least place of them, for one side; and the height for the other side, at a brick thick, in consideration of the Withs, Pargitting, and Scaffolds.

In measuring of Ceiling a foot broad, and the length of the Vallies is alwayes to be allowed, more than the whole Roof; Also the length of the Rafter feet, above or beyond the Roof.

When Rafters have their usual pitch, which is, when the breadth of the House is 12 foot, the Rafter is 9 foot long, which is 3 quarters of the Floors breadth, be it more or less; then, I say, that the Content of one Floor, and half so much, is the Area of the whole Roof in Squares; to which is to

be

be added, the Vallies and Rafter-Feet, or Eves, in Tileing.

And alſo a Deduction for Chimney-room, and Gutters, if any be.

Which work by the Line of Numbers, is done at one Operation, thus;

As *6666*, is to the length of the Houſe; So is the breadth to the Content in the Roof.

Example.

A Houſe 30 foot long, and 20 foot broad, is 900 foot, or 9 ſquare.

For the Extent, from *6666*, to 20, ſhall reach the ſame way from 30 to 900.

Alſo in meaſuring of the Roof, as to Carpenters work, by the Square, there is to be allowance for thoſe Rafters in the Dormers, and Gable-ends, on which no Tiles are laid, as over-work for a particular uſe and convenience, more than need be in a bare Covering, or Roof.

Alſo in meaſuring of Plaſterers work in Partitions and Walls; the Timbers and Quarters, are not to be deducted out of the rendring for Work only, except when the Workman finds the Work and Stuff alſo, then ſubſtract a 6th part for the Quarters in the rendring Work: But in Ceilings, the Sum-

Summers which are ſeen, are alwayes abated; and Doors and Windows alſo, unleſs by a due conſiderate (or an unconſiderate) bargain of running meaſure.

Thus you have a brief account of the uſual order, uſed among Workmen, in taking the Dimentions of a Houſe, *viz.* Brick-work by the Rod; Tileing and Carpenters-work by the Square; Chimneys uſually by the Fire; And Plaſterers and Painters-work by the Yard; Glaſiers, by the Foot.

There are many other things to be taken notice of in the Carpenters Bill, as Lintels, Mantle-trees, and Taſſels; Luthern Lights, and other Lights, both Architrave and Plain Lights, Sky-lights, or Cubiloes, Modillean Corniſh, and guttering Penthouſe Corniſh, Timber-Front-Story, Cellar-doors, and Door-caſes; the Plank and Curb at the Cellar-ſtairs, Dogleg-ſtairs, and Open-Newel-ſtairs, Canted-ſtairs, counted either by the ſtep or pair; together with the half Spaces on the Corners of the open Newel-ſtairs, the Rayles and Ballaſters, ſmall and great Corniſh, Outſide-work and Partitions, Ceiling Joyſts, and the Aſhlering, Boarded Partitions, and Chequer-work; back-Doors, and Door-caſes; Window-boards, and Wall-timber; Planks in the Foundation, Paleing, Penthouſe-floors, and

Penthouſe-

Penthouſe-roof, furring the Platform, Centerings for the Chimney, Trimmers, Girders-ends, Ends of Breſt-ſummers, and Plate; and more the like, which will come in Accompt to be remembred and ſet down according as the Building is.

Alſo, with due allowance into the Wall that way the ends of the Joyſts are entred or laid in the Wall, *as thus*;

If it be Framing Work is only meaſured, then 9 Inches ought to be allowed into each Wall, that way the Joyſts ends are laid; becauſe every Joyſt, if well laid, ſhould have 9 inches, at leaſt, hold on the Wall.

But if it be Timber, and Boarding, both to be meaſured, then 6 inches only is a competent allowance; becauſe the Timber is uſually vallued at one third part more than the Boarding is.

Alſo, As the Workman doth think on this, the Work-maſter may not forget to deduct for Stairs, and Chimneys alſo, where Work and Stuff are both meaſured; though for Work only it may be very well allowed, unleſs the better Price make an allowance for it.

Note alſo, That by the Line of Numbers, you may readily find the length of the Hips and Rafters, in a Roof of any largeneſs,

ness, at true pitch, by this following Proportion and Table.

The Breadth of the House being 40 Foot; and the Ends Square, the Length and Angles are, as in the Table, at the usual true pitch.

	feet.	100 par.
Whole breadth	40	00
Half breadth	20	00
Rafter	30	00
Hip-Rafter	36	00
Diagonal Line	56	57
Half-Diagonal	28	28
Perpendiculer	22	36

		deg.	min.
Hip Angles	at Foot	38	22
	at Top	51	38
	on the Outside	116	12
Rafter Angles	at Foot	48	10
	at Top	41	50

For any other House, by the Numbers thus: as suppose 18 Foot broad.

The Extent of the Compasses from 40, the breadth in the Table, to 18 the breadth given, shall reach the same way from 30, the Rafter in the Table, to 13-50, the Rafter required.

required. And from 36, the Hip in the Table, to 16-22, the Hip required. And from 22-36, the Perpendiculer in the Table, to 10-06, the Perpendiculer required. And from 56-57, the Diagonal in the Table, to 25-48, the Diagonal required. The Angles are alwayes the ſame in all Roofs, ſmall or great, as in the Table, being Square and true pitch.

If you would have Directions for Bevel or Taper Frames, to find the Lengths and Angles of Rafters and Hips, you may have it at large, in an *Appendix* to the *Mirrour of Architecture*; or, *Vincent Stamozzi*, Printed for *William Fiſher*, at the *Poſtern-Gate*, 1669.

By which Directions, and the Sector, you may find any thing that is there ſet down. As alſo, by the *Trianguler Quadrant*, Thred and Compaſſes.

Note alſo, That having Inches and Foot-meaſure together, you may preſently, by inſpection, find the price of one Foot, having the price of the Square, and the contrary. Alſo, having the 12 Inches on the other Foot, divided into 85 parts (near), and figured at every 8 with 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. ſhall repreſent pence and half farthings; then at any price the Rod, you have the price of one Foot, & the contrary.

As thus;

Let every Inch, represent one pound; and every 8th part, 2 shillings and 6 pence; or every 10th part, 2 shillings; because 8 half-crowns, or 10 two shillings, is 20 shillings.

Example.

Right against 6 Inches and a half, for 6 *l.* 10 *s*, on this other Line, I find 5 pence 3 farthings, the price of one Foot, at 6*l.* 10*s*. *per* Rod: And at 7 farthings *per* Foot, I find near 40 shillings, or 2 pound *per* Rod.

Also, at 40 shillings *per* Square, found at 40, on Foot-measure, is 4 pence 3 farthings; *per* Foot, found just against it on the Inches.

O CHAP.

CHAP. VIII.

The uſe of the Line of Numbers, in meaſuring of Land by Perches and Acres.

Problem I.

At any length of the Land, to find the breadth of the Acre.

IN the Anſwering of this Queſtion, it is not amiſs, but very needful to premiſe, how many Square Inches, Feet, Yards, Perches, or Chains (I mean a Chain of 66 Foot long) is contained in a Square Acre of Land; for which purpoſe, have recourſe to the Table annexed, which is drawn with great care and exactneſs for that purpoſe.

By which Table you may perceive, That 6272640 Square inches are contained in one Square Acre.

And

And (100000, or) one hundred thousand Square Links of a 4 Pole Chain, make a Square Acre.

And 43560 Square Feet, make a Square Acre.

And 4840 Square Yards, make a Square Acre.

And 1742, 4 Square Paces, make a Square Acre.

And 160 Square Perch, make a Square Acre.

And 10 Square 4 Pole Chain, make one Acre. As in the Table you may see.

And 3097 $\frac{1}{4}$ Square Ells, make one Acre of Land, Statute measure.

The Table.

	Inch	Links	Feet	Yards	Pace	Perch	Chain	Acre	Mile
Inch	1	7. 92	12	36	60	198	792	7920	63360
Link	62. 726	1	1. 515	4.56	7.575	25	100	1000	8000
Feet	144	2. 295	1	3	5	16 5	66	660	5280
Yard	1296	20. 755	9	1	1.66	5 50	22	220	1760
Pace	3600	57. 38	25	2,778	1	3. 3	13. 2	132	1056
Perch	39204	625	272. 25	30 25	10.89	1	4	40	320
Chain	627264	10000	4356	484	174.24	16	1	10	80
Acre	6272640	100000	43560	4840	1742.4	160	10	1	8
Mile	4014489600	6400000	27878400	3097600	1115136	102400	6400	640	1
Square	Inches	Links	Feet	Yards	Pace	Perch	Chain	Acre	Mile

Then as the length of the Land given in Feet, Yards, Paces, Perches, or Chains, is to the number of Square Feet, Yards, Paces, Perches, or Chains in a Square Acre; So is 1 to the breadth of the Land (in that measure the length was given) to make a Square Acre: See the *Examples* of all these measures in their order, *viz.* of Feet, Yards, Paces, Perches, and Chains.

Suppose a piece of Land be 660 Feet long, or 220 Yards, or 132 Paces, or 40 Perches, or 10 Chains in length; which several measures are all of the same quantity; I would know how much in breadth I must have to make a Square Acre?

Extend the Compasses from the length given, *viz.* 660 Feet, or 220 Yards, or 132 Paces, or 40 Perches, or 10 Chains; to 43560, for Feet; or to 4840, for Yards; or to 1742, for Paces; or to 160, for Perches; or to 10, for Chains; To the Number in the *Table* for that measure in a Square Acre; the same Extent applyed the same way from 1, shall reach to the Feet, Yards, Paces, Perches, or Chains required.

Note the Work.

1. As 660, to 43563, the Feet in a Square Acre;

So is 1, to 66, the breadth in Feet required.

2. As 220, the length in Yards, to 4840, the Square Yards in a Square Acre;
So is 1, to 22, the breadth in Yards required.

3. As 132, the length in Paces, to 1742;
So is 1, to 13-2, the breadth in Paces sought.

4. As 40, the length in Perches, to 160;
So is 1, to 4, the breadth in Perches.

5. As 10, the length in Chains, to 10;
So is 1, to 1, the breadth in Chains required.

6. As 176, the length in Elles, to 3097 $\frac{1}{7}$;
So is 1, to 17-6, the breadth in Elles required.

To work this by the Line of Lines, say;

1. As the — 43560, to = 660;
So is = 10, to 66, Latterally.

2. As

2. As the Latteral 220, to Parallel 4840;
So is Latteral 1, to Parallel 22, (single, double, or four-fold).

3. As — 132 doubled, is to = 1742 likewise doubled, because it falls near the Center;
So is — 1 quadrupled, *viz.* 4, to = 13-2 quadrupled, *viz.* 52-4.

4. As — 160, to = 40;
So is = 10 for 1, to — 4 Perch.

5. As — 10, to = 10;
So is = 1 to — 1, the breadth required.

If you would know how much breadth, at any length, shall make 2, 3, or 4 Acres;

Then say,

As the length given to the quantity of one Acre in that measure, according to the Table;
So is 2, 3, 4, or 5, to the breadth required.

Example at 30 *Perch in length.*

The Extent from 30 to 160, shall reach the same way from 4 to 21 Perch, and 34 of 100 (or 5 Foot, 06 Inches) the breadth of 4 Acres, at 30 Perches in length.

Problem II.

The length and breadth given in Perches, to find the Content in Perches, of any piece of Land.

The Extent from 1, to the breadth in Perches, shall reach the same way, from the length in Perches, to the true Content in Square Perches.

Example.

As 1 to 50, so is 179 to 8950, the Content in Square Perches.

Problem III.

The length and breadth being given in Perches, to find the Content in Square Acres.

The Extent from 160 to the breadth in Perches, shall reach the same way, from the length in Perches, to the Content in Square Acres.

Example.

As 160 to 50, so is 179 to 5-58 Acres, or 5 Acres, 2 Rood, and 13 Perches.

Problem IV.

The length and breadth of a piece of Land being given in Chains, to find the Content in Acres.

The Extent from 1, to the breadth in Chains, and 100 parts, which are Links, ſhall reach the ſame way from the length in Chains and Links, to the Content in Square Acres.

Example.

As 1 to 5 Chains, 52 Links, the breadth; So is 8 Chains, 72 Links, to 48 Acres, and 3960 Square Links.

Problem V.

Having the Baſe and Perpendiculer of a Triangle given in Chains or Perches, to find the Content in Acres.

The Extent from 2, if you uſe Chains; or from 320, if you meaſure by Perches, to the whole Baſe, ſhall reach the ſame way from the whole Perpendiculer, to the whole Content of the Triangle; or if it be a *Trapezia*, joyn both the Perpendiculers in one ſum.

Example.

Example.

As 2 (for Chains) to 3-63, the whole Perpendiculer;
So is 11-80, the whole Base, to 21 Acres, 42 Links, the Content of the whole Triangle.

Or in Perches.

As 320 to 14-55, the Perpendiculer in Perches;
So is 47-20, the length, or base Line, in Perches, to 21 Acres, 24 Links, the Content in Acres.

Problem VI.

The Area, or Content of a piece of Land given, that was measured by Statute-Perches; to find the Content of the same piece of Land in Wood-land measure, or Customary Acres, or Irish Acres.

For the better understanding of this Problem, it is necessary to describe the several kinds and quantities of Perches, which are spoken of by Authors, and used in several places; together with their proportion to the Statute Perch of 16 Foot and a half square, *London* measure.

The kinds of *Perches*, are first Statute-measure of 16 foot ½ to the Perch, according to the Standard at *Guild-Hall*, or the King's Majesties *Exchequer*. Secondly, *Woodland*-measure, a Perch whereof contains 18 Foot Square of the same *London* measure. Thirdly, *Irish* Acres, of 21 Foot to the Perch or Pole. And lastly, Three sorts of *Customary*, used in several places of *England*, of 20, 24, *Cheshire* measure, and 28 Foot square to the Perch.

As for the Proportions one to another, that is, as 16 $\frac{1}{2}$, to 18, 20, 21, 24, 28, or any the like whatsoever.

But to find their difference in Squares or Scales, the Work is thus; *By the Line of Numbers*, First appoint what Number in an Inch shall be the Scale for Statute measure, which I shall appoint a Scale of 30 in an Inch.

Then the Extent from 16 $\frac{1}{2}$, to 18, for *Woodland* measure, shall reach the contrary way from 30, being twice repeated, to 25-2; so, I say, that a Scale made to 25-2 in an Inch, shall be the Scale for a *Woodland* Perch of 18 Foot Square; and in proportion to that of 16 Foot $\frac{1}{2}$, at 30 parts in an Inch.

Again, For *Irish* Acres, which are measured by a Pole of 21 Foot to a Perch, the Extent on the Line of Numbers from 21 to 16 $\frac{1}{2}$, shall reach (being turned twice the

same

ſame way) from 30, to 16½, the quantity of the Scale for *Iriſh* Acres, to be in proportion to a Scale of 30 in an Inch for Statute-meaſure; and ſo for the reſt, or any other whatſoever, as in the following Table.

The Feet contained in a Perch, for Statute, Iriſh, or for Cuſtomary.

Feet in a Perch		Scale		Scale in an Inch	
16½	*The Scale that is to it proportionable to 30 for Statute meaſure is,*	16½	is	30—00	*Statute-Meaſure.*
18		18		25—22	*Woodland-Meaſure.*
20		20		20—42	*Cuſtomary.*
21		21		18—50	*In an Inch for Iriſh.*
22		22		16—89	*Cuſtomary.*
24		24		14—20	*Cuſtomary Cheſhire-meaſure.*
28		28		10—42	*Cuſtomary.*
30		30		09—08	*Or any other.*

So that if you have ſeveral Scales made upon a Rule (to draw the Plot of your Field withal) to theſe Proportions (which may be convenient enough for Difference between one another), then for the reducing of the

the quantity of Acres found by Statute-measure, to Woodland, Irish, or Customary, is no more but thus:

Take the Acres, measured by Statute-measure, out of the Scale of 30 in an Inch appointed for Statute measure, and measure it in the Scale of 25-22 in an Inch for Woodland; or by the Scale of 18-55 for Irish Acres; or by the Scale of 16-89 for Customary; and you shall have the quantity of Woodland, Irish, or Customary Acres required.

Example.

Suppose I have 30 Acres of Statute measure, how many Acres of Woodland, Irish, or Customary measure will they make? Take 30 from the Scale of 30 in an Inch, and on the Scale of 25-22, it shall give 25-22, for so many Woodland Acres; and on the Scale of 18-55, for Irish Acres, it shall give 18-55 for so many Irish Acres; and on the Scale of 16-89 in an Inch for Customary Acres, it shall give 16-89 for so many Customary Acres, at 22 Foot to the Perch or Pole, &c.

This being thus fully premised, to work these Questions by the Line of Numbers only: the Extent of the Compasses from

16-5,

16-5, the Feet in a Statute Perch, to 18 the Feet in a Woodland Perch (or to 21 the Feet in an Irish Perch, or to 22, 24, 28 the Feet in a Customary Perch) shall reach from 30, the Acres in Statute measure, being twice repeated, to 25-22, the Acres in Woodland measure required, &c. it being a larger Acre must needs be less in quantity.

Which work is performed by the back-Rule of Three in a duplicated proportion.

Problem VII.

Having the Plot or Draught of a Field, and its Content in Acres, to find by what Scale it was Plotted; that is, by what parts in an Inch.

Suppose a Triangle, or a Parallellagram, or long Square, do contain 4 Acres and a half, which is set down in figures thus, 4-50; which if I should measure by a Scale of 12 in an Inch, might happen to be 2-25 Chains one way, and 1 Chain, 25 Links the other way; which two sums being multiplied together, make 2-5200, whereas it should be 4-5000;

Therefore by the Line of Numbers, to gain the true Scale, do thus;

Divide the distance between 2-5200, and

4-5000,

4-5000, into two equal parts; that distance laid the right way from 12, the Scale I measured by; shall reach to 16, the Scale the Plot was made by.

For Note, That if the Scale I guessed at, gives more than I should have, then I have too many in an Inch; but if less, I must have more in an Inch, as here, which infallibly sheweth which way, which is alwayes the same way as you divided the space, from the guessed Sum or Product, to the true Product.

To this Rule may be referred the way to discover the true size of *Glasiers* Quarries; the method whereof is thus: They are usually cut to, and called by 8s, 10s, 12s, 15s, 18s, and 20s in a Foot, or any other what you please; that is to say, 8 quarries of Glass of 8s, make a Superficial Foot; and 10 quarries of 10s, make a Foot Superficial; and 12 of the 12s, &c.

Also they are cut in a Diamond form to one sort of Angle for the Square quarries; and another for the Long quarries: The acute Angle of the Square quarries being 77 degrees, and 19 minutes; and the acute Angle of the Long quarries 67 degrees and 22 minutes: The long 12s being just 6 inches long, and 4 inches broad; and the Square 10s, 6 inches long, and 4

inches

inches, and 80 parts of a 100 broad.

This being the ſtanding Rule or Method, and thoſe two ſizes being known, I would find out any other, as 13s, or 14s, or 17s, and the like.

Do thus;

Divide the diſtance on the Line of Numbers, between the Content of ſome known ſize, and the Content of the inquired ſize, into two equal parts; and that diſtance laid the right way from the ſides of the known ſize (increaſing for a bigger, and decreaſing for a leſs) ſhall give the reciprocal ſides of the ſize required.

Example.

The Sides, Ranges, Lengths, and Breadth of Square 10*s, are as in the Table following; and I would have the Ranges, Sides, Length, and Breadth of* 14*s, an unuſual Size.*

The Content of a Square quarry of Glaſs called 10s, is a juſt 10th part of a Foot, which is 1 inch and 20 parts; or one 10th part of a Superficial Foot, containing 12 long inches.

And the Content of the ſize called 14s, muſt be one 14th part of the ſame meaſure, or Foot Superficial, which is 0-85714, that is 0-857 parts of one long inch in a 1000 parts.

Then

Then, by the Line of Numbers, divide the ſpace between 1-2000, the Content of the 10s, and 0-857 the Content of the 14s into two equal parts; that Extent, I ſay, laid the ſame way from 3-76, the Ranges of Square 10s, ſhall reach to 3-18, the Ranges for 14s: And from 3-84, the ſides of Square 10s, to 3-25 the ſides of Square 14s: And from 4-80 the breadth of Square 10s, to 4-05 the breadth of Square 14s: And from 6-00, the length of Square 10s, to 5-07 the length of Square 14s, the requiſites of the unknown Size required. And the like for any other whatſoever.

The true size of Glasiers Quarries, both Long and Square,
By J. B. 1660.

Square Quarries 77 deg. 19 min.							Long Quarries 67 deg. 22 min.						
Sizes	*Rang.*	*Sides.*	*bredth*	*length*	*Content.*	*Content.*	*Size.*	*Rang.*	*Sides.*	*bredth*	*length*	*Content.*	*Content.*
	*In.*100	*In.*100	*In.*100	*In.*100	*Inc.* 100	*Tens.*		*In.*100	*In.*100	*In.*100	*In.*100	*Inc.* 100	*Tens.*
8	4 20	4 30	5 36	6 70	1 500	1 250	8	4 09	4 41	4 90	7 34	1 500	1 250
10	3 76	3 84	4 80	6 00	1 200	1 000	10	3 65	3 95	4 38	6 57	1 200	1 000
12	3 43	3 51	4 38	5 47	1 000	0 833	12	3 34	3 61	4 00	6 00	1 000	0 833
15	3 07	3 13	3 92	4 90	0 800	0 667	15	2 98	3 23	3 58	5 37	0 800	0 667
18	2 80	2 86	3 57	4 77	0 666	0 555	18	2 72	2 95	3 26	4 90	0 666	0 555
20	2 66	2 72	3 39	4 24	0 600	0 500	20	2 58	2 79	3 10	4 65	0 600	0 500

CHAP. IX.

The use of the Line of Numbers in measuring of Solid measure, as Timber, Stone, or the like Solid bodies.

Problem I.

A piece of Timber being broader one way than the other, to find the side of a Square that shall be equal thereunto, being called, Squaring the Piece.

THe side of the Square, that shall be equal to the square of the Oblong, is nothing else but a mean proportion between the length and breadth of the Oblong:

As thus;

Suppose a piece of Timber is 12 inches in depth, and 16 inches in breadth (and 10 foot in length.)

16 the breadth, and 12 the depth, mul-

tiplied together, make 192; the Square or Product of 16 and 12 multiplied.

Now the ſquare Root of 192, which is near 13 $\frac{859}{1000}$, is the ſide of a Square, equal to 12 and 16, the depth and thickneſs of the piece of Timber propounded. For if you ſhall multiply 13-859 by 13-859, you ſhall find 192-071881, the neareſt Root, you can expreſs in 5 figures, and an indifferent true mean proportion, between 12 and 16, the depth and breadth; ſo that in fine, 13-86, is the ſide of a Square, nearly equal to 12 and 16, whereas the doubling and halfing, the old falſe way, gives full 14.

To work this by the Line of Numbers, is thus;

Divide the diſtance on the Line of Numbers, between 12 and 16, into two equal parts, and you ſhall find the Point to ſtay at 13, and near 86 parts, the Anſwer required.

The way of doing it, *by the Line of Lines*, is ſhewed in the VI Chapter, and 7th Propoſition, either by the Sector, or Trianguler Quadrant, and therefore needs no repetition in this place.

Problem II.

Problem II.

At any Breadth and Depth, or Squareneſs, to find how much makes a Foot of Timber.

1. If the Timber be ſquare (or ſquared) then the way by the Line of Numbers, is *thus*;

Firſt by Foot meaſure.

Extend the Compaſſes from the ſide of the ſquare to the middle 1, the ſame Extent applyed, or turned twice the ſame way from 1, ſhall reach to the length that makes a Foot of Timber, at that ſquareneſs.

Example.

Suppoſe a piece of Timber be 50 of a 100 (or 6 inches) or half a Foot Square (which is all as one) Extend the Compaſſes from 5 to 1 (forwards) the ſame Extent being turned two times, the ſame way from 1, ſhall reach to 4, being 4 Foot, or 40 ſuch parts, whereof the ſide of the Square was 5.

2. Secondly, *The ſame again by Inches.*

The Extent from 6 to 12, ſhall reach, being turned two times the ſame way from 12, to 48, the number of inches in length that makes a Foot, at that Squareneſs; being 48 ſuch parts whereof the ſide of the Square was 6.

So that,

As the ſide of the Square, in inches, is to 12 : ſo is 12 to a 4th, and ſo is that 4th to the length of a Foot required, turning the Compaſſes twice, the ſame way as you turned from the ſide of the Square in inches to 12.

3. If the piece of Timber, or Stone, be not Square or Squared,

Then

The Extent from 1 to the depth, ſhall reach the ſame way from the breadth to a 4th Number.

Again,

The Extent from that 4th Number to 1, ſhall reach, being turned once, the ſame way from 1, to the length of a Foot in Foot-meaſure required.

Example.

Suppoſe a piece of Timber be 0-333 deep, and 0-750 broad in Foot-meaſure ; or 4 inches deep, and 9 inches broad, as with a glance of your eye on inches and foot-meaſure, you may ſee how theſe Numbers agree. The Extent, I ſay, from 1 to 0-333, ſhall reach the ſame way from 0-750 to 2-50.

Again,

I ſay, The Extent from 250 the 4th, to

1,

1, ſhall reach the ſame way, from 1 to 40, or 4 Foot, the length required, to make a Foot at that breadth and depth.

4. By Inch-meaſure, to find the length of a Foot in Inches.

As 12 to the breadth in inches, ſo is the depth in inches to a 4th; then as that 4th to 12, ſo is 12 to the length in inches required.

Example.

The Extent from 12 to 9 the breadth, ſhall reach the ſame way from 4 the depth, to 3 for a 4th.

Then the Extent from 3 the 4th to 12, ſhall reach the ſame way from 12 to 48, the inches in length required, to make a Foot.

5. The breadth and depth given in Inches, to find the length of a Foot of Timber, in Feet and Parts.

Then ſay, As 1, to the depth; ſo is the breadth to a 4th.

Again,

As that 4th to 12, ſo is 12 to the length in feet and parts.

Example.

The Extent from 1 to 4, ſhall reach the ſame way from 9 to 36, a 4th;

Then,

The Extent from 36 to 12, ſhall reach the ſame way from 12 to 4 foot, the length in feet required.

The reaſon of this Order and Method, if you conſider, you will find thus;

In the 4th way of working, you went thus;

As 12, the inches in a foot, is to the breadth in inches;
So is the depth to 3 Foot.

But in the 5th and laſt way you ſay,

As 1 foot to the depth in inches;
So is the breadth to 36 inches, which is 3 foot alſo.

But altering the Order in the beginning, alters it in the iſſue, though the ſame truth, yet in or under divers denominations; for 48 inches, and 4 foot, are the ſame; yet ſometimes one way is more convenient than another.

Problem III.

Problem III.

At any Squareneß, or Breadth and Depth given in Foot-measure, or Inches, to find how much Timber is in a Foot long, in Foot-measure, or feet, and 100 parts or inches.

1. *If the piece of Timber be Square (or Squared) then work thus for Foot-measure.*

As 1, to the ſide of the Square, ſo is the ſide of the Square to the quantity of Timber in one Foot long; which multiplied by the length, gives the whole Content required.

Example.

At 50, or half a Foot Square, how much is in a Foot long?

Extend the Compaſſes from 1 to 5, the ſame Extent turned the ſame way from 5, reaches to 25, or a quarter of a Foot; then if the Tree be 12 foot long, 12 quarters will make 3 foot, the Content.

2. *The Side of the Square given in Inches, to find the Quantity, or Content, in a Foot.*

As 12, to the ſide of the Square, ſo is the ſide

ſide of the Square to 3 twelve parts of a Foot Solid, or $\frac{1}{4}$ of a Foot.

Or,

As 1, to the ſide of the Square, ſo is the ſide of the Square to 36, 144th parts of a Foot Solid.

Example.

The Extent from 12 to 6, the inches Square, ſhall reach the ſame way from 6 to 3 inches in a Foot long, which is 3 12th parts of a Foot Solid.

Again,

The Extent from 1 to 6, the inches ſquare, ſhall reach the ſame way from 6 to 36, the number of long inches in a foot long; or pieces of 1 inch ſquare, and a foot long, 144 of which makes one foot of Timber.

3. *But if the Piece be not ſquare (or ſquared) then to find how much is in a Foot long, work thus;*

As 1 to the depth, ſo is the breadth, to the quantity in a Foot.

Example 3 wayes: At 9 and 4 breadth and depth.

1. The Extent from 1 to 0-333 ſhall reach the ſame way from 0-75, to 0-25, or a quarter of a Foot; for Foot-meaſure.

2. The

2. The Extent from 1 to 9, shall reach the same way from 4 to 36, the long inches in a foot long; for Inch-measure.

3. The Extent from 12 to 4, shall reach the same way from 9 to 3 inches, or 3 12ths, *viz.* a quarter of a Foot; for Inch-measure.

Problem IV.

The side of the Square, or the breadth and depth given in Inches, or Foot-measure, and the length in Feet, to find the Quantity, or Content of the whole Piece, in feet and parts.

1. First, for Foot-measure;

As 1, to the side of the Square, in Foot-measure, so is the length in Feet to a 4th, and then that 4 to the Content in feet and parts.

Example.

The Extent from 1 to 0-833, the side of the Square, shall reach from 10 foot 25 parts, the length to 8-54, and from thence to 7-11, the Content in feet and parts required.

2. For Inch-measure, Say,

As 12 to the side of the Square, in inches, so is the length in feet, to a 4th; and then

that

that 4th to the Content in feet and parts.

Example at 10 Inches Square, and 10 Foot, 3 Inches in length.

The Extent of the Compasses, on the Line of Numbers, from 12, to 10 inches Square, shall reach the same way from 10 foot ½, or 3 inches, to 85-4 for a 4th; and from thence to 7-11, or 7 foot 1 inch, and a third part, the Content required. As by looking for 11 on the Line of Foot-measure, right against which, on the inches, is 1 inch and a quarter, and somewhat more.

3. But if the piece of Timber be not square, and you would measure it without squaring, by the *first Problem*;

Then say first by Foot-measure, thus;

As 1 is to the breadth, so is the depth to a 4th. *Then,*

As 1 to the 4th, so is the length in feet to the true Content, in feet and parts.

Example.

Let a Timber-tree of one foot 25, or a quarter one way, and one foot 50 the other way, and 12 foot long be measured.

The Extent of the Compasses from 1 to 1-25, shall reach the same way from 150, to 18-74, for a 4th.

Then

Then the Extent from 1 to 18-74, shall reach the same way from 12 foot, the length, to 22-50, for the Content; *viz.* 22 foot and a half, the whole Content required.

4. When the breadth and depth is given in Inches, and the length in Feet, to find the Content without squaring.

As 12, to the breadth in inches;
So is the depth in inches to a 4th:
Then,
As 12 to that 4th, so is the length in feet and parts, to the Content in feet and parts required.

Example at 15 inches deep, and 18 inches broad, and 13 foot long.

Extend the Compasses on the Line of Numbers from 12 to 15 the depth; the same Extent applied the same way from 18 the breadth, shall reach to 22-50, for a 4th.

Then the Extent, from 12 to 22-50, the 4th, shall reach the same way from 13 foot, the length, to 24 foot 38 parts, or 4 inches and a half, as a glance of your eye to the Inches and Foot-measure will plainly shew.

Thus

Thus you have the *Solution* of any *Question* that may concern proper Meaſuring by Foot-meaſure, and Inches; uſing only the Center at 10 for Foot-meaſure, and at 12 for Inch-meaſure, without troubling you with 144, or 1728, or 41-57, or the like, as in the little Book of the *Carpenters Rule*, may be ſeen.

To work theſe *Queſtions* by the Line of Lines, though it may be done ſeveral ways, yet no way ſo ſoon, nor ſo exact, as by the Line of Numbers: Yet I ſhall ſhew now in this place, together by themſelves, the Three principal Queſtions, *viz. How much makes a Foot in quantity*; And, *How much is in a Foot long*; And, *By the length, breadth and depth, the Content in Feet*: In the doing whereof, you muſt conceive the 10 principal parts to be doubled, and then 10 is called 20; and conſequently 6 is called 12, the Point ſo often uſed; and 5 is called 10, the Point uſed for Foot-meaſure.

1. *To find how many Inches makes a Foot at any Squareneſs.*

As the — ſide of the Square, to = 12;
So is the = ſide of the Square again, to
a — 4th Number.

Again,

Again,

As — 12, to that = 4th Number;
So is = 12, to the — Number of Inches that goes to make a Foot of Timber.

Example, at 8 Inches Square.

Take the diſtance from the Center to 4, accounted as 8; and make it a Parallel in 6, counted as 12; or lay the Thred to the neareſt diſtance, and there keep it. Then, take the neareſt diſtance from 4 to the Thred, and that ſhall be a Latteral 4th.

Then take the Latteral diſtance from the Center to 12, according to the uſual account, and make it a Parallel in the 4th laſt found, laying the Thred to the neareſt diſtance, and there keep it; then take the neareſt diſtance from 6, counted as 12, to the Thred, and that ſhall reach Latterally from the Center to 27 Inches, the length required, to make a Foot of Timber, at 8 Inches Square.

Which work I more briefly word thus, as formerly is done.

As — 4, counted as 8, to = 6, counted for 12;
So is = 8, to a — 4th.

Then,

Then,

As — 12, to = 4th; So is = 12, to — 27, the length in inches required.

2. *If you would use Foot-measure, count the 5 in the midst for 10, or 1 Foot; and work all the rest as before: As thus for Example.*

In the same quantity, Square, exprest in Decimals:

As — 0-666, counted double, to = 5, counted double for 10;

So is = 0-666, to — 22 $\frac{1}{2}$, for a 4th.

Then,

As — 1, to = 22 $\frac{1}{2}$; So is = 5 counted for 1, to 225, which is 2 Foot $\frac{1}{4}$, as by the Foot-measure and Inches you may see.

3. *If the Piece be not square, then say thus;*

As — breadth, to = 12;

So is the = depth, to the — 4th.

Then,

As — 12, to the = 4th;

So is = 12, to — length that goes to make 1 Foot.

Example.

Example, at 9 Inches, and 4 Inches, for breadth and depth.

As — 9, to = 12;
So is = 4, to — 150, for a 4th.

As — 12, to = 4th, best taken at 75 for largeness sake;
So is = 12, to — 48 Inches.

Or else thus;

As — 9, to = 1;
So is = 4, to — 1-80, a 4th.

Then,

As — 12, to = 1-80;
So is = 12, to — 4 Foot, the length in Feet, that goes to make 1 Foot of Timber.

4. *To find how much is in a Foot-long, at any Squareness.*

As the — side of the Square is to = 1, counted double as before;
So is the = side of the Square to the — quantity in a Foot.

Example at 6 Inches, or (5) half a Foot Square.

As — 5, to = 1; ſo is = 5, to — 125 for Foot-meaſure: *Or*,
As — 6, to = 12; ſo is = 6, to — 3, for 3 inches, or $\frac{1}{4}$ of a foot.

5. *The ſide of the Square given in Inches and the length in Feet, to find the Content in Feet.*

As — ſide of the Square, to = 12;
So is the = length to a 4th.

Then,

As — 4th, to = 12; ſo is = ſide of the Square to — Content required, in feet and parts.

Example, at 9 Inches Square, and 16 Foot long.

As — 9 to = 12, ſo is = 16 to — 12.

Again,

As — 4th, *viz.* 12, to = 12;
So is = 9, the ſquare to 9, the true Content of ſuch a Piece in feet and parts required.

The

The like Work serves for Foot-measure, using of 1.

6. *The Length given in Feet, and the Breadth and Depth in Inches, to find the Content in feet and parts.*

As — breadth, to = 12;
So is = depth, to a 4th.
Then,
As — 4th, to = 12;
So is = length, to — Content in feet.

Example at 5 Inches and a half Deep, and 15 Inches Broad, and 16 Foot Long.

As — $5\frac{1}{2}$, to = 12;
So is = 15, to — 69 for a 4th (at $34\frac{1}{2}$)
Then,
As — 69 (or $34\frac{1}{2}$) that — 4th, to = 12;
So is = 16, taken at 8, to — 9 foot 2 inches; the Content required.

Thus you see the way and manner of working by the Line of Lines, either on the *Quadrant*, or *Sector*-side, for the usual *Questions*; for I have neglected to give the Content of Pieces in *Cube Inches*, for two Reasons: First, Because it is very seldom required. Secondly, Becauſe the Line of Numbers at moſt will ſhew but 4 figures, which is not ſufficient for any Piece above 6 Foot, therefore not fit for Inſtrumental Work.

And withal you may obſerve, That alwayes the Latteral Extent firſt taken, muſt be leſs than the diſtance from the Center to the parallel Point of Entrance, which in theſe *Examples* is remedied by calling 6 12: And alſo, there are ſo many *Cautions* in doubling and halfing of Numbers, to make it applicable, that without due conſideration, you may ſoon err; Alſo, the opening and ſhutting the Rule, and uſing of ſeveral Scales, makes it far inferior to the Line of Numbers, which may be eaſily enlarged.

CHAP.

CHAP. X.

To measure Round Timber, or Cillenders, by the Line of Numbers.

Problem I.

Having the Diameter of a Cillender, given in Inches, or Foot-measure, to find the length of one Foot.

1. AS the Diameter in inches, to 46-90, (at which Diameter one Inch makes a Foot); So is 1 to a 4th, and that 4th to the length in inches.

Example at 10 *Inches Diameter.*

The Extent from 10 to 46-90, being turned two times the same way from 1, shall reach to 21 inches, 8 10ths, for the length of a Foot, at that Diameter, in Inches.

Or rather work thus;

As the Inches Diameter, to 13-54;
So is 12 twice, to the Inches that make
a Foot of Timber.

Or,

The Extent from 10, to 13-54, turned twice the ſame way from 12, ſhall reach to 22 Inches.

Or,

The ſame Extent being turned two times the ſame way from 1, ſhall reach to 1-831, which is the Decimal for 22 Inches, as by looking on Inch and Foot meaſure, you may plainly ſee.

Again,

2. For the ſame Diameter in Foot-meaſure.

The Extent from 0-833 (the Decimal of 10 Inches) to 1-128, being turned twice the ſame way from 1, ſhall reach to 1-83, which is almoſt 22 Inches, as by comparing Inches and Foot-meaſure together, is plainly ſeen.

Problem II.

Problem II.

Having the Diameter given in Inches, or Foot-measure, to find how much is in a Foot long.

1. As 13-54 (the Inches Diameter that make a Foot of Timber, at one Foot long), to the Diameter in Inches;
So is 12 to a 4th, and so is that 4th, to the quantity in a Foot long.

Example at 10 Inches Diameter.

The Extent from 13-54 to 10, being repeated two times the same way from 12, shall reach to 6 Inches $\frac{1}{2}$, or, 54 of 100, being somewhat more than a half Foot, for the true Content of one Foot long.

2. But if the Timber is great, then it is more convenient to have the quantity of a Foot, in feet and parts.

Then say,

As 13-54, is to the Diameter in Inches;
So is 1, to a 4th, and that 4th to the quantity in a Foot, in feet and parts.

Example, as before, at 10 Inches.

The Extent from 13-54 to 10, the Diameter in Inches, ſhall reach, being turned twice the ſame way from 1, to 0-545, the Content of a Foot long.

Again at 30 inches Diameter.

The Extent from 13-54, to 30, being turned two times the ſame way from 1, ſhall reach to 4 foot, 93 parts; which 4-93 multiplied by the length in feet, ſhall give the whole Content of the Tree.

3. To perform the ſame, having the Diameter given in Foot-meaſure,

Do thus;

The Extent of the Compaſſes from 1-128, (the feet and 10ths Diameter that make a Foot, at one foot in length) to the Diameter in Foot-meaſure, ſhall reach, being turned twice the ſame way from 1, to the quantity in a Foot long.

Example at 1 Foot, $\frac{50}{100}$ Diameter.

The Extent from 1-128, to 1-50, ſhall reach, being turned twice the ſame way from 1, to 1-77, the true quantity in one Foot long.

Problem III.

Problem III.

1. *The Diameter of any Cillender given in Inches, and the length in Feet, to find the Content in Feet.*

As 13-54, to the Diameter in Inches;
So is the length in Feet to a 4th.

Then,

As the length, to the 4th;
So is the 4th, to the Content in Feet required.

Example at 8 *Inches Diameter, and* 20 *Foot long.*

The Extent from 13-54, to 8, being turned twice the same way from 20, the length, shall stay at 6-94, or near 7 foot.

2. *The Diameter and length of a Cillender given in Inches, to find the Content in Cube-inches.*

The Extent from 1-128, to the Diameter in Inches, being turned twice the same way from the length in Inches, shall reach to the Content in Inches.

Thus the Extent from 1-128 to 10 inches Diameter, shall reach from 24 inches, the length,

length, to 1888, the Content in inches.

3. The Diameter and Length given in Foot-measure, to find the Content in Feet.

The Extent from 1-128, to the Diameter, shall reach from the length, being twice repeated the same way, to the Content in feet required.

Thus the Extent from 1-128, to 1-50, shall reach, being turned twice the same way, from 5-30, to 9-37, the Content in feet required.

Problem IV.

Having the Circumference of a Cillender given in Inches, or Foot-measure, to find the length that makes one Foot of Solid-measure.

1. First to find the Inches in length, that makes a Foot.

As the Circumference in Inches, is to 134-50, (because at so many inches about, one of a Foot in length, is a Foot) so is 12 to a 4th, and so is that 4th to the length of a Foot in inches.

Example

Example at 30 Inches about.

The Extent from 30 to 134-50, being turned twice the ſame way from 12, ſhall reach to 24 inches, 13 parts; the inches and parts that make one Foot Solid.

2. *To find the length of a Foot in feet and parts.*

As the Circumference in Inches, to 134-50;
So is 1 to a 4th, and that 4th to the length in feet and parts, that makes 1 Foot.

For the Extent of the Compaſſes from 30 to 134-50, being turned twice from 1, the ſame way, ſhall reach to two foot, and one tenth, the length that makes one Foot Solid.

3. *When the Circumference is given in Foot-meaſure.*

As the Circumference in Feet, or Feet and parts, is to 3-54;
So is that Extent twice repeated the ſame way from 1, to the length that makes a Foot Solid.

Example

Example.

The Extent from 2-50, to 3-54, being turned two times the same way from 1, doth reach to 2 foot, 001, the length in Foot-measure.

Problem V.

The Circumference given in Inches, or Foot-measure, to find how much is in a Foot long.

1. *The Circumference of a Tree, when one Foot long makes a Foot of Timber.*

As 3 foot, 545 parts, to the feet about;
So is 1 foot to a 4th, and that 4th to the solid Content in one foot long.

Example.

The Extent of the Compasses from 3-545, to 2-50, the feet about, shall reach, being turned twice the same way from 1, to 0-497, the quantity in a foot long, *viz.* near half a foot.

2. *The Circumference given in Inches, to find the Content of one Foot in length, Solid-measure, in Inches.*

The Inches a Tree is about, when one 10th of a Foot in length, makes a Foot of Timber in quantity.

As 134-5, to the Inches about;
So is 12 to a 4th, and that 4th to the Content of one foot long.

Example

Example at 30 inches about.

Th Extent from 134-5, to 30, being turned two times from 12, ſhall reach to near 6 inches for the Content of one foot long, at 30 inches about.

3. *The Circumference of a Cillender given in Inches, to find the quantity of one Foot long in feet and inches.*

As 134-5, to the Circumference;
So is 1 to a 4th, and that 4th to the quantity of one foot long in Feet and Inches.

The Extent from 134-5, to 30, being twice repeated the ſame way from 1, ſhall reach to 0-497, or near half a foot, the Content of one foot long, at that Circumference, which being multiplied by the length in feet, gives the true Content of any Cillender whatſoever.

Problem VI.

Problem VI.

The Circumference, and length of any Cillender given in Inches, or Feet and Inches, to find the Content.

1. The Circumference given in Inches, and the length in Feet, to find the Content in feet and parts.

As 42-54 (the Circumference in Inches, that makes 1 foot long, a Foot) is to the Inches in Circumference;
So is the length in Feet to a 4th, and that 4th to the Content in Feet.

Example.

The Extent from 42-54, to 48 the inches about, being twice repeated from 12 foot the length, shall reach to 15-28, the Content in feet required.

2. The Circumference and length given in Feet, to find the Content in feet and parts.

As 3-545, (because at 3 foot and a half about, and a foot in length, is a Foot) is to the Circumference;
So is the length in Feet to a 4th, and that 4th to the Content in Foot-measure.

Example.

Example.

The Extent from 3-545, to 4-0, the Circumference, being turned two times from 12 foot the length, ſhall reach to 15-28, the Content in feet required.

3. The Circumference and length given in Inches, to find the Content in Inches.

As 3-545, to the Circumference in Inches;
So is the length in Inches to a 4th:

Then,

As the length to that 4th;
So is the 4th, to the Content in Cube-Inches.

Example.

The preciſe Extent on a true Line of Numbers, from 3-545, to 48, being turned two times from 144, the length in Inches ſhall reach to 26383, the number of Inches in a Tree 48 inches about, and 144 inches in length.

This is ſufficient for the Menſuration of any ſolid body in a ſquare, or Cillender-like form, as Timber or Stone uſually is, after the true quantity of a foot, or 1728 Cubical inches; but there is a cuſtome uſed in buying of Oaken-Timber, and Elm-Timber, when it is round and unſquared, to take a Line, and girt about the midst of th Piece; and then to double the Line 4 times, and account that 4th part of the Circumference, to be the side of the Square, equal to that Circle; but this is well known to be less than the true measure, by a fifth part of the true Content, be it more or less.

Also in measuring Elm, and Beech, and Ash, whose bark is not peeled off, as Oak usually is; to cast away 1 inch out of the 4th part of the Circumference, which may well be allowed when the Bark is 3 quarters of an inch, or more in thickness, and the Tree about 40 inches about, or the 4th part, 10 inches; but if the Bark is thinner, and the Tree less, then 8 inches-square; then an inch is too much to be allowed.

Also, if the Tree is greater than a foot-square, and the Bark thick, an inch is too little to be allowed, as by this Rule you may plainly see, by the *7th Problem* of Superficial-measure in the *7th Chapter*.

Suppose a Tree be 48 inches about, the Diameter will be 15 $\frac{1}{4}$, the 4th of 48, for the square is 12.

Now if I take away 1 inch $\frac{1}{2}$ from the Diameter, then the Tree will be but 43 inches and $\frac{1}{2}$ about, whose 4th part is under 11; so that here I may very well abate 1 inch from the 4th part of the Line;

So

So consequently, if the Rind be thinner, and the Tree less, a less allowance will serve; and if the Rind be thicker, and the Tree large, there ought to be more, as by cutting the Rind away, and then taking the true diameter, you may plainly see.

This measuring by the 4th part of the Circumference, for the side of the Square, and allowance for the Bark being allowed for, as before, I say will prove to be just one 5th part over-measure.

Especially considering this, That when it is hewed, and large wanes left, then the Tree is marked for more measure, sometimes by 10 foot in 60, than there was before it was hewed; the reason is, because when the Tree is round and unhewn, the girting it, and counting the 4th part for the side of the Square, is but very little more than the inscribed Square; and then being hewen, and that scarce to an eight Square, and measuring with a pair of Callipers, to the extremity of that, doth not then allow the Square equal to the Circle for the side of the Square, as by the working by those several Squares, will very plainly appear, which being foretold and warned of, let those whom it concerns look to it.

But this being premised, and the Parties agreeing, the difference being as 4 to 5, the best way to measure round Timber, I conceive, is by the Diameter taken with a pair of Callipers, and the length; which for the just and true measure is largely handled already.

But if this allowance be agreed on, then the Proportion for it is thus;

As 1-526, to the Diameter;
So is the length to a 4th, and so is that 4th to the Content in feet.

Example.

The Extent from 1-526 to 15-26, shall reach, being twice repeated from 10 foot, the length, to 10 foot the Content required, being all at one Point.

Or, another Example.

The Extent from 1-526, to 20 inches the Diameter, being twice repeated the same way from 10 foot, the length, shall reach to 17 foot $\frac{1}{4}$ the Content.

Or, if you have the Circumference and length.

Then the Extent from 48, to the inches about, being turned twice the same way from the length in feet, shall reach to the Content required.

The Extent from 48, to 62, the inches about, being turned twice from 10, the same way,

way, shall reach to 17 foot $\frac{1}{4}$, the Content in that measure.

Thus you have full and compleat Directions for the measuring of any round Timber by the Line of Numbers, by having the Diameter and length given, after any usual manner, there remains only one general and natural way, by finding the base of the middle, or one end, by the *7th Problem* of Superficial measure; and then to multiply that base by the length, will give the true Content in feet or inches.

Thus,

Having found the Base of the Cillender by the 7th or 10th Problem of Superficial-measure; then if you multiply that Base being found in square inches, by the length in inches, you shall have the whole Content in Cube Inches.

Example.

Suppose a Cillender have 10 inches for its Diameter, then by the 7th or 10th abovesaid, you shall find the Base to be 78-54; then if you multiply 78-54 by 80, the supposed length in inches, you shall find 2356-20 Cube Inches, which divided by 1728, the inches in a Cube Foot, sheweth how many feet there is, &c. And as to the number of figures, and the fractions cutting off, you have ample Directions in the *first*

Problem, and the *third Section* of the *sixt Chapter.*

Problem VII.

How to measure a Pyramis, or taper Timber, or the Section of a Cone.

1. *First, get the Perpendiculer length of the Pyramis or Cone, thus;*

Multiply half the Diameter of the Base, A B, by it self; then measure the side A D, and multiply that by it self; then take the lesser Square out of the greater, and the Square root of the residue is the Perpendiculer Altitude required, *viz.* D B.

Example.

Suppose the half Diameter of the Base A C, were 10-25, and the side D A 100, A B 10-25, and 10-25 multiplied together, called Squaring, makes 105, 0625; D C 100, multiplied by 100, called Squaring, makes 10000; then the lesser Square 105, 0625, taken out of 10000, the greater Square, the remainder is 9894, 9375, whose square Root found by the 8th *Problem* of the *sixt Chapter*, is 99-475, the true length of the Line D B, the length or height of the Cone.

Then if you multiply the Area or Content of the Base A C 20-5, which by the 7th

7th or 10th of Superficial measure is found to be 160-08, by 33-158, a third part of 99-475, the whole height makes 5308, cutting off the Fractions for the true Content of the Cone, whose length is 99 inches, and near a half, and whose Base is 20 inches and a half Diameter.

2. *Then Secondly, for the Segment or Section of a Cone, the shape or form of all round taper Timber, the truest way is thus;*

By the length and difference of Diameters, find the whole length of the Cone, which for all manner of Timber as it grows this way is near enough.

As thus;

As the difference of the Diameters at the the two ends, is to the length between the two ends;

So is the Diameter at the Base, to the whole length of the Cone.

Example.

The difference between the Diameters A C, and E F, is 13-70, the length, A E is 66-32. then the Extent on the Line of Numbers from 13-70, the difference of the Diameters; to 66-32, the length between, shall reach the same way from 20-50, the greater Diameter to 99 and better, the length that makes up the Cone, at that Angle of Tapering in the Timber; then if by the last Rule you measure it as a Cone of that length, and also measure the little end or point at his length and diameter; and then lastly, this little Cone taken out of the great Cone, there remains the true Content of the Taper-piece that was to be measured, *viz.* 5246-71, when 61-30, the Content of the small Cone at the end, is taken out of 5308, the Content of the whole Pyramid.

3. *If this way seem too troublesome for the common use, then use this, being more brief:*

To the Content that is found out, by the Diameter in the midst of the Timber, and the length, add the Content of a Piece found out, by half the difference of Diameters, and onethird part of the length of the whole Piece, and the sum of them two shall be the whole Content required.

4. *Or else;*

Divide the length of the Tree into 4 or 5 parts, and measure the middle of each part severally, and that cast up by his proper length, shall give the Content of each Piece; then the sum of the Contents of all the Pieces put together, is the true Content of the whole Taper Piece, very near.

Note, That this curiosity shall never need to be used, but when you meet with Timber much Taper, and Die-square, or on a Contest or Wager; for according to the usual way (and measure) of squaring the Timber, it is well, if the measure of the Square, taken with Callipers from side to side, in the middle of the length of the Piece, will make amends for half the Timber which is wanting in the wany edges of your squared Timber, and the knots, or swellings, & hollows of most round Timber, may well ballance this over-measure found by the Diameter taken in the middle of the length of the Piece. But indeed for Masts of Ships and Yards, being wrought true and smooth, where the price of a Foot is considerable, there exactness is requisite, and necessary to be used; and thus much for Solid-measure in Squares and Cillenders.

Problem VIII.

To measure Globes, and roundish Figures.

1. To measure a Sphear or Globe by Arithmetick, the ancient way, is to multiply the Diameter by it self, and then that Product, to multiply by the Diameter again; which two multiplications is called Cubing

of the Diameter; then multiply this Cube by 11, and then divide this last Product by 21, and the Quotient shall be the Solid Content of the Sphear, in such measure as the Diameter was.

Example.

Let a Sphear be to be measured, whose Diameter is 10 inches: First, 10 times 10, is 100; and 10 times 100, is 1000; the Cube of 10, that multiplied by 11, makes 11000; which being divided by 21, makes 523-81, for the Solid Content.

Which by the Line of Numbers, you may work thus;

2. The Extent from 1, to the Diameter, shall reach the same way from the Diameter to the Square of the Diameter.

Then again,

The Extent from 1, to the Square of the Diameter, shall reach the same way from the Diameter, to the Cube of the Diameter.

Then,

The Extent from 1, to the Cube of the Diameter, shall reach the same way from 11, to the Product of the Cube of the Diameter, multiplied by 11.

Lastly, This Extent from 21, to this last Product, shall reach the same way from 1, to the Solid Content of the Sphear required.

Or else more briefly thus ;

3. The Extent from 1, to the Diameter, being turned three times the same way from 0-5238, shall stay at the Solid Content of the Sphear, or Globe, required.

Example at 12 Diameter.

The Extent from 1 to 12, being turned three times the same way from 0-5238, shall reach to 905-143, the Solid Content required.

3. *The Diameter given, to find the Superficial Content.*

Square the Diameter, and multiply that by 3-1416, and the Product is the Superficial Content.

Or, by the Line of Numbers ;

The Extent from 1, to the Diameter, being turned twice the same way from 3-1416, shall reach to the Superficial Content, of the out-side round about the Gobe, *viz.* at 12 Diameter, 452-44.

4. *Having the Superficial Content, to find the Diameter.*

The Extent from 1 to 0-3183, shall reach the same way from the Superficial Content, to the Square of the Diameter, whose

Square-root is the Diameter required. As at 452-44, gives 144.

5. *Having the Solid Content, to find the Diameter of a Globe.*

The Extent from 1 to 1-90986, ſhall reach from the Solid Content to the Cube of the Diameter, as at 905-143 Solidity gives 1728, the Cube of 12.

6. *Having a Segment of a Sphear, to find the Superficial Content.*

The Extent from 1, to the Chord of the half Segment, ſhall reach, being twice repeated, from 3-1416, to the Superficial Content of the round part of the Segment, A B C.

Example.

Let the Segment be the half Sphear, A B C; A C being 12, then B C which is the Chord of the *Peripheria*, B C is 8-485, whoſe Square is 72.

Then,

The Extent of the Compaſſes from 1, to 8-485, being turned twice the ſame way from 3-1416, ſhall reach to 226-22, the Superficial Content of the round part of the Segment, or half Sphear or Globe, to which if you add the Content of the Circle or Baſe, you have the whole Superficies round about.

7. To

7. *To find the Solid Content of a Segment of a Globe.*

First, you must find the Diameter of that Sphear, of which the given Segment to be measured is part.

Thus;

Add the Square of the Altitude, and the Square of the Diameter of the Segment together, and the sum divide by the Altitude of the Segment, the Quotient shall be the whole Sphears Diameter.

Then,

Taking the Altitude of the Segment given, from the whole Diameter, there remains the Altitude of the other Segment.

Then,

Extend the Compasses from the whole Diameter of the Sphear, to 1; the same Extent applied the same way from the Altitude of the given Segment, shall reach to a 4th Number, on a Line of Artificial Solid Segments joyned to the Line of Numbers, which 4th Number keep.

Then,

The Extent from 1, to the whole Content of the Sphear, shall reach the same way from the 4th Number on the Line of Numbers, to the Solid Content of the Segment required.

Example.

Example.

Let the whole Diameter of a Sphear be 14, then the whole Solid Content by the former Rules, you will find to be 1437 $\frac{1}{3}$, a Segment of that Sphear whose Altitude or Depth is 4, the Solidity is required.

Extend the Compasses from 14, the whole Sphears Diameter, to 1; that Extent applied the same way from 4, the Altitude of the Segment, shall reach to 2-86 on the Numbers, or to 19-88, on the Line of Solid Segments joyn'd to the Line of Numbers, which 19-88, is the 4th Number to be kept.

Then secondly,

The Extent from 1 to 1437, the whole Content of the whole Sphear, shall reach the same way from 19-88, to 284 $\frac{2}{3}$, the Content of the Segment required to be found.

If you want the *Line of Segments*, the Table annexed will supply that defect:

Thus;

Look for the 4th Number, found on the *Line of Numbers*, among the figures on the *Table*, and the number answering it in the *first Column*, is the Solid Segment, or 4th to be kept; as here, on the Numbers, you find 2-86; seek 2-86 in the *Table* annexed, and

and in the *first Column*, you find near 20 for the 4th in Segments.

8. *To perform the same by Arithmetick after the way set forth by Mr.* Thomas Diggs, 1574.

To find the Superficial Content of a Globe or Sphear.

Multiply the Diameter by the Circumference, the Product shall be the Superficial Content round about the Globe.

9. *Or,*

Multiply the Content of a Circle, having like Diameter, by 4, the Product shall be the Superficial Content.

10. *And*

If you multiply the Superficial Content, by a 6th part of the Diameter, the Product shall be the Solid Content of the Sphear.

A Table of Segments.

Num.	Seg	Num.	Segm.	Num.	Segm.	Num	Segm.
	059		335		506		679
	084		342		513		686
	104		349		520		694
	122		356		527		703
5	137	30	363	55	534	80	712
	152		371		540		720
	164		378		547		728
	176		385		554		737
	188		392		560		746
10	197	35	399	60	567	85	753
	207		406		574		763
	218		413		580		772
	228		420		587		782
	237		426		594		793
15	245	40	433	65	601	90	803
	254		440		608		812
	263		447		615		825
	272		453		622		836
	280		460		629		848
20	288	45	466	70	637	95	865
	297		473		644		878
	306		480		651		896
	314		487		658		916
	321		494		665		941
25	328	50	500	75	672	100	1000

11. *For the Segment work thus ;*

Multiply the whole Superficial Content of the whole Globe, by the Altitude of the Segment, and divide the Product by the Sphears whole Diameter, the Quotient ſhall be the Superficial Content of the Convexity or round part of the Segment.

12. *But for the Solid Content, work thus ;*

Firſt, find the difference between the height of the Segment, and the half Diameter of the Sphear ; then multiply this difference (being found by ſubſtracting the leſs from the greater) by the Superficial Content of the Baſe of the Segment, and the Product ſubſtract from the Product of the Sphears ſemi-Diameter, and the Convex Superficies of the Segment ; then a third part of the remainder ſhall be the Solid Content of the Segment required.

Example as before.

The Sphears Diameter is 14, the Segments Altitude is 4, the Segments Altitude taken from 7, the half Diameter, the remainder is 3, which multiplied by 126, the Superficial Content of the Baſe of the Segment, makes 378 ; then having multiplied 7, the Sphears half Diameter, by the Convex Superficies of the

the Segment 176, the Product is 1232; from which number take 378, the Product last found, and the remainder is 854, whose third part $284\frac{2}{3}$, is the Solidity of the Segment required.

There are other fragments of Sphears, as Multiformed and Irregular, Cones or Pyramids, and Solid Angles; but the Mensuration of these I shall not trouble my self, nor the Learner with, for whom I only write, intending the Mensuration of things that may come in use only.

13. But yet to conclude this Chapter, take these Observations along with you, concerning the Proportion of a Cube, a Prisma, and a Pyramid, a Cillender, Sphear, and Cone; whose Shapes and Proportions are as in the Figures.

If a Cube be made or conceived, whose side is 12 inches, then the solidity thereof is 1728 Cube inches; and a Prisma, having the same Base and Altitude, contains 864 Cube inches; and a square Pyramis, of the same Base and Altitude, contains 576 Cube inches; and a Trianguler Pyramid, as before, contains 249-6 Cube inches; A Cillender contains $1357\frac{5}{7}$, being the same Height and Diameter of 12 inches: A Sphear, whose Axis is 12 inches, contains

tains 905 $\frac{1}{7}$ Cube inches; and a Cone, of the ſame Diameter and Altitude, contains 452 $\frac{4}{8}$.

The Superficies of the Cillender about, (excepting the top and bottom) is equal to the Superficial Content of a Globe.

Cube ———	1728
Cillender ———	1357 $\frac{5}{7}$
Sphear ———	905 $\frac{1}{7}$
Priſma ———	864
Piramis ———	576
Cone ———	452 $\frac{4}{8}$
△ Tetrahed. —	2496
Octahed. — whoſe Triangle Side is 12 —	814-6

By the foregoing Proportions, it is evident that a Cube is double the Priſma, and treble to the Square Pyramis of equal Baſe and Altitude, or as 3, 2, 1; for 3 times 576 is 1728, and 2 times 864 is 1728.

Alſo a Cillender is $\frac{11}{14}$ of a Cube; and a Globe is $\frac{11}{21}$ of a Cube, or $\frac{2}{3}$ of a Cillender, whoſe Sides and Diameters are equal; and a Cone is $\frac{1}{3}$ of a Cillender; ſo that the Proportion between the Cone, Sphear, and Cillender, is as 1, 2, & 3; for 3 times 452 $\frac{4}{7}$, makes 1357 $\frac{5}{7}$; and two thirds

S of

of 452 $\frac{4}{6}$ makes 905 $\frac{1}{7}$; the Content of a Sphear. The Trianguler Pyramid is little more than $\frac{1}{7}$ of a Cube; so that if any one have frequent occasions for these proportions, let Centers be put in the Line of Numbers, at these proportional Numbers, and then work with those Points from the Cube and Cillender, as is directed before, for the Circumference, and Diameter, and Squares, equal and inscribed in *Chap.* 8. Prob. 6.

So much for the measuring of regular ordinary Solids; for the extraordinary and irregular, the best Mechanick way is by Weights or Waters to measure their Crassitudes or Solidities, either by Weight or Measure.

A fur-

A further improvement of the Trianguler Quadrant, *as I have made it several times, with a sliding Cover on the in-side, when made hollow, to carry Ink, Pens, and Compasses; then on the sliding Cover, and Edges, is put the Line of Numbers, according to Mr.* White's *first Contrivance for manner of Operation; but much augmented, and made easie, by* John Brown.

1. THe description thereof for *one side*, being the same with the Line of Numbers on the outter-Edge, except that the first part is sometimes (when required for that particular purpose) divided into 12 parts, for inches, instead of 10' that is to say; The space between the first 1, and the middle 1, on the Rule; (the space I say) between every Figure, on the first half part, is cut into 12 parts, instead of 10, to answer to the 12 inches in a Foot; and the other half, as the Line of Numbers on the Edge. And in the same manner are *White's*

ſliding Rules made, only for this particular purpoſe.

2. On the *other ſide*, is the Line of Numbers drawn double, the one Line to the other, for the ready meaſuring of ſolid Meaſure at one Operation; the deſcription whereof in brief is thus;

Firſt, The diviſions on the ſliding-piece in hollow-Rules, or on the right-ſide in ſliding-Rules; when the figures of the Timber-ſide ſtand fit to read, I call the right-ſide, or ſingle-ſide, being alwayes toward the right hand, and a ſingle Radius.

The diviſions therefore on the fixed-edge of the Rule, muſt needs be the left-ſide, and is alſo divided to a double Radius, or one Radius twice repeated.

So alſo in ſliding-Rules, the double Radius is on the left-ſide alſo. *See the Figure thereof, with right and left-ſide expreſt upon it.*

For the right reading thoſe Lines, the Method is thus;

The Figures on the right or ſingle-ſide, do uſually begin at 3 or 4, and ſo proceed with 4, 5, 6, 7, 8, 9, 10, 11, for ſo many inches of a Foot.

Then 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, &c. for ſo many whole Feet.

The

The ſmaller Cuts between the firſt Figures, from 3 inches to 1 foot, being quarters of inches; And the ſmall Diviſions between the Figures, that repreſent Feet, are only every whole Inch; The halfs, and quarters of Feet, alſo noted by a longer ſtroke as in ſuch work is neceſſary and uſual.

3. On the ſame *right-ſide* alſo, for more eaſe and readineſs in the uſe, are noted ſeveral Gage-points (as it were); *As*,

Firſt, At 1 Foot is the word *ſquare.*

Secondly, At 1 Foot, 1 inch and $\frac{1}{2}$, is a ſpot; and cloſe to it is ſet t.d. for true Diameter of a round ſolid Cillender.

Thirdly, At 1 foot 3 inches $\frac{4}{5}$ is another ſpot, and near to it is ſet D, for the Diameter of a rough piece of Timber, according to the uſual allowance for unhewed Timber, according to the fourth part of a Line girt about and counted for the ſide of the ſquare.

Fourthly, At 3 foot 6 inches and $\frac{1}{20}$, and near to it is ſet t: r: for the true Circumference of a round Cillender.

Fiftly, At 4 foot juſt is ſet R, for the Circumference, according to the former allowance.

Sixtly, At 1 foot 5 inches $\frac{1}{7}$ *ferè*, is a ſpot; and cloſe to it the letter W, as the Gage-point for a Wine-gallon.

Seventhly, At near 19 inches, or 1 foot 7 inches, is another ſpot; and cloſe to it the letter a, as the Gage-point of an Ale-gallon.

Eightly, At 2 foot 8 inches $\frac{8}{10}$ is a ſpot, and cloſe to it is ſet B, for the Gage-point of a Beer Barrel; and at 2 foot 7 inches is ſet A, for an Ale Barrel. *The Uſes whereof in order follow.*

The Figures on the *left-ſide*, or fixed-edges, are read and counted as thoſe on the right: For the ſmall, 1,2,3,4,5,6,7,8,9, 10,11, are to repreſent inches, and the cuts between, quarters of inches; Then the 1,2, 3,4,5,6,7,8,9,10 Figures next, ſomewhat bigger, as to repreſent ſo many feet, and the cuts between, are whole inches: Then 20, 30,40,50,60,70,80,90,100,150, for tens of feet, and the parts between, for ſingle feet, for the moſt part; or elſe whole and half feet, as is uſual. The Uſes follow.

Uſe I.

A piece of Timber being not Square, (or having its breadth and depth unequal) to make it Square, or find the Square equal;

Set the breadth of the Piece, counted on the right-ſide, to the ſame breadth counted on on the left-ſide; then right againſt the depth found on the left-ſide, on the right or ſingle-ſide, is the inches and quarters ſquare required.

Example, at 15 inches broad, and 9 inches thick, or deep.

Set 9 inches on the right-ſide, to 9 on the left; then right againſt 15 inches, or 1 foot 3 inches, found out on the double or left-ſide, on the right or ſingle-ſide, is 11 inches and $\frac{5}{8}$ the Square equal required.

Alſo, if you ſet 15 to 15, then right againſt 9, found out on the left-ſide, on the right-ſide is 11 inches $\frac{5}{8}$, the Square equal required.

Uſe II.

The Side of the Square given, to find how much in length will make 1 Foot.

Set the inches (or feet & inches) found out on the right-ſide, to 1 foot on the left; then right againſt 1 foot on the right, is the inches, or the feet and inches required, to make a Foot of Timber.

Example at 9 inches ſquare.

Set 9 inches, found out on the right-ſide, to 1 foot on the left-ſide; then right againſt 1, on the right-ſide, is 1 foot 9 inches $\frac{2}{3}$ on the left.

If the Square be so big, that the 1 on the right falls beyond the End at the beginning, then right against 10 foot on the right-side, is on the left, the hundredth part of a foot, that makes a Foot of Timber.

Example, at 4 Foot Square.

Set 4 foot, found out on the right, to 1 foot on the left; then right against 10 foot on the right-side, is 0-063 on the left-side, or against 12 foot you have 9, 12 parts of 1 inch, the length that goes to make 1 foot of Timber required.

Use III.

At any (bigness or) Inches, or feet and inches square, to find how much is in 1 Foot long.

Just as the Rule stands even, that is 1 foot on the right, against 1 foot on the left, seek the inches, or feet and inches, the Piece is square on the right or single-side; and just against it on the left or double-side is the Answer required; in inches, or feet and inches.

Example at 19 inches square.

Just against 1 foot 7 inches, or 19 inches, (which is all one) found out on the right-

side,

ſide, on the left-ſide is 2 foot 6 inches, the quantity of Timber in 1 foot long, at 19 inches ſquare; which Number of 2 foot 6 inches, multiplied by the length in feet, gives the true Content of the whole Piece of Timber required.

Note, That this is a moſt excellent way for great Wood, and very exact.

Alſo Note, That here, by inſpection, you may ſquare a ſmall Number, or find the ſquare-root of a ſmall Number.

As thus;

The ſquare of 8 & $\frac{1}{2}$, is near 72;

Or,

The ſquare-root of 72, is near 8 & $\frac{1}{2}$.

Uſe IV.

The ſide of the Square, and the length of any Piece being given, to find the Content in feet and parts.

Set the word *ſquare*, or 1 foot, alwayes to the length, found out on the left-ſide; then right againſt the inches, or feet and inches ſquare counted on the right, on the left is the Content required.

Example, at 20 foot long, and 15 inches ſquare.

Set 1 foot on the right, to 20 on the left, then right againſt 1 foot 3 inches on the right,

right, is 31 foot 2 inches and $\frac{1}{2}$, the Content.

Note, That if the Piece be very ſmall, call the feet on the left-ſide, inches; and the parts between 12ˢ of inches; then the Anſwer will be found on the left-ſide in 144ˢ of a Foot.

Example, at 2 inches ſquare, and 30 foot long, how much is there?

Set 1 foot on the right, to 30 foot on the left; then right againſt 2 foot on the right, counted as 2 inches, is 120 parts of a foot divided into 144 parts, being juſt 10 inches, for 10 times 12 is 120.

But if it be a great Piece of Timber, then work thus;

Set 1 foot, or the word *ſquare*, to the length on the left, counting the ſingle feet 10ˢ of feet; then right againſt the feet and inches ſquare are the 100ˢ of feet required.

Example, at 40 foot long, and 4 foot ſquare.

Set 1 foot to 4 foot, counted as 40, on the left; then right againſt 4 foot on the right, is 640, the true Content, increaſing the 10ˢ to 100ˢ. Thus mnch for ſquare Timber.

Though

Though there be many other wayes and manner of workings, ſome whereof you may find in a Book ſet forth under the name of *The Carpenters Rule*, 1666, by *J.Brown*, and well known abroad already.

Uſe V. *For Round Timber.*

The middle Diameter of any Piece given, to find how much is in a Foot long, at true meaſure.

Set the ſpot by t.d. to 1 foot on the left, then juſt againſt the inches, or feet and inches Diameter, found on the right, is the quantity of Timber in 1 foot long on the left-ſide required.

Example, at 2 foot 9 inches Diameter.

Suppoſe a piece of Stone-Pillar, or Garden-Roul, be two foot 9 inches Diameter, ſet the ſpot by t.d. juſt againſt 1 foot, then right againſt 2 foot 9 inches, found on the right, on the left is 6 foot, the quantity of ſolid meaſure in one foot long; which being multiplied by the length in feet, gives the true Content of the whole Piece.

Note, That if you would have the uſual allowance, ſet D to 1, inſtead of t.d.

Use VI.

The Diameter of any Piece of Timber given, to find how much in length will make one Foot.

Set 1 foot on the left, to the inches Diameter counted on the right; then right against t.d. for true measure; or D for the usual allowance, is the Answer required, found on the left-side.

Example, at 9 inches Diameter.

Set 1 on the left to 9 on the right; then just against t.d. is 2 foot 3 ½ on the left; and right against D, is 2 foot 11 inches on the left, the length required to make a foot solid at true measure, or the usual allowance, when the 4th part of the girt about, is counted the side of the Square, equal to the round piece of Timber.

Note, That for great Timber, you must set the left 1 foot, to the feet and inches Diameter as before; but count the t.d. or D, as far beyond 12 foot, as it is placed beyond 12 inches, and you shall have the Answer in 144s of a foot.

Example, at 5 foot Diameter.

If you set 1 on the left to 5 foot on the right, and count so much beyond 12 foot

on

on the right, as t.d. is beyond 12 inches, you shall find 7¼, that is, 7 144s and a quarter, to make 1 foot true measure, and 9-12s and ½ for the usual allowance.

But for small Timber, set 1 foot, 2 foot, &c. on the right, counted as 1, 2, 3, 4, and inches, to 1 on the left; then right against t.d. or D, is a Number, that multiplied by 12, is the Number of feet required.

Example, at 2 inches Diameter, how much makes 1 Foot?

Set 2 foot on the right, counted as 1 inch, to 1 on the left; then right against t.d. is 3 foot 10 inches, calling the inches feet, and the feet 10s of feet; which 3 foot 10 inches, multiplied by 12, make 46 foot, for the length of 1 foot of Timber at 2 inches Diameter, the thing required for true measure.

Use VII.

The Diameter and Length given, to find the Content.

Set t.d. or D, for true measure, or usual allowance, alwayes to the length counted on the left; then right against the inches Diameter counted on the right, on the left-side is the Content required.

Example,

Example, at 6 inches Diameter, and 30 Foot long.

Set t.d. to 30; then right againſt 6 inches, counted on the right; on the left is 5 foot 11 inches, the Content required.

Note, If the Piece be ſmall, then count every foot on the right as inches, and you have the Anſwer in 144s of a foot, which is eaſily counted by having 1 ſet at 12, 2 at 24, 3 at 36, 4 at 48, 5 at 60, &c. to 12 at 144, which little ſmall figures are counted as inches of 12s of a foot.

Example, at 2 inches Diameter, and 20 foot long.

You ſhall find 64; 144s; that is, 5 in ½ true meaſure.

But for great Pieces, ſet t.d. or D, to the length, counting 1 foot, or the left for 10 foot, then have you the Anſwer in 100s of feet.

Example, at 5 foot Diameter, and 30 foot long.

Set t.d. to 3 inches, counting 30 foot for the length; then right againſt 5 foot on the right, on the left is 592 foot, the Content required.

Uſe VIII.

Use VIII.

To meaſure round Timber, by having the Girt, or Circumference about, and length given.

This being the ſame in operation with the Diameter, I ſhall paſs it over more briefly; which way of wording, may ſerve for the Square and Diameter alſo; only I labour to be plain and brief.

The Circumference given, to find how much in a Foot long.

Set t.r. or R, for true round, or allowance to 1 foot on the left; then againſt the inches about, on the left is the Anſwer required.

Example.

At 2 foot about, you will find 3 inches and $\frac{10}{12}$ in a foot long true meaſure; or juſt 3 inches at the uſual allowance.

The Circumference given, to find how much makes a Foot.

Set the inches, or feet and inches about, to 1 foot on the left; ſo is t.r. or R, to the length to make a foot.

Example

Example; at 18 inches about;

As single 18, to double 1;
So is t.r. to 5 foot, 7 inches $\frac{1}{2}$ *Or,*
So is R, to 7 foot 2 inches, for the usual allowance.

The Circumference and length, to find the Content.

As t.r, or R, to the length;
So is the feet and inches about, to the Content.

Example, at 3 foot about, and 30 foot long, true measure.

As t.r. to 30; so is 3 foot, to 21 foot 5 inches, the Content.
For great things, call 1 foot on the left, 10 foot, as before.
For small things, call 1 foot on the right, 1 inch, as before.

Use IX.

To Gage small Cask by the mean Diameter and Length.

Set the spot by W, for Wine-gallons, alwayes to the length of the Vessel, given in inches, counted on the left-side; then right against the inches, or feet and inches Diameter,

meter, counted on the right; on the left is the Content in Gallons required.

Example, at 24 inches, or 2 foot mean Diameter, and 30 inches long.

As the ſpot at W, to 30 inches counted at 30 foot;
So is 24 inches, or 2 foot, to 58 Gallons 3 quarters, the Content required.

For greater Veſſels, count the feet on the left for 10' of inches in the length, and you have your deſire.

Example, at 60 inches long, and 38 inches mean Diameter.

As W, to 6 foot on the left for 60 inches;
So is 3 foot 2 inches, or 38 inches to 295 Gallons, the Anſwer required.

If you would have it in Ale-gallons, uſe the mark at a.

Uſe X.

To gage Great Brewers Veſſels, round Tuns.

The Diameter and length being given in feet and inches, to find the Content in Beer-Barrels, at one Operation.

Set the ſpot at BB, to the depth of the

Tun, counted on the left, in feet and inches; then right against the mean Diameter, found out on the right, on the left is the Content in Barrels required.

Example, a Tun 4 foot deep, and 10 foot mean Diameter.

As the ſpot at BB, to 4 foot;
So is 10 to 53 Barrels, and 2 third parts.

If you would have the Content in Ale-Barrels, uſe the mark at AB.

Thus much for the Timber-ſide, the uſe of the other, or board-ſide, is the ſame with that by the Compaſſes before treated of, and therefore needs here no repetition, unleſs as to the bare manner of working with it.

The ſliding-Rule is only two Rules, or Pieces fitted together, with a ſhort Grove, and Tenon, and two Braces at each end, to keep it from falling aſſunder; and even ſo alſo is the ſliding-Cover, and two edges on the inſide of the *Trianguler Quadrant*; and the Numbers graduated thereon, are cut a-croſs the middle Joynt, having the ſame diviſions on both ſides; that is to ſay, on each Rule, or on the Cover and Edge of the inſide of the Rule.

The reading and deſcription is the ſame with that in *Chap.* I. *Page* 12, 13, 14, 15, 16;

36; and the general Method in uſe is *thus*;

That ſide or part of the Rule, on which you count the firſt term in the Queſtion, is called alwayes the firſt-ſide; then the other muſt needs be the ſecond.

Then for Multiplication, thus;

As 1, on the firſt-ſide, to the Multiplier on the ſecond, or other-ſide;
So is the Multiplicand, on the firſt-ſide, where 1 was, to the Product on the ſecond.

For Diviſion, alwayes thus;

As the Diviſor found on any one ſide, is to 1 on the ſecond, or other-ſide;
So is the Dividend on the ſame firſt-ſide, to the Quotient on the ſecond.

For the Rule of Three.

As the firſt term on the firſt-ſide, to the ſecond on the other;
So is the third term found on the firſt-ſide, to the 4th term on the ſecond-ſide.

For Superficial Measure, by Inches and Feet.

As 12, to the breadth in inches on the ſecond;
So is the length in feet, to the Content on the ſecond.

For any thing elſe, the ſame Rules and Precepts you find in *Chap.* VII. will give you ample and plain directions.

The Lines being fitted, as much as may be, to ſpeak out the Anſwer to the Queſtion, as by well conſidering the *Figure*, you may ſee.

CHAP. XI.

To make and measure the Five Regular Platonical Bodies, with their Declinations and Reclinations.

1. *For the Cube, being the Foundation of all other.*

IT is a Square Solid Body, every way alike, and ſpoken of largely before, as to the Menſuration thereof,and obvious enough to every indifferent Workman, as to the making thereof, and needs no repetition in this place.

2. *For the Tetrahedron.*

It is a Figure, comprehended of 4 equilatteral plain Triangles, or a Trianguler Pyramid, laſt mentioned, the beſt and neareſt way, as I conceive of making, is thus. According to directions of Mr. *John Leak.*

On any rough Piece, make one side plain and flat, so large, as to contain the Triangle which you intend shall be one side of the *Tetrahedron*; then set a Bevel to 70 degrees 31 minutes and 42 seconds; and plain another side, to fit the former side, and the Bevel (*secundum Artem*); then mark this last plained side, according to the former, and cut away the residue, plaining them away just to the strokes, and fit to the Bevel formerly set, and you shall constitute the *Tetrahedron* required.

The *Superficial* Content is the Area of the 4 equilatteral Triangles mentioned before, and the solid Content is found by multiplying the Area of one Triangle by one third part of the Altitude of the Pyramid, or Tetrahedron, from the midst of one Plain to the Apex, or top of the opposite Solid Angle.

If the measuring the sides, Perpendiculer, and Altitude of the Tetrahedron with Compasses, Callipers, and Scale, serve not to exactness; then proceed thus;

First, for the Perpendiculer, the Triangle being equilatteral.

Multiply one side given by 13, and divide the Product by 15; the Quotient is the Perpendiculer.

Example.

Example.

If the ſide of the Tetrahedron be 12, that multiplied by 13, gives 156; which divided by 15, leaves 10-4, for the length of the Perpendiculer in the equilatteral Triangle, whoſe ſide is 12.

Then for the Perpendiculer Altitude, work thus, by the Artificial Numbers and Sines.

As the Sine of 90, to the ſine of 70 deg. 31 min. 42 ſec;
So is 10-4, the Perpendiculer, to 9-80, the perpendiculer Altitude required.

Or by the Sector, work thus;

Take 12, the ſide of the Tetrahedron, from (any Scale, or) the Line of Lines, and ſet the Sector to 60 degrees, by making the Latteral 12, a Parallel 12, then the neareſt diſtance from 12, to the Line of Lines, is the true Perpendiculer; which meaſured on the ſame Line of Lines, will be found to be 10-4, as before; then make this 10-4 a Parallel Sine of 90, and 90 the Sector ſo ſet, take out the Parallel-ſine of 70-31-42, and meaſure it on the ſame Scale, and it ſhall be 9-8, as before.

But if you use the Quadrant-side, then first lay the Thred to 60 degrees, counted from the Head; then take the nearest distance from 12, on the Line of Lines, to the Thred, and it shall be the Perpendiculer of the Triangle, 10-4; then set this Perpendiculer in 90, and lay the Thred to the nearest distance, and there keep it; then take the nearest distance from 70-31 in the Sines to the Thred, and that shall be 9-80, for the Perpendiculer Altitude of the Trianguler Pyramid, or Tetrahedron.

Then,

Lastly, This perpendiculer Altitude being multiplied by the Area of the Base, gives a Number, whose third part is the Solid-Content of the Tetrahedron required.

For 12 the side, and 5-2 the half Perpendiculer, makes 62-4, the Superficial-Content of one Triangle, or Base; then 62-4, the Base, multiplied by 9-8, the perpendiculer Altitude, gives 611-52, and a third part of 611-52 is 203-86, the solid Content required.

The three Triangles recline from the Perpendiculer upright, 19 degr. 28 min. and 18 sec. and decline when the edge is South 60, South East, and South West, and the opposite Plain a just North; but if you make

make one South, then the other two are are North-east and North-west 60 deg.

3. *For the Octahedron,*

Which is a solid body, comprehended under 8 equilatteral Triangles: The way of making which, *is thus*;

Make a plain Parallelepipedon, or long-Cube, if the breadth both wayes be 1000, let the length be 1-414; or if the length be 500000, the breadth both wayes must be 3-53553; then to these Measures square it exactly; then divide the length and breadth just in the midst, and draw Lines both wayes on all the 6 sides; then draw the *Diagonal*-Lines from the midst of the length, to the midst of the breadth; and cut by these *Diagonal*-Lines, and the *Octahedran* will appear to be truly made.

For the Mensuration thereof, it is the same as in the *Tetrahedron*; For, supposing the side of one of the Triangles 12; the Base is 144 in Content, and the Triangles Perpendiculer is 10-4, as before: But the Perpendiculer Altitude is just half the length, *viz.* 8-49; for if the breadth be 12, then the length must be 16-98, whose halfs are 6 and 8-49; Then if you multiply 144 the Base, by 8-49 the perpendiculer Altitude,

Altitude, the Product will be 1222-56, whose third part is 407-52, the half of the *Tetrahedron*, and 815-04 is the whole solid Content of the *Tetrahedron* required, as near as we can see by Instrumental Operation; but if you work to a Figure more, you shall find the total Area to be but 814-656 more exact.

To find this Perpendiculer Altitude by the Sector, work thus;

First, The Triangles Perpendiculer being 10-4, as before; Take the Latteral 10-4, from the Line of Lines, make it a Parallel in 90, lay the Thred exactly to the nearest distance, and there keep it; then the Parallel-distance from the Sine of 54 deg. 44 min. 45 sec. the Reclination shall be 8-486, the true Perpendiculer Altitude required.

Then if the *Octahedron* stand on one Triangle, you have one Horizontal Plain, and one South and North Reclining and Inclining 19 deg. 28 min. 18 sec. as the *Tetrahedron* was; and two South, and two North, declining 60, and reclining and inclining 19 deg. 28 min. 18 sec. as afore.

But if it stand on a Point, then you have 4 direct or declining 45, and reclining 54, 44, 45; and 4 incliners, inclining as much

much and direct, or declining, as you shall please to set them.

4. *For the Dodecahedron,*

Which is a regular solid body, contained under (or made up of) 12 Pentagonal Pyramids, or Pyramids whose Base hath 5 equal sides, and the perpendiculer Altitudes of those 12 Pyramids equal to half the Dodecahedrons Altitude, standing on one side, or equal to the semi-diameter of the inscribed Sphear.

To cut this Body, take any round Piece, and if the Diameter be 100000, the length must be 0-81005, or as 4-906 to 3-973, then the Piece being turned round, and the two Ends flat to the former measures of Length and Diameters (which are near according to the Sphear inscribed, and to the Circle circumscribing) being measured by Compasses, Callipers, and Line of Lines very carefully and exactly. Then divide the Circumference of the two Ends of the Cillender into 10 equal parts, and draw Lines Perpendiculer from end to end, and plain all away between the Lines flat and smooth, so that the two Plains on both ends will become a regular ten-sided Figure.

Then making the whole Diameter abovesaid, 10000 in the Line of Lines, take out

0-309,

0-309, and with this meaſure (as a Radius on the Center) at both ends deſcribe a Circle; and if you draw Lines, from every oppoſite Line of the 10 firſt drawn, you ſhall have Points in the laſt deſcribed circle, to draw a *Pentagon* by; which is the Baſe of one of the 12 Pentagonal Pyramids, contained in the body. This Work is to be done at both Ends; but be ſure that the Angle of the Pentagon at one end, be oppoſite to a ſide of the Pentagon at the other end; then theſe Lines drawn, the two ends are fully marked.

Then to mark the 10 *Sides, do thus;*

Count the firſt length 1000, *viz.* the meaſure from the top to the bottom, or from Center to Center; and fit this length in 10 and 10, of the Line of Lines; the Sector ſo ſet, take out 0-3821, and lay it from the two ends, and either draw, or gage Lines round about from each end; and in the midſt between the two Lines will remain 0-2358; then Lines drawn Diagonally on the 10 ſides, will guide to the true cutting of the Dodecahedron.

If you ſet a Bevel to 116 deg. 33 min. 54 ſec. and apply it from the two ends, you may try the truth of your Work.

The

The Declination and Reclination of all the 10 Pantagonal Plains, are as followeth.

First, You have 1 North, reclining 26 deg. 34 min; and 1 South, inclining as much.

Secondly, You have 2 North declining 72, and reclining 26, 34; and 2 South, declining 72, and inclining 26, 34.

Thirdly, You have 2 North, declining 36, and inclining 26, 34; and 2 South, declining 36, and reclining 26, 34; And 1 Horizontal Plain, and his opposite Base to stand on.

As for the measuring of this Body, the Plain and Natural way is thus;

First, find the Superficial Content of the Base of one of the Pentagons, by multiplying the measure from the Center to the middle of one of the Sides, (which is the contained Circles semi-diameter) and half the sum of the measure of all the sides put together; and then to multiply this Product by one third part of half the Altitude of the body, and the Product shall be the Content of one Pentagonal Pyramid, being one twelft part of the Dodecahedron; and this last, multiplied by 12, gives the solid Con-

Content of the Dodecahedron; or 12 times the Superficial Content of one side, is the Superficial Content thereof.

Example.

Suppose the side of a Dodecahedron be 6, then the sum of the sides measured is 30, the contained Circles semi-diameter is 4-12; then 15 the half of 30, and 4-12 multiplied together, make 61-80; and 12 times this, makes 741-60, for the Superficial Content of the Dodecahedron.

Then for the Solid Content, multiply 61-80, the Superficial Content of one side by 2-233, one 6th part of 13-392, the whole Altitude of the body; the Product is 137-99940: Again, this multiplied by 12, the number of Pyramids, makes 1655, 9928, the Solid Content, as near as may be, in such a Decimal way of Computation.

5. *For the Icosahedron,*

Which is a regular solid body, made up of, or contained under 20 Triangular Pyramids, whose Base (or one of whose Sides) is an equilatteral Triangle; and the perpendiculer Altitude of one of these 20 Pyramids, is equal to half the perpendiculer Altitude of the Icosahedron, from any one side, to his opposite side, or equal to the semi-diameter of the inscribed Sphear.

To

To cut this body, take any round Piece; and if the Diameter thereof be 10000, let the length thereof be turned flat and even to 8075; or if the true Round and Cillendrical Form in Diameter be 4910, let the true length, when the ends are plain and flat, be 3964; then divide the Cillendrical part into 6 equal parts, and plain away all to the Lines, so that the two ends may be two 6-sided-figures; then making 5000, the former semi-diameter, 1000 in the Line of Lines, take out 616, and on the Center, at each end, describe a Circle; and by drawing Lines to each opposite Point, make a Triangle, whose circumscribing Circle may be the Circle drawn at each end; but be sure to mark the side of one Triangle opposite to the Point of the other Triangle at the other end, as before in the Dodecahedron; thus both the ends shall be fully and truly marked.

Then making the length a Parallel in 1000, of the Line of Lines, take out -379, and -095, and prick those two measures from each end, and by those Points (draw or gage) Lines round about, on the 6 sides. Then Diagonal Lines drawn from Point to Line, and from Line to Point round about, shews how to cut the Body at 12 cuts.

Note,

Note, That if you set a Bevel to -138-11-23, and apply it from each end, it will guide you in the true plaining of the sides of the Icosahedron. And a Bevel set to 100 degrees, will fit, being applied from the midst of one side, to the meeting of two sides.

The Reclination of the three Triangles, whose upper sides are adjacent (or next) to the three sides of the upper Horizontal-Triangle is 48 11 23, from the Perpendiculer, or 41 48 37, from the Horizontal, and when one corner stands South, the Declination of one of these 3, *viz.* that opposite to the South-corner a direct North; th'other two decline 60 degrees, one South-east, the other South-west; the other 6, about the corners of the Horizental-plain, do all recline 19 deg. 28 min. 16 sec. the two that behold the South, decline 22 deg. 14 min. 29 sec. and those two that behold the North, decline 37 deg. 45 min. 51 sec. toward the East and West; the other two remaining, recline as before, and decline one North-east, and the other North-west 82 deg. 14 min. 19 sec.

The other Nine under-Plains, opposite to every one of these, decline and incline, as much as the opposite did recline and decline, as by due consideration will plainly appear.

For

For the measuring of this body, do as you did by the Dodecahedron, find the Area of one Triangle, and multiply it by 20, gives the Superficial Content; and the Area of one Triangle, multiplied by one sixt part of the Altitude of the body, gives the solid Content of one Trianguler Pyramid; and that Product multiplied by 20, the number of Pyramids, gives the whole Solid Content of the Icosahedron.

Example.

Suppose the side of an Icosahedron be 12, first square one side (*viz.* 12, which makes 144); then multiply that Square by 13, and then divide the Product by 30, the Quotient and his remainder is the Superficial Content of the Equilatteral Triangle, whose side is 12; namely, 62-400; or more exactly, the Square-root of 3888, which is near 62-354; 20 times this, is the Superficial Content, namely, 1247-08.

Then for the Solid Capacity or Content, multiply 3-023, the sixt part of the bodies Altitude, or one third of the Pyramids Altitude, by 62-354, the Area of one Trianguler Base, and the Product will be 188-493229. Lastly, this multiplied by 20, the number of Pyramids in the Body, the Product is 3769-864380, the true solid Content of the Icosahedron.

Thus you have the way of cutting, and the Declinations and Reclinations and Measures, Superficial and Solid, of the 5 Regular Bodies, as near as by Decimal Accompt to 100 part of an Integer may be, the exact measuring whereof, requires the help of *Algebra*, whereof I am ignorant.

The Measures of the Containing, and Contained Sphears, Circles, and Diameters, Sides and Axis's, Diagonal-lines and Altitudes of the five Regular Bodies, gathered in a *Table* to a Containing Sphear, whose Diameter was 10 inches (or Integers) found out by *Geometry*, according to this *Scheam*, taken from Mr. *Tho. Diggs*.

Let the Line A B be 10 of some Diagonal Scale, representing the Diameter of the Containing Sphear. Which Line A B, you must divide into two parts at C, and into three parts at E; A E being one third part, and on the Points C and E, raise two Lines Perpendiculer to A B; and with 5 of your Diagonal Scale, on the Center C, describe the semi-Circle A F D B, and note the Points F and D, in the semi-Circle, with F and D, drawing Lines from either of them to A, and from F to B;

Then,

Then, Divide AF by extream and mean Proportion; the greater Segment being AG, (by the 10th Problem of the 6th Chapter) then extend the Line AF to H, making FH equal to FG, and draw the Line HB; and from F, draw another Line Parallel to HB, cutting the Diameter in I, and from I, draw a Line Parallel to CD, as IK; then make IL a third part of IB, and draw ML Parallel to IK; also, draw the Line MB, and divide it into two parts at N, and into 4 parts at S; then divide the 4th part, MS, by extream and mean Proportion, whose greater Segment (or part) let be SV; then divide FB in 4 parts, making FO the half, and FR the quarter; divide likewise FE in two parts, and at the middle set P: The *Figure* being thus made, then with your Compasses, and Diagonal Scale, you may measure all the Diameters, Sides, and Altitudes, of all the 5 Regular Bodies.

As thus;

AB is in all of them, the Contained Spheres Diameter.

EC, OF, RO, NC, NC, the Contained Spheres Semi-Diameter.

EF, OB, OF, NB, MB, the Containing Circles Semi-Diameter.

EP, OC, CO, VN, MN, the Contained Circles Semi-Diameter.

FB, AF, AD, AG, KB, the Length of the Sides of each Body.

EB, AF, FA, MA, AM, the Altitude of the Bodies.

AD, FB, VB, SB, the Perpendiculer Line of the Bases.

FB, AF, the Diagonal-Line of the Bases, as in the *Table*.

These Measures and Proportions are for a Sphear of 10 inches Diameter.

If you would have the like for any other, then say by the Line of Numbers, or Line of Lines, or Rule of Three, thus;

As the side (Diameter or Altitude) for 10, as in the *Table*, is to the given Side, Diameter, or Altitude;
So is any other Number, in the *Table*, for Diameter, Side, or Altitude, to his Proportional Measure required.

Example

Example.

I have a Dodecahedron, whose Side is 6, What shall all his other Spheres, or Circles, Diameters, or Altitude be?

The Extent of the Compasses from 3-570, the Dodecahedrons-side in the *Table*, to 6 the side given, shall reach from 10, the Containing Spheres Diameter in the *Table*, to 16, the Containing Spheres Diameter, for a Dodecahedron, whose side is 6: And from 7-970, the Contained Spheres Diameter, to 12-643, the Contained Spheres Diameter. And so for any other whatsoever.

The Table.

The Names of the Bodies.	*Tetrahedron*	*Cube.*	*Octahed.*	*Dodecahed.*	*Icosahed.*
Containing Sphear. The containing Sphears Diameter, that comprehends the body in it, is for ever one of them.—	AB 10.000	AB 10.000	AB 10.000	AB 10.000	AB 10.000
Contained Sphear. The contained SphearS Diameter that is contained in the body, called also Axis, is ——	2.332	8.1648	4 0824	7.970	7.970
The half thereof, is ——	EC 1.666	OP 4.0824	RO 2.0412	NC 3.983	NC 3.985
Containing Circle. The containing Circles Diameter, (or the Diameter of that Circle which comprehends one side or base of the body, is —	9.420	9.648	9.644	6.070	MB 6.070
The half thereof, is ——	EF 4.710	OB 4.0824	OF 4.0824	NE 3.035	3.035
Contained Circles. The contained Circles Diameter, comprehended in the Base of one side, is ——	4.410	5.7840	3.7840	4.910	MN 3.035
The half thereof, is ——	EP 2.355	OC 2.8920	OC 2.8920	VN 2.455	1.5175
Sides, The length of one side of the Triangle Square, or Pentagon, being the base of the figure, is	FB 8.1647	AF 5.774	AD 7.071	AG 3.570	KB 5.260
The half therof, ——	4.08235	2.887	3.5355	1.785	2.630
Altitude, The Altitude from side, to the side opposite, or from Side to the Point opposite, ——	EB 6.666	AF 5.774	AF 5.774	MA 7.960	MA 7.960
The half thereof, ——	3.3333	2, 887	2. 887	3.980	3.980
Perpendicular, The Length of the Perpendiculer Line of any One Side or Base, ——	AD 7 073	FB 8.1647	6.123	VB 5.485	SB 4.556
The half thereof,— ——	3 5365	4.0823	3.0625	2.7425	2.278
Diagonal-Line. The Diagonal-Line, from Corner to Corner of the same Base, is, ——	none—	FB 8.1647	none—	AF 5.774	none—
The half thereof is,——		4.0823		2.887	

This *Table* was gathered from this *Geometrical Figure*, drawn on a Slate, by a good Diagonal Scale of 6 parts in a Foot, whereby I could very well come to the 100th part of an Integer; and is true enough for any Mechanick Operation, for whose use I only do it, and I hope it may be as kindly accepted, as it was carefully Calculated, and offered to Publick view.

CHAP. XII.

The use of the Line of Numbers, in Gaging of Vessels, close or open.

GAging of Vessels, is no other than the Measuring of Solid Bodies; and the former directions for solid Measure, conveniently and aptly applied, is fully sufficient; only observing this difference, That the result or issue of the Question is to be rendred in proper terms, according to the demand

demand of the Question, as thus; in measuring of Timber or Stone, the Question is, *How many Feet, or Inches, is there in the Solid Body?* But in Gaging, the Question is, *How many Gallons, Kilderkins, or Barrels is there in the Vessel to be measured?* For which purpose there are fit Numbers, or Gage-Points, requisite to be known, for the more speedy attaining the Answer to the Question, of which in their order, as followeth;

First, You are to remember, That the solid capacity of a Wine-Gallon, is 231 Cube Inches; a Corn-Gallon 272 $\frac{1}{4}$ Cube inches; an Ale or Beer-Gallon, is 282 $\frac{1}{4}$ Cube inches; or as some say, 288 Cube inches; So that when you have found the Content of any Vessel in Cube inches, if you divide that sum in inches, by the respective Number for the Gallons you would have, the Quotient shall be the Content in Gallons required.

Problem I.

To measure a Square Vessel.

From hence it follows, That to measure any Square or Oblong Vessel, you must multiply the length and breadth taken in inches, and tenth parts, together; that is to say,

ſay, The one by the other; and the Product ſhall be the Content of the Baſe in inches, ſuperficially: Then multiply this Superficial Content of the Baſe, by the inches and tenth parts deep, and the Product ſhall be the ſolid Content in Cube inches; then divide this Product by 282, gives the Content in Ale-Gallons in the Quotient, and the remainder, if any be, are Cube inches.

But if you divide by 10161, the Cube inches in a Beer Barrel; or, by 9032 the Cube inches in an Ale Barrel; the Quotient ſheweth the Number of Beer or Ale Barrels, (and the remainder Cube inches.)

Example of a Brewers Cooler.

The length let be 78 inches and 1 tenth, the breadth let be 320 inches and 5 tenths, and the depth 9 inches and 5 tenths, or half an inch; by multiplying and dividing, as above, you will find 843 Gallons, and 68 Cube inches, to be the ſolid Content of that Cooler; which work is very readily done by the Line of Numbers, in this manner;

Extend the Compaſſes from 1, to the breadth or length; and the ſame Extent ſhall reach from the length or breadth to a 4th, which is the Superficial Content of the Baſe, or bottom, in Superficial inches.

Then,

Then,

The Extent from 282¼, to the laſt Number found, ſhall reach the ſame way from the inches, and tenths deep, to the Content in Gallons.

Example.

The Extent from 1, to 78-1, ſhall reach the ſame way from 320-5, to 25031; then the Extent from 282¼, to 25031, ſhall reach the ſame way from 9-5, to 842-68, the Solid Content in Gallons required.

Indeed, you muſt *Note*, You cannot ſee ſo many Figures on the Line, as the Product of 4 figures multiplied by 3; yet by the Rules (in *Chap.6. Sect.3.*) you have directions as to the number of Figures, which here is 7; the two laſt (next the right hand) being Fractions, or parts of an Inch, and is therefore neglected.

Again,

In dividing the Product of 25031, and 9-5, multiplied together, which makes 6 Figures beſide the Fraction by 282, there muſt needs be three Figures in the Quotient, which are the Gallons: This artificial help you have, beſide the preſent view of the Veſſel, which will direct you not to call 842 Gallons, only 84½, nor 8420, as you muſt needs do, if you miſtake as to the denomination.

Again,

Again,

You need not to trouble your ſelf, to know what the 4th Number is; but having found the Point repreſenting it, keep the Compaſs-point fixed there, and open the other to 282 $\frac{1}{4}$, where you may have a Braſs Center-pin for more readineſs; but let your account go as 282 $\frac{1}{4}$ to the 4th, for methods ſake, and not as the 4th to 282 $\frac{1}{4}$; for then you muſt ſay, ſo is the depth the contrary way to the Content in Gallons. All this is hinted for plainneſs and caution ſake, in benefit to young Learners.

Alſo Note, That if you would have had the Anſwer in Ale or Beer Barrels; then inſtead of 282 $\frac{1}{4}$, you muſt uſe the Point at 9032, for Ale Barrels; or the Point at 10161 for Beer Barrels, being the number of Cube inches in thoſe Barrels, as 282 $\frac{1}{4}$ is the number of inches, in a Gallon of Ale or Beer.

Example for the ſame Cooler.

The Extent from 1 to 78-1, ſhall reach from 320-5, the ſame way to 25031; *then,* the Extent from 10161 to 25031, ſhall reach the ſame way from 9-5 to 23 $\frac{1}{2}$, the true number of Beer Barrels required. *Or,*

The Extent from 9032, to 25031, ſhall reach to 26 Barrels, and near 1 third: which

is

is as quick and ready a way as can be for Square Vessels.

Problem II.

To Gage or Measure any round Tunn or Vessel.

The plain and natural way for measuring of a round Tun, is this; Measure the Diameter in inches and tenths, and set down half thereof; Measure also, the Compass round about the inside, and set down the half of that also, in inches and tenth parts; and multiply those two Numbers together, the Product shall be the Content of the Base, or bottom, in Superficial inches; then this Product multiplied by the depth in inches, gives the solid Content in inches; then lastly, this Product divided by 282, or by 10161, or by 9032, gives the solid Content in Gallons, or Beer, or Ale Barrels, as before.

For, half the Diameter, and half the Circumference, doth reduce the round Vessel to an Oblong Vessel, equal to that round Vessel.

Which Vessel, when it is brought to a Square, by taking of half the Diameter, and half the Circumference; then the Rule

last

last mentioned, for Square Veſſels, performs the work exactly, to Gallons, or Barrels, as you pleaſe.

But when the Veſſel is Taper, that is to ſay, the bottom and top of different Diameters, as generally they all are; then the chief care is to come by the true Diameters, which is beſt done by a ſliding Rule applied to the inſide, whoſe regular equal computation is thus to be ordered;

When the Veſſel is taper, and the Sides go ſtreight, like the Segment of a Cone; then you may add the Diameters at top and bottom together, and count the half ſum for the mean Diameter of that taper Veſſel, and multiply half that Diameter, and half his proportional Circumference, as before; and multiply and divide, to get the ſolid Content in Gallons, or Barrels.

But when the Staves are bending, as moſt of your cloſe Cask are, then the readieſt way to come to a mean Diameter, is thus;

Say,

As 10 to 7, or as 10 to 6 & $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$; according as you ſhall find moſt true for ſeveral Cask:
So is the difference of Diameters to a 4th Number, which is to be added to the leaſt of the Diameters, to make up a mean Diameter.

As

As for Example.

If the Sides be round or arching, and the leſs Diameter be 30 inches, and the greater 40 inches; then, As 10 to 7; So is 10, the difference to 7 inches; which makes (being added to 30, the leaſt Diameter) 37, for a mean Diameter.

But Note, It is hinted by Mr. *Dary*, That Veſſels, uſually, are between a Spheroid and a Parabolick Spindle; then, if as 10 to 7, be too much to add to the leaſt Diameter; *You may ſay*,

As 10, to $6\frac{1}{2}$; *Or*, As 10, to $6\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$;
So is the difference of Diameters to a 4th Number, which you muſt add to the leaſt Diameter, to make a mean Diameter.

Having thus gained a mean Diameter, you may work as before; or rather thus more readily and eaſily, by the Line of Numbers, *thus*;

As the Gage-point is to the mean Diameter;
So is the Length to a 4th, and that 4th to the Content required.

The

The Gage-point for Wine, and Oyl-gallons, at 231 Cube inches in a Gallon, is ———————— 17–15

The Gage-point for Ale-gallons, at 282 $\frac{1}{4}$, is, ———————— 18–95

The Gage-point for Ale, or Beer-gallons, at 288, is, ———————— 19–15

The Gage-point for a Corn-gallon, at 272 $\frac{1}{4}$, is, ———————— 18–62

The Gage-point for a Beer Barrel, at 10161, is, ———————— 35–96

The Gage-point for an Ale Barrel, at 9032, is, ———————— 33–91

The Extent of the Compasses, on the Line of Numbers, from the Gage-point to the mean Diameter of a Vessel; being turned two times the same way from the length of a Vessel, shall reach to the Content of the Vessel, in Gallons or Barrels, according to the nature of the Gage-point.

Example.

A mean Diameter being 30, and the Length 40, the Content is in Wine-gallons 123, near.

In the lesser Ale or Beer-gallons, 100-$\frac{1}{2}$.

In the greater Ale-gallons, at 288-098 gallons and a half.

In

In Corn-gallons, at 272-$\frac{1}{4}$, — 104 Gallons.

In Beer Barrels, by his Gage-point you will find 2-78, or 2 three quarters: 2—78.

In Ale Barrels, you will find 3 and 11 of a hundred: 3 — 11.

And the like for any other Meaſure, whoſe Gage-point is known.

Problem III.

To find the Gage-Point of any Meaſure.

The Gage-point of any Solid Meaſure, is only the Diameter of a Circle, whoſe Superficial Content is equal to the Solid Content of the ſame Meaſure.

As thus more plainly;

The Solid Content of a Wine Gallon is 231 Cube inches: Now if you have a Circle that contains 231 Superficial inches, the Diameter thereof will be found to be 17 inches, and 15 of a hundred; as by the 7th *Problem* of the 7th *Chapter*, is well ſeen.

Theſe Directions may ſerve for any round Veſſel, either cloſe or open; yet Mr. *Oughtred*, a very able Mathematition, hath a

way

way accounted ſomewhat more exact, and conſequently more tedious and troubleſome to uſe either by the Pen or Compaſſes, which is this;

You muſt meaſure the Diameters at head and bung, or the tcp and bottom in inches and 10ths, the length alſo by the ſame meaſure; then find out the Superficial Content of the Circles, anſwerable to thoſe two Diameters, and take two thirds of the greateſt, and one third of the leaſt, and add them together in one ſum; which ſum you muſt multiply by the length in inches and tens, and the Product ſhall be the Content in Cube inches; which Product divided by 282, gives Ale Gallons; or by 231, gives Wine Gallons, as before.

By the Line of Numbers, this way is more eaſie and ready thus;

The Extent from 1, to 0-5236, a Number fit for 2 thirds, of the Circle at the bung;
So is the Square of the Diameter at the bung to a 4th.

Then again;

is 1, to 0-2618, the half of the former Number, and fit for one third of the Circle at head;

So is the Square of the Diameter at head to a 4th.

These two 4ths add together, then say;

As 231 (for Wine, or 282 $\frac{1}{4}$ for Ale-Gallons), is to the sum of the two 4ths added together;

So is the length to the Content in Wine-Gallons.

Example, at 18 inches at head, and 32 at bung, that old Example.

The Square of 32, is 1024;

The Square of 18, is 324:

Then,

The Extent of the Compasses from 1, to 0-5236, shall reach from 1024, the Square of 32, to 536-4, two thirds of the bung-Circle.

Again,

The Extent from 1, to 0-2618, shall reach from 324, the Square of 18, the Diameter at head, to 84-9, the sum of 536-4, and 84-9, is 621-3.

Then lastly,

The Extent from 231, to 621-3, shall reach

reach from 40 the length, to 107-58, or 107 Gallons and a half, and better, the Content in Wine-gallons, as briefly as can be done this way.

But if you take the Diameters at head and bung, with a Line called *Oughtred's* Gage-line; and set the measure found at the bung by that Line, down twice; and the measure found at the head, found by the same Line, once, and bring them into one sum; then multiply that sum by the length of the Vessel in inches, and 10 parts, and then the Product shall be the Content in Wine-gallons required.

As if I should measure a Cask of 18, and 32, as before: right against 18 inches on *Oughtreds*-Line, you find 0-367; and right against 32, you shall find 1-161; this last set down twice, and 0-367 once; added, makes 2-689; and then this sum multiplied by 40, makes 107-56, being very near to the former operation, but differing about 2 Gallons, from the way set before by the mean Diameter and Gage-point, by reason of the extream swelling of the Cask; But if this way should prove the truest in the Book of *the Carpenters Rule*, you have a *Table* to rectifie this difference, which you will very seldom have occasion to use.

Note also, That this Line, called *Oughtred*'s Gage-Line, is very excellently improved to find the Content of Great Vessels, either in the whole, or inch by inch; which you will find at large in the Book before mentioned.

Also, The use of the Lines called Diagonal-Lines, and Lines to find the emptiness of Cask, and to measure Corn-measures by, to which I shall, for the present, refer you.

Problem IV.

The Diameter and content of a Vessel being given, to find the length of the Vessel.

Extend the Compasses from the Diameter to the Gage-point, the same Extent twice repeated from the Content, shall give the length required.

Example.

If the Content be 60, and the Diameter 24, then extend the Compasses from 24, to 17-15, the Gage-point for Wine; this Extent turned twice the same way, from 60 the Content, shall reach to 30 inches, and 6 tenths, and a half, the length required.

Problem V.

The Length and Content of a Vessel given, to find the Diameter.

Divide the ſpace on the Line of Numbers, between the Length and the Content, into two equal parts; the Compaſſes ſo ſet, ſhall reach the ſame way from the Gage-point to the Diameter of the Veſſel.

Example.

The half diſtance between 31-65 the length, and 60 the Content, ſhall reach the ſame way from 17-15 the Gage-point, to 24 the Diameter required.

Theſe two laſt Problems may be uſeful for *Coopers*, to make Cask of any length, diameter, and quantity.

Problem VI.

To find what is wanting in any cloſe Cask, at any number of inches and parts, (the Cask lying after the uſual manner, with the bung-hole uppermoſt) from the bung-hole to the ſuperficies of the Liquor given, two wayes.

This Problem I ſhall reſolve *two wayes*, either of which is experimented to come near the truth, and will very well ſerve, till a better comes to light.

The *One*, by a Line of Segments, jöyned to the Line of Numbers, as before in the meaſuring the Fragments of a Globe; But,

The *Other*, is by a way found out by Mr. *Bennit*, a Cooper, that hath long exerciſed the way of Gaging, which is by comparing a Cask known, and its quantity of emptineſs, to a Cask unknown, and its inches of emptineſs, as followeth.

Firſt, by the Line of Numbers, and Artificial Line of Segments, to find the quantity of Gallons that any Veſſel wants of being full, at any number of Inches, from the inſide of the bung-hole, to the ſuperficies of the Liquor, which is uſually called Inches dry.

Extend the Compaſſes, on the Line of Numbers, from the inches and tenths diameter at the bung, to 100 on the Line of Segments, the ſame extent applied the ſame way from the inches and parts dry, ſhall reach to a 4th Number, on the Line of Artificial Segments; which 4th Number you muſt keep. (Or, if you will, you may uſe the inches wet, laying the ſame extent from the inches wet, and that alſo will on the Segments,

ments give a 4th Number, which you must likewise keep.)

Then secondly,

As the Extent from 1, to the whole Content of the Vessel in Wine or Ale-gallons; So is the 4th Number kept to the Gallons of emptiness, or fullness, that it wants of being full, or the quantity of Gallons in the Vessel.

Example, of a Canary-Pipe, whose Diameter at bung, is 28 inches and 7, and full Content is Gallons 116 $\frac{1}{2}$, at 12 inches dry, or 16 inches, and 7 tenths wet.

The Extent of the Compasses from 28-7, to 100, (at the end of the Line of Segments) shall reach the same way from 12, the inches dry, to 39 $\frac{1}{2}$ on the Line of Segments for a 4th; or from 16-7 wet, to 60 $\frac{2}{10}$ on the Segments, for his 4th also, which two 4ths keep.

Then secondly,

The Extent from 1, to 116 $\frac{1}{2}$, the whole Content in Gallons, shall reach from 39, the dry 4th, on the Line of Numbers, to 46 $\frac{3}{10}$, for the gallons dry or wanting: or the same extent shall reach the same way, on the Line of Numbers, from 60 $\frac{2}{10}$, the 4th Number for wet, to 70 gallons, and 2 tenths

 in

in the Veſſel, at 16 inches and 7 tenths wet; which two Numbers put together, makes up 116 gallons and a half, the full Content.

The like manner of working ſerves for any Cask whatſoever, and the nearer the Veſſel wants of being half empty, the more near to the truth will your work be, and the moſt errour in very round and ſwelling Cask, when the emptineſs is not above one or two inches; but in Veſſels near to Cillenders, it will give the Anſwer very true, and as readily as any way whatſoever.

Obſerve alſo, That if you uſe the Segments in taking the wants, you muſt abate of the gallons found, till you come to the 2 thirds of the half diameter; that is to ſay, the Rule ſayes, there is more wanting than indeed there is; and that ſomewhat conſiderable about the firſt 6 inches in a veſſel of 30 inches diameter: So that I find a *Table* made as a mean between the Superficial and ſolid Segments, would do the work the trueſt and beſt of any other; Or elſe, uſe the mean diameter and mean parts of emptineſs; found thus.

Take the equaded diameter, from the diameter at the bung; and note the difference: then half this difference taken from the inches and parts empty gives the mean

emptineſs;

emptineſs ; then uſe the mean diameter, and mean emptineſs, inſtead of the other, and the work is more exact.

The other way of Mr. Bennits *invention is thus ;*

First, you are to fill an ordinary Cask, of a competent magnitude, as 60 or 100 gallons, of a mean form, between a Spheriord (or roundiſh form) and a Cillenderical form ; or elſe fill two Casks of each form, and learn the true Content, and Diameter of that mean Veſſel, or rather of both thoſe Veſſels ; and the Veſſel being full, draw off with a true gallon-meaſure, and on the drawing off every gallon, take the exact quantity of inches and 10th parts, that the drawing off of every gallon makes in the emptineſs or drineſs of that mean Veſſel, or rather both thoſe Veſſels, at leaſt until you have drawn off the half quantity of the Veſſel, which number of gallons drawn off, and the inches and tenth parts of emptineſs, or fulneſs, or drineſs or wetneſs, you muſt draw into a *Table*, or inſert them on a Rule, making the inches as equal parts, and the gallons, and his proportional part of a gallon, the unequal parts ; then with the Line of Numbers, and this mean *Table*, or rather two *Tables* or *Scales*, which you may put

put on a Rule, as Mr. *Bennit* hath done, you may find out the wants of any Cask whatsoever; either Spherioid, or Cillender-like, as followeth.

This measured Cask on the Scale, or Table, for methods sake, and avoiding tautologie, I shall call the first Cask, and the Vessel or Cask, whose wants you would know, I shall call the second Cask; then the proportion is thus.

As the Diameter at the bung of the second Cask, is to the bung diameter of the first Cask (which is always fixed);
So is the inches dry of the second Cask to a 4th (on the Line of Numbers) which 4th Number sought on the inches of your Table, or Scale, on the opposite part of your Scale or Table, gives a 5th Number, which you must keep.

Then,

As the whole Content of the first Cask, is to the whole Content of the second Cask;
So is the first Number kept, to the Number of Gallons the Vessel wants of being full, at so many inches dry.

Example.

There is such a Scale made on purpose for Victuallers use, to measure what they want of a Barrel of Ale, being put into a Beer-barrel,

barrel, which Scale I ſhall here uſe, to try this former *Example* by.

Suppoſe, as before, a Canary Pipe want 12 inches of being full, and the Content 116 $\frac{1}{2}$ gallons, and 28 inches and 7 tenths diameter at bung; The Extent on the Line of Numbers from 28-7, to 22-5, ſhall reach from 12, to 9-4; then juſt againſt 9 inches and 4 tenths, on that Barrel Scale, I find 14 gallons of Beer, which is 17 gallons and a half of Wine, being the 5th Number to be kept.

Then the Extent from 44, the Content of a Barrel in Wine-gallons, to 116 $\frac{1}{2}$, the Content of a Canary-Pipe in the ſame gallons, ſhall reach the ſame way from 17$\frac{1}{2}$ the Number kept, to 46, and near a half, the gallons wanting at 12 inches dry, in the Canary Pipe, and 46 gallons, and 3 quarts, is the Number Mr. *Bennit* finds in a Canary-Pipe, by meaſuring at 12 inches dry.

Thus you have an account of the two eaſie Mechanick wayes, to diſcover the wants of Cask, very applicable, and ready, and experimented to be *Propè verum.*

The

The Gallons wanting in a Barrel, at every inch and quarter.

	Beer Gall.			Wine Gall					
	gal.	pi.	100	gal.	pi.	100	gal.	1000	
	0	0	40	0	0	49	0	0612	
	0	1	20	0	1	47	0	184	22
	0	2	10	0	2	57	0	321	
1	0	3	10	0	3	80	0	475	
	0	4	33	0	5	30	0	663	
	0	6	00	0	7	35	0	920	21
	0	7	60	1	1	29	1	161	
2	1	1	86	1	4	00	1	500	
	1	3	90	1	6	56	1	821	
	1	6	10	2	1	22	2	153	20
	2	0	66	2	4	34	2	543	
3	2	3	50	2	7	98	2	998	
	2	6	16	3	3	10	3	388	
	3	0	70	3	6	20	3	772	19
	3	3	80	4	2	00	4	250	
4	3	6	50	4	5	30	4	663	
	4	1	80	5	1	35	5	169	
	4	5	25	5	5	60	5	700	18
	5	0	42	6	1	45	6	182	
5	5	3	90	6	5	70	6	713	

	Beer Gall.			Wine Gall.					
	gal.	pi.	100	gal.	pi.	100	gal	1000	
	5	7	20	7	1	70	7	213	
	6	2	80	7	6	20	7	777	17
	6	6	50	8	2	65	8	333	
6	7	2	20	8	7	20	8	900	
	7	5	50	9	3	20	9	400	
	8	1	10	9	7	70	9	960	16
	8	4	80	10	4	20	10	525	
7	9	0	70	11	1	00	11	125	
	9	4	50	11	5	40	11	806	
	10	0	40	12	2	20	12	275	15
	10	4	30	12	7	00	12	876	
8	11	0	50	13	4	10	13	513	
	11	4	30	14	0	80	14	110	
	12	0	30	14	5	80	14	725	14
	12	4	29	15	2	80	15	350	
9	13	0	30	15	7	70	15	926	
	13	4	30	16	4	60	16	577	
	14	0	40	17	1	60	17	200	13
	14	4	60	17	6	60	17	827	
10	15	0	50	18	3	40	18	425	
	15	4	48	19	0	30	19	037	
	16	0	80	19	6	50	19	815	12
	16	5	50	20	3	25	20	407	
11	17	2	20	21	1	00	21	225	

	Beer Gall. gal.	pi.	100	Wine Gal. gal.	pi.	100	gal.	1000	
	17	7	90	22	0	00	21	000	
	18	5	49	22	6	98	22	644	11
	19	2	00	23	4	31	23	391	
12	19	6	16	24	1	48	24	184	
	20	3	00	24	7	30	24	961	
	20	7	40	25	4	60	25	575	10
	21	3	10	26	1	36	26	170	
13	21	7	40	26	6	38	26	799	
	22	3	00	27	3	36	27	130	
	22	7	00	28	2	18	28	174	9
	23	3	00	28	5	18	28	648	
14	23	7	30	29	2	19	29	275	
	24	3	70	29	7	40	29	926	
	24	7	40	30	3	90	30	488	8
	25	3	60	31	1	00	31	125	
15	25	7	50	31	5	80	31	726	
	26	3	30	32	2	60	32	325	
	26	7	00	32	7	00	32	875	7
	27	3	00	33	3	80	33	475	
16	27	6	40	34	0	30	34	037	
	28	2	20	34	4	80	34	600	
	28	5	80	35	0	80	35	100	6
	29	1	40	35	5	34	35	668	
17	29	4	80	36	1	80	36	225	

	Beer Gall.			Wine Gall.					
	gal.	pi.	100	gal.	pi.	100	gal.	1000	
	30	0	40	36	6	29	36	788	
	30	4	10	37	2	29	37	287	5
	30	7	50	37	6	54	37	820	
18	31	3	00	38	2	39	38	299	
	31	6	10	38	6	64	38	833	
	32	1	80	39	2	70	39	338	4
	32	5	00	39	6	00	39	752	
19	32	7	80	40	1	80	40	225	
	33	2	10	40	4	90	40	614	
	33	4	80	41	0	10	41	012	3
	33	7	40	41	3	65	41	457	
20	34	2	00	41	6	77	41	848	
	34	4	30	42	1	43	42	180	
	34	6	20	42	4	00	42	500	2
	35	0	10	42	6	70	42	840	
21	35	2	00	43	0	64	43	055	
	35	3	60	43	2	70	43	338	
	35	4	80	43	4	19	43	524	1
	35	6	00	43	5	42	43	678	
22	35	6	80	43	6	51	43	816	
	35	7	40	43	7	50	43	938	
	36	0	00	44	0	00	44	000	

CHAP. XIII.

The use of the Line of Numbers, in Questions of Interest and Annuities.

Problem I.

At any rate of Interest per annum *for a hundred pounds, to find what the Interest of any greater or lesser sum comes to in one year.*

EXtend the Compasses from 100 to the increase of 100 *l.* in one year, the same Extent shall reach from the sum propounded, to its increase for one year, at that rate propounded.

Example.

What is the increase or profit of 124 l. 10 s. for one year, at 6 per cent. per annum?

The Extent of the Compasses, from 100 to 6, being laid the same way from 124 *l.* 10 *s.*

10 *s.* (which is at 124-5) ſhall reach to 7-47, which is 7 *l.*— 9 *s.* — 4 *d.* the profit of 124-10 *s* in one year.

Problem II.

Any ſum of Money, and the rate of Intereſt propounded, to find what it will increaſe to, at any number of years, counting Intereſt upon Intereſt.

The Extent of the Compaſſes from 100, to the increaſe of 100, being turned as many times from the ſum propounded the ſame way, as there be years propounded, ſhall at laſt ſtay at the Principal and Intereſt required.

Example.

To what ſum ſhall 143 pounds 10 ſhillings, amount to in 10 years, counting Intereſt upon Intereſt, at 6 per cent ?

The Extent of the Compaſſes from 100, to 106, being turned 10 times from 143 $\frac{1}{2}$, ſhall reach to 257 *l.* 0 *s.* the ſum of Principal and Intereſt at 10 years end.

Note, That in doing this, you ought to be very preciſe, in taking the firſt Extent from 100, to 106; but to cure the uncertainty thereof, you have this very good remedy: If you have a Diagonal Scale, equal to the Radius of the Line of Numbers, then uſe that; if not, uſe the Line of Lines on the Sector-ſide, which ſhould be made fit to (or the double, or the half of) the Radius of the Line of Numbers.

As thus;

Take the Extent from the Line of Numbers, between 100, and 106; this Extent meaſured on the Line of Lines, will be 0253058, could you ſee ſo many Figures, but 02531, will ſerve your turn very well; which Number you muſt note, is the Logarithm of 106, neglecting the Caracteriſtick; then this Number multiplied by 10, the Number of years, is 25310; this Extent taken from the Center, on the Line of Lines, and laid increaſing from 143 ½, ſhall reach to 257 *l.* 0 *s.* 0 *d.* the true Number of the Uſe and Principal of 143 *l.* 10 *s.* put out, or forborn for ten years.

Problem III.

A ſum of Money being due at any time to come, to find what it is worth in ready Money to be paid preſently, at any rate propounded.

This Problem is the contrary to the laſt, for if you ſhall turn the Extent between 100 and 106, ten times backward from

257, it will stay at 143 $\frac{1}{2}$, the worth in ready Money.

Or, to make use of the former remedy;

Multiply 0253058, the Logarithm of 106 by 10; then this Extent taken and laid the decreasing way from 257, shall reach to 143 $\frac{1}{2}$.

For Note, That the Line of Lines is the Scale of equal parts, that makes the Line of Numbers, and 10, or 7, or 15, or any other Number multiplied by the Logarithm of 106, taken from that Scale of Lines all at once, is equal to so many repetitions; and consequently more exact, because of the difficulty of taking the 10, 12, or 15th part of any Number whatsoever; and observe, That so much as you err in the first, it will be 10, 12, or 15, or 20 times so much at last, which may be considerable in this.

Problem IV.

A yearly Rent, or Annuity being forborn a certain number of years, to find what the Arrears thereof will amount unto, according to any rate propounded.

First, you must find out the Principal-Money, that answers to the Rent, or Annuity in question; then find the sum of that Principal and Use, at the end of the term given, at the rate propounded; then the Principal taken out of this sum, both of Arrears and Principal, the Arrears do remain, which is the sum you look for.

Example.

Suppose a Landlord live far from his Tennant, and yet judging his Tennant honest, and able, is content to take his Rent once in every fourth year, which should be paid every year, or every quarter of the year; and suppose the Rent be 10 *l. per annum*, and the rate of profit, for the forbearance, be 8 *per cent.*

First, to find the Principal for 10 *l. per annum*, at the rate of 8 *l. per cent.* Say, If 8 *l.* have 100 for his Principal, what shall 10 *l.* have? The Answer will be 125; for the Extent from 8 to 100, shall reach from 10, the same way, to 125; then by the 2d Problem of this Chapter, 125 *l.* forborn for four years, will come to 170 *l.* which is 170 *l.* 0 *s.* 0 *d.* from which sum, if you substract 125 *l.* there remains 45 *l.* the Arrears for 10 *l. per annum* forborn four years, at the rate of 8 *per cent.*

But if you would have the profit of these Arrearages, supposing 2 *l.*—10 *s.* the 4th part of 10 *l. per annum* to be paid quarterly, and to count Use upon Use at the rate abovesaid;

ſaid, then you will find the Principal and Arrears to be 171 l. 10 s. For if you multiply 0086, the log. of 102 l. the Intereſt and Principal of 100 l. for a quarter of a year by 16, the quarters in four years, it will be 1376, which Number taken from the Line of Lines, and laid from 120, on the Line of Numbers, ſhall reach to 171 $\frac{1}{2}$, or 171 l. 10 s. being 30 s. more than the former ſum, when 150 l. the Principal is taken away, the reſidue Arreares is 46 l. 10 s.

Or,

If you turn the diſtance on the Numbers between 100 and 102, 16 times from 125, which you may help thus; turn firſt 4 times, then take them 4 times in one Extent, and turn 3 times more, and you will ſtay at 271 $\frac{1}{2}$, the Anſwer required.

Problem V.

A yearly Rent, or Annuity propounded, to find the worth thereof in ready Money, at any rate whatſoever.

Firſt, by the 4th Problem, find the Arrears that ſhall be due at the end of the term, and at the rate propounded; then by the 3d Problem, find what thoſe Arrears are worth in ready money, which ſhall be the worth of the Annuity, or Rent required.

Example.

There is a Leaſe of a Houſe or Land worth 12 l. per annum, and there is 16 years yet to come; which Leaſe a man would buy, provided he may lay out his money to gain after the rate of 10 l. per cent: the queſtion is, What is it worth?

Firſt, by the laſt, if 10 l. have 100 for his Principal, What ſhall 12? the Anſwer is 120; Then by the ſecond part of the ſecond, 120 l. forborn 16 years, comes to 551 l. the Principal and Intereſt: from which ſum, taking 120 l. the Principal, there remains 431 the Arrears. Then by the third Problem find what 431 due 16 years to come, is worth in ready money; and the Anſwer will be at 10 in the 100, 93 l. 14 s.

Alſo herein obſerve, That if there be any Reverſion of a Leaſe to be expired, before it may be injoyed; then you are to find the worth of 431 l. after ſo many years more; as ſuppoſe it be 5 years before the Annuity begin; then find the worth of 431, forborn 21 years, which will be 58 l. 4 s.

Problem VI.

A sum of Money is propounded, and the rate whereby a man intends to Purchase, to find what Annuity, and how many years to continue, that sum of money will buy.

Take any known Annuity at pleasure, and find by the last, the value of that in ready money, then this proportion holds;

As the value found, is to the Annuity supposed;
So is the sum of money to be improved, to the Annuity required.

Example.

What Annuity, to continue 16 years, will 500 l. Purchase, whereby a man may gain after the rate of 10 l. per cent?

By the last Problem I find, That 93 l. 14 s. will purchase 12 l. a year, for 16 years, at 10 per cent.

Therefore,

The Extent of the Compasses from 93 l. 7, to 12 l. *per annum*, shall reach the same from 500, to 64 l. *per annum*. For such an Annuity, to continue 16 years, will 500 l. purchase, to gain 10 l. *per annum, per cent.* for your Money.

Problem VII,

Or, first rather;

Lands or Houses, sold at any certain number of years Purchase; to find what the value of the whole will be?

The usual way of valuing Land or Houses, is by the years Purchase, and Land Fee-simple is usually vallued at 20 years Purchase; Coppy-hold-Land, at 15 or 16 years Purchase; and good, strong, and new Houses, at 12, 13, or 14 years Purchase for Fee-simple.

But a Lease of a House of 21 years about 7 years Purchase; and a Lease of 31 years, about 8 years Purchase, rather less than more; and a Lease of 60, or 100, not worth above 8 $\frac{1}{2}$ years Purchase.

Again,

The usual profit allowed for Land in Fee-simple, is not above 5 l. in the 100 *per annum*, because of the certainty thereof; for Coppy-hold Land, full 6 l. in the 100 *per annum*; for the best Houses, 7 and 8 l. in the 100 Fee-simple.

But in laying out Money on Leases, either of Land or Houses, Men shall hardly be savers, if they gain not 8, 9, or 10 in the 100 *per annum*, for their Money; The reason and

and demonstration whereof, you may read at large in Mr. *Phillips* his *Purchasers Pattern*.

Thus the number of years Purchase agreed on, (which ought to be cleer, from Quit-rent, and Taxes, and the like; the Rent is usually various, according to the place, and time where, and wherein, the Purchase shall happen to be) then to find the quantity of the whole Purchase,

Say,

As 1, to 20, 18, 15, 14, 12, 10, or 8, the number of years Purchase, for Fee-simple, or Coppy-hold Land, or Houses Fee-simple, or Coppy-hold; For Leases of 60, 50, 40, or 30 years, or 21 years;
So is the yearly Rent to the whole value.

Example.

A Parcel of Land worth 10 *l. per annum* Fee-simple, valued at 20 years Purchase, will amount to 200 l.

For,

The Extent from 1, to 20, will reach the same way from 10 to 200, the whole price of 20 years Purchase, at 10 *l. per annum*.

CHAP. XIV.

The Use of the Line of Numbers IN Military Questions.

Problem I.

Any Number of Souldiers being propounded, to order them into a Square Battel of Men; that is, as many in Rank as in File.

FInd the Square-root of the Number of Souldiers, and that shall be the Number of Men in Rank and File required.

As suppose it were required to order 1770 Men, in the order abovesaid, you shall, by the 8th Probl. of the 6th Chapt. find, that the Square-root of 1770 is 42, and 6 over, which here is not considerable.

Problem II.

Problem II.

Any number of Souldiers propounded, to order them into a double Battel of Men; that is to say, twice as many in Rank as File.

Find the Square-root of half the Number of Men, and that is the Number of Men in File, and the double the Number in Rank.

As for Example.

If 2603, were so to be placed, the half of 2603, is 1301; whose Square-root by the 8th of 6th, is 36; the number of Men in File: and 72, the double thereof, is the number in Rank. For if you shall multiply 72 by 36, the Product is 2592, almost the number of Men propounded.

Problem III.

Any Number of Souldiers being propounded, to order them into a Quadruple Battel of Men; viz. 4 *times as many in Rank as File.*

Find the Square-root of a 4th part of the Number of Men, and that shall be the Number in File; and 4 times so many the Number in Rank.

So the 4th part of 2603, is 650; whose Square-root is 25 $\frac{1}{2}$, and 4 times 25 is 100, the Number in Rank.

Problem IV.

Any Number of Souldiers being given, together with their Distance one from another in Rank and File, to order them into a Square Battel of Ground.

As suppose I would order 3000 Men so, that being 7 foot asunder in File, and 3 foot apart in Rank, the Ground whereon they stood should be Square.

Extend the Compasses from 7 foot, the distance in File; to 3 foot, the distance in Rank; then that Extent applied the same way from 3000, the Number of Souldiers, reaches to 1286, whose greatest Square-root is 35-7; that is, 35, the Number of Men to be placed in File.

Then,

If you divide 3000, the whole Number, by 35-7, the Quotient is 84, the Number in Rank, to use and imploy a Square plat of ground to stand in.

As 7, to 3; so is 3000, to 1286, whose Square-root is 35-7. *Then,*

As 35-7, to 1; so is 3000, to 84.

Problem V.

Problem V.

Any Number of Souldiers propounded, to order them into Rank and File, according to the ratio *of any two Numbers given.*

This Question is all one with the former;

For,

As the Number given for the distance in File, is to that for the distance in Rank;

So is the whole Number of Souldiers to a 4th, whose Square-root is the Number of Men in Rank.

Then.

The whole Number divided by the Number in Rank, the Quotient is the Number to be placed in File.

Example.

Suppose 3000 Souldiers were to be ordered in Rank and File: As 5 is to 10, or as 5 is to 9; that is to say, that the Men in Rank, might be in Proportion to them in File, as 9 is to 5.

Say thus;

As the Extent from 5, to 9;

So is 3000, to 5400, whose Square-root is 73 $\frac{1}{3}$, the Number of Men in Rank.

Then,

Then,

As $73\frac{1}{2}$, to 1;
So is 3000, to near 41, the Number in File.

Problem VI.

There are 8100 *Men to be ordered into a Square Body of Men, and to have so many Pikes, as to arm the main Square Body round about, with* 6 *Ranks of Pikes; the Question is, How many Ranks must be in the whole Square Battel? And, How many Pikes and Musquets?*

First, the Square-root of 8100, is 90, the Number of Men in, and Number of Ranks and Files; now in regard that there must be 6 Ranks of Pikes round about the Musquetiers, there will be 12 Ranks less of them, both in Front and Flank, than in the whole Body; therefore substracting of 12 from 90, rest 78, whose Square is 6084, the number of Musquetiers; which taken from 8100, there remains 2061, the number of Pikes.

Problem VII.

Problem VII.

To three Numbers given, to find a fourth in a doubled Proportion.

For as much as like Squares, are in double the Proportion of their anſwerable ſides; therefore you muſt work by their Squares, and Square-root.

But by the Line of Numbers, in this manner.

If a Fathom of Rope, of 6 inches compaſs about, weigh 6 pound, 2 ounces, (or 6-$\frac{125}{1000}$) what ſhall a Fathom of Rope of 12 inches compaſs weigh?

Here Note alwayes, That when the two Numbers of like denomination, which are given, are of Lines, or ſides of Squares, or Diameters of Circles; then the Extent of the Compaſſes upon the Line of Numbers, from one Line to the other, or from one ſide to the other ſide; that Extent turned twice the ſame way from the given Area, or Content, ſhall reach to the other required:

So here, the Extent of the Compaſſes from 6 to 12, being turned two times the ſame way from 6-125, ſhall reach to 24-50, for 24 pound and a half, the weight required.

But if the two terms given of one denomination, are of Squares, or Superficies, or Areas; then the half distance, on the Line of Numbers, between one Area and the other, being turned the same way on the Line, from the given Line or Side, it shall reach to the Side, or Line, required.

For the half-distance, between 24-50, and 6-125, shall reach from 12 to 6; or the contrary, from 6 to 12.

An *Example* whereof, you have in the 4th and 5th Problems of the 12th Chapter: Also, in the 6th and 7th Problems of the 8th Chapter, which treates of Superficial-measure, in measuring of Land.

Note also, That if you have three Lines of Numbers, *viz.* a Great, a Mean, and a Less; after Mr. *Windgates* way; then these Questions are wrought without doubling, or halving, and very neatly and speedily.

As thus;

The Extent on the mean Line, from 24-50, to 6-125, the weight of the two Ropes, shall reach on the great Line, from 12 to 6; or from 6 to 12, the inches in compass about of each Rope.

Problem VIII.

Problem VIII.

To three Numbers given, to find a fourth in a tripled Proportion.

For as much as like Solids, are in a tripled Proportion to their anſwerable ſide; the Cubes of their ſides are proportional one to another; therefore, to work theſe Queſtions by the Line of Numbers, *do thus*;

When the two given terms, of like denomination in the Queſtion, are of Sides, Lines, or Diameters; then the Extent of the Compaſſes, on the Line of Numbers, from one ſide to the other, that is, from the ſide, whoſe Cube or Solidity is alſo given, to the other; the ſame Extent, turned three times from the given Cube, or Solidity, ſhall reach to the inquired Cube, or Solidity.

As for Example.

If a ſide of a Cube, being 12 inches, contain in Solidity 1728 cube inches; How many inches is there in a Cube, whoſe ſide is 8 inches? The Extent from 12 to 8, being turned three times from 1728, ſhall reach to 512, the Solidity required of the Cube, whoſe ſide is 8 inches every way.

Again, on the contrary.

When the two terms of the ſame denomination, are Cubes or Solidities, then divide the ſpace on the Line of Numbers, between the two Solidities, into three equal parts, and lay that Extent the ſame way, as the reaſon of the Queſtion doth require, either increaſing or diminiſhing, from the given Side or Line, and it ſhall reach to the inquired Side, or Line.

Example.

If 1728, be the Cube of 12, the Root or ſide; what ſhall be the Root or ſide of 864, the half of 1728, being half a foot of Timber? The Extent between 1728, and 864, being divided into three parts, and that third part, laid decreaſing from 12, ſhall reach to 9-525, the ſide or root required of half a foot of Timber, though not exactly, yet very near.

Again for another Example.

If an Iron Bullet, of 6 inches Diameter, weigh 30 pound; what ſhall a Bullet of 7 inches Diameter weigh? The Extent from 6 to 7, ſhall reach, being turned three times, to 47-7.

Again,

If a Ship, whoſe Burthen is 300 Tun, be

75 Foot by the Keel; what shall that Ship be, whose Keel is 100 Foot? The Extent between 75 and 100, turned three times from 300, shall reach to 713 Tun Burthen.

Again,

If a Ship of 29 Foot and a half at the beam, be 300 Tun Burthen, what shall a Ship of 713 Tun burthen be? The third part of the distance between 300 and 713, shall reach from 29 $\frac{1}{2}$ to 39-35, its measure at the beam.

Again,

If a Ship of 300 Tun be 13 Foot in hold, what shall a Ship of 713 Tun be in hold? The third part between 300 and 713, shall reach from 13 foot, to 17-35, the Feet in the Hold of a Ship of 713 Tun.

If you have a treble Line, then you may save the dividing, by taking from the little-Line, and measuring on the great-Line, and the contrary, as the nature of the Question doth require.

Lastly,

Know, that by adding of twelve Centers and Points, the Line may be made to speak, as it were, and so made more fit for any mans more particular occasions.

A Brief Touch of the Use of the Logarithms, or Tables, of the Artificial Numbers, Sines, and Tangents.

See more in Gunter's *Works.*

IT may happen, that some may meet with this Book, that had rather use the *Tables of Logarithms*, from whence these Lines are framed, than the Lines on the Rule; or out of curiosity to prove the truth of their work; for whose sakes I have added these following plain Precepts, without *Examples*.

1. *To multiply one Number by another.*

Set the Logarithm of the Multiplicator, and Multiplicand, right under one another, and add them together, and the sum is the Logarithm of the Product.

2. *To divide one Number by another.*

Set down first the Logarithm of the Dividend, and then right under it the Logarithm of the Divisor, and then substract the log. of the Divisor, from the log. of the Dividend,

Dividend, and the remainder is the Log. of the Quotient required.

3. *To find the Square-root of a Number.*

Half the Logarithm of the Number given, is the whole Logarithm of the Square-root of it.

4. *To find the Cubick-root of a Number.*

One third part of the Logarithm of the given Number, is the full Logarithm of the Cubick-root of the given Number, as a third of 1431 3637; the logarithm of 27 is 0-4771212, the Log. of 3, the Cube-root of 27, required.

5. *To work the Rule of Three direct, or three Numbers given, to find a 4th by the Logarithms.*

Set down the Logarithms of the 1, 2, & 3 Numbers, one right over another; then add the logarithms of the second and third together; and from the sum, substract the logarithms of the first, and the remainders is the logarithms of the 4th required.

6. When in common Arithmetick the second term is divided by the first, and the Quotient multiplied by the third.

Then by Logarithms,

Take the Logarithm of the first term, from the Logarithm of the second; and add the difference to the log. of the third, and the sum is the log. of the 4th.

7. When in common Arithmetick the second term is divided by the first, and the third by the Quotient.

Then take the log. of the second, from the log. of the first term; and take the difference out of the log. of the third, and the remainder is the log. of the 4th term required.

8. Between two extream Numbers, to find a mean Proportional.

Add the logarithm[illegible]o extream Numb[illegible] sum is the [illegible]equired.

9. *To*

9. *To work the Rule of Three in the Logarithms of Artificial Numbers, Sines, and Tangents.*

1. *When Radius is the first term.*

Add the Logarithms of the ſecond and third terms together, and Radius, or a unite, in the firſt place, taken from the ſum, there ſhall remain the logarithm of the 4th term required; according to the 5th Precept.

2. *When Radius is in the ſecond place, or term.*

Then the firſt term (and ſecond virtually) taken from the third, cutting off a unite in the firſt place for Radius, is the 4th term.

3. *When Radius is in the third place.*

Then ſubſtract the logarithm of the ſecond term, from the log. of the firſt term, cutting off a unite for Radius, and the remainder is the 4th term.

4. *If Radius be none of the three terms.*

Then add the Logarithms of the ſecond and third terms; and from the ſum, ſubſtract the logarithm of the firſt term, and the remainder is the logarithm of the 4th term.

5. *Or elſe.*

Set down the Arithmetical complement of the firſt term, and the logarithms of the ſecond and third term, and add all together, and the ſum cutting off Radius, is the 4th term.

10. When the Number is not to be found in the Canon of Logarithms of Numbers, Sines, or Tangents; take the next neareſt, or for more exactneſs uſe the part proportional.

11. Though Numbers and Sines, or Numbers and Tangents are uſed together, the work is all one, as with Sines and Tangents, as to the Precept in working.

12. In uſing the Logarithms, great regard is to be had to the Index, or Characteriſtick, to rule in the Number of places; the Characteriſtick being one unite leſs than the Number of places to expreſs that Number; thus the Characteriſtick of 3-5932861, the logarithm of 3920 is 3, being one leſs than the Number of places in 3920, which conſiſts of 4 figures.

CHAP.

CHAP. XV.

The use of the Trianguler Quadrant, in Geometry, and Astronomy.

Use I.

The Radius of a Circle, or Line being given, to find readily, any required Sine, Tangent, or Secant, or Chord, to that Radius.

And first, to do it by the Quadrantal-side.

By th'Trianguler-Quadrant.

FIrst, If your Radius happen to be equal to the greater Scale of (Altitudes or) Sines, issuing from the Center, then the measure of any degree or minuite, from the Center toward the head, shall be the Sine, the measure from the Center-point at 60|0, on the degrees, to any degree and minuit required, shall be the *Sines.*

Tangent. the Tangent to the ſame Radius; and the meaſure from the Tangent to the Center, *Secant.* ſhall be the Secant, to the ſame Radius. And if you have an Index, or a Bead upon your Thred, and ſet the Bead, when the Thred is drawn ſtreight, to the Center at 60|0 on the degrees or Tangents, or to *Chord.* the Sine of 90; then if you lay the Thred to any Number of degrees and minuits, counted from 90, and there keep it; then the extent from the Sine of 90, to the Bead, ſhall be the Chord of the Angle the Thred is laid to, to the Radius of the greater Scale of Sines, iſſuing from the Center.

But if this happen to be too large, then the other leſſer Line of Sines, iſſuing upwards from the Center, being about one third part of the other, hath firſt it ſelf for *Sines.* Sines; ſecondly, the degrees on the looſe-*Tangents.* piece for Tangents, counting from the Center at 60; thirdly, the meaſure from the *Secants.* Tangent, to the Center, for a Secant; *Chords.* fourthly, the Bead and Thred, for a Chord, as before; all at once to one Radius, clearly and diſtinctly, without any interruption, to 75 degrees of the Tangent, or Secant.

But if any other Radius be given, then they will not be had ſo readily altogether, but thus in order one after another, and firſt

first for the Sine, by the *Trianguler Quadrant.*

Take the Radius between your Compasses, set one foot in 90, with the other lay the Thred to the nearest distance, and there keep it; then take the nearest distance from the Sine of any Ark or Angle you would have, and that shall be the Sine of the Ark or Angle required to the given Radius.

1. But by the Sector-side work thus, being near alike, fit the given Radius in the Parallel-sine of 90 & 90; then take out the Parallel-sine, of the Ark or Angle required, and you have your desire. *By the Sector-side.* *Sine.*

2. Also the Sector being so set, if you take out any Parallel Tangent under 45, you have that also to the same Radius. *Tangent.*

3. Also, if you would have any Tangent under 76, as the Sector stands, take out the parallel Tangent thereof, and that shall be the 4th part of the Tangent required to the same Radius, and is to be turned 4 times for that greater Radius. *Tan. to 76.*

4. Also, if you want a Secant under 60 degrees; at the same Radius take out the parallel Secant of the Ark or Angle required; and that shall be the half of the Secant required; for note, the Secant of one degree is more than Radius; and why *Secant.*

I use a half, rather than a 4th part, in time you may well see.

By the Lines on the Edge. By the Artificial Numbers, Sines, and Tangents, this cannot properly be done; only thus you may do by them, counting your given Radius (be it great or little) 10000 parts, you may by them find out readily how many of them parts will go to make the Sine, Tangent, or Secant, to any Number of degrees and minuts. *As thus;*

Sines. Take the distance from the Sine of 90, on the Artificial Sines, to the Sine of any degree and minuit required; and set the same distance, the same way, from 10 on the Line of Numbers, reading it as a Scale of equal parts, and that shall be the Natural Sine of the degree and minuit required.

Or,

If you lay a Square to the Sine given, on the Numbers, it cuts the Natural Sine required.

Example.

Right against the Artificial Sine of 30, on the Line of Numbers, you find 5000, which is the Natural Number thereof.

But, if you measure this distance from 10, in the Line of Lines, it will give the Logarithmal Sine thereof, *viz.* 69897.

Tangent. And the like for the Tangent also, under 45, in the same manner.

But

But for the Secants, and, for the Tangents above 45, you must count thus; *Secants & Tangents beyond 45 degrees.*

Measure, as before from 90, to the Cosine of the Angle, required, for a Secant; and from 45, to the co-Tangent of 45, for a Tangent; This extent laid the contrary way from 1, in the Numbers, shews how many Radiusses, and also how much above Radius, you must have to make up the Natural Tangent, or Secant required, in Numbers.

Example.

The Secant of 50 degrees, and the Tangent of 57 degrees, 16 minuts, being near alike, is 1 Radius 5556; for the Natural Number thereof: and this distance measured on the Line of Lines, gives Radius because above 45, and 1919; more for the Artificial Tangent of 57-16, or the Secant of 50 degrees. *Artificial-logarithms*

This have I hinted, in the first place, that thereby you might see the nature of the Lines, and the making of the Instrument, with its great convenience in the Contrivance of the Work on both sides, and the harmony, and proportion, the Natural way hath to the Artificial; also hereby you may readily prove the truth of your Instrument, being an equilatteral Triangle, whether you use the greater or the lesser Sines; For the

measure

The proof of the truth of the Instrument.

measure from the Center, where the Thred is fastened, to the Center-point of Brass on the moveable-leg, and loose-piece at 60 on the degrees, ought to be equal to each Line of Sines; and also to the Tangent of 45 on the Tangent Line; The measure from the Center to the rectifying-point on the Head, at the meeting of the Lines for the Hour and Azimuth, and the Lines for the Sines and Lines, is equal to the Tangent of twice 30 on that piece.

Again,

The measure from the Center, to the rectifying-point on the end of the Head-leg, shall reach from thence to 30 on the loose-piece; and being turned twice, reaches to 0|60 on the loose-piece: Also, the Radius, or Tangent of 45, turned twice from 0|60 on the loose-piece, shall reach to 75; as by comparing the Natural Numbers together, will most exactly appear; Though perhaps without this hint, it might not have beeen observed by an ordinary eye.

Having been so large, and plain, in this first *Use*, I shall be, I hope, as plain, though far more brief in all the rest; for if you look back to *Chapt.* VI. *Probl.* I. *Sect.* 3. you shall there see the full explaining of Latteral and Parallel, and Nearest-distance, and

and how to take them; the mark for Latteral being thus —; The mark for Parallel thus =; Nearest-distance thus ND, &c.

Use II.

The Sine of any Ark or Angle given, to find the Radius to it.

Take the Sine between your Compasses, and setting one foot of the Compasses in the given Sine; and with the other Point lay the Thred to the nearest-distance, and there keep it; then the nearest-distance from the Sine of 90 to the Thred, shall be the Radius required. *Quadrant.*

Make the given Sine a Parallel Sine, and then take out the Parallel Radius, and you have your desire. *Sector.*

The Artificial Sines and Tangents, are not proper for this work, further then to give the Natural Number thereof, as before; therefore I shall only add the use of them when it is convenient in the fit place.

Use III.

The Radius, or any known Sine being given, to find the quantity of any other unknown Sine, to the same Radius.

Take the Radius, or known Sine given, *Quadr.*

and make it a Parallel in the Sine of 90 for Radius, or in the Sine of the known Angle given, and lay the Thred to ND. Then, take the unknown Sine between your Compasses, and carry one Point along the Line of Sines, till the other foot being turned about, will but just touch the Thred; then the place where the Compasses stayes, shall be the Sine of the unknown Angle required, to that Radius or known Sine.

Sector. Make the given Radius a Parallel Radius, or the given Sine a = Sine, in the answerable Sine thereof: Then, taking the unknown Sine, carry it parallelly along the Line of Sines till it stay in like parts, which parts shall be the Numerator to the Sine required.

Use IV.

The Radius being given, by the Sines alone to find any Tangent or Secant to that Radius.

Quadrant: Take the Radius between your Compasses, and set one Point in the Sine complement of the Tangent required, and lay the Thred to the ND; then the ND from the Sine of the Tangent required, to the Thred, shall be the Tangent required: And the ND from 90, to the Thred, shall be the Secant required.

Make

Make the given Radius a = in the co-Sine of the Tangent required; then the = Sine (of the inquired Ark or Angle) shall be the Tangent required; and = 90 shall be the Secant required to that Radius. *Sector.*

Use V.

Any Tangent or Secant being given, to find the answerable Radius; and then any other proportionable Tangent, or Secant, by Sines only.

First, if it be a Tangent that is given, take it between your Compasses, and setting one foot in the Sine thereof, lay the Thred to N D, then the = Co-sine thereof shall be Radius; *Quadr.*

But, if it be a Secant, take it between your Compasses, and set one foot alwayes in 90, lay the Thred to the N D, then the nearest distance from the Co-sine to the Thred (or the = Co-sine) shall be the Radius required.

Take the given Tangent, make it a = in the Sine thereof; then the = Co-sine thereof shall be Radius. *Sector.*

Or, if it be a Secant given, then

Take the given Secant, make it a = in 90, then the = Co-sine thereof, shall be the Radius required.

Then having gotten Radius, the 4th Use shewes how to come by any Tangent, or Secant, by the Sines only.

Use VI.

To lay down any Chord, to any Radius; less then the Sine of 30 degrees.

Quadr. Take the given Radius, set one Point in the Sine of 30, lay the Thred to the N D (and for your more ready setting it again, note, what degree and minuit the Thred doth stay at, on the degrees) and there keep it. Then the N D from the Sine of half the Angle you would have, shall be the Chord of the Angle required.

Sector. Take the given Radius, and make it alwayes a = in 30, and 30 of Sines; the = Sine of half the Chord, shall be the Chord required.

Use VII.

To lay down any Chord to the Radius of the whole Line of Sines.

Quadr. Take the Radius between your Compasses, and setting one Point in 90 of the Sines, lay

lay the Thred to the ND, obſerving the place, there keep it.

Then taking the = Sine of the Angle required, with it ſet one Point in the Line to which you would draw the Angle, as far from the Center as the Radius is; then draw the Convexity of an Ark, and by that Convexity, and the Center, draw the Line for the Angle required.

Example.

Let A B be a Radius of any length, under or equal to the whole Line of Sines: Take A B between your Compaſſes, and ſetting one Point in 90; lay the Thred to N D, then take out the = Sine of 38, or any other Number you pleaſe, and ſetting one Point in B, the end of the Radius from A the Center, and trace the Ark D C, by the Convexity of which Ark, draw the Line A C for the Angle required.

Take the given Radius A B, make it a = in 90, and 90 of Sines; then take out = 38, and ſetting one foot in B, draw the Ark DC, and draw AC for the Angle required. *Sector.*

Or elſe work thus;

Take AB, the given Radius, (having drawn the Ark BE) and make it a = in

 the

the Co-ſine of half the Angle required; and lay the Thred to N D, (or ſet the Sector).

Then,

Take the = N D, from the right-ſine of the An:le required, and it ſhall be B E, the Chord required to be found.

Note, That the contrary work finds Radius.

Uſe VIII.

To lay off any Angle by the Line of Tangents, or Secants, to prove it.

Sector. Having drawn the Ground-Line, A B, at the Point B, raiſe a Perpendiculer, as the Line B C extended at length, then make A B, the Radius, a = Tangent in 45 and 45; then take out the = Tangent of the Angle required, and lay it from B to C in the Perpendiculer, and draw the Line A C for the Angle required.

Alſo, If you take out the Secant of the Angle, as the Sector ſtands, and lay it twice in the Line A E, it will reach juſt to C, the Point required.

Alſo Note, That if you want an Angle above 45 degrees, as the Sector ſtands, take the ſame from the ſmall Tangent that proceeds to 75, and turn that Extent 4 times from B, and it ſhall give the Point required in the Line B C.

Use IX.

To lay down, or protract any Angle by the Tangent of 45 only.

First, make a Geometrical Square, as Quadr. ABCD, and let A be the Anguler Point; then making AB Radius, make AB a = Co-sine of the Angle you would have, and lay the Thred to the nearest distance, then the ND from the right Sine of the Angle to the Thred, shall be the Tangent required.

Example.

I make AB Radius a = in 50, the co-Sine of 40, then the = Sine of 40 shall be BE.

Again,

If I make AD equal to AB, the = co-Sine of 30, *viz.* 60; and then take out the = Sine of 30, and lay it from D to F, it shall be an Angle of 60 from AB, or 30 from D to F.

But by the Sector this is more easie;

For making AB, or AD, the side of the Square Radius, lay off the = Tangents of any Angle under 45 from B toward C, and the complements thereof above 45 from D, toward C, calling 40, 50; & 30, 60; and 20, 70; 10, 80, &c.

Use X.

To take out readily, any Tangent above 45, by the Tangent to 45 on the Sector-side.

Sector. Take the given Radius, make it a = in the co-Tangent of the Tangent required; then the = Tangent of 45, shall be the Tangent required.

Example.

I would have a Tangent to 80 degrees; take the given Radius, make it a = in 10, the complement of 80; then the = Tangent of 45, shall be the Tangent of 80 required.

But if your Radius be so big, that you cannot enter it, then take the half, or a quarter of your Radius, and then = 45 will be the half, or the quarter of the Tangent required.

Use XI.

How to work Proportions, in Sines alone, by the Natural Sines.

There are 4 Varieties in this Work, that include all Proportions, *viz.*

1. *When*

1. When the Sine of 90 is the first term, then the work is thus;

Lay the Thred to the ſecond term, counted on the degrees from the Head, toward the looſe-piece; and count the third term on the Line of Sines, from the Center downwards; and taking the neareſt diſtance from thence to the Thred, and that diſtance meaſured from the Center downwards, on the Line of Sines, gives the 4th term required.

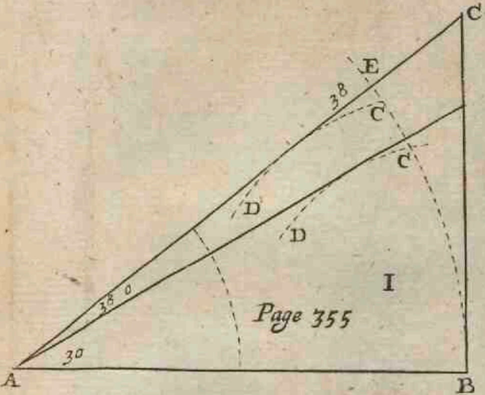
Example.

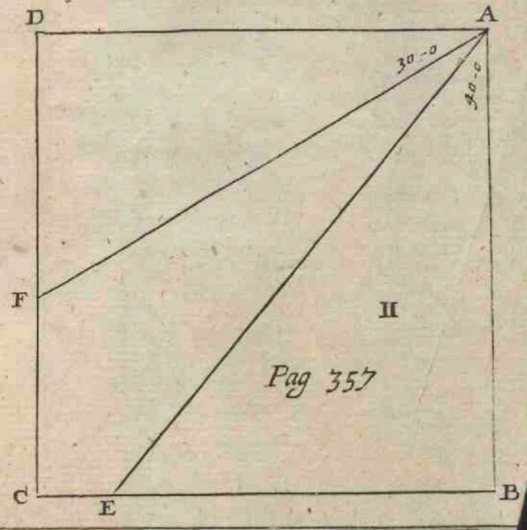
As Sine 90, to Sine 23-30;
So is 30, to 11-31.

Take the Latteral ſecond term, make it a = Sine of 90; then take out the = third term, and meaſuring it from the Center, it gives the 4th term required.

2. When the Sine of 90 is the third term, then work thus;

Take the — Sine, of the ſecond term, from the Center downwards, and make it a = Sine in the firſt term, laying the Thred to ND; then on the degrees, the Thred ſhall give the 4th term required.





 Example.

Example.

As the Sine of 30, to 23-31;
So is the Sine of 90, to Sine of 52-56.

But by the Sector,

Sector. Take the — Sine of the second, make it a = Sine of the first term; then take out = 90, and measure it from the Center, and it shall give the 4th term required. *Example* as before.

3. *When the Radius, or Sine of 90, is in the second place, work thus;*

Quadr. Take — 90 from a lesser Scale, as the uppermost Sine above the Center, or the Line of Right-Ascentions, or the Azimuth-Scale, or the like; and make it a = in the Sine of the first, laying the Thred to N D, then the = third term, taken and measured on the same Scale that 90 was taken from, shall give the 4th term required.

Example.

As — 90, on the Line of Right-Ascention, is to = 30;
So is = 20 to 43-12, measured on the same Line that 90 was taken from.

Or

Or else secondly, work thus;

As — 30, to = 90;
So is — 20, to = 43-12.

By carrying the Compasses till it so stayes, as that the foot turned about, will but just touch the Thred, at the nearest distance.

Or else thus, thirdly;

By transposing the terms, when the third is not greater than the first: *thus*;

As the first, to the third;
So is the second term, to the 4th:

Where the Radius being brought to the third place, it is wrought by the second Rule, as before.

By the Sector.

Take a smaller — Sine of 90, make it a = in 30; then the = Sine of 20, taken and measured on the small Sine, gives 43-12, as before.

Again,

As — 90, to = 30;
So is = 20, to — 43-12.

Again,

As — 30, to = 90;
So is — 20, to = 43-12:

Lastly

Laſtly, by tranſpoſing.

As — 20, to = 30;
So is = Radius, to — 43-12; as before.

4. *When Radius is none of the given terms.*

Quadr. Then when the firſt term is greater than the ſecond and third, *work thus*;

Take the — ſecond term, make it a = in the firſt, laying the Thred to the N D; then the neareſt diſtance, from the third term, to the Thred meaſured from the Center downward, give the 4th Sine required.

Example.

As 20, to 12; ſo is 18, to 10-50.

By the Quadrant.

As — 12, to = 20;
So is = 18, to — 10-50.

When only the ſecond term is greater than the firſt, then tranſpoſe the terms, and work as before: Or elſe uſe a double Radius, which is on this Inſtrument very eaſily done, having ſeveral Radiuſſes.

Or,

Or,

Laſtly, uſe a Parallel entrance, or anſwer rather, as before, which being carefully wrought, will do very well.

By the Sector.

The ſame manner of work, is as before by the Quadrant, and the ſetting the Sector, is all one to the laying the Thred, as will be largely ſeen in all the following Propoſitions, wrought both by the Artificial and Natural Lines, of Numbers, Sines, and Tangents, as followeth.

Uſe XII.

Having the day of the Month, or Suns place given, to find his Declination.

Lay the Thred on the day of the Month in the Kalender, and in the Line of degrees, on the Moving-leg, you have his Declination, either Northward, or Southward, according to the time of the year, counting from 60|0, toward the Head, for North-declination; or toward the End, for South-declination. *By the Quadr. particularly.*

By

By the Artificial Sines and Tangents on the Edge of the Instrument.

Extend the Compaſſes from the Sine of 90, to the Sine of 23 degrees 31 minuts, the Suns greateſt Declination: The ſame Extent applied the ſame way, from the Sine of the Suns place, or the Suns diſtance from the next Equinoctial-point, ſhall cauſe the Moving-point to fall, on the ſine of the Suns declination; This being the general way of working.

Example.

The Extent from the ſine of 90, to the ſine of 23-31, ſhall reach from the ſine of 30, to 11 deg. 31 min. the Suns declination, in ♉ *Taurus* 30 degrees from ♈ *Aries*, the next Equinoctial-point, and from 60 degrees, the Suns diſtance in ♊ *Gemini* 60 degrees, from ♈ 20 deg. 12 min. the Suns declination then. This being the manner of working by theſe Lines, by extending the Compaſſes from the firſt to the ſecond term: I ſhall for the reſt wave this large repetition of extending the Compaſſes, and render it only thus by the words of the Cannon-general in all Books;

As Sine 90, to Sine 23-31;
So is the Sine of 30, to Sine 11-31.

Lay

Lay the Thred to 23-31, on the degrees on the Moveable-piece, counted from the Head toward the End; then count the Suns place from the next Equinoctial-point, on the Line of Sines from the Center downwards, and take the N D from thence to the Thred; then this distance being measured from the Center downwards, shall be the sine of the Suns declination, required for that distance, from the next Equinoctial-point; (*by the 1st Rule abovesaid*). *Quad. Generally.*

By the Sector.

Take — 23-31, from the Sines, make it a = in the sine of 90; then the = sine of the Suns distance from the next Equinoctial-point, shall be the — sine of the Suns declination; *Example* as before (*Rule the 1st*).

Use XIII.

The Suns Declination being given, to find his true place or distance from ♈ or ♎, the two Equinoctial Points.

Lay the Thred to the Declination counted in the degrees from 60|0, and in the Line of the Suus place, is his true place required. *Quadr. Particularly.*

Example.

Example.

When the Suns declination is 12 degrees Northward, the dayes increasing, then the Sun will be 31 deg. and 23 min. from ♈, or 1 deg. 23 min. in ♉, his true place required.

Artificial-S. & T. As Sine of 23-31, the Suns greatest declination, to Sine of 90;
So Sine of 12-00, the Suns present declination, to Sine of Suns distance from ♈ or ♎ 31-23.

Which, by considering the time of the year, gives his true place, by looking on the Months and Line of Suns place on the Quadrantal-side.

Quad. Generally. Take the — Sine of the present declination, make it a = Sine in the greatest declination, laying the Thred to N D; and on the degrees the Thred shall give the Suns distance from ♈, or ♎, required. *Example* as before.

Sector. Make — Sine of the given Suns declination, a = Sine in the Suns greatest declination, then = Sine of 90, measured from the Center, is the = Sine of the Suns distance, from ♈ or ♎, required; or count 30 deg. for one sign, and the Center for the next Equinoctialquinoctial-point, and 90 for the two Tropicks of *Cancer*, and *Capricorn*. ♋. ♑.

Use XIV.

The Suns place, or Day of the Month, and greatest Declination given; to find his Right Ascention from the same Equinoctial.

Lay the Thred to the day of the Month, or place given, and in the Line of the Suns Right Ascention, you have his Rignt Ascention in degrees, or hours and minutes, counting 4 minuts for every degree. *Particular Quadr.*

Example.

On the 9th of *April*, near night, the Sun being then entring ♉, the Suns Right Ascention will be 1 hour 52 min. or 28 degrees of Right Ascention, distant from ♈.

As the Sine of 90, to the Sine complement of the Suns greatest declination (or C.S.) of 23-31, counting backwards from 90, which will be at the Sine of 66-29′.) *Artificial. S. & T.*

So is the Tangent of the Suns distance from the next Equinoctial-point; to the Tangent of the Suns Right Ascention from the same Equinoctial-point.

Take

Quad. Generally.

Take the — co-ſine of the greateſt declination from the Center downwards, being the — ſine of 66-29′. make it a = ſine of 90, laying the Thred to N D; and note what degree and minuit it cuts, for this is fixed to this Proportion: Then take the Tangent of the Suus diſtance from the next Equinoctial-point, from the Center at 60|0, on the degrees toward the End, and lay it on the ſines, from the Center downwards, and note the Point where it ſtayeth, for the N D from thence to the Thred, ſhall be the Tangent of the Suns Right Aſcention required.

Note, That if the Suns diſtance from ♈, or ♎, be above 45 degrees, then the Tangents on the looſe-piece, are to be uſed inſtead of the Tangents on the moveable-leg.

Or, by Sines only thus;

Or, Take — Sine of the preſent *Suns* declination, make it a = in the Sine of the *Suns* greateſt declination, and lay the Thred to N D; then take = Co-ſine of the *Suns* greateſt declination, and make it a = in Co-ſine of the *Suns* preſent declination, and lay the Thred to N D, and in the degrees it cuts the *Suns* Right Aſcention, required.

Make

Make = Co-sine of 23-31, *viz.* the right Sine of 66-29, a = sine of 90, then the = Tangent of the Suns distance from ♈, or ♎, is the = Tangent of the Suns Right Ascention from the same Point of ♈, or ♎; as at 30 from ♈, it is 28 degrees, or 1 hour and 52 minuts from ♈, (neer). *Sector.*

Use XV.

Having the Suns Right Ascention, and greatest Declination, to find the Angle of the Ecliptick and Meridian.

As Sine 90, to Sine 23-31; *Art. Sine.*
So is the Co-sine of the Suns Right Ascention, to the Co-sine of the Angle of the Ecliptick and Meridian.

Lay the Thred to 23-31, counted on the degrees from the Head; then count the Co-sine of the Right Ascention, from the Center downward, or the Sine from 90 upwards, and take the N D from thence to the Thred, and measure it from the Center, and it shall reach to the Co-sine of the Angle required. *Quadr.*

Example.

The Right Ascention being 30 degrees, or 2 hours, the Angle shall be 69-50.

Sector. Make the — right sine of 23-31, a = sine of 90; then the = co-sine of 30, *viz.* = 60, shall make the — sine of 69-50, the Angle of the Ecliptick and Meridian.

Use XVI.

Having the Latitude, and Declination of the Sun or Stars, to find the Suns or Stars Amplitude, at rising or Setting.

Partic. Quadr. Take the Suns declination, from the particular Scale of Sines, and lay it from 6, in the hour or Azimuth-line, and it shall give the Amplitude from South, as it is figured; or from East, or West, counting from 90; observing to turn the Compasses the same way from 90 or 6, as the declination is Northward, or Southwards.

Example.

The Suns declination being 10 degrees Northward, the Suns Amplitude, or Line, is 106-12, from the South, or 16-12 from the East-point.

Art. Sines. As co-sine of the Latitude, to S. 90;
So is S. of the Suns declination, to S. of the Amplitude.

Quadr. generally. Take the — Sine of the Suns declination, make it a = in the co-sine of the Latitude,

titude, and lay the Thred to the nearest distance, and on the degrees the Thred shall shew the true Amplitude required. *Sector.*

Make the — right Sine of the Suns declination, a = in co-sine latitude, then = 90, taken and measured from the Center, gives the Amplitude or Line.

Use XVII.

Having the same Amplitude, and Declination, to find the Latitude.

As S. of the *Suns* Amplitude, to S. the *Suns* Declination; *Art. Sine.*
So is S 90, to Co-sine Latitude.

Take the — sine of the *Suns* declination; set one Point in the Sine of the *Suns* Amplitude, lay the Thred to N D, and on the degrees it sheweth the complement of the Latitude required. *Quadr.*

Example.

The Declination being 20 degrees, and the Amplitude 33-15, the complement of the Latitude will be 38-28 —, counting from the Head, toward the End.

Make the right Sine of the *Suns* Declination, a = sine in the *Suns* Amplitude; then the = sine of 90, shall be the — co-sine of the Latitude required. *Sector*

Use XVIII.

Having the Latitude, and Suns Declination, to find his Altitude at East or West, commonly called the Vertical-Circle; or Azimuth of East or West.

Partic.Q. Take the *Suns* Declination from the particular Line of Sines, set one Point in 90 on the Azimuth-line, and lay the Thred to the N D, and on the degrees it sheweth the Altitude required; counting from 60|0 toward the End.

Artific. S. As S. latitude S. of 90;
So S. of *Suns* declination, to S. *Suns* height, at East or West.

Gen.Quad. Take the — sine of the *Suns* declination, make it a = in the sine of the latitude, and lay the Thred to N D, and on the degrees it shall shew the *Suns* Altitude, at East and West required.

Example.

Declination 10. Latitude 51-32; the Altitude is 12 degrees, and 50 minuts.

Sector. As — S. of the *Suns* Declination, to = S. of Latitude;
So is the = S. of 90, to — S. of Vertical Altitude.

Use XIX.

Having the Latitude, and Suns Declination, to find the time when the Sun will be due East or West.

Having gotten the Altitude by the last Rule, take it from the particular Sine; then lay the Thred to the Suns declination, counted on the degrees; then setting one Point in the Hour-line, so as the other turned about, shall but just touch the Thred, and the Compass-point shall stay at the hour and minuit of time required. *Part. Q.*

As Tangent latitude, to Sine 90; *Artificial*
So is the Tangent of the *Suns* declination, to Co-sine of the hour. *S. & T.*

Or,

As sine 90, to Tangent *Suns* declination;
So is Co-tangent-latitude, to Co-sine of the hour from noon.

Example.

Latitude 51-32, declination 10, the *Sun* will be due East at 6-32, and West at 5-28.

Take the — Tangent of the Latitude (on the loose-piece, counting from 60 toward the moveable-leg; or else from 60|0, on the *Gen. Quad.*

moving-leg, or degrees, according as the Latitude is above or under 45 degrees) and lay it from the Center downwards, and note the Point where it ends. Then take from the ſame Tangent, the Tangent of the *Suns* declination, and ſetting one foot in the Point laſt noted, lay the Thred to N D, then the = ſine of 90, ſhall be the — ſine of the hour from 6.

Or by the Sines only work thus ;

Take the — ſine of the *Suns* declination, make it a = in ſine of the latitude ; lay the Thred to N D, then take N D from the Co-ſine latitude to the Thred ; then ſet one foot in the Co-ſine of the *Suns* declination, lay the Thred to N D, and on the degrees it gives the hour from noon, as it is figured, or the hour from 6, counting from the head, counting 4 minuts for every degree.

Sector. Make the ſmall Tangent of the Latitude, if above 45, taken from the Center, a = ſine of 90 ; then the — Tangent of the *Suns* declination, taken from the ſame ſmall Tangent, and carried Parallelly till it ſtay in like Sines, ſhall be the Sine of the hour from 6.

Or,

Or, as before, by Sines only.

Make — ſine Declination, a = ſine Latitude; then take = Co-ſine Latitude, and make it a = Co-ſine of the *Suns* Declination; then take = 90, and lay it from the Center, it gives the Sine of the hour from 6.

Uſe XX.

Having the Latitude, and Suns Declination, to find the Aſcentional Difference, or the Suns Riſing and Setting, and Oblique Aſcention.

Lay the Thred to the Day of the Month, (or to the *Suns* Declination, or true Place, or to his Right Aſcention; for the Thred being laid to any one of them, is then alſo laid to all the reſt) then in the Azimuth-line, it cuts the Aſcentional difference, if it you count from 90, or the *Suns* Riſing, as you count the morning hours; or his Setting, counting the afternoon hours. *Partic. Q.*

The Oblique Aſcention is found out for the ſix Northern ſigns, or Summer half-year, by ſubſtracting the *Suns* difference of Aſcentions, out of the *Suns* Right Aſcention. But for the other Winter-half year, or ſix Southern ſigns, it is found by adding the *Oblique-Aſcention.*

Suns difference of Aſcentions to his Right Aſcention; this ſum in Winter, and the remainder as above-ſaid in Summer, ſhall be the *Suns* Oblique Aſcention required.

Artificial S.& Tan. As Co-tangent Lat. to ſine 90;
To is the Tangent of the *Suns* declination, to the ſine of the *Suns* Aſcentional difference.

G.Quad. Take the — co-tangent latitude, from the looſe or moveable-piece, as it is above or under 45 degrees, make it a = in ſine 90, lay the Thred to ND, then take the — Tangent of the *Suns* declination from the ſame Tangents, and carry it = till it ſtay in the parts, that the other foot, turned about, will but juſt touch the Thred, which parts ſhall be the Sine of the *Suns* Aſcentional difference required.

Or thus, by Sines only;

Make the — ſine of Declination, a = Co-ſine of the Latitude; lay the Thred to N D, then take the = ſine of Latitude, make it a = in Co-ſine of the declination, and lay the Thred to ND, and on the degrees it ſhall cut the Suns Aſcentional-difference required; which being turned into time, by counting 4 minuts for every degree, and

and added to, or taken from 6, gives the *Suns* Rising in Summer, or Winter.

Make the — Co-tangent Latitude, a = sine of 90; then take — Tangent of the *Suns* declination, and carry it = till it stay in like parts, *viz.* the Sine of the *Suns* Ascentional difference required. *Sector.*

Example otherwise;

As — sine 90, to = Tangent 38-28;
So is = Tangent of 23-31, the *Suns* greatest declination, to the — sine of the *Suns* greatest Ascentional difference, 33 deg. and 12 min.

Use XXI.

The Latitude and Suns Declination given, to find the Suns Meridian Altitude.

When the Latitude and Declination is both alike, *viz.* both North, or both South; then substract the Declination out of the Latitude, or the less from the greater, and the remainder shall be the complement of the *Suns* Meridian Altitude.

But when they be unlike, then add them together, and the sum shall be the complement of the Meridian Altitude: The contrary work serves when the complement of the Latitude and Declination is given, to find the Meridian Altitude.

Lay the Thred to the Declination, counted on the degrees from 60|0, the right way, toward the Head for North, and toward the End for South declination.

Then,

Take the neareſt diſtance, from the Center-prick at 12, in the Hour-line, to the Thred; this diſtance meaſured on the Particular-line of Sines, ſhall ſhew the *Suns* Meridian Altitude required.

Uſe XXI.

The Latitude, and Hour from the midnight Meridian given, to find the Angle of the Suns Poſition, viz. *the Angle between the Hour and Azimuth-lines in the Center of the Sun.*

Artificial- S. & T. As Sine 90, to Co-ſine of the Latitude; So is the Sine of the Hour from Midnight, to the ſine of the Angle of Poſition.

Example.

As Sine 90, to Co-ſine Latitude 38-28; So is the Co-ſine of the Hour from midnight, 120, for which you muſt uſe 60, to 32-34, the Angle of Poſition.

Partic, Q. Take the diſtance from the Hour to the 90 Azimuth on the Hour-line, and meaſure it in the particular ſines, and it ſhall ſhew the

the Angle of Position required. This holds in the Equinoctial.

Take — Co-sine Latitude, make it a = in sine 90; then take out the = Co-sine of the Hour from the Meridian, and it shall be the — sine of the *Suns* Position. *Gen. Quad*

Make — Co-sine Latitude a = sine 90; then = Co-sine of the Hour, shall be — sine of the *Suns* Position. *Sector.*

Note,

The Angle of the *Suns* Position may be varied, and it is generally the Angle made in the Center of the *Sun*, by his Meridian or Hour-circle, being a Circle passing thorow the Pole of the World, and the Center of the Sun; and any other principal Circle, as the Meridian, the Horizon, or any Azimuth, the Anguler-Point being alwayes the Center of the Sun.

Use XXII.

The Suns Declination given, to find the beginning and end of Twi-light, or Day-break.

Lay the Thred to the Declination on the degrees, but counted the contrary way, *viz.* South-declination toward the Head; and North-declination toward the End; then take 18 degrees from the particular Scale of *Partic. Q.*

Sines

Sines for Twi-light, or 13 degrees for Day-break, or clear light; Then carry this distance of 18 for Twi-light, or 13 for Day-break, along the Line of Hours on that side of the Thred next the End; till the other Foot, turned about, will but just touch the Thred, then shall the Point shew the time of Twi-light, or Day-break, required.

Example.

The Suns Declination being 12 degrees North, the Twi-light continues, till 9 hours 24 minuts; or it begins in the morning at 38 minuts after 2; but the Day-break is not till 22 minuts after 3 in the morning, or 38 minuts after 8 at night, and last no longer.

To work this for any other place, where the Latitude doth vary, do thus;

Find the Hour that answers to 18 degrees of Altitude, in as much Declination the contrary way, and that shall be the time of Twi-light; or at 13 degrees for Day-break, according to the Rules in the 26th Use, where the way how is largely handled to the 33d Use, both wayes generally.

Use XXIII.

Use XXIII.

To find for what Latitude your Instrument is particularly made for;

Take the nearest distance from the Center on the Head-leg, to the Azimuth-line on the moveable-leg; this distance measured on the particular Scale of Sines, shall shew the Latitude required; or the Extent from 0 to 90, on the Azimuth-line, shall shew the complement of the Latitude, being measured as before. *Particular Quadr.*

Use XXIV.

Having the Meridian Altitude given, to find the time of Sun Rising or Setting, true Place, or Declination.

Take the Suns Meridian Altitude from the particular Scale, and setting on Point in ☉ on the Azimuth-line; lay the Thred to the N D, and on the Hour-line it sheweth the time of Rising or Setting; and on the degrees, the Declination; and the rest in their respective Lines. *Particular Quadrant.*

Example.

The Meridian Altitude being 50, the Sun riseth at 5, and sets at 7.

Use XXV.

The Latitude and Declination given, to find the Suns height at 6.

Particular Quadrant. Lay the Thred to the Day of the Month, or Declination, then take the N D from the Hour-point of 06, and 6 to the Thred, and that distance measured on the particular Scale of Sines, shall be the Suns Altitude at 6 in Summer time, or his depression under the Horizon in the Winter time.

Artificial-S. & T. As sine of 90, to sine of the Suns Declination;
So is sine Latitude, to sine of the Suns Altitude at 6.

Gen.Quad. Count the Suns declination on the degrees from 90, toward the End, and there lay the Thred; then the least distance from the sine of the Latitude to the Thred, measured from the Center downwards, shall be the sine of the Suns Altitude at 6.

Sector. Make the — sine of the Declination a = sine of 90; then the = sine of the Latitude, shall be the — sine of the Suns height at 6.

Example.

Latitude 51-32, Declination 23-31, the height at 6, is 18 deg. 13 min.

Use XXVI.

Having the Latitude, the Suns Declination and Altitude, to find the Hour of the Day.

Take the *Suns* Altitude, from the particular Scale of Sines, between the Compasses; then lay the Thred to the Day of the Month, or Declination; then carry the Compasses along the Line of Hours, between the Thred and the End, till the other Point (being turned about) will but just touch the Thred, and then the fixed Point shall shew the true hour and min. required, in the Fore, or After-noon; if you be in doubt which it is, then another Observation presently after, will determine it. *Particular Quadrant.*

Example.

May 10th, at 30 degrees of Altitude, the hour will be 32 minuts after 7 in the Morning, or 28 minuts after 4 in the Afternoon.

This Work being somewhat more difficult than the former, I shall part it thus;

1. *First, to find the Hour the Sun being in the Equinoctial.*

Take the — sine of the Suns Altitude, *Gen. Quad.*
make

make it a = Co-ſine of the Latitude; lay the Thred to ND, and on the degrees it ſhall give the Hour from 12, as it is figured, counting 15 degrees for an hour, or from 6, counting from the Head at 90.

Example.

Latitude 51-30, Altitude 20, the hour is 8 & 12′ in the forenoon, or 3-48′ in the afternoon.

The ſame by Artificial Sines & Tangents.

As Co-ſine Latitude, to ſine 90;
So is the ſine of the Suns Altitude, to ſine of the hour from 6.

Sector. Make — S. ☉ Altitude, a = S. in ☉ Latitude; then take out = S. 90, and it ſhall be the — ſine of the hour from 6.

2. The Latitude, Declination, and Altitude given, to find the Hour at any time.

Gen.Quad Firſt by the 25th Uſe, find the Suns Altitude or depreſſion at 6; then in Summer-time, lay this diſtance from the Center downwards; and in Winter-time, lay it upwards from the Center toward the End of the Head-leg; and note that Point for that day, or degree of Declination; for by taking the diſtance from thence to the Suns Altitude, on the General Scale, you have added,

added, or substracted the Altitude at 6, to, or from the present Altitude.

(For by taking the distance from that noted Point, over, or under the Center, to the Suns present Altitude, you have in Summer the *difference* between the Suns present Altitude, and his Altitude at 6. And in Winter you have the *sum* of the present Altitude, and the Altitude at 6.)

This Operation is plainly hinted at, in the 4th Chapter, and 9th and 10th Section, which being understood, take the whole Operation in shorter terms, *thus*;

Count the Suns Declination from 90, toward the end, and thereunto lay the Thred; the nearest distance from the sine of the Latitude to the Thred, is the Suns height, or depression at 6: In Winter use the *sum* of, in Summer the *difference* between, the Suns Altitude at 6, and his present Altitude; with this distance between your Compasses, set one Point in the co-sine of the Latitude; lay the Thred to N D, then take the N D from 90, to the Thred; then set one foot in the Co-sine of the Suns declination, and lay the Thred to N D, and on the degrees it gives the hour required; from 6 counting from 90, or from 12, as it is figured.

Example.

On *April* 20, at 30 deg. 20 min. of Altitude, Latitude 51-32, the hour will be found to be just 2 hours from 6, or just 8.

Again,

On the 10th of *November*, at 8 deg. 25 min. high, it is just 3 hours from 6, or 9 a clock in the forenoon, or 3 afternoon.

Or somewhat differing thus;

Take the — sine of the *sum*, or *difference*, of the Suns present Altitude, and Altitude at 6, and make it a = in the co. sine of the Latitude, and lay the Thred to the nearest distance; then take out the = Secant of the declination beyond 90ᵈ, and make it a = sine of 90; and laying the Thred to the nearest distance, on the degrees it shall shew the hour from 6 required.

By Artificial Sines & Tang. First, by Use 25, find the Suns height at 6, or depression in Winter; then by the former 2d, find the *sum* or *difference* between the Altitude at 6, and the Suns present Altitude; but if you have Tables of Natural Sines and Tangents; then in Winter, add the Natural Sines of the two Altitudes together; and in Summer, substract the lesser out of the greater, and find the Ark of difference more exactly.

Then,

Then,

As the Co-sine of the Latitude, to the Secant of the Declination (counted beyond 90, as much forward as from 90 to the Co-sine of the Suns Declination);
So is the Sine of the *sum*, or *difference*, to the hour from 6, required.

Or else thus;

As the Co-sine of the Latitude, to the Sine of the *sum*, or *difference*;
So is sine of 90, to a 4th.

Then,

As the Co-sine of the Suns declination, to that 4th;
So is sine 90, to the hour from 6.

By the Sector.

Take the — secant of the Suns declination, make it a = in the co-sine of the Latitude; then take out the = sine of the *sum* or *difference*, and turn it twice from the Center latterally, and it shall be the sine of the hour from 6, required.

Example.

April 20, the Suns Declination is 15 degrees; and the Suns Height at 6, then is, 11 deg. 42 min. now the Natural ſine of 11-42, 20278, taken from the Natural ſine of 30 deg. 20 min. 50502, the Suns preſent Altitude, the reſidue is 30224, the ſine of 17 deg. 35 min. and a half.

Then,

The — Secant of 15 made a = ſine of 38-28, and the Sector ſo ſet, the = ſine of 17-35 ½, turned latterally twice from the Center, ſhall reach to 30, the ſine of 2 hours from 6, the hours required.

Uſe XXVII.

Having the Latitude, the Suns Declination, and Altitude, to find the Suns Azimuth.

Particular Quadrant. Take the Declination from the particular Scale of Sines, for the particular Latitude the Inſtrument is made for; Then, count the given Altitude on the degrees from 60 ſo toward the looſe-piece, and ſometimes on the looſe-piece alſo; and thereunto lay the Thred, then carry the Compaſſes, ſo ſet, along the Azimuth-line on the right-ſide of the Thred in Northern-declinations, and on on the left-side in Southern-declinations, till the other foot, turned about, will but just touch the Thred; then the fixed-point shall stay at the Suns true Azimuth required.

Take two or three Examples.

1. First, When the Sun is in the Equinoctial and hath no Declination, then there is nothing to take between your Compasses, but just to lay the Thred to the Suns Altitude, counted from 60|0 on the loose-piece toward the End; then on the Azimuth-line, it cuts the Azimuth from the South required.

Example.

At 00 degrees high, the Azimuth is 90 from South; and at 10 degrees high, it is 77-5; at 20 high, the Azimuth is 62-45; at 30 degrees high, it is 43-30; at 34 degrees high, it is 32 degrees of Azimuth from South; and at 38-28 degrees high, it is just South.

2. Secondly, at 10 degrees of Declination Northward, and 20 degrees of Altitude, take 10 degrees from the particular Scale, and lay the Thred to the Suns present Altitude, as before, and carry the Compasses on the right-side of the Thred on the Azimuth-line, till the other foot, being turned about, will but just touch it; then

ſhall the Point reſt at 80 degrees, 42 min. of Azimuth from the South.

3. But if the Declination be the ſame to the Southwards, and the Altitude alſo the ſame; then carry the Compaſſes on the left-ſide of the Thred, on the Azimuth-line, till the other foot, turned about, will but juſt touch it, and you ſhall find the Point to ſtay at 41 deg. 10 min. the true Azimuth from the South required.

Note, That any thing, as thick as the Rule, laid by the Rule, and the Thred drawn over, it will keep the Thred ſteady, till you get the neareſt diſtance more truly.

Firſt, by the 18th Uſe, find the Suns Altitude in the Vertical Circle, or Circle of Eaſt and Weſt, thus;

General-Quadr. Take the ſine of the Suns Declination, and ſet one foot in the ſine of Latitude, lay the Thred to ND, and in the degrees you ſhall have the Altitude at Eaſt and Weſt required.

Which Vertical Altitude in Summer or Northern Declinations, you muſt ſubſtract out of the Suns preſent Altitude; or take the leſſer from the greater, to find a difference; but in Winter, you muſt add this depreſſion in the Vertical Circle, to the Suns pre-

present Altitude to get a *sum*, which must be done on a Line of Natural Sines, or by the TABLE of Natural Sines, as before, in the Hour, by laying it over or under the Center, and taking from that noted Point to the Suns present Altitude all that day. Then take the distance from the Center to the Tangent of the Suns present Altitude on the loose-piece, which is the Secant of the Suns present Altitude, and lay it from the Center on the Line of Sines, and note the place; then take the distance from 60, on the loose-piece, to the co-tangent of the Latitude (by counting 10, 20, 30, &c. from 60, toward the moveable-leg) between your Compasses; then setting one Point on the Secant of the Suns Altitude last found, and noted on the Line of Sines; and with the other, lay the Thred to the nearest distance, and there keep it, (by noting what degree, day of the month, or hour & minut, or Azimuth it cuts).

Then take the — distance on the Sines, from the sine of the Suns Vertical Altitude, to his present Altitude, for a *difference* in Summer; *Or*,

The distance from a Point made beyond the Center, (equal to the sine of the Suns Vertical depression) to the Suns present Altitude, for a *sum* in Winter.

Then having this — distance of *sum* or *difference*, for Winter or Summer, between your Compasses; carry one Point parallelly on the Line of Sines, till the other, being turned about, shall just touch the Thred at the N D, the place where the Point stayeth, shall be the Azimuth from East or West, as it is figured from the Center; or from North or South, counting from 90.

Which work in brief, may be sufficiently worded thus;

As — co-tangent of the Latitude, to the = secant of the Suns present Altitude, laying the Thred to N D;
So is the — sine of the *sum*, or *difference*, of the Suns present Altitude, & Vertical depression in Winter, or the *difference* between his Vertical and present Altitude in Summer; to the = sine of the Suns Azimuth, at that Altitude and Declination.

Yet again, more short.

As — C.T. Lat. to = Sec. ☉ Alt.
So — S. of *sum* or *difference*, to = S. ☉ Azimuth.

But

But note, That in Latitudes under 45, when the complements of the Latitude are too large, then work thus;

As the — co-ſine of the Suns Altitude, to = Tangent of the Latitude, taken from the degrees on the moveable-leg, laying the Thred to N D, then the — ſine of the *ſum* or *difference*, carried parallelly, ſhall ſtay at the Suns Azimuth required.

If the Tangents are too ſmall, on the Sector-ſide is a larger; and if the Sines are too great, on the Head-leg there is a leſs.

Find the Vertical Altitude by Uſe 18, and the *ſum* or *difference* of the preſent and A
Vertical Altitude by the Table, or Line of Si
Natural Sines, as before ſhewed; then the Ta
Canon or Proportion runs *thus*;

As the Co-ſine of the Suns Altitude, to the Tangent of the Latitude;
So is the ſine of the *ſum* or *difference*, to the ſine of the Azimuth, from Eaſt or Weſt.

Or,

As Co-tangent Latitude, to Secant of the Suns Altitude;
So is the ſine of the *ſum* or *difference*, to the ſine of the Azimuth.

Make

By the Sector. Make the —— Secant of the Suns Altitude, a = Co-tangent of Latitude; then the —— sine of the *sum* or *difference*, shall be half the —— sine of the Azimuth; or being turned twice from the Center, the whole sine.

Or else thus;

Make the — Tangent of the Latitude, a = Co-sine of the Suns Altitude; then the = sine of the *sum* or *difference*, shall be the —— sine of the Azimuth, measured on the Sine, equal to the Radius of the Tangents first taken.

Example.

In Latitude 51-32, Declination North and South 13-15, the Vertical Altitude or Depression being 17-01, and the present Altitude 20; the Azimuth for South-declination will be found to be 31-45, from South, the Depression at East and West being 17-01; and the *sum* of the present Altitude and Depression 39-25.

Again,

For North-declination, or Summer-time, the *difference* between the Vertical and present Altitude, is 2-54; and the Azimuth from South, will be found to be 86 degrees and 15 minuts.

Use XXVIII.

Uſe XXVIII.

To make a Scale, whereby to perform all theſe Propoſitions, by the former Rules, agreeable to the Triangular Quadrant, being added chiefly as a Demonſtration of the Inſtrument, and former Operations.

Firſt, Draw an Equilatteral Triangle, as A B C, at any largeneſs you pleaſe, by drawing firſt the Line A B; then take the Extent A B between your Compaſſes, ſet one Point in A, and with the other draw a touch of an Ark about C, then removing the Point to B, croſs the former Arch in the Point C; then the drawing the Lines A C, and B C, will conſtitute the Equilatteral-Triangle. Then conſider whether one Radius of your Scale ſhall be double, tripple, or quadruple one to the other, and accordingly divide the Line A B into 3, 4, or 5 parts, as here it is into 3 parts, to make one double to the other, (and for Sea-Inſtruments into 5 parts is beſt to make the Scales quadruple one to the other) whereof A D is one; Then make B H equal to B D, and A G and G I equal to DA; Alſo, make D 90, equal to D F, the neareſt diſtance from D to A C; and D 90, equal to D E, the neareſt diſtance from D to C B. Alſo, make

make B E the half of B H, and A F the half of A G. Again, make F 45, equal to F D, and E 45 equal to E D, at neareſt diſtance. Further, if you lay the Radius D F once from I, which is 60, it ſhall reach to 69-54 near C; and being repeated again, it ſhall reach to 75; for 1-0000 the Radius or Tangent of 45, once added to 1-732, the Tangent of 60, makes 2-732, the Tangent of 69-54; but if you add it twice, it makes 3-732, the Tangent of 75 juſt.

Then making D E Radius, deſcribe the Circle 90 E I, and divide it into 180 equal degrees; Alſo, draw the leſſer Circle 90 F to the Radius D F, then a Rule laid to the Center D, and every one of the 180 degrees, ſhall divide the Tangent Lines A C, and B C, into 180 degrees; and if you work right, you will meet with all the former Points, F, G, 45, I, 69-54, 60-45, H, and E, in their true places, as firſt drawn.

Alſo, Perpendiculers let fall from every degree in the Circle 90 E I to the Line D B, ſhall divide the Line of Sines, D 90, to the the greater Radius; and the like Perpendiculers from the degrees in the leſſer Circle, to the Line D A, ſhall divide the leſſer Line of Sines; Alſo, the Extent from the Center D, to the Tangent of any Ark or Angle in the

the Line A C, counting from F, ſhall be the Secant to that Ark or Angle, to the leſſer Radius; and the meaſure from the Center D, to the Tangent of any Ark or Angle in the Line C B (but counted from E) ſhall be the Secant to that Ark or Angle, to the greater Radius.

This little *Inſtrument* thus made, and a Thred faſtened at D, will perform any *Propoſition* by the Rules here inſerted, and is the very making of the *Triangular Quadrant*; or you may put theſe Lines on a Rule as a plain Scale, and uſe them thus:

As for Example, for the Azimuth laſt treated on.

Firſt draw a ſtreight Line, as A B, repreſenting the Line A B in the *Triangular-Quadrant*; then appoint in that Line any Point for a Center, as C; then for this Propoſition of finding the Azimuth, the Sines and Tangents being on a ſtreight Scale, work *thus*;

Firſt, to find the Suns Altitude, or Depreſſion in the Vertical-Circle.

Take the Sine of the Latitude, and lay it from C to 51-30; then take out the Sine of of 13-15, between your Compasses, and setting one Point in the Point 51-30, last made in the Line A B, and strike the touch of the Arch at D, and draw the Line C D; also, on the Line C B, lay down from C the sine of 90 out of the Scale, then the nearest distance from the Point for 90 in C B, to the Line C D shall be the Sine of the Suns Altitude in the Vertical, in Summer or Northern declination, or his depression in Winter, *viz.* 17-01.

Then, as before, on the Line of Sines, find a *sum* for Winter, or a *difference* in Summer, between the Vertical and present Altitude; Now supposing the Altitude 15, the *sum* is 33-30, or the *difference* is 1-58, which you must remember.

Then take the Secant of 15, the Suns present Altitude from the Scale, lay it from C to E; then take out the Co-tangent of the Latitude between your Compasses, set one Point in E, and strike the touch of an Ark, as at F, and draw the Line C F; then take the sine of 33-30, the *sum*, if it be Winter, or 1-58, if it be Summer, between your Compasses, carry one Point in the Line C B, higher or lower, till the other foot, turned about, will but just touch the Line C F; then the measure from thence to the Point C, shall be the Sine of the Azimuth

muth required, *viz.* in Winter 43-50; and in Summer 92-30, from the South, because the present Altitude is less than the Vertical, or East and West.

But when the Co-tangent of the Latitude is too large for a Parallel entrance, then prick off first the Tangent of the Latitude, and take the Co-sine of the Suns Altitude to work in a Parallel way, which will remedy the inconveniences; Thus you see that by drawing three Lines only this work is done; yet not so soon by far, as by the Instrument with the Thred, which represents those Lines more certainly and exactly, after the same way of Operation.

To find the Suns Azimuth in Southern Declinations.

As the Co-sine of the Latitude, to the Sine of the Suns present Altitude; *By Artifi. S. & T.*
So is the Sine of the Latitude to a 4th sine; which 4th sine is to be added to the Suns Amplitude, for that time, on a Line of Natural sines, and the *sum* observed, as a 5th.

Then,

As the Co-sine of the present Altitude, is to t..e sine of the *sum* last found;

So

So is the ſine of 90, to the ſine of the Suns Azimuth, from Eaſt or Weſt, required.

For the Amplitude, work thus ;

As Co-ſine Lat. to S. Suns declination ;
So is S. 90, to the ſine of Amplitude.

Uſe XXIX.

Having the Latitude, Suns Altitude, and Vertical Altitude, to find the Azimuth.

And firſt for Northern-Declinations.

Firſt, find the Vertical Altitude by the former Rule, and find the difference between it and the preſent Altitude, by the Line of Sines: then take this difference from the general Sines between your Compaſſes, and ſetting one foot in the Co-ſine of the Latitude, lay the Thred to the N D, then take the N D from the ſine of the Latitude to the Thred ; having this diſtance, ſet one one foot in the Co-ſine of the Suns Altitude, and lay the Thred to N D, and on the degrees it ſhall ſhew the Suns true Azimuth at that Altitude and Declination required.

Example.

Example.

The Suns Declination being 7, the Virtical Altitude is 8-57; the Suns present Altitude being 30, the difference or residue in Sines will be 20-13, and the Suns Azimuth found thereby will be 60-12′.

The same by Artificial Sines and Tangents, in Summer.

As Co-S. Lat. to S. of residue;
So is S. 90, to a 4th sine.

Then,

As Co-S. ☉ Alt. to the 4th sine;
So is S. 90, to S. of ☉ Azimuth, from East or West.

Secondly, in Southern-Declinations, work thus;

First, find the Suns Amplitude for that Declination, *thus*; Take the — sine of the Declination, make it a = in the Co-sine of the Latitude; lay the Thred to N D, and on the degrees it gives the Suns Amplitude for that Declination, which you must remember.

Then,

Take the — sine of the Suns present Altitude, make it a = in the Co-sine of

Latitude, lay the Thred to the N D, then take the N D from the ſine of the Latitude to the Thred, and as the Compaſſes ſo ſtand, ſet one foot in the ſine of the Suns Amplitude firſt found, and turn the other foot onward toward 90; then take from thence to the Center. Thus have you added the Amplitude, and laſt found diſtance together on Sines, then this added Latteral-diſtance, muſt be made a Parallel in the Suns Co-altitude, and the Thred laid to the neareſt diſtance in the degrees, gives the Azimuth required.

Example.

At 15 degrees of Declination, and 10 degrees of Altitude, the Azimuth will be found to be 49 degrees 46 minuts from the South, and the Amplitude 24-30 in 51-32 of Latitude.

The ſame, work by Artificial Sines and Tangents, in Winter.

As co-S. of Lat. to S. of ⊙ preſent Alt.
So is S. of Lat. to a 4th; which you muſt add to the Suns Amplitude on Natural Sines, and keep it as a *ſum*;

Then,

As co-S. of ⊙ Alt. to S. of the *ſum*;
So is S. 90, to S. Azim. from Eaſt or Weſt.

Use XXX.

Having the Latitude, the Suns Declination, his Meridian and present Altitude given, to find the Hour.

Make the — Secant of the Latitude, a = in the Co-sine of the Suns declination, laying the Thred to ND (and note the place); then take the — distance on the sines, between the Suns Meridian and present Altitude, and lay it from the Center toward 90; then the ND from that Point to the Thred (as before laid) shall be the versed Sine of the Hour, measured on a Line of versed Sines, equal in Radius to the Line of Secants first taken, as the Sines above 90 are. *Gen. Quad.*

Make the — Secant Latitude, a = Sine of Co-declination; then the — distance between the *Suns* Meridian and present Altitude, laid on both Legs from the Center latterally, and the = distance between, measured on versed Sines, equal to the Secants, shall give the hour required; as the great Line of Sines on the Sector are, by turning the Compasses twice, because the Line of Secants is half the Radius of those Sines, as at first was hinted. *By the Sector.*

and reckon the excess above 90, from the Center toward 90, and take from thence to the Center, and add this distance on the Sines to the *Suns* declination toward 90, and take from thence the nearest distance to the Thred, and that shall be the versed Sine of the *Suns* Azimuth from noon.

But when the complements are under 90, then the ND from the noted place to the Thred, shall be the versed Sine of the Azimuth required.

But in Winter, when the *sum* of the complements are above 90, and are counted backwards, from the Center, towards 90; take the — distance from thence to the Sine of the *Suns* declination, the lesser from the greater, and set this distance, or residue from the Center downwards; then the nearest distance from thence to the Thred, shall be the versed Sine of the Azimuth.

But when the Latitude is less than the *Suns* Declination, and the same way; then take the — distance (on the Sines) from the *sum* of the *Suns* Altitude, and Co-latitude, found by Addition, when under 90, and counted from the Center to the declination, and lay that from the Center, as before is shewed.

But if the *sum* of the *Suns* Altitude, and the complement of the Latitude, be above 90;

50; then, having counted forwards from the Center to 90, count the excess from 90 toward the Center, and take the — distance from thence, to the Sine of the *Suns* declination, and lay it from the Center, as before; then the ND from thence to the Thred, shall give the versed Sine of the *Suns* Azimuth on the small Sines beyond the Center.

The very same manner of Operation that serves for the *General-Quadrant*, serves also for the *Sector*, and this way being more troublesome than the rest, I shall say no more to it, but proceed to others. *Sector.*

Use XXXII.

The Suns Altitude, the Latitude, and Declination given, to find the Hour.

Add the Co-latitude, Co-altitude, and Suns distance from the Elevated Pole together, for a *sum*; and find the half *sum*, and the difference between the half *sum* and the Co-altitude. *Artificial S. & Tan.*

Then say;

As Sine 90, to Co-sine Latitude;
So is the Sine of *Suns* distance from the Pole, to a 4th Sine.

and reckon the excess above 90, from the Center toward 90, and take from thence to the Center, and add this distance on the Sines to the *Suns* declination toward 90, and take from thence the nearest distance to the Thred, and that shall be the versed Sine of the *Suns* Azimuth from noon.

But when the complements are under 90, then the N D from the noted place to the Thred, shall be the versed Sine of the Azimuth required.

But in Winter, when the *sum* of the complements are above 90, and are counted backwards, from the Center, towards 90; take the — distance from thence to the Sine of the *Suns* declination, the lesser from the greater, and set this distance, or residue, from the Center downwards; then the nearest distance from thence to the Thred, shall be the versed Sine of the Azimuth.

But when the Latitude is less than the *Suns* Declination, and the same way; then take the — distance (on the Sines) from the *sum* of the *Suns* Altitude, and Co-latitude, found by Addition, when under 90, and counted from the Center to the declination, and lay that from the Center, as before is shewed.

But if the *sum* of the *Suns* Altitude, and the complement of the Latitude, be above 90;

90; then, having counted forwards from the Center to 90, count the excess from 90 toward the Center, and take the — distance from thence, to the Sine of the *Suns* declination, and lay it from the Center, as before; then the ND from thence to the Thred, shall give the versed Sine of the *Suns* Azimuth on the small Sines beyond the Center.

The very same manner of Operation that serves for the *General-Quadrant*, serves also for the *Sector*, and this way being more troublesome than the rest, I shall say no more to it, but proceed to others. *Sector.*

Use XXXII.

The Suns Altitude, the Latitude, and Declination given, to find the Hour.

Add the Co-latitude, Co-altitude, and *Suns* distance from the Elevated Pole together, for a *sum*; and find the half *sum*, and the difference between the half *sum* and the Co-altitude. *Artificial S. & Tan.*

Then say;

As Sine 90, to Co-sine Latitude;
So is the Sine of *Suns* distance from the Pole, to a 4th Sine.

Again,

As the 4th Sine, to the Sine of the half-*ſum*;

So is the Sine of the *difference*, to the verſed Sine of the Hour, if you have them on the Rule; if not, to a 7th Sine, whoſe half-diſtance on the Sines towards 90, gives a Sine, whoſe complement doubled, and turned into time, is the Hour from South required.

Example, at 36 deg. 42 min. Altitude, and 23 deg. 31 min. Declination, Latitude 51-32 North.

53-18 the Co-altitude; 38-28, the Co-latitude; and 66-29, added together, makes 158-15 for a *ſum*; then the half-*ſum* is 79-07, and the difference between 79-07 and 53-18, is 25-49 for a *difference*.

Then,

The Extent from ſine 90, to the Sine of 38-28, will reach the ſame way from the ſine of 66-29, to the ſine of 34 47, for a 4th Sine.

Again,

The Extent from Sine 34-47, to Sine 79-7, ſhall reach the ſame way from the

Sine

Sine of 25-49, the *difference*, to the Sine of 48-34, a 7th Sine, right against which, on the versed Sines, is 60, *viz.* 4 hours from noon.

Or else,

The half-distance, between Sine 48-34, and the Sine of 90, is the Sign of 60 degrees, whose complement, *viz.* 30 doubled is 60 degrees, or 4 hours in time, from noon.

Use XXXIII.

To find the Suns Azimuth, having the same things given, viz. *Co-latitude, Co-altitude, and Suns distance from the Pole.*

Add, as before, the three Nunbers together, and thereby find the *sum*, and half-*sum*, and the difference between the half-*sum*, and the *Suns* distance from the Elevated Pole, *Artificial S. & T.*

Then say,

As the sine of 90, to the Co-sine of the Latitude;
So is the Co-sine of the Altitude, to a 4th sine.

Again,

Again,

As the Sine of the 4th, to the Sine of the half-*sum*;
So is the Sine of the *difference*, to the versed Sine of the *Suns* Azimuth, from South, (or to a 7th sine, whose half-distance, toward 90, gives a sine, whose complement doubled, is the Azimuth from South).

Example, Latitude 51-32, *Altitude* 41-53, *Declination North* 13.

The 3 Numbers *viz.* 38-28, 49-7, and 77-0, added together, makes 163-35; whose half is 81-47 ½, and the difference between the half-*sum*, and the *Suns* distance from the Pole, is 4-47 ½.

Then,

As sine 90, to sine 38-28;
So is sine 48-7, to sine 27-36.

Then,

As sine 27-36, to sine 81-47;
So sine 4-47 ½, to (V.S. of 130, the Azimuth from the North:) the sine of 10-15, a 7th sine, whose half-distance toward 90, is 25, whose complement 65 doubled, is 130, the Azimuth from the North, whose complement to 180, *viz.* 50, is the Azimuth from South.

Having

Having the same complements, to find the Hour, and Azimuth, by the General-Quadrant and Sector; and first for the Azimuth.

First, of the complements of the Latitude, and *Suns* present Altitude, by substraction find the difference. *By the General-Quadrant & Sector.*

Secondly, Count this difference on the Line of Natural Sines from 90, toward the Center, as the smaller figures are counted.

Thirdly, Take the distance on the Sines, from thence to the — sine of the *Suns* declination. But note, That when the Latitude and Declination differ, *viz.* one North, and the other South, as it is with us in Winter; you must count the *Suns* Declination beyond the Center, and call it the *Suns* distance from the Elevated Pole, and take from thence.

Fourthly, Make this — distance, a = in the Co-sine of the Latitude, laying the Thred to N D, or keeping the Sector at that opening. *Then*,

Fiftly, Take out the = sine of 90, *And*

Sixtly, Make it a = sine in the *Suns* Co-altitude, setting the Sector, or laying the Thred to the (nearest distance) N D.

Seventhly, Take out the = sine of 90.

And,

And,

Eightly, Meaſure it from the ſine of 90, towards (and if need be beyond) the Center, and it ſhall reach to the verſed ſine of the *Suns* Azimuth from North or South, when you count from 90; or from Eaſt or Weſt, if you count from the Center, on a Line of Sines, or middle of the Line of verſed Sines.

Note, That if the general Sines are too big, you have a leſs adjoyning, whereon to begin and end the Work; as ſometime the Hour-Scale, and ſometimes the Line of Right Aſcentions.

Example.

In the Latitude of 51-32, the *Suns* Declination 18-30, the *Suns* Altitude 48-12, you ſhall find the *Suns* Azimuth to be 130 from the North, or 50 from South.

Secondly, for the Hour, by the ſame data, *or things given.*

1. Firſt of the complement of the Latitude, and the *Suns* diſtance from the Elevated Pole, find the *difference* by Subſtraction.

2. Count it on the Line of Sines from 90 toward

toward the Center, (or beginning of the Sines).

3. Take the — diſtance from thence, to to the ſine of the *Suns* preſent Altitude.

4. Make this — diſtance, a = in the Co-ſine of the Latitude, ſetting the Sector, or laying the Thred to the N D, and there keep it.

5. Then take out the = ſine of 90;

And,

6. Make that a = in the Co-ſine of the *Suns* Declination, laying the Thred to ND.

7. Then take out the = ſine of 90 again,

And,

8. Meaſure it from 90, toward the Center, and it ſhall ſhew the verſed ſine of the Hour from Mid-night, or the contrary from noon; or from 6, if you count from the Center of the Sines, or the middle on verſed Sines.

Example.

Latitude 51-32, Declination North 20-14, Altitude 50-55, you ſhall find the Hour to be 150 from North, *viz.* 10 in the fore-noon, or 30 degrees ſhort of South.

Use XXXIV.

Having the Latitude, Suns Altitude, and distance from the Elevated Pole, to find the Hour, by the Line of versed Sines, on the Sector.

First, By Addition, find the *sum* of, and by Substraction, the *difference* between the complement of the Latitude, and the *Suns* distance from the Elevated Pole.

Secondly, Count this *sum* and *difference* from the Center, or the versed Sines on the Sector, (or the beginning of the Azimuth-Line, if you use that, or any other, which is not drawn from a Center) and with Compasses take the — distance between them.

Thirdly, Make this — distance, a = versed Sine of 180.

Fourthly, Take the — distance between the versed sine of the *sum*, and the complement of the *Suns* Altitude, and carry parallelly till it stay in like versed Sines, which shall be the versed Sine of the Hour from the North Meridian, or mid-night.

Or,

If you take the — distance from the *difference* to the Co-altitude, and carry that = till it stay in like sines, it shall be the hour from noon; counting the Center 12

at

at noon, the middle at 90, the two sixes and 180 at the end, for 12 at night.

Use XXXV.

Having the Latitude, the Suns Altitude, and Distance from the Elevated Pole, to find his true Azimuth from South or North, by Natural versed Sines.

First, Of the Co-altitude, and Co-latitude, find the *sum* and *difference*, by Addition and Substraction.

Secondly, Count the *sum* and *difference* from the Center, and take the — distance between them with Compasses on the versed Sines.

Thirdly, Make it a = versed sine of 180, and so keep the Sector.

Fourthly, Take the — distance, between the *sum*, and the *Suns* distance from the Pole, (counting the Center the Elevated Pole, and 90 the Equinoctial) and carry it = till it stay in like parts, which shall be the Azimuth from South.

Or,

If you take the — distance from the *difference*, to the *Suns* distance from the Pole, and carry it as before, it shall stay at the versed sine of the Azimuth, from the North part of the Horizon.

These

These five general wayes of finding the Hour and Azimuth, are not all needful to be learned by every one, but to delight the ingenious, and to hold forth the usefulness of the Instrument, and to supply defects that at some times may happen by Excursions, and as a four-fold Testimony, to shew the harmony in several wayes of Operation; the first particular way, and this last by versed Sines, being most easie and comprehensive of any other.

Use XXXVI.

To work the last without the Line of versed Sines.

Note, That if for want of room, the versed Sines be set but on one Leg, then it is to be laid at the nearest distance instead of like parts, after the manner of using the Thred on the *General Quadrant*.

Also, If you have it not at all, then the Azimuth-line for the particular Latitude; and if that be too large, the little Line of Sines, beyond the Center, will supply this defect very well thus;

First, Turn the Radius, or whole length of that Line of Sines, two times from the Center downwards, (which in Sea-Instruments, will most conveniently stay at 30 on the

the large Line of Sines, or general Scale, as was hinted in the 28th Use, being just 4 times as much one as the other). For a Point representing 180 of versed Sines, to set the Compasses in, when you lay the Thred to N D, and to take any versed sine above 90 degrees; this being premised, the Operation is thus:

Example.

Lat. 51-32, ☉ Dist. from Pole 80, *Gen. Quad.*
☉ Alt. 25, to find the Hour;
The *sum* of Co-lat. 38-28, and 80, is 118-28;
And the *difference* is 41-32.

Now in regard the *sum* is above 90, count the Center 90, 10 on the smaller Sines 100, and 20 on the same Sines, 110, and 28 deg. 28 min; 118 deg. 28 min. turn this distance the other way from the Center downwards, and note that place, for the Point, representing the *sum* on the versed sines.

Then,

The — Extent between this *sum*, and the *difference* 41-32, as the smaller figures reckon it, being taken between your Compasses, set one Point in 180, the Point first found, and lay the Thred to N D, and there keep it (or observe where it cuts), then taking the — distance between the versed sine of the *difference*, counted as the small figures are reckoned, and the sine of the *Suns* Altitude 25, as the greater figures are reckoned from the Center toward the End; and carrying this Extent parallelly along the greater Line of sines, till the other Point will but just touch the Thred at ND; Then, I say, the measure from that Point to the Center, measured on the small sines, as versed sines, shall be the versed sine of the Hour required, *viz.* 62 from South, or 7 hours 52 minuts from mid-night.

This Rule, or Use, is longer far in wording, than the Operation need be in working; for if you shall approve of this way, the adding of two brass Center-pins will shew you the two Points most used very readily, and the Thred is sooner laid, than the Legs can be opened or shut, and the Instrument keeps its Trianguler form as it is in, during the time of Observation.

Use XXXVII.

Having the Latitude, Suns Declination, and Hour, to find his Altitude.

This Problem being not of such use as the contrary, *viz.* having the Altitude, to find

find the hour, it ſhall ſuffice to hint only *two wayes*, the moſt convenient. And,

Firſt by the Particular Quadrant.

Lay the Thred to the Day, or Declination, then the N D from the Hour to the Thred, meaſured in the particular Scale of Altitudes, ſhall ſhew the *Suns* Altitude required.

Secondly, by the verſed Sines.

1. Firſt, of the Co-latitude, and *Suns* diſtance from the Pole, find the *ſum* and *difference*.

2. Take the — diſtance between them, and make it a = verſed ſine of 180 by ſetting the Sector, or laying the Thred to N D.

3. Then take the = verſed ſine of the Hour, and lay it latterally from the *ſum*, and it ſhall give the complement of the Altitude required

This work is the ſame, both by the Sector, or General Quadrant, as is ſhewed in Uſe the 36th, and is nothing elſe but a backward working; but the Altitude at any Azimuth, is not ſo to be done.

To do the same by the Natural-Sines.

First, having the Latitude, and the *Suns* Declination, find the *Suns* Altitude, or Depression at 6; and note the Point, either below, or above, or in the Center, as is largely shewed in Use the 26th, where the Altitude is given, to find the hour in any Latitude.

Then,

Lay the Thred to the Hour, counted in the degrees either from 12, or 6;

Then,

Take the N D from the Co-sine of the *Suns* Declination, and make it a = in the sine of 90, laying the Thred to the N D; then the N D from the sine complement of the Latitude to the Thred, shall reach from the noted Point, for the *Suns* Altitude or Depression at 6, to the *Suns* Altitude required.

Example.

Latitude 51-32, Declination 23-31, at 8 or 4, *viz.* 2 hours from 6 Southwards the Altitude will be found to be 36-42′.

1. *For the Altitude at 6, at any time of the year, say;*

As the ſine of 90; to ſine of the Latitude;
So is the ſine of the *Suns* Declination, to the ſine of the *Suns* Altitude at 6.

By Artificial Sines & Tang.

2. *For the Suns Altitude, at any hour or quarter, in* Aries *or* Libra, (*the Equinoctial*).

As ſine 90, to Co-ſine Latitude;
So is the ſine of the *Suns* diſtance from 6 in degrees, to ſine of the *Suns* Altitude.

3. *For the Suns Altitude at all other hours, or times of the year.*

As ſine 90, to Co-tangent Latitude;
So is ſine of the *Suns* diſtance from 6, to the Tangent of a 4th Ark, in the Tangents.

Which 4th Ark being taken from the *Suns* diſtance from the Elevated Pole, then the reſidue is the 5th Ark; but for hours before and after 6, add the 4th Ark, and the *Suns* diſtance from the Pole together, to make a 5th Ark.

Then say,

As the Co-sine of the 4th Ark, to the sine of the Latitude ;
So is the Co-sine of the residue (or *sum*) being the 5th Ark, to the sine of the *Suns* Altitude at that hour.

Use XXXVIII.

The Latitude, Suns Azimuth and Declination given, to find the Altitude, or height thereof.

First, to find the Suns Altitude at all Azimuths in the Equinoctial.

By Artifi. S. & T. As sine 90, to Co-tangent Latitude ;
So is the Co-sine Azimuth from South, to the Tangent of the *Suns* Altitude in *Aries.*

Or,

As sine 90, to the Co-sine of the Azimuth from South ;
So is Co-tangent Lat. to the Tangent of the *Suns* Altitude, at that Azimuth in the Equinoctial, which you must gather into a Table for every single degree.

Then,

Then,

As the ſine Lat. to the ſine of the *Suns* Declination;
So is the Co-ſine of the *Suns* Altitude in Equinoctial, to the ſine of a 4th Ark.

Then,

When the Latitude and Declination are alike, as both North, or South; then add the 4th Ark and the Altitude (in the Equator) together, and the *ſum* is the Altitude required.

But in Winter-time, when the Latitude and Declination is unlike, take the 4th Ark out of the reciprocal Altitude in the Equator, and the reſidue is the *Suns* Altitude required.

Alſo, in all Azimuths from Eaſt and Weſt Northwards, in Summer-time alſo, you muſt uſe Subſtraction alſo, and not Addition; as the Rule before-going ſuggeſts.

By the Particular Quadrant, work thus;

Take the *Sun* or *Stars* Declination from the particular Scale, and ſetting one Point in the *Suns* Azimuth, on the Azimuth Line, and with the other lay the Thred to the N D, the right way, and on the degrees the Thred cuts the Altitude required.

By the General Quadrant.

As the — Co-tangent Latitude, taken from the Moving-leg, or Loose-piece, to = sine of 90, laying the Thred to ND;

So is the = Co-sine of the *Suns* Azimuth from South, to the — Tangent of the *Suns* Altitude in the Equator, at that reciprocal Azimuth.

Which being remembred, or gathered into a Table together, then say;

As the — Co-sine of the *Suns* Altitude in the Equator, to the = sine of the Latitude, laying the Thred to the ND;

So is the = sine of the *Suns* Declination, to the — sine of the 4th Ark.

Which 4th Ark is to be added, or substracted, as immediately before is directed, and the *sum* or residue, shall be the true Altitude required.

Example.

At 60 degrees of Azimuth from South, the Equinoctial Altitude will be found to be 21-40, for *London* latitude of 51-32; and the 4th Ark in ♋ or ♑ is 28-16.

Then,

Then,

21-40, the *Suns* Altitude at 60 in ♈, and 28-16, the reciprocal 4th Ark in ♋ added, makes 49-56, the *Suns* Altitude at 60 degrees from the South in ♋.

The same way of working serves for the *Sector*, as is used for the *General Quadrant*, only observing to set the Sector, instead of laying the Thred to the nearest distance, as the Ingenious will soon perceive. *By the Sector.*

Use XXXIX.

Having the Latitude, Declination, Azimuth, and Altitude, to find the Hour.

As the — Co-sine of the *Suns* Altitude, to = Co-sine of the *Suns* Declination; *General Quadr. Or, Sector.*

So is the = sine of the *Suns* Azimuth, to — sine of the Hour.

Or else thus;

First find the Altitude, at that Azimuth; and then at that Altitude, and Declination, the Hour. *Particular Quadrant.*

As — Co-sine of Declination, to = sine of the Azimuth;

So is the — Co-sine Altitude, to = sine of the Hour.

By the Artificial-Sines and Tangents. As Co-sine Declination, to the Sine of the Azimuth;
So is Co-sine Altitude, to Sine of the Hour.

Use XL.

Having the Latitude, Declination, Hour, and Altitude, to find the Azimuth.

General-Quadr; As — Co-sine of Declination, to = Co-sine of the *Suns* Altitude;
So is = sine of the Hour, to — sine of the Azimuth.

Particular Quadrant. First, find the Altitude at that Hour, and then the Azimuth at that Altitude, as before.

Artificial-S. & T. As Co-sine of Altitude, to sine of the Hour;
So is Co-sine Declination, to sine of the Azimuth from South, or North, as the Hour is counted; *that is to say*, from South, if the Hour is between 6 at morning, and 6 at night; and from the North if the contrary; *that is to say*, between 6 at night, and 6 next morning, or next to midnight.

Use XLI.

Having the Latitude, and the Suns Declination, to find the Suns Azimuth at 6.

As the sine of 90, to the Co-sine of the Latitude; *Artificial-S. & T.*
So is the Tangent of the *Suns* Declination, to Co-tangent of the *Suns* Azimuth from the North, at the hour of 6.

First find the *Suns* height at 6, and then the *Suns* Azimuth at that Altitude. *Partic. Q.*

Make the — Tangent of the Declination, a = sine of 90, laying the Thred to ND, then the = Co-sine Latitude shall be the — Co-tangent of the *Suns* Azimuth from the North at 6. *Gen. Quad*

Use XLII.

To find the Amplitude, Azimuth, Rising, Setting, and Southing of the fixed Stars, having the Latitude, Altitude, and Declination, or time of the year given.

First for the Amplitude, Take the Stars Declination, out of the particular Scale of Altitudes, and measure it from 90 in the Azimuth- *Particular Quadrant.*

Azimuth-line; and count the same way, and the other Point shall shew the Stars Amplitude required.

Example.

The Declination of the *Bulls Eye*, being 15-48; if you take 15-48 from the particular Scale, and lay it from 90 in the Azimuth-line, it will reach to 26 degrees, counting from 90 towards either end, the same as for the *Sun* in Use 16. But in other Latitudes, work as you do for the *Sun* by the Rules in the 16th Use abovesaid.

For a Stars Azimuth.

The work here is the same as for the *Sun*, *thus*; Take the Stars Declination from the particular Scale of Altitudes, or Sines, between your Compasses, and lay the Thred to the Stars Altitude, counted from 60|0 toward the Loose-piece; then carry the Compasses, or the right-side of the Thred, for Northern-stars; and on the left-side for Southern-stars, along the Azimuth-line, till the other foot, being turned about, will but just touch the Thred; then the fixed Point on the Azimuth-line shall shew the Stars Azimuth, from the South, required.

Example.

Example.

The *Bulls Eye* being 30 degrees high, ſhall have 77 degrees and 10 minuts of Azimuth from the South:

If you be in other Latitudes, uſe the general wayes, as for the *Sun* in all reſpects, having the ſame Declination that the Star hath North or South.

To find the Stars Riſing, or Setting.

Count the Stars Declination on the degrees, as you count the *Suns*, North, or South, and there lay the Thred; and in the Line of Hours is the Stars Riſing, or Setting, when the Stars Right Aſcention and Declination are equal.

But at other times, you muſt reckon thus;

Firſt, find the *Suns* Right Aſcention, by Uſe 14, and ſet down the complement thereof to 12 Hours, and the Stars Right-Aſcention, and the hour of Riſing the Thred cuts, and add them into one ſum, and the ſum, if under 12, is the time of his Riſing in common hours; or if you add the hour of Setting that the Thred ſheweth, it ſhall give his ſetting.

Example.

Example.

If you lay the Thred to 15-48, the Declination of the *Bulls Eye*, in the Hour-line it cuts 4 hours 36 min. for Rising; or 7-24, for his Setting; then if you work, for *April* the 23d, the *Suns* Right Ascention, then is 2-44, and the complement thereof to 12, is 9-16; and the Stars Right-Ascention is 4 hours and 16 minutes; and the Hour cut, is 4-36 for Rising; and the three Numbers, *viz.* 9-16, the complement of the *Suns* Right Ascention, and 4-16, the Stars Right Ascention, and 4-36, the Hour of Rising the Thred cuts, being added, makes 18-8; from which, taking 12, rest 6-8, the time that the *Bulls Eye* Riseth on *April* 23; and if you add 7-24, the time of Setting that the thred cuts, there comes forth 8-56, *viz.* one hour and 32 min. after the *Sun*.

To find the time of a Stars coming to South.

Substract the Right Ascention of the *Sun*, from the Right Ascention of the Star, increased by 24, when you cannot do without, and the remainder, if less than 12, is the time between 12 at noon, and 12 at night;

night; but if the remainder be more than 12, it is the time between mid-night, and mid-day, following.

Example.

The *Lyons-Heart*, whose Right Ascention is 9-50, will come to the South on *March* 10, at 9-48, the *Suns* Right Ascention, being then only 2 minuts.

By the Line of 24 hours (say, or) work thus;

Extend the Compasses from the *Suns* Right Ascention, to the *Stars* Right Ascention; that distance laid the same way from 12 at the middle, or at the beginning, shall reach to the time of the *Stars* coming to South.

To find the time of the Stars continuance above the Horizon.

First find what the *Suns* semi-diurnal Ark is, having the same declination, and that doubled, is the whole time of continuance; Or, if you shall add and substract it to, or from the time of the *Stars* coming to South, you shall find the time of Setting or Rising.

Or else,

By laying the Thred to the *Stars* Declination, it sheweth the Ascentional difference in this Latitude, which added in those Stars that have North declination, or substracted in Southern to 6 hours, gives the semi-diurnal Ark of the Star above the Horizon.

Example.

The *Bulls Eye*'s Ascentional-difference, is one hour and 24 minuts; which added to 6 hours, because of Northern declination, makes 7-24, for the semi-diurnal-Ark, or $14^h\ 48'$, for the whole time of being above the Horizon.

Note, That to work this for other Latitudes, the *Suns* Ascentional-difference is to be found for that Latitude you are in, and the Operation is general for all places.

To find a Meridian Line by the Sun.

On any flat *Horizontal-Plain*, set up a streight Wyre in the Center of a Circle; or hold up a Thred or Plummet, till the shadow of the Thred cut the Center, and any where in the Circumference, which two Points you must note; then immediately take the *Suns* Altitude, and find the *Suns* Azimuth, and count so many degrees in the Circle the right way, as the *Suns* Azimuth comes to, from the Points of the shadow marked in the Circumference, and draw that Line for a true Meridian-line. This Work is best done before 10 in the

morning,

morning, and after two afternoon; or in the night, by two Plumb-lines, ſet in a right-Line with the North-Star, at a right ſcituation.

Uſe XLIII.

To find the Hour of the Night by the Fixed Stars.

Firſt, find the Stars Altitude, by looking along the Fixed or Moveable-leg, to the middle of the Star, letting the Thred, with a weighty Plummet, play evenly by the degrees, between your Thumb and Fore-finger, to the end you may command the Thred, and know whether it playeth well or no by feeling.

Then,

Take the Altitude found, from the particular Scale of Sines, and laying the Thred over the Stars declination, which for readineſs ſake is marked with 1,2,3,4,5,6,7,8, 9,10,11,12, according to the Figures ſet to the 12 Names of the 12 Stars on the Rule; and then carrying the Compaſſes as you do in finding the hour by the *Sun*, you ſhall find how much the Star wants, or is paſt the Meridian, which is called the Stars-Hour; And note, That if the Star be paſt the South, it is an afternoon hour; if not

come

come to the South a morning hour, which you must remember.

Also, knowing the *Suns* Right Ascention, set one Point of the Compasses in the *Suns* Right Ascention, (counted in the Line of twice 12, or 24 hours, on the outward-leg of the fixed-piece, next to the particular Scale of Sines) and open the other to the Stars Right Ascention, noting which way you turn the Compasses; for the same Extent, applied the same way, from the Stars hour last found, shall shew the true hour of the night required.

Example.

Suppose on the 10th of *January*, I should observe the Altitude of the *Bulls Eye* to be 20 degrees; if you take 20 degrees, the Altitude, from the particular Scale, and lay the Thred on 15-48, the Stars declination Northward, and measure from the Hour-scale the nearest distance to the Thred, you shall find the Compass-point to stay at 6-49 on the East-side of the Meridian; (suppose)

Also, The *Suns* Right Ascention, the same day, is 8 hours and 12 minnts.

Then,

The Extent from 8 hours 12 minuts (on the Line of twice 12 hours) the *Suns* Right Ascention, to 4-16, the Stars Right Ascen-

tion,

tion, ſhall reach the ſame way from 6-49, the Stars hour, to 2-53, the true hour.

Uſe XLIV.

To find the Hour of the Night by the Moon.

First, by an *Almanack*, or *Ephemerides*, find the Moons Age, and true Place for the preſent time; then, by laying the Thred on the Moons place, you may have her Right Aſcention, and alſo the Suns Right Aſcention; and by the Moons Altitude, taken from the particular Scale, and the Thred laid over the Moons place, you find what the Moon wants, or is paſt coming to South, which is called the Moons hour.

Then, by the Line of 24 Hours, ſay;

As the Suns Right Aſcention, is to the Moons Right Aſcention;

So is the Moons hour laſt found, to the true hour.

Example.

Suppoſe that on the 8th of *January*, about 40 min. after 3, there is a New Moon; then note, That the Suns true place, is the Moons true place; and conſequently, their Right Aſcentions; and the Moons Hour and Altitude is the ſame with the Suns.

 Therefore,

Therefore;

As 8 hours 04 min. the Suns Right Aſcention, is to 8-04, the Moons Right Aſcention;

So is the Moons hour at any Altitude, to the Suns true hour.

Again,

Suppoſe that on the 1ſt Quarter-day, the Moon being gone 90 degrees from the Sun, to find her place;

Then do thus;

Set one Point in the Moons place the Change-day, and open the other to the beginning or the end of the Line of 24 hours,

Then,

The ſame Extent applied the contrary way from 6 hours, or 7 dayes and a half, the Moons Age, ſhall give 28 deg. 58 min. ♈; to which you muſt add 7 degrees and 30 minuts (the Suns place) between, and the *ſum* ſhall be the Moons true place required, *viz.* 6-28 degrees in ♉.

Example.

If the Moon Change on the 8th day, the Firſt Quarter being 7 dayes and a half after, will be on the 15th day later at night; then the difference between the Sun and Moons Right Aſcention, will be found to be near 6 hours; for the Suns Right Aſcention,

January

January 15, is 8-32; and the Moons Right Aſcention, the ſame day, being about 8 degrees and a half in ♉, is 2 hours and 28 minuts; if you take the diſtance between them, on the 24 hours, it is near 6 hours; which is the difference of time between the Moon and the Suns hour.

Again,

For the Full Moon; on the 22 day, near 4 hours after noon, the Moons Age being 14 dayes $\frac{3}{4}$; if you add 12 hours, or 6 ſigns, to the Moons place at the Change, you ſhall find ♋ 29-0; to which if you add 14-45, the dayes between the New and Full, you ſhall find ♌ 13 deg. 45 min. for the Moons place; the Suns Right Aſcention the 22 day is 9 hours, and the Moons the ſame day at 1 afternoon, is 9 hours alſo (or rather 12 difference) ſo that the Suns hour and the Moons is equal; only one is North, and the other South.

Again,

For the Laſt Quarter 22 $\frac{1}{4}$ dayes, or 18 hours added, and 22 degrees alſo together, makes ♏ 22 deg. 11 min. for the Moons place, by help of which, to find the Moons hour by her Altitude above the Horizon found by obſervation.

 Or,

Or,

Without regarding the Sun or Moons Right Afcention, having her true Age, and Hour,

Say thus;

As 12 on the Line of 24 hours, is to the
Moons Age in the Line of her Age;
So is the Moons hour, to the true hour.

For,

The Extent from 12 in the middle, to the Moons Age under or over the middle, fhall reach the fame way, on the fame Line, from the Moons hour, to the true hour.

The like work ferves to find the hour of the night by any Planets, as *Saturn*, *Mars*, or *Jupiter*, which are feen to fhine very brave and bright in Winter evenings; and having learned their Place by their diftance from the fixed Stars, or by the *Ephemerides*, then their Altitude and Place will find their hour from the Meridian, and the compa-ring their Right Afcentions with the Suns, gives the true hour, as before, in the Fixed-Stars.

Use XLV.

To find the Moons Place and Declination, without the Ephemerides, somewhat near.

First, observe when the Moon is in the Meridian, and then find her Altitude, and take the same from the particular Scale between your Compasses; then set one Point in the hour 12, and lay the Thred to N D, and on the degrees it shall shew the Moons declination; and in the Line of the Suns Place, the Moons present Place, counting her Progress orderly from the last Change-day, or New Moon, when she was with the Sun.

Otherwise thus;

Observe what Hour the Moon sheweth on any Sun-dial, at the same instance by the Fixed Stars, or other wayes, find the true Hour;

Then,

The Extent from the Moons Hour, to the the true Hour, shall reach the same way from 12, to the Moons Age, right against which is her coming to South, at which time you may find her true Altitude, and so come by her Declination.

Yet again, for her Age and Place, according to Mr. Street, *and Mr.*Blundevil.

Add the Epact, the Month, and Day of Month in one *sum*, counting the Months from *March*, by calling *March* the first Month, *April* the second, &c. then that *sum*, if under 30, is the Moons Age; but if the *sum* be above 30, then substract 30, and the remainder is the Moons Age, when the Month hath 31 dayes; but if the Month hath but 30, or less than 30 dayes, then substract but 29, and the remainder is the Moons Age.

Or thus;

Add to the Epact for the present year, and in *January* 0, in *February* 2, in *March* 1, in *April* 2, in *May* 3, in *June* 4, in *July* 5, in *August* 6, in *September* 8, in *October* 8, in *November* 10, in *December* 10; and the sum, if under 30, or the excess above 30, added to the day of the Month, abating 30, if need be, gives the Moons Age that day; but substracted from 30, leaves the day of her Change in that Month, or from the beginning of that Month.

Example.

Example. July 10. 1668.

The Epact that year is 26, and the Number for *July* is 5, the Excess above 30, is 1; which added to any day of the Month as to 10, gives 11, for the Moons Age, *July* 10. 1668.

Then for the Moons Place.

Multiply the Moons Age by 4, and the Product divided by 10, the Quotient giveth the signs; and the remainder multiplied by 3, gives the degrees, which you must add to the Suns place that day, to find out the Moons place for that day of her Age.

Example.

On *July* 10. 1668, the Moons Age is 11, which multiplied by 4, makes 44; and 44 divided by 10, gives 4 signs in the Quotient; and 4, the remainder, multiplied by 3, makes 12 degrees more; which added to *Cancer*, 29 degrees, the Suns place on the 10th day of *July*, makes 11 degrees in *Sagittarius*, the Moons place the same day, *propè verum*.

Or

Or rather by the Rule thus, on the Line of 24 hours by particular Scale, having the Moons place, to find her Age by the Line of 24 hours.

The Extent from the Suns true place, to the Moons true place, ſhall reach the ſame way, from o day, to the day of her Age.

Or contrarily, having the Moons true Age, to find her true Place.

The Extent from o day old, to the Moons true Age, ſhall reach the ſame way from the Suns true Place to the Moons.

Or, having the Moons true Place at the New Moon, to find her Place any day of her Age after.

The Extent from ♈, to the Moons true Place at the Change, ſhall reach the ſame way, from the day of her true Age, to her true Place, adding as many degrees to the Number found, as the Moon is dayes old.

Then,

Having her Place, and Age, it is eaſie to find the Moons Hour, and then her true Hour; but I fear I ſpend herein too much time on an uncertain ſubject.

Uſe XLVI.

Use XLVI.

The Right Ascention and Declination of any Star, with the Suns Right Ascention, and the Hour of the Night given, to find the Altitude and Azimuth of that Star, and thereby to know the Star, if you knew it not before.

Set one Point of the Compasses in the Stars Right Ascention, found in the Line of twice 12 hours; and open the other to the Suns Right Ascention, found in the same Line; then this Extent shall reach, in the same Line, from the true hour of the Night, to the Stars hour from the Meridian; then laying the Thred to the Stars Declination, the N D from the Stars hour, in the Line of hours, to the Thred, measured on the particular Scale of Altitudes, gives the Stars Altitude; then by his Declination and Altitude, you may soon find his Azimuth, by Use 27.

And if the Instrument be neatly fixed to a Foot, to set North and South, and turn to any Azimuth and Altitude, you may find any Star, at any time convenient and visible.

Use XLVII.

The Altitude and Azimuth of any Star being given, to find his Declination.

Lay the Thred to the Altitude on the degrees, counted from 60|0 toward the end, then ſetting one Point on the Stars Azimuth, counted in the Azimuth Line, and take the ND from thence to the Thred; which diſtance meaſured from the beginning of the particular Scale of Altitudes, ſhall give the Declination.

If the Compaſſes ſtand on the right-ſide of the Thred, then the Declination is North; if on the left, it is South; according as you work for the Suns Azimuth in a particular Latitude.

Uſe XLVIII.

The Altitude and Declination of any Star, with the Right Aſcention of the Sun, and the true Hour of the Night given, to find the Right Aſcention of that Star.

Firſt, by the 43d Uſe, find the Stars Hour, *viz.* How many hours and minuts it wants of coming to, or is paſt the Meridian; then the Extent of the Compaſſes (on the

the Line of 24 hours on the Head-leg) from the Stars hour to the true hour, ſhall reach the ſame way from the Suns Right Aſcention, to the Stars Right Aſcention, on the Line of twice 12, or 24 hours.

Uſe XLIX.

To find when any Fixed Star cometh to South, by the Line of twice 12, or 24 hours.

In Uſe 42, Section 4, you have the way by Subſtraction, with its Cautions: But by the Line of twice 12, or 24 hours, *work thus*;

Count the Suns Right Aſcention on that Line, and take the diſtance from thence to the next 12 backward, *viz.* that at ♈, at the beginning of the Line, when the Suns Right Aſcention is under 12 hours; or, to the next 12 in the middle of the Rule at ♎, when the Suns Right Aſcention is above 12 hours, (which is nothing but a rejecting 12 for more conveniency).

Then,

The ſame Extent laid the ſame way from the Stars Right Aſcention, ſhall reach to the Stars coming to South.

Or,

Or,

The Extent from the Sun, to the Stars Right Aſcention, ſhall reach the ſame way from 12, to the Stars coming to South.

Example, for the Lyons-Heart, Auguſt 20.

The Suns Right Aſcention the 20th of *Auguſt*, is 10 hours 36 minuts; the Right Aſcention of the Lions-Heart, is 9 hours and 50 min.

Therefore,

The Extent from 10 hours 35 min. to the beginning, ſhall reach the ſame way from 9 hours 50 min. (by borrowing 12 hours) becauſe the Suns Right Aſcention is more than the Stars) to 11 hours 13 min. of the next day, *viz.* at a quarter paſt 11; or, at 11 hours and 13 min. the ſame day; where you may obſerve, that the remainder being above 12, if you add 24 hours, the time of Southing is between mid-night, and mid-day next following.

Uſe L.

To find what two dayes in the year are of equal length, and the Suns Riſing and Setting.

Lay the Thred on any one day in the upper Line of Months and Dayes, and at the ſame time the Thred cuts in the lower-Line of Months the day that is anſwerable to it in length, riſing, ſetting, and declination, and other requiſites.

Example.

The 1ſt of *April*, and the 21 of *Auguſt*, are dayes of equal length; and the Suns Riſing and Setting is the ſame on both thoſe dayes; only in the upper-Line, the dayes are increaſing in length, and in the lower-Line they are decreaſing.

Uſe LI.

To find how many degrees the Sun is under the Horizon at any Hour, the Declination and Hour being given.

Count the Suns Declination on the degrees, the contrary way, *viz.* for North Declination, count from 60|0 toward the end; and count for Southern Declination toward the Head, and thereunto lay the Thred; then

then take the nearest distance from the hour given to the Thred; this distance measured in the particular Scale of Altitudes, shall shew the Suns Depression under the Horizon at that hour.

Example.

January *the* 10*th at* 8 *at Night, how many degrees is the Sun under the Horizon.*

On that Day and Hour, the Suns Declination is about 20 degrees South; then if I lay the Thred to 20 degrees of Declination North, and take the nearest distance from 8 to the Thred, that distance, I say, measured in the particular Scale, gives 34 degrees and 9 min. for the Suns Depression under the Horizon of 8 afternoon.

To do this in other Latitudes, you are to find the Suns Altitude at 8 in Northern Declination, by Use 37.

CHAP.

CHAP. XVII.

The use of the Trianguler Quadrant, in finding of Heights and Distances, accessable or inaccessable.

Use I.

To find an Altitude at one Station.

FIrst, The *Trianguler Quadrant* being rectified, and fixed to a Ball and Socket and three-legged-staff, being necessary in these Operations to perform them exactly, especially for Distances; look up to the object as you would to a Star; and observe what degree and minut the Thred cuts, and set it down: Also, observe the place where you stand at the time of Observation, and the distance from your Eye to the ground, and the place on the object that is level with your eye also; as the playing of the Thred and Plummet will plainly shew.

Also, you must have the measure from the place where you stood observing, to the Point exactly right under the object, whose height you would have in Feet, Yards, Perch, or what you please, to Integers, and Fractions in Decimals, if it may be.

Also Note, That in all Right-Angle-Triangles, one Acute Angle is alwayes the complement of the other; so that observing or finding one by Observation, by consequence you have the other, by taking that from 90.

These things being premised, the Operation followes, by the Artificial Numbers, Sines and Tangents, and also by the Natural.

Note also by the way, That in regard the complement of the Angle observed is frequently used, if you count the degrees the contrary way, that is to say from the Head, you shall have the complement required; as hath been oftentimes hinted before.

Then,

As the sine of the Angle, opposite to the measured side, is to the measured side, counted on the Numbers;

So is the sine of the Angle found, to the Altitude or Height required on Numbers.

Figure I.

Example at one station.

Standing at C, I look up to B the object, whose Height is required, and I find the Thred to fall on 41 degrees and 45 minuts; but if you count from the Head, it is 48-15, the complement thereof, as in the *Figure* you see. Fig. 1

Also, the measure from C to A, is found to be 218 foot.

Then,

As the sine of 48-15, the Angle at B, being the complement of the Angle at C, is to 218 on the Line of Numbers;
So is the sine of the Angle at C, 41-45, to 195 the Altitude of A B the height required, found on the Line of Numbers.

To which you must add the height of your eye from the ground, in the time of Observation; or on rising grounds from a mark on the Building, or any other object that is level with your eye in time of Observation.

A second Example standing at D.

Fig. 1. But if I were ſtanding at D, 129 foot and a half from A, and would find the height A B, the complement of the Angle at D, that is to ſay, the Angle at B is 33-30.

This being prepared, then ſay;

As the ſine of 33-30, the Angle at B, to the meaſured-ſide D A, 129 ½ counted on the Numbers;
So is 56-30, the ſine of the Angle at D, to 195, the Altitude required, A B, and 5 foot more, the uſual height of the eye from the Level to the ground, makes 200, the whole height required.

To work this by the Triangular Quadrant, ſay thus;

As — 129 ½, taken from any Scale, is to the = ſine of 33 deg. 30 min. laying the Thred to the neareſt diſtance;
So is the = ſine of 56-30, the Angle at D, to the — meaſure of 195 on the Scale you took 129 ½ from.

The

The like manner of work is by the Sector, as thus, in the foregoing Example.

As 218, taken from the Line of Lines, to the = sine of 48 deg. 15 min.
So is the = sine of 41-45, to 195 on the Line of Lines latterally.

And yet further,

So is the = sine of 90, to 291, the Line CB.

Use II.

To find an Altitude at two stations.

But if you cannot come to measure to the foot of the object, then you must observe at two places. Fig. I.

As thus for Example.

First, as before, find the Angle at D, or rather the complement thereof, *viz.* 33-30; then go further backward in a right Line with the object and first station, any competent Number of feet, as suppose 88 ½ to C; there also observe the Altitude or Complement, *viz.* the Angle ABC, 48-15.

Then,

Find the difference between 48-15, and 33-30, and it is 14-45.

Then,

As the sine of the *difference* last found, *viz.* the Angle C B D, 14-45, to 88 ½, on the Line of Numbers;
So is the sine of the Angle at C, 41-45, to the measure of the side D B, 233, on the Line of Numbers.

Again, for the second Operation.

As the sine of 90, the Angle at A, to the Hypothenusa D B, 233;
So is the sine of 56-30, the Angle at D, to 195, the Altitude required.

The same by the Trianguler Quadrant, or Sector.

As — 88 ½, the measured distance C D, to the = sine of 14-45, C B D;
So is = sine of 41-45, to the — measure of 233, the opposite-side D B.

Again,

As — 233, taken from the Line of Lines, to = sine of 90;
So is the = sine of 56-30, the Angle at D, to — 195, on the Line of Lines, the height required.

Use III.

Use III.

Another way to save one Operation from I C.

First, observe the complement of the Angle at D, and also the complement of the Angle at C; then count these two complements on the Line of Natural Tangents, on the loose-piece, or moving-leg, and take the distance between them, and measure it on the same Tangent-line from the beginning thereof, and note what Tangent the Compass-point stayeth at, and count that for the first term, in degrees and minuts. Fig. I.

Then,

As the Tangent of this first term, to the measured distance CD, 88 $\frac{1}{2}$, on the Line of Numbers;
So is the Tangent of 45, to the Altitude required.

Thus in our Example;

The distance measured is 88 $\frac{1}{2}$, the two complements 33-30, and 48-15; the distance between them makes the Tangent of 24-34, to be used as a first term.

Then,

As the Tangent of 24-34, the first term last found, to 88 $\frac{1}{2}$ on the Numbers;
So is the Tangent of 45, to 195, *ferè*, on the Numbers, the height required.

But if the distance from D *or* C, *to* A, *the foot of the Object, were required, then the manner of Calculation runs thus;*

As the Tangent of the *difference* of the Co-tangents first found, 24-34, is to the distance between D and C 88 $\frac{1}{2}$;
So is the Co-tangent of the greater Ark 48-15, to the greater distance C A 218.

Or,

So is the Co-tangent of the lesser Ark 33-30, to the lesser distance D A, 129 $\frac{1}{2}$.

But if the Hypothenusaes be required, then reason thus;

As the Tangent of the *difference* first found is 24-34, to the distance between the stations D and C, 88 $\frac{1}{2}$;
So is the Secant of the Angle at B the greater, *viz.* 48-15, counted beyond 90, to C B 291.

Or,

So is the Secant of 33-30, the lesser Angle at B, to 233 the lesser distance D B, the Hypothenusa required.

To

To work these two last by the Trianguler Quadrant.

First, prick off the Tangents and Secants to be used parallelly, from the loose-piece, on the greater general Scale; and note those Points for your present use.

As thus;

The Tangent of 24-34, taken from the loose-piece from 60, counted as 00 will reach to the sine of 10-40, on the general Scale.

Secondly,

The Secant of 33-30, being the measure from the Tangent of 33-30, on the loose-piece (counting from 60) to the Center, will reach on the general Scale from the Center, to 28-50.

Thirdly,

The measure from the Tangent of 48-15, on the loose-piece, to the Center, being the Secant of 48-15, will reach from the Center to 32-5, on the general Scale.

This being prepared, the work is thus;

As — distance between the two stations, to = Tangent, of the first term, at 10-40;

So is = Tangent of 45, to the Altitude required.

Again,

Again, for the Distance.

As — distance between the two stations, to the = Tangent of the first term;

So is the = Tangent of the greater Angles complement, at 26-36, to the greatest distance C A 218. *Or,*

So is the = Tangent of the lesser Angles complement, at 15-25, to the lesser distance D A, 129 ½; *Or,*

So is the = Secant of the greater Angles complement, at 32-5, to the greater Hypothenusa C B, 291. *Or,*

So is the = Secant of the lesser Angles complement, at 28-50, to the lesser Hypothenusa D B, 233.

Use IV.

Another way for Altitudes, by the Line of Shadows, either accessable or unaccessable, by one or two stations.

If this way be desired, it may be put on this, as well as any other *Quadrants.*

Then the use is thus; Figure II.

Suppose that A B be the height of a Tree, or other Object to be found; go so far back from

from it, as suppose to C, till looking up by the two Pins put for sights, the Thred falls on 45 degrees on the Quadrant, or on 1 on the Line of Shadows; then, I say, that the height A B, is equal to the distance C A, more by the height of your eye from the ground.

But if you go further back still to D, till the Thred falls on 2 on the Line of Shadows; that is to say, at 26 deg. 34 min. the Altitude will be but half the distance from A; but if you remove to E, the Thred falling on 3 on the Shadows, the Altitude will be but one third part of the distance E A.

From hence you may observe, that observing at C, and at D, where the Thred falls on 1, and on 2, the distance between C and D, is equal to the Altitude; so likewise at D and at E, and so by consequence at $1\frac{1}{2}$ and $2\frac{1}{2}$ and $3\frac{1}{2}$, or any other equal parts. This is an excellent easie way.

The like will be if you observe at D and C, looking up to F, where the Altitude A F is twice the distance A C.

Use V.

Another way, by the Line of Shadows, at one station.

Measure any distance, as feet, yards, or Fig. II.

the

the like from any object; as suppose from A to D were 200 foot, and looking up to B, the Thred cuts the stroke by 2 on the Line of Shadows.

Then by the Line of Numbers, say;

As 2, the parts cut, is to 1;
So is 200, the distance measured, to 100 the height.

Or,

Suppose I measured any other uneven Number, and the Thred fall between 00 on the Loose-piece, and 1 on the Shadows, commonly called contrary Shadow.

The Rule is alwayes thus;

As the parts cut by the Thred, are to 1;
So is the measured distance, to the height required, being less than the measured distance.

But when the Thred falls between 1 *and* 90 *at the Head, called right Shadow; then the Rule goes thus;*

As 1, to the parts cut by the Thred;
So is the measured distance, to the height, being alwayes more than the measured distance from the foot of the object, to the station.

Use VI.

Use VI.

Another way by the Line of Shadows, and the Sun shining.

When the Sun shineth, find his Altitude, and also as the Thred lies, see what division on the Line of Shadows is cut by the Thred, and then straightway measure the shadows length on the ground; and if the Sun be under 45 degrees high, the shadow is longer than the length of that object which causeth the shadow; but if the Sun be above 45 degrees high, then the object is longer than the shadow; and the Operation is thus by the Line of Numbers, only with a pair of Compasses.

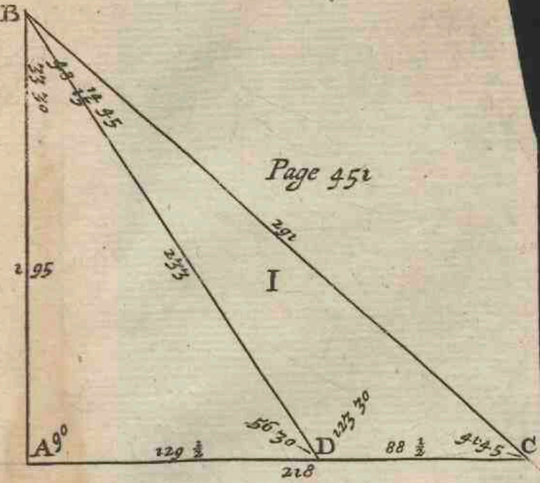
The Height of the Sun being under 45, say;

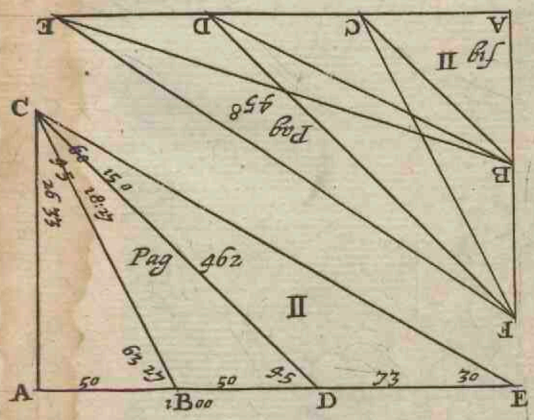
As the parts cut by the Thred on the Shadows, is to 1;
So is the Shadow measured, to the height required.

The Height of the Sun being above 45, say;

As 1, to the parts cut by the Thred on the Line of Shadows;
So is the measure of the shadow, to the height in the same parts.

Use VII.





Use VII.

To find an inacceſſable Altitude, by the Quadrat and Shadows, otherwiſe.

Obſerve the Altitude at both ſtations, and count the obſerved Altitudes at both ſtations, on the Quadrat or Shadows, according as it happens to be either above or under 45 degrees; and take the leſſer out of the greater, noting the remainder for the firſt term; and the Diviſor to divide the diſtance between the ſtations, increaſed with Cyphers, if need be; and the Quotient is the Anſwer required.

But by the Line of Numbers, work thus;

The Extent from the *difference* to 1, ſhall reach the ſame way from the meaſured diſtance, to the height required.

Example. Figure II.

Let ABCDE repreſent the Object, and three Stations; let the Line AC repreſent the Altitude; the Point B one ſtation, 50 foot from A; D another ſtation, 100 foot from A, or 50 from B; and E another ſtation, 73 foot from D, or 173 foot from A; all which meaſures you need not know before, but only BD, and DE; Alſo, the

Angle

Angle at B, 63-27, and his complement; counting the other way, being the Angle at C 26 degrees 33 minuts; the Angle at D 45, and his complement ſo alſo; the Angle at E 30, and his complement 60. Now mind the Operation by either of theſe, Firſt lay the Thred on 26-33, and in the Quadrat it cuts 50; lay the Thred on 45, and in the Shadows, or Quadrat, it cuts 100, or 1; or, if you lay the Thred to 60, then in the ſhadows it cuts 173.

The *difference* between 173, and 100, is 73.

Then,

As 73, the difference in Tangents between the two obſervations, is to the diſtance in feet, 73;
So is Radius 100, or the ſide of the Quadrat, to 100, the hight required.

Again, for the two neareſt Obſervations, whoſe difference of Tangents, is 50.

As 50, the difference in Tangents, to 50 foot the meaſured diſtance;
So is 100, the ſide of the Quadrat, to 100 the height.

Again,

Again, lastly by the observations at B *&* E, *the difference of Tangents being* 123.

As 123, the difference in Tangents, to 123, the measured distance;
So is 100, the Radius or side of the Quadrat, to 100, the height required.

Or,

In the *first Figure*, the Angles at the top being 33-30, and 48-15; and the measured distance 88 foot and a half, the difference in Tangents will be 45-8.

Then,

As 45-8, to 100, the side of the Quadrat;
So is 88 $\frac{1}{2}$, the measured distance to 194, the Altitude required.

This way is general for any Station, though both of right shadow, or both of contrary, or mixt of right and contrary, and done by the Line of Numbers, or by Multiplication and Division.

Also Note, That you may find this difference in Tangents or Secants, by the Natural Tangents, or Natural Secants on the Sector, and the Scale of equal parts belonging to them.

Thus

Thus ;

Take the diſtance between the comple-plement of the two obſervations, on the greater or leſſer Line of Tangents, (as is moſt convenient) and meaſure this diſtance in the Line of Lines, or equal parts equal to that Radius ; and that ſhall be the difference in Tangents required. The like for the Secants.

Alſo, By the Artificial Numbers, Sines, and Tangents, you may come by this differences in Tangents, or Secants, very well *thus* ;

Juſt right againſt the Tangent of the Co-altitude, counted on the Line of Tangents, in the Line of Numbers, is one Number ; and againſt the Tangent of the complement of the other Angle, is the other Number ; only with this Caution, That if the Tangent be above 45, then take the diſtance from 45 to the Tangent, as it is counted backward, with Compaſſes, and ſet the ſame the increaſing way from 1, on the Numbers, to the other Number required ; then the leſſer taken from the greater, leaves the difference in Tangents that was required. In the ſame manner, the Sines counted from 90, and laid the contrary way from 1 increaſing, will give the difference in Secants, to meaſure the Ba e, and Hypothenuſa by N unbers only.

Use VIII.

Another pretty way by Scale and Compaſs, without Arithmetick, from T. S.

F V. On any plain Boards end, or Trencher, draw a right Angle, and in the meeting-Point, and on one Lines-end, knock two Pins, or ſmall Nails, as near as you can upright; then on the Pin that ſtands in the right Angle, hang a Thred and Plummet; then lift up the Board, with the right Angle toward the Object, whoſe height you would have, till the two Pins and the Object are brought to a ſtreight Line, the Plummet playing even and truly.

Then draw the Line, that the Thred maketh, on the Board;

Then meaſure from your ſtanding, to the foot of the Object, and take the number of feet, or yards from any Scale, and lay it from the right Angle on the other Line, and raiſe a Perpendiculer from thence to the Plumb-line made by the Thred, and that ſhall be the Altitude required, being meaſured on the ſame Scale.

Example.

Let A B G D repreſent the Boards end, or Trencher, and on that, let A B be one ſtreight Line, and A G another Perpendiculer

culer to it; in the Point A, knock in one Pin; and in B, or any where toward the end, another; On the Pin at A, hang a Thred and Plummet; and ſtanding at I, any convenient ſtation, look up by the two Pins at B and A, till they bourn in a right Line with the Point H, the object whoſe height is to be meaſured; then the Plummet playing well and even, make a Point juſt therein, and draw the Line AD, as the Thred ſhewed.

Then, having meaſured the diſtance from G the foot of the Object, to I the ſtation, take it from any firſt Scale, and lay it from A to G; then on the Point G, raiſe a Perpendiculer to A G, till it interſect the Plumb-line A D; then, I ſay, the diſtance C D, meaſured on the ſame Scale you took A C from, ſhall be equal to the Altitude G H, which was required.

Uſe IX.

The ſame work at two ſtations.

But if you cannot come to meaſure from I, the firſt ſtation to G; then meaſure from I to K; and having obſerved at I, and drawn the Plumb-line A D, take the meaſure between I and K, the two ſtations, from any fit Scale of equal parts, and lay it on the Line Fig. V.

Line A C, from A to C, *viz.* 79 parts; and in the Point C, knock another Pin, and hang the Thred and Plummet thereon, and observe carefully where this last Plumb-line doth cross the other, as suppose at E; then from E, let fall a Perpendiculer to the Line A C, which Line A C shall be the height G H required; (or thus, the nearest distance from E to A C is the height required) *viz.* 120 of the same parts that I K is 79; Note the *Figure*, and behold that A C F E, the small *Figure* on the Board is like and proportional to A A, G H, the greater *Figure*.

Fig; VI. Other wayes there be, as by a Bowl of Water, or a Glass, or a Plash of Water, or a Square; but these set down, are as convenient and ready as any whatsoever; As in the next *Figure* you may see the way by the Glass, and Square.

As thus;

Let C represent a Glass, a Bowl, or Plash of Water, wherein the Eye, at A, sees the picture or reflection of the Object E.

Then, by the Line of Numbers;

As C B, the measure from your foot to the Glass, is to A B, the height from your eye, to the ground at your foot;

So

So is the meaſure from C to D, to the height D E. *See Figure* VI.

Again, to find a diſtance by the Square, that is not over-long.

Let C repreſent the upper-corner of a Square, hung on a ſtaff at F; then the one part of the Square directed to E, and the other to A.

The Proportion will hold, by the Line of Numbers.

As F A 11-37, to F C 50;
So is F C 50, to F E 220.

That is,

So many times as you find A F in F C;
So many times is F C in F E, and the like.

Note, That you muſt conceive A F E to be the Ground, or Baſe-line in this Operation by the Square; C being the top of an upright Staff, 5 foot long, called 50 for Fraction ſake.

Uſe X.

To find a Diſtance not approachable by the Triangular Quadrant.

Let A repreſent the place of ſtanding, and A C be the diſtance required. Fig. III.

 First,

First, I plant my *Trianguler Quadrant*, ſet upon a three legged Staff and Ball ſocket, right over the place A ; and then bring the Index with two ſights in it, laid or faſtened to the Center of the *Trianguler Quadrant*, right over the Lines of Sines, and Lines cutting 90 at the Head ; the Index and ſights ſo placed, hold it there, and bring it and the Inſtrument together, till you ſee the mark at C, through the two ſights, by help of the Ball-ſocket, and then there keep it ; then remove the Index only to 0-60 on the looſe-piece, which makes a right Angle ; and ſet up a mark in that Line, at any convenient diſtance ; as ſuppoſe at B, 102 foot from A ; then remove the Inſtrument to B, and laying the Index on the Center, and 0-60 on the looſe-piece, direct the ſights to A, the firſt ſtation, by help of a mark left there on purpoſe ; Then remove the ſights till you ſee the mark at C, and note exactly on what degree the Index falleth, as here on 60, counting from 0|60 on the looſe-piece ; or on 30, counting from the Head, which is the Angles at B, and at C.

Then

Then by the Artificial Numbers, Sines and Tangents on the edge, ſay;

As the ſine of 30, the Angle at C, to 102, the meaſured diſtance counted on the Numbers;
So is the ſine of 60, the Angle at B, to 117, on the Numbers, the diſtance required.
So alſo is 90, the Angle at A, to 206, the diſtance from B to C.

Or, by the Lines and Sines on the Quadrant-ſide, as it lies, thus;

As the — meaſure of 102, taken from any Scale, as the Line of Lines doubling, to the = ſine of 30, laying the Index, or a Thred, to the neareſt diſtance;
So is the = ſine of 60, to 117, meaſured latterally on the ſame Line of Lines.

And,

So is the = ſine of 90, to 206, the diſtance from B to C.

So alſo, If you obſerve at B, and at D only, you muſt be ſure to ſet your Inſtrument at one ſtation, at the ſame ſcituation, as at the other, as a looking back from station to station will do it, and the same way of work will serve.

For,

As the Sine of 20, to 110;
So is the Sine of 40, to 206.

And,

So is the Sine of 120, to the Line DC 278, &c.

Use XI.

To find a Breadth and a Distance at any two Stations.

Fig.IV. Let A B be two marks, as two corners of a House or Wall, and let the breadth between them be demanded, and their distance from C and D, the two stations; *First*, set up two marks at the two stations, then setting up the Instrument at C, set the fiducial Line on the Rule to D, the other mark; then direct the sights exactly to B, and to A; observe the Angles DCB 45, and DCA 113-0, as in the *Figure*.

Secondly, Remove the Instrument to D, the other station, and set the fiducial-Line of the Quadrant (*viz.* the Line of Lines and Sines) directly to C; then fix it there, and remove the Index and sights to A, and to B, to get the Angles CDA 42-30, and CDB

CDB 109-0; Then observe, that the 3 Angles, of every Triangle, being equal to 180 degrees; having got the Angles at C 113, and the Angle at D 42-30, by consequence, as you take 155, the sum of the Angles at C and D, out of 180, then there remains 24-30, the Angle at A.

So also, Taking 109 and 45 from 180, rests 26, the Angle at B; then also, taking 45, the Angle BCD, out of 113, the Angle DCA, rests 68 degrees; the Angle BCA, in like manner, taking 42-30 from 109, the Angles at D, rests 66-30, the Angle ADB; and let the distance measured, between the two stations, be 100, *viz.* CD. These things thus prepared by the Artificial Numbers, Sines and Tangents on the edge,

Say,

As the Sine of 24-30, the Angle at A, to 100, on the Numbers, the measured side CD;

So is the Sine of the Angle at D 42-30, to 164, on the Numbers, the side CA.

So is the Sine of 113, the Angle ACD, to 222, on the Numbers, the distance from C to B.

Also,

Also, for the other Triangle, at the other Station D.

As the Sine of 26, the Angle C B D, to 100, on the Numbers, the measured distance CD;
So is the Sine of 45, to 161, on the Line of Numbers, the distance from D to B;
So is the Sine of 109, the Angle C D B, to 216, on the Numbers, the distance from D to A.

Then, having the Sides DB 161, *and* AD 222, *and* ADB *the Angle included* 66-30, *to find the Angles* DAB, *or* ABD, *use this Proportion.*

As the *sum* of the two sides given, is to the *difference* between the two sides;
So is the Tangent of half the *sum* of the two Angles sought, to the Tangent of half their *difference.*

Example.

222, and 161, make 383 for a *sum*; and 161, taken from 222, rest 61 for a *difference.*

Again,

Again,

66-30, taken from 180, rest 113-30, for a *sum* of the Angles sought, whose half 56-45, is the third Number in the proportion.

As 383, the *sum* of the two known sides, is to 061, the *difference* on the Numbers;
So is the Tangent of 56-45, the half-*sum* of the two Angles sought, to the Tangent of half the *difference* 13-40; which half-*difference*, 13-40, added to 56-45, makes 70-25, the greater Angle required at B, *viz.* A B D.

Then also,

If you take 13-40, from 56-45, the half-*sum* of the Angles inquired, rest 43-05, the Angle B A B; the like may you do with the other Triangle A B C, being needless in our Proposition.

Thus having found the Angles, and one side, the Sines of the Angles, as proportional to their opposite sides.

As the Sine of 44-33, the Angle A B C, is to the side A C 146, on the Numbers;

So

So is the Sine of 68, the Angle at C, to 217, the diſtance between the marks required.

Or,

As Sine 43-05, the Angle at A, to 161;
So is Sine of 66-30, the Angle at D, to 217, the diſtance between the marks required.

Alſo note,

That if this manner of Calculation be tedious or difficult, then on a Slate, or ſheet of Paper, you may do it by protraction, by the Line of Lines and Chords, or half Sines, very near the matter with care;

Thus: Draw CD the Station-Line, or meaſured-diſtance; and make A D 100, from any fit Scale. Then, on C and D draw a Circle, and on that Circle lay off from C and D the Angles, found by obſervation, and draw thoſe Lines, and where they croſs one another, as at A and B, draw the Line A B: thoſe Lines and Angles meaſured on the ſame Scales and Chords, ſhall be the Sides, breadth, and diſtances required; as you ſee in the Figure.

Uſe XII.

Another way for a long Diſtance.

Fig. VII. Let C be your ſtanding place to ſet your Inſtru-

Inſtrument, and let E be the mark afar off, whoſe diſtance from you C would know: firſt, move in a right Line between C and E to A, any number of Yards or Perches, as ſuppoſe 50 Perch, and ſet a mark at A; Then move in a Perpendiculer-Line to C E, from A to B any diſtance, and there ſet up a mark at B, as ſuppoſe 66 Perches from A.

Then come back again to C, and remove in a Perpendiculer-Line to C E, till you ſee the mark ſet up at B, and the enquired point at the diſtance E in a Right-Line; and note that place at D, getting the exact diſtance thereof from C, ſuppoſe 76.

Then ſubſtract the meaſured diſtance A B from the meaſured diſtance C D, and note the difference 10. Then, by the Line of Numbers, or by the Rule of Three, ſay,

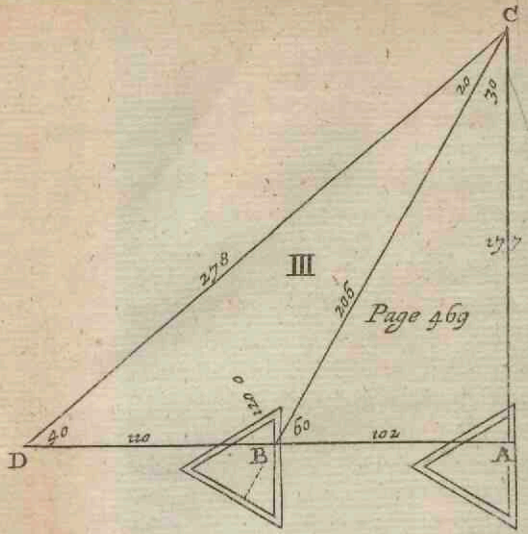
As the Difference between A B and BC, 10, is to the diſtance between A & C 50:
So is the meaſured diſtance C D 76, to the diſtance C E 380.

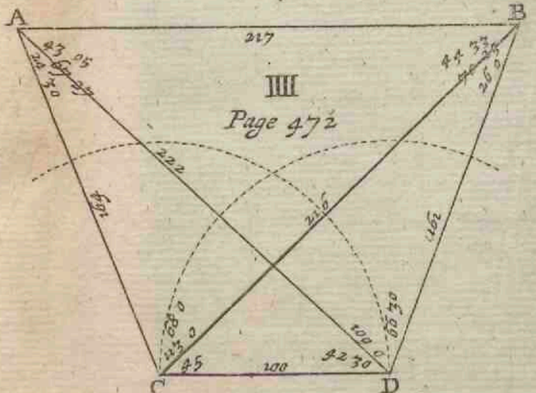
Or,

So is the meaſured diſtance A B 66, to the diſtance AE 330, the diſtance required.

Note, That you muſt be careful and exact in meaſuring the Diſtances A C, A B, & CD, and the Anſwer will be the more exact accordingly.

Uſe





Uſe XIII.

To find an Altitude of a Houſe or Tower, by knowing part of it.

Fig. VIII. If you divide the inſide-edge of the Looſe-peice into inches, or any equal parts, ſuch as the neareſt diſtance from the Rectifying-Point to that inſide-edge may be 1000, and for this uſe two ſmall ſliding ſights may be convenient: Then the uſe is thus for any Angle under 30 degrees;

Fix the Inſtrument to the Ball-ſocket and Staff, and turn it toward the Object, cauſing the Plummet to play on 30 degrees; for then the Looſe-piece is perpendiculer. Then one pin or ſight ſet in the Rectifying-Point, ſlip on a ſight along the inner-edge of the Looſe-piece, till you ſee the Object at the upper part of the Altitude, and another ſight at the lower part of the Altitude known; and obſerve the preciſe diſtance in parts between the two ſights, on the Looſe-piece; Or, the ſeveral parts cut by the *Index* at each Obſervation:

Then,

As the diſtance between the two ſights, is
 to the diſtance between the remoteſt ſight
 from the middle of the Looſe-piece;
So is the height of the known part, to the
whole

whole height required above the level of the eye.

Example.

Let C I represent the Altitude of a Pyramid on the Tower of a Steeple 30 foot high, and, standing at B, I would know the height of I A from the level of the eye upward.

Fix the *Triangular Quadrant* on the Staff and Ball-socket, with the Head-Center at B, with the Plummet playing on 30 degrees, and the Loose-piece perpendiculer: Then slip two sights on the Loose-piece, one in a Right-Line to C, the other to I; and note the parts between, and the parts the furthest sight cuts, from the middle stroak on the Loose-piece, from whence the parts are numbred, which in our Example let be 500, the sight of H, and the sight at G to cut 359; then the distance between the sights will be 143, and the remotest from the middle of the Loose-piece to be 500; and the known Altitude, being part of the whole, to be 30 foot.

Then, by the Line of Numbers, say,

As 143, the distance betwixt the sights at G & H, to 500 the remotest sight from the level or middle, *viz.* F H:

So is 30 foot, part of the Altitude known, C I, to 105, the whole Altitude unknown, A C.

Or,

Or,

As 357, the parts cut at G, to 500 the parts cut at H, the remoteſt ſight:
So is 75, the height of the lower part, to 105 the whole height A C.

Or,

As 143, the diſtance between the ſights, to I C the part of the height known 30:
So is 357, the parts cut between F and G, to 75 the height A I unknown, &c.

Uſe XIV.

Having the Height, to find a Diſtance.

Fig. VIII.

Let C A be the Altitude given, and A B the diſtance required. Then I ſtanding at C, obſerve the Angle C A B, by ſetting the end of the Head-leg to my eye, and the Head-end downwards, and ſet down, as the Thread cuts, numbring both wayes, for the Angle at C and at B his complement.

Then ſay,

As the Angle at B, 30 deg. 40 minutes, counted on the Sines, to 105 the height of the Tower:
So is 59 deg. 20 min. the Angle at C on the Sines, to 176 the diſtance required on the Numbers.

Alſo note by the way,

That if you take an Altitude at two ſta-

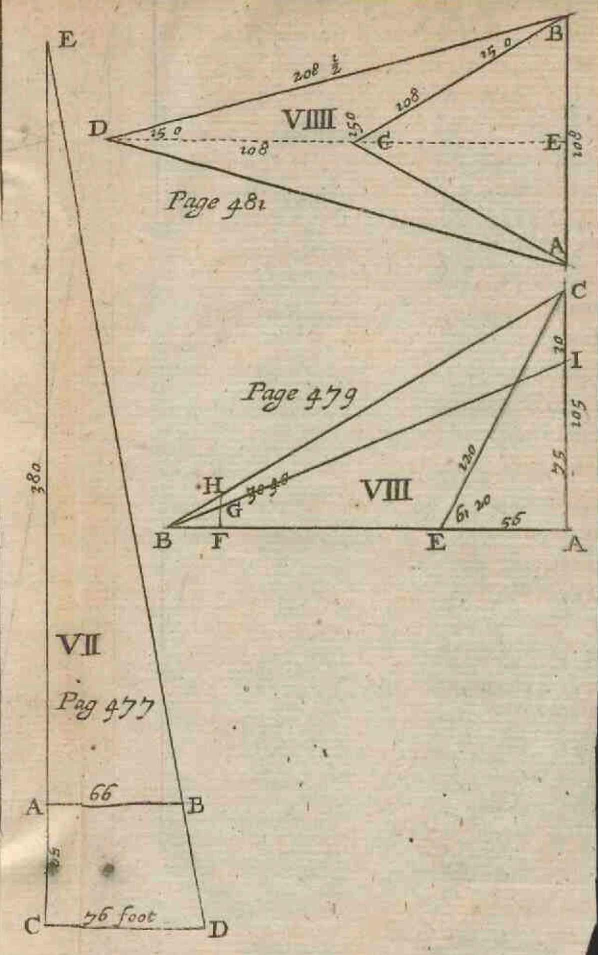
tions,

tions, as suppose at E and at B; if the Angle observed at B, be found to be the half of the Angle at E; as here in *Figure* VIII, the Angle at E, being 61-20, and the Angle at B 30-40, the just half thereof; then, I say, that the distance between the two stations, is equal to the Hypothenusa E C, at the first station, *viz.* E B is equal to E C; which being observed, *say*;

As the sine of 90, to 120, on the Numbers.
So is 61-20 on the Sines, to 105, the height required on the Numbers.

A further proof hereof, take in this following Figure IX.

Let A B be a breadth of a Wall, or Fort, not to be approached unto; then by the degrees on the in-side of the loose-piece, to find that breadth one way, is thus; Put two Pins into the two holes in the Head and Moving-leg, (or set the sights there in large Instruments); then move nearer or further from the objects, till your eye, fixed at the rectifying Point, can but just see the marks A and B by the two Pins in each Leg, which will only be at the mark C, at an Angle of 60 degrees; for so the Rule is made to that Angle: then the Instrument being still fixed at C, look backward in a right Line from the middle of the loose-piece, and rectify-



ing Point toward D, putting up a mark either in, or over, or beyond the Point D; and also be ſure to leave a mark at C, the firſt place of obſervation; Then remove the ſights to 15 degrees, the half of 30, counting from the middle, and go back in a right Line from C, toward D, till you can juſt ſee the marks by the two ſights ſet at 15 degrees each way; for then, I ſay, that the meaſure between the two ſtations, C and D, ſhall be exactly equal both to A B, the breadth required, and alſo to C B, or C A, the Hypothenuſaes; then, having the ſides C B, and C D, and the Angles B C E, and C B E, and B D C, it is eaſie to find all the other Sides and Angles, by the Rules before rehearſed, by the Lines of Artificial Nmmbers and Sines.

For,

As the Sine of 15 degrees, the Angle at D, *viz.* B D C, to 108 on the Numbers;

So alſo is the Angle at B, *viz.* D B C 15, to 108 on the Sines and Numbers.

So alſo is the Sine of 150, the Angle at C, *viz.* D C B, to B D 208 ½ on the Numbers.

Note alſo, That if the Angles of 60 and 30 be inconvenient, then you may make uſe of 52 and 26, or 48 and 24, or 40 and 20, or any other, and the half thereof; and then

then the meaſured diſtance, and the Hypothenuſa B C, at the neareſt ſtation, will alwayes be equal; but not equal to the breadth at any other Angle, except 30 and 60, as in the *Figure*. But having the Angles, and thoſe Sides, you may ſoon find all the others by the Artificial Numbers, Sines and Tangents, by the former directions.

The End of the Firſt Part.

The *Table* or *Index* of the *things* contained in this Book.

Page

Explanation of Terms particularly belonging to this Instrnment.

Of Terms used in Dialling:

Of Terms in Astronomy.

To

The

Page

Page

To make and measure the 5 regular bodies, with the Declination and Reclination of every side, at any scituation of them.

—*Artificially*

FINIS.