Mathematical elements of natural philosophy, confirm'd by experiments; or, an introduction to Sir Isaac Newton's philosophy

Mathematical
ELEMENTS
OF
Natural Philoſophy,
Confirm'd by EXPERIMENTS;
OR, AN
INTRODUCTION
TO
Sir ISAAC NEWTON's Philoſophy.

VOL. I.

Written in LATIN

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Tranſlated into ENGLISH

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MDCCXXXI.

TO

Sir *Isaac Newton*, K$^{nt.}$

PRESIDENT

OF THE

ROYAL SOCIETY, &c.

SIR,

IN dedicating this Tranſlation to You, I do no more than what my learned Friend, the ingenious Author, would have done, if Cuſtom and Gratitude had not obliged him to offer the firſt Philoſophical Work he has publiſhed ſince his being Profeſſor, to the Governors of the Univerſity that gave him the Chair. And as there are more Admirers of your wonderful Diſcoveries, than there are Mathematicians able to underſtand the two firſt Books of your *Principia*; ſo I hope You will not be diſpleaſed, that both my Author and myſelf have, by Experiments, endeavoured to explain ſome of thoſe Propoſitions, which were implicitly believed by many of your Readers; at the ſame Time that the greateſt Part of your third Book, and ſeveral of the *Corollaries* and *Scholia* in the other two, gave them the higheſt Satisfaction that an inquiſitive Mind is capable of receiving. Mathematicians of the firſt Rank, who want no ſuch Helps in reading your incomparable Works, take a freſh Pleaſure in ſeeing thoſe Experiments performed which you have made yourſelf: And

And though ſome of ours may not always prove, but ſometimes only illuſtrate a Propoſition; yet, ſuch Mathematicians, as are of a communicative Temper, will be glad to uſe them, as a new Set of Words, to give Beginners ſo clear a Notion of the Syſtem of the World, as to encourage them to the Study of the higher Geometry; whereby they may know how to value your Solutions of the moſt difficult *Phænomena*, and learn from You, that a whole Science may be contained in a ſingle Propoſition.

For my own Part, ſince I cannot enough acknowledge the Advantages which I have had by being admitted to your Converſation, and your generous Way of gratifying me for ſuch Experiments as I have made by your Direction; therefore I ſhall here forbear to pay that Tribute which is due to

You from all Lovers of Knowledge; and rather choose to be thought singular, than, by praising, to offend You, I am,

SIR,

Your most Obedient,

AND

Very Humble Servant,

J. T. DESAGULIERS.

THE

TRANSLATOR

TO THE

READER.

THIS Book will ſufficiently recommend it ſelf; therefore, I need not ſay any thing in Praiſe of it, either on Account of the uſeful Subject that it treats of, or the excellent Method and familiar Way in which our Author has handled it: only I thought proper to obſerve to the Reader, that the Numbers in the Margin expreſs ſo many Propoſitions, which are referred to, as you go forward in the Book, to avoid Repetitions and Tautology. If what is printed in Italic *Characters be read by it ſelf, it will appear to be a Compendium of the whole Book; or the Doctrinal Part of it, without the*

the Experiments and Demonſtrations. I have endeavoured to anſwer this End in my English *Tranſlation, where you will find, that whatever is in* Italic *Characters, makes up the Senſe of the reſt of the Book; which alſo readily makes Senſe by it ſelf, though taken from Places where it ſeems irregularly diſperſed.*

The firſt Edition of this Tranſlation had ſome Errors of the Preſs, and Faults in the Plates, which were occaſioned by the Haſte in which it was printed off, to prevent the Injury that muſt have been done to Dr. 's Graveſande, *by a Tranſlation that ſome Bookſellers endeavoured to get out before mine; which was ſo ill done, that no Body that had read the* Latin *Book, would be able to know it again in their* Engliſh.

I have, therefore, in this Fourth Edition, carefully review'd and corrected every Error, both in the Book and Plates.

THE

THE

PREFACE.

IF we compare the Writings of different Philosophers concerning PHYSICS, *we may easily see that they call different Sciences by the same Name, tho' they all profess to explain the true Cause of Natural* * Phænomena. *And no Wonder if they disagree among themselves, since even Mathematicians, who deal in Certainties, can hardly be kept from wrangling.*

But that Diversity of Opinions should not deter us from searching after Truth; since Labour and Study will find it out; and the more we are in love with it, the less we are liable to Errors, excepting such as human Frailty renders unavoidable.

We must proceed cautiously in Physics, *since that Science considers the Works of the Supreme Wisdom, and sets forth,*

* Appearances.

* What Laws JEHOVAH to himſelf preſcrib'd,
And of his Work the firm Foundation made,
When He of Things the firſt Deſign ſurvey'd.

How the whole Univerſe is govern'd by thoſe Laws, and how the ſame Laws run thro' all the Works of Nature, and are conſtantly obſerv'd with a wonderful Regularity.

We muſt take care not to admit Fiction for Truth; for by that Means we ſhut out all further Examination. No true Explanation of Phænomena can ſpring out of a falſe Principle: And what a vaſt Difference there is betwixt learning the Fictions of whimſical Men, and examining the Works of the moſt wiſe God! Since an Enquiry into Divine Wiſdom, and the Veneration inſeparable from it, is to be the Scope of a Philoſopher, we need not enlarge upon the Vanity of reaſoning upon fictitious Hypotheſes.

Nature herſelf is therefore attentively and inceſſantly to be examined with indefatigable Pains. That Way indeed our Progreſs will be but ſlow, but then our Diſcoveries will be certain; and oftentimes we ſhall even be able to determine the Limits of Human Underſtanding.

What has led moſt People into Errors, is an immoderate Deſire of Knowledge, and the Shame of confeſſing our Ignorance. But Reaſon ſhould get the better of that ill grounded Shame; ſince there is a learned Ignorance that is the Fruit of Knowledge, and which is much preferable to an ignorant Learning.

Natural Philoſophy is placed among thoſe Parts of Mathematicks, whoſe Object is Quantity in general. *Mathe-*

* ——— quas dum primordia rerum
Pangeret, omniparens legens violare Creator
Noluit, æternique operis fundamina fixit.

Mathematics are divided into Pure and Mixed. Pure Mathematics enquire into the General Properties of Figures and abstracted Ideas. Mixed Mathematics examine Things themselves, and will have our Notions and Deductions to agree both with Reason and Experience.

Physics belong to mixed Mathematics. The Properties of Bodies, and the Laws of Nature, are the Foundations of Mathematical Reasoning, as all that have examined the Scope of this Science will freely confess. But Philosophers do not equally agree upon what is to pass for a Law of Nature, and what Method is to be followed in Quest of those Laws. I have therefore thought fit in this Preface to make good the Newtonian *Method, which I have followed in this Work. What that Method is, I have briefly set down in the first Chapter.*

Physics do not meddle with the first Foundation of Things. That the World was created by God, *is a Position wherein Reason so perfectly agrees with Scripture, that the least Examination of Nature will shew plain Footsteps of Supreme Wisdom. It is confounding and oversetting all our clearest Notions, to assert that the World may have taken its Rise from some general Laws of Motion, and that* it imports not *what is imagined concerning the first Division of Matter.* And that there can hardly be any thing supposed, from which the same Effect may not be deduced by the same Laws of Nature; *and that for this Reason : That* since Matter successively assumes all the Forms it is capable of by means of those Laws, if we consider all those Forms in Order, we must at last come to that Form wherein this present World is framed; so that we have no Reason in this Case to fear any Error from a wrong Supposition. *This Assertion, I say, overthrows all our clearest Notions, as has been fully proved by many Learned Men; and is indeed so* *unreasonable, and so injurious to the Deity, that it will seem unworthy of an Answer to any one that does not know that it has been maintained by many ancient and modern Philosophers, and some of them of the first Rank, and far removed from any Suspicion of Atheism.*

Then first laying it down as an undoubted Truth, that God *has created all Things, we must afterwards explain by what Laws every thing is governed; and to mention only the Moon, we must explain, why*

* The Silver Moon runs with unequal Pace,
Which yet Astronomers could never trace,
Or fix in Numbers her uncertain Place:
What Force her *Apsides* has forward driven,
And made her *Nodes* recede i'th' Starry Heaven.
What is her Pow'r to agitate the Sea,
Whose various Tides her Presence still obey;
When th'*Ocean* swells, its topmost Banks to lave,
Or ebbs from weedy Shores with broken Wave,
Leaving the Sands, the Sailor's Terror, bare:
———

In Order to explain more fully which Way we trace out the Laws of Nature, we must begin by some previous and preparatory Reflexions.

What Substances are, is one of the Things hidden from us. We know, for Instance, some of the Pro-

———— * Quâ causa argentea Phœbe
Passibus haud æquis graditur; cur subdita nulli
Hactenus Astronomo numerorum fræna recuset.
Cur remeant Nodi, curq; Auges progrediuntur.
———quantis refluum vaga Cynthia pontum
Viribus impellit, dum fractis fluctibus ulvam
Deserit, ac nautis suspectas nudat arenas;
Alternis vicibus suprema ad littora pulsans.

Properties of Matter; but we are absolutely ignorant, what Subject they are inherent in.

Who dares affirm that there are not in Body *many other Properties, which we have no Notions of? And who ever could certainly know, that, besides the Properties of* Body *which flow from the Essence of Matter, there are not others depending upon the free Power of* God, *and that extended and solid Substance (for thus we define Body) is endowed with some Properties without which it could exist? We are not, I own, to affirm or deny any thing concerning what we do not know. But this Rule is not followed by those, who reason in Physical Matters, as if they had a compleat Knowledge of whatever belongs to Body, and who do not scruple to affirm, that the few Properties of Body, which they are acquanted with, constitute the very Essence of Body.*

What do they mean by saying, that the Properties of Substance constitute the very Substance?

Can those Things subsist when joined together, that cannot subsist separately? Can Extension, Impenetrability, Motion, &c. *be conceived without a Subject to which they belong? And have we any Notion of that Subject?*

We must give up as uncertain what we find to be so, and not be ashamed to confess our Ignorance. Tho' we need not fear being too bold in affirming, that a Subject altogether unknown to us may perhaps be endowed with some unknown Properties. And those Men who at the same Time that they say, conformably to this Axiom, That we must not reason about Things unknown, lay it down as a Foundation of their Reasonings, that nothing relating to Body is unknown to us, are beholden to meer Chance, if they are not mistaken.

The Properties of Body cannot be known à priori; *We must therefore examine Body itself, and nicely* *consider all its Properties; that we may be able to determine what natural Effects* do *flow from those Properties.*

Upon a farther Examination of Body, we find there are some general Laws, according to which Bodies are moved. It is past Doubt, for Instance, That a Body once moved continues in Motion: that Reaction is always equal and contrary to Action. *And several other such Laws concerning Body have been discovered; which can no Way be deduced from those Properties that are said to constitute Body; and since those Laws always hold good, and upon all Occasions, they are to be looked upon as general Laws of Nature. But then we are at a Loss to know, whether they flow from the Essence of Matter, or whether they are deducible from Properties, given by* God *to the Bodies, the World consists of; but no Way essential to Body; or whatever finally those Effects, which pass for Laws of Nature, depend upon external Causes, which even our Ideas cannot attain to.*

Who dares affirm any thing upon this Point concerning all, or any Laws of Nature, without incurring the Guilt of Rashness? Besides, whoever examines the Phænomena of Nature will be fully persuaded, that many of its Laws are not yet discovered, and that many Particulars are wanting towards the compleat Knowledge of others.

The Study of Natural Philosophy is not however to be contemned, as built upon an unknown Foundation. The Sphere of human Knowledge is bounded within a narrow Compass; and he, that denies his Assent to every thing but Evidence, wavers in Doubt every Minute; and looks upon many Things as unknown which the Generality of People never so much as call in Question. But rightly to distinguish Things known, from Things unknown, is a Perfection above the Level of human Mind. Though many

Things

Things in Nature are hidden from us; yet what is ſet down in Phyſics, as a Science, is undoubted. From a few general Principles numberleſs particular Phænomena or Effects are explained, and deduced by Mathematical Demonſtration. For, the comparing of Motion, or in other Words, of Quantities, is the continual Theme; and whoever will go about that Work any other Way, than by Mathematical Demonſtrations, will be ſure to fall into Uncertainties at leaſt, if not into Errors.

How much ſoever then may be unknown in Natural Philoſophy, it ſtill remains a vaſt, certain, and very uſeful Science. It corrects an infinite Number of Prejudices concerning natural Things, and divine Wiſdom; and, as we examine the Works of God *continually, ſets that Wiſdom before our Eyes; and there is a wide Difference, betwixt knowing the divine Power and Wiſdom by a Metaphyſical Argument, and beholding them with our Eyes every Minute in their Effects. It appears then ſufficiently, what is the End of Phyſics, from what Laws of Nature the Phænomena are to be deduced, and wherefore, when we are once come to the general Laws, we cannot penetrate any further into the Knowledge of Cauſes. There remains only to diſcourſe of the Method of ſearching after thoſe Laws; and to prove that the three* Newtonian *Laws, delivered in the firſt Chapter of this Work, ought to be followed.*

The firſt is, That we ought not to admit any more Cauſes of Natural Things, than what are true, and ſufficient to explain their Phænomena. *The firſt Part of this Rule plainly follows from what has been ſaid above. The other cannot be called in Queſtion by any that owns the Wiſdom of the Creator. If one Cauſe ſuffices, it is needleſs to ſuperadd another; eſpecially, if it be conſidered, that an Effect from a double Cauſe is never exactly* *ly the same with an Effect from a single one. Therefore we are not to multiply Causes, 'till it appears one single Cause will not do the Business.*

In order to prove the following Rules, we must premise some general Reflections.

We have already said, that Mathematical Demonstrations have no Standard to be judged by, but their Conformity with our Ideas; and when the Question is about Natural Things, the first Requisite is, that our Ideas agree with those Things, which cannot be proved by any Mathematical Demonstration. And yet as we have Occasion to reason of Things themselves every Moment, and of those Things nothing can be present to our Minds besides our Ideas, upon which our Reasonings immediately turn; it follows, that God *has established some Rules, by which we may judge of the Agreement of our Ideas with the Things themselves.*

All Mathematical Reasonings turn upon the Comparison of Quantities, and their Truth is evidenced by implying a Contradiction in a contrary Proposition. A rectilineal Triangle, for Instance, whose three Angles are not equal to two right ones, is a Thing impossible. When the Question is not about the Comparison of Quantities, a contrary Proposition is not always impossible. It is certain, for Instance, that Peter *is living, though it is as certain that he might have died Yesterday. Now there being numberless Cases of that Kind, where one may affirm or deny with equal Certainty; there follows, that there are many Reasonings very certain, tho' altogether different from the Mathematical Ones. And they evidently follow from the Establishment of Things, and therefore from the pre-determined Will of* God. *For by forcing Men upon the Necessity of pronouncing concerning the Truth or Falshood of a Proposition; he plainly shews they must assent to Agreements, which their Judgments ne-*

ceſſarily acquieſce in; and whoever reaſons otherwiſe, does not think worthily of God.

To return to Phyſics: *We are in this Science to judge by our Senſes, of the Agreement that there is betwixt Things and our Ideas. The Extenſion and Solidity of Matter, for Inſtance, aſſerted upon that Ground, are paſt all Doubt. Here we examine the Thing in general, without taking notice of the Fallacy of our Senſes upon ſome Occaſions, and which Way Error is then to be avoided.*

We cannot immediately judge of all Phyſical Matters by our Senſes. We have then Recourſe *to another juſt Way of Reaſoning, though not Mathematical. It depends upon this Axiom; (viz.)* We muſt look upon as true, whatever being denied would deſtroy civil Society, and deprive us of the Means of Living. *From which Propoſition the ſecond and third Rules of the* Newtonian *Method moſt evidently follow.*

For who could live a Minute's Time in Tranquillity, if a Man was to doubt the Truth of what paſſes for certain, whatever Experiments have been made about it; and if he did not depend upon ſeeing the like Effects produced by the ſame Cauſe?

The following Reaſonings, *for Example, are daily taken for granted as undoubtedly true, without any previous Examination; becauſe every Body ſees that they cannot be called in Queſtion without deſtroying the preſent Oeconomy of Nature.*

A Building, this Day firm in all its Parts, will not of its ſelf run to Ruin to Morrow. *Thus, by a Parity of Reaſon, the Coheſion and Gravity of the Parts of Bodies, which I never ſaw altered, nor heard of having been altered, without ſome intervening external Cauſe, will not be altered to Night, becauſe the Cauſe of Coheſion and Gravity will be the ſame to Morrow as it is to Day. Who does not ſee, that the Certainty of this Reaſoning depends*

pends only upon the Truth of the fore-mentioned Principle?

The Timber and Stones of any Country, which are fit for a Building, if brought over here, will ſerve in this Place, except what Changes may ariſe from an external Cauſe; and I ſhall no more fear the Fall of my Building, than the Inhabitants of the Country, from whence thoſe Materials were brought, would do, if they had built a Houſe with them. *Thus the Power which cauſes the Coheſion of Parts, and that which gives Weight to Bodies, are the ſame in all Countries.*

I have uſed ſuch Kind of Food for ſo many Years, therefore I will uſe it again to Day without Fear.

When I ſee Hemlock, I conclude it to be poiſonous, tho' I never made an Experiment of that very Hemlock I ſee before my Eyes.

All theſe Reaſonings are grounded upon Analogy; and there is no Doubt, but our Creator has in many Caſes left us no other Way of Reaſoning, and therefore it is a right Way.

Which being once proved, we may afterwards make uſe of the ſame Method in other Matters, where no abſolute Neceſſity forces us to reaſon at all. When an Argument is good in one Caſe, there is no Reaſon why we ſhould refuſe our Aſſent to it in another. For who can conceive, that Things proved the ſame Way are not equally certain? Beſides, tho' we conclude in general, that this Method of Reaſoning is right from the Neceſſity of uſing it; yet it does not follow that particular Reaſonings depend upon that Neceſſity. I conclude from Analogy, that Food is not poiſonous; but is that Argument only good, when I am hungry?

In Phyſics then we are to diſcover the Laws of Nature by the Phænomena, then by Induction prove them to be general Laws; all the reſt is to be handled

dled Mathematically. Whoever will ſeriouſly examine, what Foundation this Method of Phyſics is built upon, will eaſily diſcover this to be the only true one, and that all Hypotheſes are to be laid aſide.

So much for the Method of philoſophiſing. I have now a Word to ſay of the Work itſelf, of which this is the firſt Tome.

The whole Work is divided into four Books. The firſt treats of Body in general, and the Motion of Solid Bodies. The Second of Fluids. What belongs to Light is handled in the Third. The fourth explains the Motions of Celeſtial Bodies, and what has a Relation to them upon Earth. The two firſt Books are contained in this Tome.

In order to render the Study of Natural Philoſophy as eaſy and agreeable as poſſible, I have thought fit to illuſtrate every thing by Experiments, and to ſet the very Mathematical Concluſions before the Reader's Eyes by this Method.

He that ſets forth the Elements of a Science, does not promiſe the learned World any thing new in the main Therefore I thought it needleſs, to point out where what is here contained is to be found. I have made my Property of whatever ſerved my Purpoſe; and I thought giving Notice of it once for all, was ſufficient to avoid the Suſpicion of Theft. I had rather loſe the Honour of a few Diſcoveries, diſperſed here and there in this Treatiſe, than rob any one of theirs. Let who will then take to himſelf what he thinks his own: I lay claim to nothing.

As to Machines which ſerve for making the Experiments, I have taken Care to imitate ſeveral from other Authors, have altered and improved others, and added many new ones of my own Invention. And no Wonder I ſhould be forced to that Neceſſity, having made Experiments upon many

Things never tried perhaps by any one before. For Mathematicians think Experiments superfluous, where Mathematical Demonstrations will take Place: But as all Mathematical Demonstrations are abstracted, I do not question their becoming easier, when Experiments set forth the Conclusions before our Eyes; following therein the Example of the English, *whose Way of teaching Natural Philosophy gave me Occasion to think of the Method I have followed in this Work. I shall always glory in treading in their Footsteps, who, with the Prince of Philosophers for their Guide, have first opened the Way to the Discovery of Truth in Philosophical Matters.*

As to the Machines, I will say thus much more by Way of Advertisement, That most of them have been made by a very ingenious Artist of this Town, and no unskilful Philosopher, whose Name is John van Musschenbroek, *and who has a perfect Knowledge of every thing that is here explained. Which Advertisement, I suppose, will not be displeasing to those who may have a Fancy to get some of those Machines made for themselves.*

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MATHEMATICAL ELEMENTS OF *Natural Philosophy*, CONFIRMED BY EXPERIMENTS.

BOOK I.

PART I. *Of BODY in General.*

CHAP. I.
Of the Scope of Natural Philosophy, and the Rules of Philosophizing.

NATURAL PHILOSOPHY treats of *Natural Things*, and their *Phænomena*.

DEFINITION I, and II.

Natural Things *are all Bodies: And the Assemblage or System of them all is called the* Universe.

DEFINITION III.

2 Natural Phænomena are *all Situations and all Motions of Natural Bodies, not immediately depending upon the Action of an intelligent Being; and which may be observed by our Senses.*

We do not exclude, out of the Number of Natural Phænomena, those which happen in our Bodies by our Will; for they are produced by the Motion of our Muscles, and their Action depends upon another Motion: In these, there is only that Motion which arises from the immediate Action of the Mind, and is entirely unknown to us, which is not a Natural Phænomenon.

All these Motions are performed by certain Rules, and always subject to the same Laws.

The Sun rises and sets daily; and the Time of his Rising and Setting may always be determined, according to the Time of the Year, and Latitude of the Place. Plants of the same Kind, under the same Circumstances, are always produced and grow in the same Manner: And so on in other Cases. Nay, even in those things which appear to be wholly fortuitous and uncertain, certain Rules are without doubt observed.

3 Natural Philosophy *explains Natural Phænomena*; that is, gives an Account of their Causes.

In enquiring after those Causes, BODY in general is to be examined; and then the Rules which the Creator has established, according to which, Motions are to be perform'd. These *Rules* are called *Laws of Nature.*

DEFINITION IV.

4 A Law of Nature then is, *the Rule and Law, according to which God resolved that certain Motions should always, that is, in all Cases, be performed.*

Every

Every Law does immediately depend upon the Will of God.

Alſo in reſpect to us, we call a Law of Nature, every Effect which in all Occaſions is produced after the ſame Manner; although its Cauſe is unknown to us, and we do not ſee that it flows from any Law known to us.

For we make no Difference between a thing which immediately depends upon the Will of God, and what it produces by the Intermediation of a Cauſe of which we have no Idea.

The Laws of Nature are only deduced from an Examination of Natural Phænomena.

By Help of the Laws, thus diſcovered, other Phænomena muſt be explained.

In order to find out the Laws of Nature, Sir *Iſaac Newton*'s following Rules are to be obſerved.

RULE I.

We are not to admit more Cauſes of Natural 5
Things than ſuch as are true, and ſufficient for explaining their Phænomena.

RULE II.

Natural Effects of the ſame Kind have the ſame 6
Cauſes.

Such Qualities of Bodies, whoſe Virtue cannot 7
be increaſed and diminiſhed, and which belong to all Bodies upon which Experiments may be made, muſt be looked upon as Qualities of all Bodies.

CHAP. II.

Of BODY in General.

8 WHAT we first consider in Body is *Extension.*

What is meant by Extension, no Body is ignorant of. Its Idea is most simple, and almost always obvious to our Mind; from whence it is very intelligible, tho' we want Words to describe it.

Every Body has Extension; without Extension there is no such thing as Body. And yet all that has Extension is not always a Body; although it is impossible to determine, how Body differs from mere Space, till the other Properties of Body shall first be ascertained.

9 The Second Thing to be examined in Body is *Solidity*. Body, having no Power to remove itself, will consequently exclude every other Body from the Place possessed by it; and the most fluid, as well as the hardest Bodies, have this Property.

10 The Third Property of Body is *Divisibility*; because if a Body be extended, it is also divisible; for you may always conceive one Extension less than another. From whence we see, that there are Parts in all Extensions; which Parts in a Body may be separated from each other; Because,

11 Body hath a Fourth Property, that is, that it may be carried from one Place to another; whence it is said to be *Moveable.*

All Obstacles being removed, a Body yields to the least Blow: Nevertheless there is a greater Force required to move a Body with a greater Celerity than with a less, as also to move a greater than a smaller Body, allowing their Velocity to be equal. There is also a greater Force required

required in the ſame Caſe to ſtop the Velocity of different Bodies in Motion. Hence it is, that Bodies at Reſt, and Bodies in Motion, endeavour to continue in their State.

This ariſes from the Inactivity of Matter, (*Inertia*) which in all Bodies is ever proportionable to their Quantity of Matter, becauſe it equally belongs to all the Particles of Matter.

All Bodies have ſome Figure; whence *Figurability* (that is, to be of ſome Shape or Figure) is commonly eſteemed one of the eſſential Properties of Body, though it ſeems rather to be derived from other Properties. 12

If a Body be divided on every Side, and thoſe Parts removed; what remains in the Middle is terminated on every Part, and conſequently has a certain Figure. The ſame Body is capable of having different Figures; becauſe it may be divided into Parts, and thoſe Parts placed in different Order in reſpect to each other. Neither does it imply a Contradiction to ſay, that a Body, that ſhould have no Figure, would be an infinite Body.

CHAP. III.

Of Extenſion, Solidity, and Vacuity.

HERE the Queſtion (ſo often handled by the Learned) concerning a *Vacuum*, is to be conſidered; namely, Whether there be an Extenſion void of all Matter; for this Extenſion is called a *Vacuum*, an *Emptineſs* or *mere Space*. 13

That there is really a Vacuum, is proved from Phænomena: This Propoſition therefore ſhall be hereafter more fully treated of.

The Poſſibility of a Vacuum appears from the bare Examination of Ideas. For whatever we conceive to be poſſible may exiſt.

The Queſtion therefore amounts to this, *viz.* Whether we have an Idea of an Extenſion that is not ſolid?

We acquire an Idea of Solidity by the Touch: We feel that ſome Bodies reſiſt us; and indeed thoſe Bodies reſiſt us every Moment, that hinder us from deſcending to the loweſt Places: From which Reſiſtance it appears, that a Body excludes every other Body from the Place which itſelf takes up; that is, it appears that a Body is ſolid; which Idea of Solidity we transfer to thoſe more ſubtile Bodies, which, by reaſon of the Smallneſs of their Parts, eſcape our Senſes; and we find by Experience, that even thoſe reſiſt other Bodies, as well as the hardeſt.

14 *Experiment.*] The Air, in which we live, does almoſt always eſcape our Sight and Touch; yet in a Syringe, that is cloſe ſhut at the End, it reſiſts the Piſton, ſo that it can be puſh'd to the Bottom of the Syringe by no Force.

The Idea of Solidity is not indeed contained in the Idea of Extenſion; that only follows from Contact, but this may be had without it; for if a Man had never touch'd a Body, he would have no Notion of Solidity.

Let any one obſerve an Image projected in the Air, or repreſented between a Concave Mirror and the Object; ſuch an Image does not reſiſt, and yet it ſeems to be a Body as denſe as the Object itſelf; for the Colours may appear more vivid in the Image than in the Object itſelf: If a Man had never ſeen any thing elſe but ſuch Images, and his own Body was like ſuch an Image, could he have any Idea of Solidity? It does not appear that he could; and yet he would certainly have an Idea of Extenſion.

As

As we are here disputing of Ideas, we shall not consider what the above-mentioned Image is; it is enough that there is such a Thing.

All the Difference between *Space* and *Body* does 15
not consist in a Privation of Solidity.

That Space is infinite, and can be contained by no Limits, is plain to any one that attentively considers it.

We plainly see that Space has Parts, but they cannot be separated from one another, being immoveable as Space itself.

The Idea of Space is very simple; that of Body is more complex, it may be moved, its Parts are separated, and what is finite is easily conceived.

Solidity is by some call'd *Impenetrability*; and 16
they endeavour to deduce it from the Nature of Extension: For Example, one cannot add one cubic Foot of Extension to another cubic Foot of Extension, without having two cubic Feet; for each of them has all that is required to constitute that Magnitude; therefore one Part of Space excludes all others, and cannot admit them.

Answer. This is all true, because the Parts of Space are immoveable; but it would be false, if it was not that it would imply a Contradiction, to suppose one Part of Space conveyed to another Place: And the Consequence follows only from the Immobility, not from the Impenetrability or Solidity of the Parts of Space.

CHAP. IV.

Of the Divisibility of Body in infinitum; *and of the Subtilty of the Particles of Matter.*

THE Extension of a Body implies its Divi- 17
sibility; that is, one may consider Parts in it.

 Yet

Yet the Diviſibility of Body differs from the Diviſibility of Extenſion; for its Parts may be ſeparated from one another. But as this Property depends upon Extenſion, it muſt be examined under the Conſideration of Extenſion. And then we may eaſily transfer to Body what is demonſtrated.

18 Body is diviſible *in infinitum*; that is, you cannot conceive any Part of its Extenſion ever ſo ſmall, but ſtill there may be a ſmaller.

Let there be a Line AD perpendicular to BF, (*Plate* I. *Fig.* 1.) and another as GH at a ſmall Diſtance from A, alſo perpendicular to the ſame Line, with the Centers C, C, C, *&c.* and Diſtances CA, CA, *&c.* deſcribe Circles cutting the Line GH in the Points *e*, *e*, *&c.* The greater the Radius AC is, the leſs is the Part *e*G; the Radius may be augmented *in infinitum*, and therefore the Part *e*G may be diminiſhed in the ſame Manner; and yet it can never be reduced to Nothing; becauſe the Circle can never coincide with the Right Line BF.

Therefore the Parts of any Magnitude may be diminiſh'd *in infinitum*, and there is no End of ſuch a Diviſion.

The ſame Thing may be proved by a great many other Mathematical Demonſtrations.

19 The chief Objections are, — That an Infinite cannot be contained by a Finite; — That it follows from a Diviſibility *in infinitum*, that all Bodies are equal, or, that one Infinite is greater than another.

But theſe are eaſily anſwer'd; for to an Infinite may be attributed the Properties of a finite and determined Quantity. Who has ever proved that there could not be an infinite Number of Parts infinitely ſmall in a finite Quantity; or that all Infinites are equal? The contrary is demonſtrated

ſtrated by Mathematicians in innumerable Inſtances.

There are alſo other Objections propoſed, ſuppoſing that we affirm an actual Diviſion of a Body into an infinite Number of Parts ſeparated from one another. We neither defend nor conceive ſuch a Diviſion: We have demonſtrated, that however ſmall a Body is, it may ſtill be farther divided; and, upon that Account, we believe that we may call that a Diviſion *in infinitum*, becauſe what has no Limits is call'd *infinite*.

There are no ſuch Things as Parts infinitely 20
ſmall; but yet the Subtilty of the Particles of ſeveral Bodies is ſuch, that they very much ſurpaſs our Conception; and there are innumerable Inſtances in Nature of ſuch Parts that are actually ſeparated from one another.

Mr. *Boyle* has proved it by ſeveral Arguments.

He ſpeaks of a ſilken Thread 300 Yards long, 21
that weighed but two Grains and a Half.

He meaſured Leaf-Gold, and found by weigh- 22
ing it, that 50 ſquare Inches weighed but one Grain: If the Length of an Inch be divided into 200 Parts, the Eye may diſtinguiſh them all; therefore there are in one ſquare Inch 40000 viſible Parts; and in one Grain of Gold there are two Millions of ſuch Parts; which viſible Parts no one will deny to be farther diviſible.

A whole Ounce of Silver may be gilt with 23
eight Grains of Gold, which is afterwards drawn into a Wire thirteen thouſand Foot long.

In odoriferous Bodies we can ſtill perceive a 24
greater Subtilty of Parts, and which are ſeparated from one another; ſeveral Bodies ſcarce loſe any ſenſible Part of their Weight in a long Time, and yet continually fill a very large Space with odoriferous Particles: Whoever will be at the Pains to make Calculations concerning thoſe ſub-

tile

tile *Effluvia*, will find the Number of Parts to be amazing.

25 By Help of Microſcopes, ſuch Objects as would otherwiſe eſcape our Sight, appear very large: There are ſome ſmall Animals ſcarce viſible with the beſt Microſcopes; and yet theſe have all the Parts neceſſary for Life, Blood, and other Liquors: How wonderful muſt the Subtilty of thoſe Particles be, which make up ſuch Fluids!

We cannot end this Chapter more aptly than by the following *Theorem*, which is eaſily deduced from what has been ſaid of the Subtilty of Matter.

THEOREM.

26 ANY Particle of Matter, how ſmall ſoever, and any finite Space, how large ſoever, being given; it is poſſible that that ſmall Sand, or Particle of Matter, ſhall be diffuſed thro' all that great Space, and ſhall fill it in ſuch Manner that there ſhall be no Pore in it, whoſe Diameter ſhall exceed a given Line.

CHAP. V.

Concerning the Coheſion of Parts; where we ſhall treat of Hardneſs, Softneſs, Fluidity, and Elaſticity.

27 ALL Bodies, that are perceived by our Senſes, conſiſt of very ſmall Parts, no one of which is indiviſible in itſelf; but all of them are in reſpect to us: For all the Diviſion we can make, is only a Separation of Parts.

When a great Force is required to make ſuch a Diviſion, a Body is ſaid to be *Hard*.

If the Parts yield more eaſily and fall in by being preſs'd, ſuch a Body is ſaid to be *Soft*.

But

But this great and lesser Force, in the common Signification, determine nothing; for a Body that is Hard, in respect to one Man, seems Soft to another.

Definition I.

A Body is said to be *Hard*, in a Philosophical
Sense, *when its Parts mutually cohere, and do not at* 28
all yield inwards, so as not to be subject to any Motion in respect to each other, without breaking the Body.

Definition II.

A Body is said to be *Soft*, in a Philosophical Sense,
when its Parts yield inwards, and slip in upon one 29
another, even tho' it may require a Blow with a Hammer to do it.

Definition III.

A Body whose Parts yield to any Impression, and 30
by yielding are easily moved, in respect to each other, is call'd a Fluid.

All these Things depend upon the Cohesion of Parts; the closer a Body is, the nearer it approaches to perfect Hardness.

But the Hardness of the smallest Parts does not differ from their Solidity; it is an essential Property of a Body, which is no more to be explained, than why a Body is extended, or a Mind thinks.

I do not know whether all Bodies consist of Parts that are equal and alike: And there are also several Things very difficult, in Relation to the Cause of the Cohesion of the small Parts of Bodies.

The Laws of Nature, which are admitted here, are deduced from *Phænomena*.

It is a particular Law of Cohesion, that all the 31
Parts have an *attractive Force.*

DEFINITION IV.

By the Word *Attraction* I underſtand, *any Force by which two Bodies tend toward each other*; tho' perhaps it may happen by Impulſe.

But that *Attraction* is ſubject to theſe Laws; *That it is very great, in the very Contact of the Parts; and that it ſuddenly decreaſes, inſomuch that it acts no more at the leaſt ſenſible Diſtance; nay, at a greater Diſtance, it is changed into a* repellent Force, *by which the Particles fly from each other.*

By Help of this Law, ſeveral Phænomena are very eaſily explained; and that Attraction and Repulſion is fully proved by a vaſt Number of chymical Experiments. That there is ſuch a Thing, appears alſo from the following experiments.

32 *Experiment* 1.] WE ſee that in all Liquors all the Parts attract one another, from the Spherical Figure that the Drops always have; and alſo becauſe there is no Liquor whoſe Parts are not ſticking to one another, which is evidently true even in *Mercury* itſelf.

33 *Experiment* 2.] BUT this mutual Attraction of Particles is much better proved; becauſe in all Liquids, two Drops, as A and B (*Plate* I. *Fig.* 2.) as ſoon as they touch one another ever ſo little, they immediately run into one larger Drop, as F. All which Things, as they alſo happen in liquified Metals, it follows, that the Parts of which they are compounded do attract one another, even when they are disjoined by the Motion of the Fire.

Theſe Appearances do not depend upon the Preſſure of the Air, becauſe they alſo happen where there is no Air; neither do they depend upon the Preſſure of any other Matter equally from all Sides; for though ſuch a Preſſure is able to keep the Drops

Drops to their Spherical Figure, it can by no means bring them to it at firſt.

In the Oval Drop *acbd*, (*Plate* I. *Fig.* 3.) the 34 Preſſures upon the Surfaces *ad* and *cb* are leſs than the Preſſures upon the Surfaces *ac*, *db*; for the Drop is ſuppos'd to be preſs'd equally from all Parts, therefore the Preſſure is leſs in a leſs Space: Yet the Drop can never become round, till thoſe leſſer Preſſures overcome the greater, which is abſurd.

On the contrary, in Attraction, the greater the Number is of the Particles which attract one another between two Particles, the greater is the Force with which they are carried towards one another; which produces a Motion in the Drop, till the Diſtances between the oppoſite Points in the Surface become every where equal; which can only happen in a Spherical Figure.

Several Bodies act upon other extraneous Bodies by this Attraction. I ſhall give a few Examples, in which the Effects of it are moſt remarkable.

Experiment 3.] Immerge in Water the Ends of 35 ſmall Glaſs Tubes open at both Ends, in the Manner repreſented in *Plate* I. *Fig.* 4. The Water will ſpontaneouſly aſcend in them, and ſo much the higher as the Diameter is leſs. It is not required that the Tubes be extremely ſmall; for the Experiments will ſucceed in Tubes whoſe Bore is the ſixth Part of an Inch. That this is not to be attributed to the Preſſure of the Air, appears from the following Experiment.

Experiment 4.] Having fixed the ſmall Tubes A 36 to a Piece of Cork, and ſuſpended them with the Braſs Wire AB, (*Plate* I. *Fig.* 5.) pump out the Air from the Recipient R, which ſtands upon the Braſs Plate of the Air-Pump; then by moving the Wire AB, AB, the Tubes may be immersed in the Water which is contained in the Glass CD, and the Water in this Case will rise up into the Tube just as it did in the foregoing Experiment. How the Wire may be moved, without letting the Air into the Recipient, will be explained hereafter.

37 *Experiment* 5.] ABCD are two Glass Plates, or Planes, (*Plate* I. *Fig.* 6.) touching one another at AB, but a little separated at CD, by thrusting a thin Plate of any Kind between them; they are sustained by the wooden Frame HILM, in such Manner, that the Side DC is always at the same Height; the Planes may be brought to make any Angle with the Horizon, by raising the End AB where they are joined, the Cylinder NO likewise sustaining the Plane in any Position. The Screw P makes fast the Cylinder at any Height.

A Drop of Water or Oil, G, is put between the Planes, so as to touch both the Planes, which must before-hand be made wet with the same Liquor; this Drop is attracted by both Planes, but the Attraction has a greater Effect upon the Drop, where their Distance is the less: that is, a greater at *e* than at *f*, therefore the Drop is moved towards *e*; that is, ascends, and moves upwards the faster, in Proportion as it is higher, the Surfaces in which the Drop touches the Glasses growing very much, where the Distances between the Planes is diminished. The Angle of Inclination of the Planes may be so increased, that the Gravity of the Drop shall balance the Attraction, and then the Drop is at rest; and in that Case, if you raise the End AB of the Planes still higher, the Drop will descend by its Gravity, which will then overcome the Attraction.

Experi-

Experiment 6. Let two Glaſs Planes, ABCD, 38
(*Plate* I. *Fig*. 7.) touch one another at AB, and at CD let them be a little ſeparated by the Interpoſition of ſome thin Plate, and then let their Ends be immers'd into Water tinged with ſome Colour, in ſuch Manner, that the Sides AB and CD may be in a vertical Poſition, the Planes having been moiſtened with the ſame Liquor beforehand. The Water will riſe between thoſe Planes by their Attraction, and riſe higheſt where the Planes are neareſt together; and as their Diſtance continually decreaſes from CD to AB, the Water riſes up to different Heights in every Place, and makes the Curve Line *e f g*.

Experiment 7.] Quickſilver unites itſelf to Tin 39
and Gold; alſo Water and Oil ſtick to Wood and clean Glaſs.

We have Inſtances of Repulſion between Water 40
and Oil, and generally between Water and all unctuous Bodies; between *Mercury* and Iron; as alſo between the Particles of any Duſt.

Experiment 8.] If any greaſed Body, lighter 41
than Water, is laid upon the Surface of Water, or Piece of Iron upon *Mercury*, the Surface of the Fluid will be depreſſed about the Bodies laid upon it, as it appears about the Ball A (*Plate* I. *Fig*. 8.) And after the ſame Manner, where the Attraction obtains, the Surface of the Liquor is higher about the floating Bodies, as about the Ball B, and does not run to a Level by its Gravity; ſo here where the repellent Force exerts its Action, Liquors, notwithſtanding their Gravity, do not run down to fill up the Cavities which are made round the floating Bodies.

Upon this depend all the Phænomena of very 42
light Glaſs Bubbles (*Plate* I. *Fig*. 18.) which ſwim

upon

upon Water; when they are clean, the Water riſes about them all round, as at B; but, when they are made greaſy, the Water makes an Hollow all round them, as at A; in the Glaſs Veſſel where the Experiments are made, the Water alſo ſtands higher all round next to the Glaſs, as at C and D; but when the Glaſs is ſo fill'd that the Water runs down from all Parts, then, by the mutual Attraction of the Parts of the Water, it ſtands higher in the Middle than at the Sides, and forms the convex Surface ABC: (*Plate* I. *Fig.* 9.) From theſe Principles only can the following Experiments be explain'd.

Experiment 9, 10, 11, 12 and 13.] When a Glaſs is not quite full of Water, a clean Glaſs Bubble always runs to the Side, and there ſticks, provided it be not laid on too far from it. The Bubble is preſs'd every Way by the Water, when it comes to the Side of the Veſſel; the ſame Force, by which the Water is raiſed there, does in part take off that Preſſure; ſo the Preſſure on the other Side overcomes, and the Bubble is moved towards the Side of the Glaſs.

When the Glaſs is ſo full as to be ready to run over, the Bubble goes off it ſelf from the Side to the Middle of the Glaſs, for the ſame Reaſon; becauſe the Force, by which the Water is raiſed in the Middle, does alſo diminiſh the Preſſure upon the Bubble towards the Middle.

Juſt the Reverſe happens when the Bubble is greaſy, becauſe that Force, by which the Water and the Bubble repel one another, is greateſt where the Water is higheſt.

Two clean Bubbles, or two greaſy ones, run towards each other. As to clean Bubbles, we have juſt given the Reaſon; when they are made greaſy, there is a Cavity round each of them;

 and

and where the Cavities join, the Preſſure is diminiſh'd, and the Bubbles run that Way.

If one Bubble be clean, and the other greaſy, they fly from one another, for the Reaſons before given.

The Particles of any Salts attract one another 43 with a very great Force, as appears by ſeveral Experiments: The following will be ſufficient to prove, that that Attraction exerts it ſelf at a very ſmall Diſtance, and the repellent Force at a greater.

Experiment 14.] Diſſolve Salt in Water, and, when that Water is reduced into Vapour, the ſmall ſaline Particles will unite together and form greater Lumps; which proves the Attraction.

Theſe Particles are all equal, and of the ſame Figure: Whence it follows, that the leaſt Parts, of which they are form'd, had every where the ſame Situation in reſpect to each other; that is, were every where diffuſed in the Water at equal Diſtances; which cannot be, unleſs they all repel one another with equal Forces.

The Elaſticity of Bodies, namely, that Property 44 *whereby they return to their former Figure, when it has been alter'd by any Force, is eaſily deduced from what has been ſaid*: For if a compact Body be dented in without the Parts falling into that Dent, the Body will return to its former Figure, from the mutual Attraction of its Parts.

We ſhall alſo in its proper Place ſhew, that 45 that Property of the Air, which is call'd its *Elaſticity*, ariſes from the Force whereby its Parts repel one another.

And leſt any one ſhould imagine, becauſe we 46 don't give the Cauſe of the ſaid Attraction and

Repulsion, that they must be look'd upon as *occult Qualities*: We say here with Sir *Isaac Newton*, 'That we consider those Principles not as 'occult Qualities, which are supposed to arise 'from the specifick Forms of Things; but as universal Laws of Nature, by which the Things 'themselves are form'd: For the Phænomena of 'Nature shew that there are really such Principles, 'tho' it has not been yet explain'd what their 'Causes are. To affirm that the several Species of 'Things have occult specifick Qualities, by which 'they act with a certain Force, is just saying nothing. But from two or three Phænomena of 'Nature to deduce general Principles of Motion, 'and then explain in what Manner the Properties 'and Actions of all things follow from those 'Principles, would be a great Progress made in 'Philosophy, tho' the Causes of those Principles 'should not yet be known.

PART

PART II.
Of the Motion of Solid Bodies.

CHAP. VI.
Of Motion *in general; where we shall speak of* Place *and* Time.

THE Subject we are now entring upon has a large Scope in Physicks: All that happens in natural Bodies belongs to Motion, and even what has been said of the Cohesion of Parts, has a Relation to it: For though the Parts are not moved in their Cohesion itself, yet that Cohesion cannot be explain'd, nor can what is said about it be confirm'd by Experiments, without Motion.

Motion *is a Translation from one Place to ano-* 47
ther Place, or a continual Change of Place. Every Body has an Idea of it; and Philosophers have in vain laboured to find a Definition of the simple Idea, and proved with a great deal of Pains, that one may come to be ignorant of a Thing, which otherwise every Body knows.

Place *is the Space taken up by a Body*; of which 48
may be said, what has just been said concerning *Motion.*

It is Twofold; *True* or *Absolute*, and *Relative.*

DEFINITION I.

True Place is that Part of immoveable Space, 49
which a Body takes up.

DEFINITION II.

50 Relative Place, which only can be diſtinguiſh'd by our Senſes, *is the Situation of a Body in reſpect of other Bodies.*

True Place is often changed, whilſt relative Place is not, and ſo reciprocally.

Whence ariſes a *True* and *Abſolute* Motion, and another Sort call'd a *Relative* Motion.

Whilſt a Body moves, Time goes on.

51 Time alſo is Twofold; *True* or *Abſolute*, and *Relative*.

True Time has no Relation to the Motion of Bodies, nor to the Succeſſion of Ideas in an Intelligent Being, but by its Nature it always flows equally.

DEFINITION III.

52 Relative Time *is Part of the true Time meaſured by the Motion of Bodies*; and this is the only Time that our Senſes perceive.

All Motion may become ſwifter, as likewiſe a Body may move ſlower than it did before; and it is very likely that there is no Motion of Bodies wholly equable; whence it follows, that *relative* Time differs from *true* Time, which never flows faſter nor ſlower.

DEFINITION IV.

53 *That Affection of Motion, by which a Body runs thro' a certain Space in a certain Time, is call'd* Celerity *or* Velocity; which is greater or leſs, according to the *Bigneſs* of that *Space, to which it is always proportionable.*

DEFINITION V.

54 The greater Force is impreſs'd upon a Body to make it change its Place, the greater is its Motion;

Motion; and that Force is called the *Quantity* or *Momentum of Motion.*

Definition VI.

The Direction of Motion *is in a Right Line, which we suppose drawn towards the Place where the moving Body tends.* 55

Definition VII.

A Power *is any Force acting upon a Body to move it.* 56

Definition VIII.

The Greatness of that Force is call'd, the *Intensity of the Power.* 57

CHAP. VII.

Of Motions compared together.

AXIOMS.

I.

THE Quantity, *or* Momentum of Motion, *follows the Proportion of the Cause producing the Motion.* 58

II.

The whole Effects of Motions, produced at the same Time, have the same Relation to each other, as the Momenta *of those Motions.* 59

If two equal Motions act in a contrary Direction, they destroy each other; and the one can never overcome the other. 60

Bodies in Motion may differ in two Respects, either in Respect to the Quantity of Matter in each, or in Respect of the Space gone thro' in the same Time, that is, in Respect of the Velocity, * and there is no other Difference: These 61

*53.

two Things therefore, and only these two, are to be considered in comparing of Motions.

When the Velocities are equal, nothing is to be considered but the Quantities of Matter; and if it be double in one Body, the Quantity of Motion in that Body will also be double; because such a Body is made up of two Bodies, each equal to the least Body, and moved with the same Celerity as the little one. The same may be said of all other Relations between two Bodies; whence we deduce this general Rule:

62 *In equal Bodies that move with the same Velocity, the Quantity of Motion is as the Quantity of Matter in each.*

When the Quantities of Matter are equal, the Velocities only are to be considered: And then

63 *In equal Bodies, the* Momenta *are as the Velocities*: That is, as the Spaces gone thro' in the
53. same Time. For the going thro' those Spaces are the whole Effects of the Motions produced in that Time, and are to one another as those Spaces; therefore the *Momenta* also are in the same Pro-
59. portion.

In order to determine the Relation between two
64 Motions, *when the Velocities are unequal, and the Bodies different in Quantity of Matter; you must find two Quantities that are to one another as the Masses and as the Velocities. Multiplying the Velocities of each Body by its Mass or Quantity of Matter, the Products will be to each other in the said Proportion.*

When for Example, the Velocity is double, and the Mass triple, a double Quantity of Motion must be tripled; therefore it will be sextuple: This is the Case when in one Body the Velocity is two, and four in another; and the Mass of the first Five, the Mass of the other being

ing Fifteen; multiplying each Mass by its Velocity, the Products are 10 and 60, the last of which is six Times the first.

This is called a Ratio, compounded of the Ratio of the Masses and the Celerities.

A greater Body may move more slowly than a less, in such a Manner, that the lesser Body may have an equal Quantity of Motion with, or a greater than the other.

When the Velocity in the lesser Body is to the Velo- 65
city in the greater, as the Mass of the greater to the Mass of the lesser; the Quantities of Motion are equal in the two Bodies.

As much as the Quantity of Motion in the lesser Body is less, in Respect of its Mass, so much is it greater, in Respect of the Velocity: Whence an Equality arises. Likewise in that Case, the Products of the Mass of each Body by its Velocity are equal; and the Celerities are said to be in an inverse Ratio of the Masses, or reciprocally as the Masses.

When such Momenta *of Motion act contrariwise,* 66
*they destroy each other.** *60.

CHAP. VIII.

How to compare the Actions of Powers.

THE Actions of Powers, acting upon Bodies, may be compared together, in the same Manner as the Quantities of Motion; and the same Rules serve for both.

We shall hereafter shew, that a Body once in Motion will continue in that Motion, tho' the Cause that first gave it, ceaseth: So that if a Body should be continually acted upon by any Power, the Motion would become swifter every Moment.

We do not here consider such an Action of Powers; but we take notice only of Powers that act against an Obstacle, in such Manner, that by the Resistance of the Obstacle the Action of the Power is continually destroyed, which is to be observed; for in another Case, the following Demonstrations do not obtain. When therefore we speak here of an Obstacle to be removed by any Power, we speak of the *greatest Obstacle* that can be moved by that Power; for otherwise the Obstacle would not destroy the whole Action of the Power.

The Actions of Powers may differ from one another, both in respect of the Greatness of the Obstacles, and in Respect of the Spaces run thro' by the Obstacles; (that is, by the Points to which the Powers are applied:) These two Things alone are to be consider'd in comparing of Powers.

The Obstacles, which may be removed by Powers, are to one another as the Intensities of
57. the Powers.

From whence it follows, *that the Actions of*
67 *Powers, equal in their Intensities, are to one another as the Spaces run thro'.* For they only differ in that Respect, because the Obstacles are equal.

68 *When the Spaces run thro' are equal, those Actions are as the Intensities.*

69 *When both the Spaces run thro', and the Intensities, are different, the Actions of the Powers are to one another, in a Ratio compounded of the Intensities and the Spaces gone through.*

70 When the Spaces gone thro' are in an inverse Ratio of the Intensities, the Actions are equal.

Definition.

71 We call the *whole Force of Power* its *Action, in Respect to Time*; and therefore the whole Forces of

of Powers are to one another, as the Actions produced in the same Time.

These Things may be demonstrated in the same Manner, as what has been said in the foregoing Chapter.

CHAP. IX.

General Things concerning Gravity.

PHENOMENON I.

ALL Bodies near the Earth, if hinder'd by no Obstacle, are carried towards the Earth. 72

DEFINITION I.

The Force, by which Bodies are carried towards the Earth, is called Gravity. 73

DEFINITION II.

That Force, in Respect to a Body acted upon by it, is call'd the Weight *of a Body.* 74

PHENOMENON II.

The Force of Gravity acts equally, and every Moment of Time, near the Earth's Surface. 75

There is indeed a small Difference of Gravity in different Countries, which we shall take Notice of hereafter, but it is too small to be consider'd here, especially because it is wholly insensible in neighbouring Countries.

When the Descent of a Body is hinder'd by an 76
Obstacle, it continually presses that Obstacle equally, tending towards the Earth's Center; therefore it may be look'd upon as a Power acting upon an Obstacle; and therefore what has in the foregoing Chapter been demonstrated, concerning Powers, does obtain here also.

PHENO-

PHENOMENON III.

77 *Bodies which descend by the Force of Gravity, (if all Resistance be taken off) fall with the same Velocity.* Which is prov'd by an

Experiment.] Pump out the Air from the tall Recipient AB (*Plate* II. *Fig.* 1.) which is made up of two Glasses, and is about three Foot high: Then from the Top of the Glass within, by moving the Handle CD, let fall a Piece of Gold and a very light Feather just at the same Time, and they will always come down to the Brass Plate of the Pump upon which the Receiver stands, at the same Instant of Time.

78 For making the Experiment readily, the Top of the Recipient is cover'd with a Brass Plate laid upon it. A thin Plate, bent into the Figure E, is fixed to the covering Plate at *e*, by Help of two Screws H, that go thro' two lesser Plates, one of which you see at *gf*, and are joined to the other Plate E.

The Ends of the Plate spring together, and so hold the Feather and the Gold, whilst the Receiver is exhausting.

A Brass Wire runs thro' the Cover, which, by Means of the Handle CD, may be turned round without admitting the external Air; which we shall explain, when we come to speak of the Air-Pump.

The Wire goes thro' an Hole in the upper Part of the Plate *e*, and the End of the Wire, which comes down between the springing Plate, may be seen at L: It is square and hollow, that the Oval Plate I may be joined to it.

You must observe, that the small Diameter of the Oval be small enough for this little Plate to be contained between the Springs E, when their Ends come together.

Now when the Braſs Wire, and by it the Plate I*i* is turned round; by Reaſon of the Difference of Diameters in the Oval, the Ends of the Springs open; and then the Bodies, that are ſuſpended, are let go at the ſame Moment of Time.

The ſame Phænomenon is alſo deduced from another Experiment, which we ſhall mention hereafter. * * 160

79 Hence it follows, *that Gravity in all Bodies*, that is, their Weight, *is proportionable to their Quantity of Matter.** *62

80 Therefore all Bodies conſiſt of Matter that is equally heavy; and the Reaſon that Bodies do not appear equally heavy, is becauſe ſome have more Matter than others under the ſame Bulk; that is, in the ſame Space.

81 When Weight is conſidered as a Power, the Intenſity of the Power is proportional to the Quantity of Matter in the heavy Body; and the Direction of the Power is towards the Center of the Earth.

CHAP. X.

Of the ſingle Pulley, Balance, and of the Center of Gravity.

Definition. I.

82 A Single Pulley *is a little Wheel moveable about its Axis, over which goes a drawing* or *running Rope*, dce (*Plate* I. *Fig.* 10.)

By this Engine, the Direction of the Power is changed, neither is it of any other Uſe when fix'd; for in that Caſe, *if the Force of Power apply'd to the drawing Rope, as* M, *be equal to the Obſtacle* P, *it balances that Obſtacle** ; *for in that Caſe the Power can't* *71 *move*,

move, unless the Obstacle does at the same Time go through an equal Space.

Weights are found, that is, the Quantities of
*79 Matter in Bodies are compared together *, by a Balance, or a Pair of Scales, which is a well known Instrument.

Definition II.

84 The Axis of a Balance *is that Line about which the Balance moves, or rather turns round.*

Definition III.

85 When we consider the Length of the *Brachia*, or of the Beam, then the Axis is to be looked upon as a Point, and called *the Center of the Balance.*

Definition IV.

86 We call Points of Suspension, or Application, *those Points where the Weights really are, or from which they hang freely; or the Scales in which the Weights are placed.*

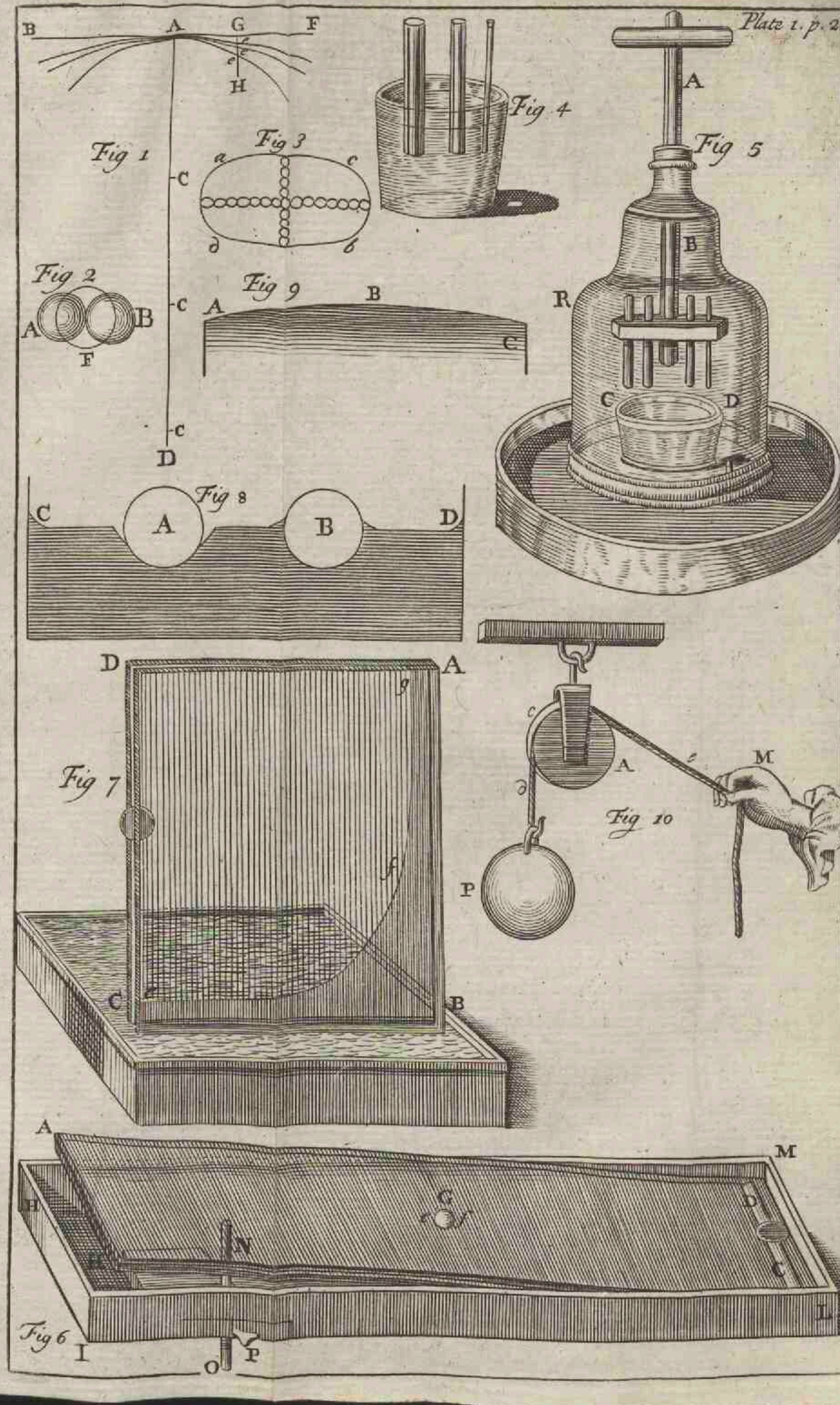
Concerning this Machine, we are to observe,
87 *That the Weight does equally press the Point of Suspension, at whatever Height it hangs from it, and in the same Manner as if it was fixed at that very Point.*

For the Weight, at all Heights, equally stretches
74,75 the Rope by which it hangs. This is also proved by

Experiment 1.] In the Balance AB, the Weight P, by Means of the Rope BD (*Plate* II. *Fig.* 2.] is suspended at different Heights: And the Position of the Balance is not changed by it.

The

Plate 1. p. 26



The Action of a Weight to move a Balance is by so much greater, as the Point pressed by the Weight is more distant from the Center of the Balance; and that Action follows the Proportion of the Distance of the said Point from that Center. 88

When the Balance moves about its Center, the Point B describes the Arc B*b* (*Plate* II. *Fig.* 3.) whilst the Point A describes the Arc A*a*, which is the biggest of the two; therefore, in that Motion of the Balance, the Action of the same Weight is different, according to the Point to which it is applied, and it follows the Proportion of the Space* gone through by that Point: At A there- *67,76
fore it is as A*a*, and at B as B*b*; but those Arcs are to one another, as CA, to CB.

Experiment 2.] The *Brachia* of the Balance AB (*Plate* II. *Fig.* 4.) are divided into equal Parts; and one Ounce applied to the ninth Division from the Center, is as powerful as three Ounces at the third; and two Ounces at the sixth Division act as strongly as three at the fourth, &c.

The Construction of a Balance, for this and some other following Experiments, is plain enough from the Figure, adding to it what is said at *Numb.* 102. Hence it follows, that the Action of a Power to move a Balance is in a Ratio compounded of the Power it self, and its Distance from the Center*; for that Distance is * 69
as the Space gone through in the Motion of the Balance.

Definition V.

A Balance is said to be in Æquilibrio, *when the Actions of the Weights upon each* Brachium, *to move the Balance, are equal; so that they mutually destroy each other*; as appears by the foregoing Experiment. 89

Defi-

Definition VI.

When a Balance is in Æquilibrio, the Weights on each Side are ſaid to *æquiponderate.*

90 *Unequal Weights can æquiponderate.* For this, it is requiſite, that the Diſtances from the Center
89,70 be reciprocally as the Weights. In that Caſe, if each Weight be multiplied by its Diſtance, the Products will be equal.

Experiment 3.] In the above-mentioned Balance (*Plate* II. *Fig.* 4.) one Ounce at the Diſtance of the ninth Diviſion from the Center, æquiponderates with three Ounces at the third Diviſion.

91 The Steel-yard, or *Statera Romana*, which weighs every Thing with one Weight, is made upon this Principle.

Experiment 4.] The Steel-yard AB (*Plate* II. *Fig.* 5.) has two Brachia very unequal; a Scale hangs at the ſhorteſt; the longeſt is divided into unequal Parts: Apply ſuch a Weight to it, that, at the firſt Diviſion, it ſhall æquiponderate with one Ounce laid in the Scale; then the Body to be weighed is put into the Scale, and the above-mentioned Weight is to be moved along the longeſt Brachium, till you find the Æquilibrium; the Number of Diviſions, between the Body and the Center, ſhew the Number of Ounces that the Body weighs, and the Subdiviſions the Parts of an Ounce.

92 Upon this Principle alſo is founded the deceitful Balance, which cheats by the Inequality of the Brachia.

Experiment 5.] Take two Scales of unequal Weights, in the Proportion of 9 to 10 (*Plate* III. *Fig.* 1.) and hang one of them at the tenth Diviſion

Division of the Balance above described, and the other at the ninth Division, so that there may be an Æquilibrium: If then you take any Weights, which are to one another as 9 to 10, and put the first in the first Scale, and the other in the other Scale, they will æquiponderate.

Several Weights, hanging at several Distances on one Side, may æquiponderate with a single Weight on the other Side. To do this, it is required, that the Product of that Weight, by its Distance from the Center, be equal to the Sum of the Products of all the other Weights, each being multiplied by its Distance from the Center. 93

Experiment 6.] Hang three Weights of an Ounce each, at the second, third and fifth Divisions from the Center, and they will æquiponderate with the Weight of one single Ounce applied at the tenth Division of the other Brachium (*Plate* II. *Fig.* 6.) And the Weight of one Ounce at the sixth Division, and another of three Ounces at the fourth Division, will æquiponderate with a Weight of two Ounces on the other Side at the ninth Division.

Several Weights, unequal in Number, on either Side, may æquiponderate. In that Case, if each of them be multiplied by its Distance from the Center, the Sums of the Products on either Side will be equal; and if those Sums are equal, there will be an Æquilibrium.

Experiment 7.] Hang on a Weight of two Ounces (*Plate* II. *Fig.* 7.) at the fifth Division, and two others, each of one Ounce, at the second and seventh; and on the other Side hang two Weights, each also of one Ounce, at the ninth and tenth Divisions; and these two will æquiponderate with those three.

DEFI-

Definition VII.

The Center of Gravity *is a Point in a Body, about which all the Parts of the Body (whatever Poſition it is in) are* in æquilibrio.

98 When two or more Bodies are joined, whether they are contiguous or ſeparated, they have a common Center of Gravity.

When the Center of Gravity is ſuſtained, the Body remains at reſt.

Experiment 8.] The Body A (*Plate* III. *Fig.* 2.) is ſuſtained and at reſt, becauſe its Center of Gravity is ſuſtained by the Prop F.

When the Center of Gravity is not ſuſtained, the Body moves till that Center comes to be ſuſtained.

Experiment 9.] The Body A, laid upon the Table, will fall, and the Body B will not remain in its Poſition, becauſe their Centers of Gravity are not ſuſtained.

99 Hence may be known, why ſome Bodies, laid upon inclined Planes, will roll off, whilſt ſome only ſlide off.

Experiment 10.] The Body A ſlides, becauſe its Center of Gravity is ſuſtained by an inclined Plane; (*Plate* III. *Fig.* 4.) that is, the Vertical Line, which goes through that Center, *c*, cuts the inclined Plane within the Body. But the Body B rolls, becauſe the Vertical Line, thro' its Center of Gravity, cuts the inclined Plane without the Body.

100 From what has been ſaid it follows alſo, that a Body deſcends, when its Center of Gravity deſcends, that is, is moved towards the Center of the Earth.

Some-

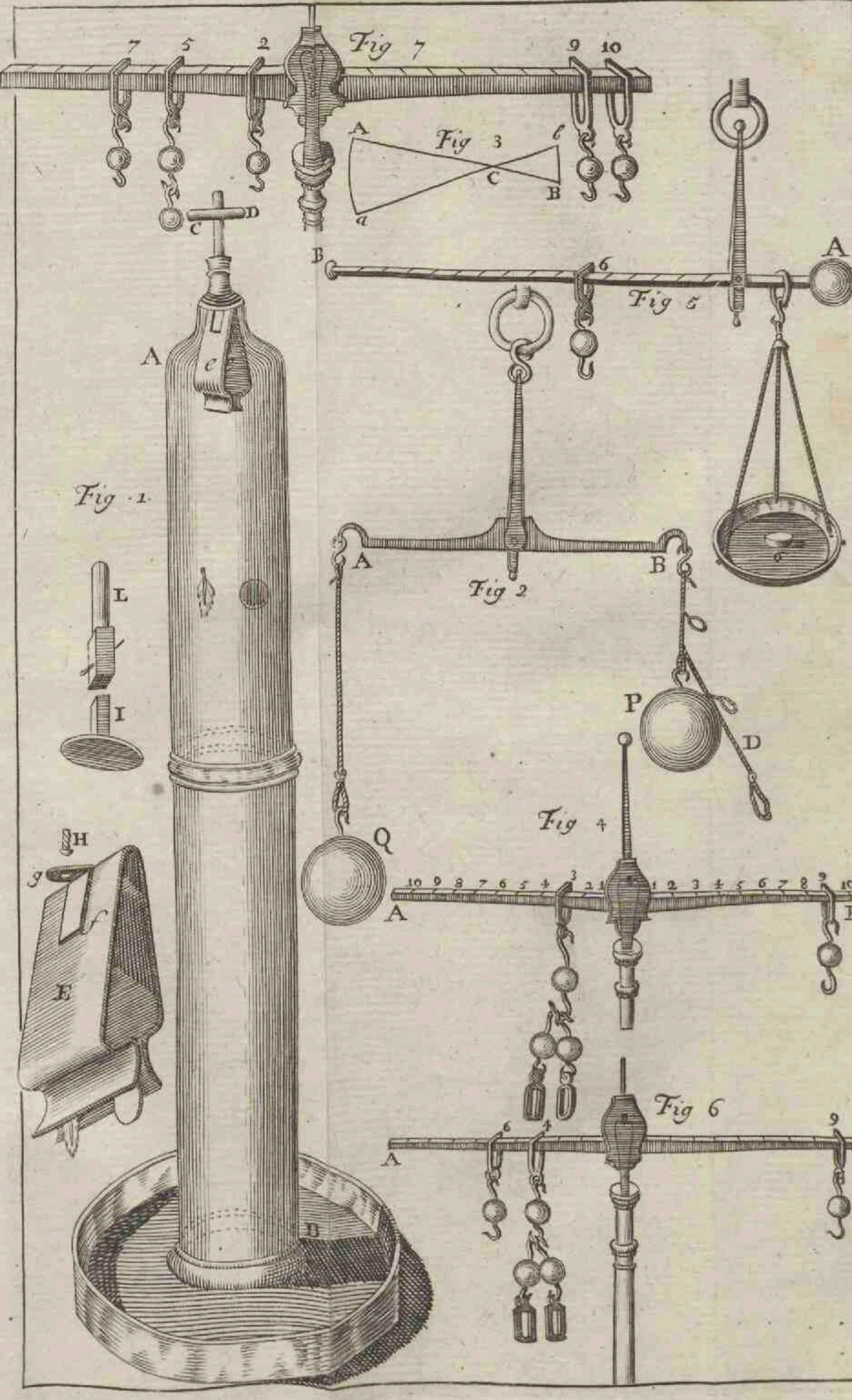


Fig 7
Fig 3
Fig 5
Fig 1.
Fig 2
Fig 4
Fig 6
A
B
C
D
E
H
I
L
P
Q

Sometimes in that Case a Body seems to ascend, and oftentimes it does really ascend; but as all Bodies descend by Gravity, that is, their Centers of Gravity descend; it follows, that a Body seems to ascend by its Gravity, and can really ascend.

Experiment 11.] The Wheel A (*Plate* III. *Fig.* 6.) whose Axis is made of two Cones, the Bases of which join to the Wheel, when put between two Planes, whose Sides D G, F H continued, make the Angle F C D, which has the Base, (supposing a Triangle made) higher than the Vertex, will from H G, the lower Part of the Planes, roll towards F D, the highest Part of the Planes.

By reason of the greater Distance between the Planes at F D, the Wheel A, whose Axis is a Cone both Ways, descends more between the Planes, when it moves towards that Part, and is so carried thither by its Gravity, provided that Descent be greater than the Ascent from the Inclination of the Angle F C D with the Horizon.

Experiment 12.] The Wooden Cylinder A (*Plate* III. *Fig.* 5.) has within it, near the Side, a Leaden Cylinder; their common Center of Gravity is in a Section parallel to the Base, which divides the Cylinder into two equal Parts, and in a Point answering to the Point *c* of the Base.

Whatever Position this Cylinder be laid in, it will move until the above-mentioned Center of Gravity be in the lowest Place which it can come to.

If it be laid upon an inclined Plane, in the Position described in the Figure; the Center of Gravity will descend whilst the Body rises along the Plane, if it be inclined in a fit Manner.

The Body afcends by rolling towards the upper Part of the Plane, but care muft be taken, that, whilft it is endeavouring to roll up, it does not flide down along the Plane; and therefore you muft ufe a Rope, which goes in part round the Cylinder; one End of which is joined to the Cylinder at *f*, and the other is fixed to the Plane at *d*.

101 From what has been faid of the Center of Gravity, it is farther deduced, that whatever Point of a Body or Machine fuftains the Center of Gravity of any Weight, that Point fuftains the whole Weight: So that the whole Force, by which any Body tends towards the Earth, is, as it were, collected to that Center.

Experiment 13.] If the Body AB (*Plate* III. *Fig.* 7.) whofe Center of Gravity is laid upon the Brachium of a Balance, does in any Pofition æquiponderate with any Weight P; it will in any other Pofition, as *a b*, *a b*, æquiponderate with it, provided the Center of Gravity be ftill at C.

102 That a Balance may be perfect, it is required, 1. That the Points of Sufpenfion of the Scales or Weights be exactly in the fame Line as the Center of the Balance. 2. That they be precifely equidiftant from that Point on either Side. 3. That the Brachia of the Balance be as long as they conveniently can. 4. That there be as little Friction as poffible in the Motion of the Beam and Scales. 5. And laftly, that the Center of Gravity of the Beam be placed a little below the Center of Motion.

CHAP.

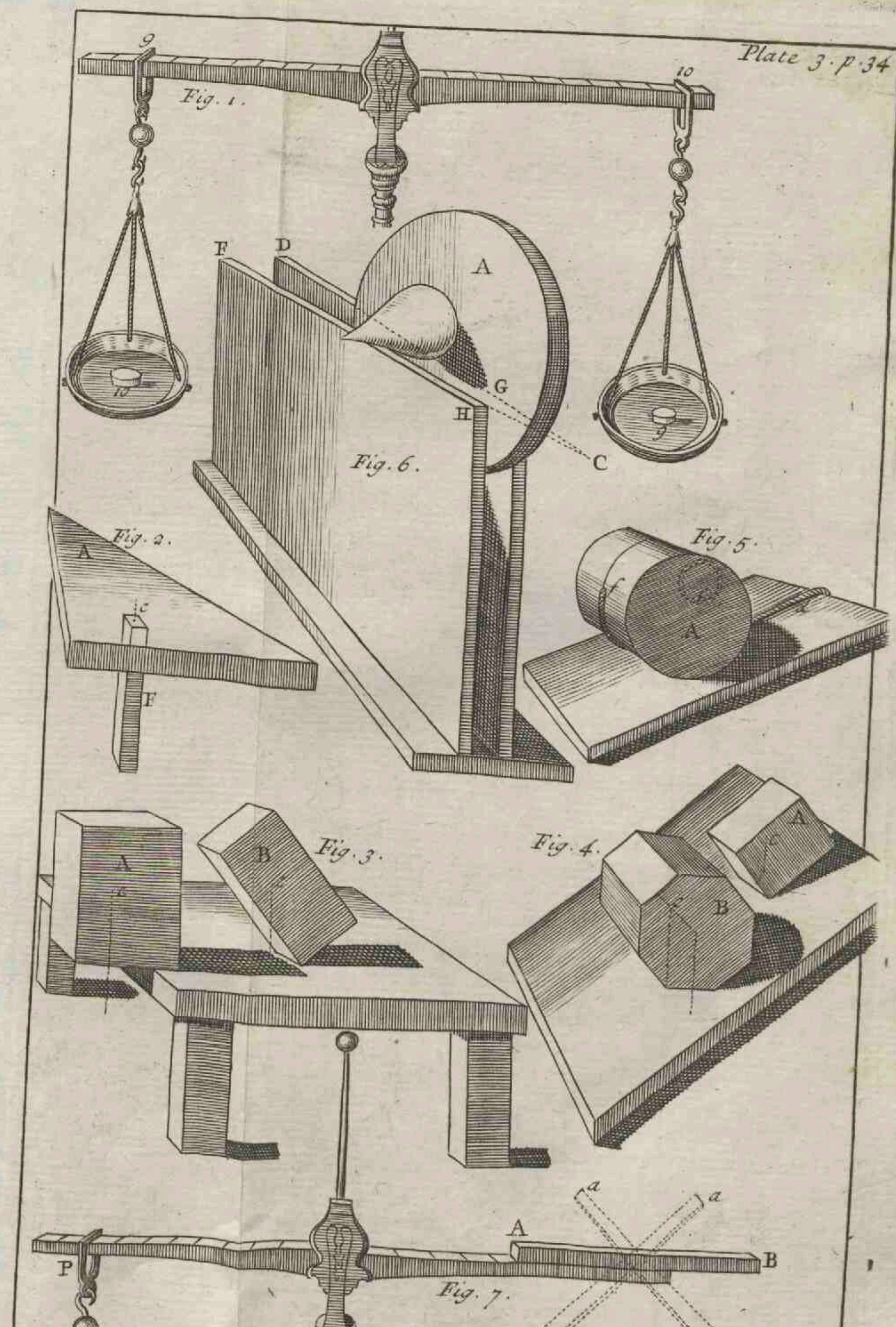


Plate 3 · p · 34
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.

CHAP. XI.

Of the LEVER.

DEFINITION I.

A Lever is called by Mathematicians, *an inflexible Right Line, made uſe of to raiſe Weights, either weighing nothing itſelf, or of ſuch Weight as may be balanced.* (*Plate* IV. *Fig.* 1.) 103

It is the firſt of thoſe that we call Simple Machines (or Mechanical Powers) as being the moſt ſimple of all; and it ſerves, when Weights are to be raiſed but to a ſmall Height.

There are four other Simple Machines, which we ſhall treat of in the three following Chapters.

Concerning the Lever, three things are to be conſidered.

1. The Weight to be raiſed or ſuſtained, as P.
2. The Power, by which it is to be raiſed or ſuſtained, which here is repreſented by the Weight M, tho' commonly it is the Action of a Man.
3. The *Fulcrum*, or Prop, by which the Lever is ſuſtained, or rather on which it moves round, whilſt the ſaid Point F remains fixed.

The Lever is three-fold.

1. Sometimes the Fulcrum is placed between the Weight and the Power. (*Plate* IV. *Fig.* 1.) 104

2. Sometimes the Weight is between the Fulcrum and the Power. (*Plate* IV. *Fig.* 2.)

3. And often alſo the Power acts between the Weight and the Fulcrum. (*Plate* IV. *Fig.* 3.)

The ſame Rules ſerve in all theſe Caſes, which follow from what has been ſaid of the Balance*: *88
And this ſhews the Analogy between the Lever and the Balance. The Lever of the firſt Kind is, as it were, a Steel-yard to raiſe Weights.

The Action of a Power and the Reſiſtance of the Weight increaſe, in proportion to their Diſtance from 105

*88 *the Fulcrum**; and therefore, *that a Power may be able to ſuſtain a Weight, it is required, that the Diſtance of the Point in the Lever, to which it is applied, be to the Diſtance of the Weight, as the Weight*
*90 *to the Intenſity of the Power**; *which, if it be ever ſo little increaſed, will raiſe the Weight.*

Experim. 1, 2, and 3.] This Rule is confirm'd by Experiments, in reſpect of the three Levers, as it appears from the firſt, ſecond and third Figures of the fourth Plate; for there is an Equilibrium, when the Weights P, and the Weights M, which repreſent the Powers, and alſo the Diſtances from the *Fulcra*, bear thoſe Proportions to each other, as the Numbers written in the Figures expreſs. A Sight of the Figures ſo plainly ſhews the Conſtruction of the Machines wherewith the Experiments are made, that a farther Explanation would be needleſs.

Workmen alſo make uſe of a Lever to carry Weights; and there are ſeveral remarkable Caſes of thoſe Levers, the Demonſtration of which may be deduced from what has been ſaid.

106 In all Caſes this is generally to be obſerved, that *the Intenſity of the Power, or the Intenſities of the Powers taken all together (when there are more than one) muſt act as ſtrongly as the Gravity of the Weights to be carried or ſuſtained.*

107 If a Weight is to be carried or ſuſtained by two Powers, it muſt be placed between the two Powers, and the Diſtances of the Powers on each Side of the Weight muſt be in an inverſe Ratio of the Intenſities of the Powers.

Experiment 4.] This Propoſition is confirm'd by the Experiment of *Fig.* 4. which requires no farther Explanation.

Expe-

Experiment 5.] When two Weights are to be 108
ſuſtained by one Power, that Power muſt be placed between the Weights; and then what has been ſaid before of the Powers, muſt be applied to the Weights. See *Fig.* 1. *Plate* V.

Several Weights are often carried or ſuſtained by one or more Powers. In which Caſes it is 109
to be obſerved, that *all Weights, in whatever Poſition, have one common Center of Gravity*; which Center is ſuch, that if, on either Side, each Weight be multiplied by its Diſtance from that Point, the Sums of the Products on each Side will be equal.* *95,99

Let the Powers alſo be diſpoſed in any Poſition, they have a common Center of Gravity; for they may be repreſented by Weights*; and here the *76
Intenſity of each Power is to be multiplied by its Diſtance from the Center, and the Sums of the Products will then be equal on both Sides: *That the Powers may be able to ſuſtain the Weights, it is required that the Center of Gravity of the Powers and the Weights be the ſame.*

Experiments 6, and 7.] What has been ſaid ſufficiently explains the Figures, (*Plate* V. *Fig.* 2, and 3.) where C denotes the Center of Gravity common to the Weights and the Powers.

Experiment 8.] What has been ſaid is true, if 110
the Lever is drawn (*Plate* V. *Fig.* 4.) by Powers on each Side; which we ſee in the Lever of *Fig.* 4. which is drawn horizontally on each Side; where the Æquilibrium only depends upon what has been laid down in the Rules above-mentioned.

We may alſo make uſe of a compound Lever 111
for raiſing Weights: In which Caſe, inſtead of a Power, a ſecond Lever is applied to the firſt,

and a third to that, and so on as far as you will, and a Power is applied to the last Lever; and then *the Ratio of the Power to the Weight* (when it sustains it) *is compounded of the Ratio's of the Power to the Weights in each Lever, when they are used separately.*

Experiment 9.] The three Levers A, B, D, are so disposed (*Plate* IV. *Fig.* 5.) that the Power M sustains the Weight P. In the Lever A, if it were used singly, the Power would be to the Weight as 1 to 5; the Lever B, as 1 to 4; and in the Lever D, as 1 to 6. The Ratio, compounded of all these, is as 1 to 120. For one Ounce M does here sustain the Weight P, of 120 Ounces. Observe, that in the Motion of this Engine, the Spaces gone thro' by the Weight and the Power are to one another, as 1 to 120; that is, in the said inverse Ratio, which is requir'd
70 to make an Æquilibrium.

CHAP. XII

Of the Axis in Peritrochio, and Wheels with Teeth.

THE Lever, as was said in the Beginning of the foregoing Chapter, serves to raise Weights to a small Height; when they must be raised higher we use an *Axis in Peritrochio.*

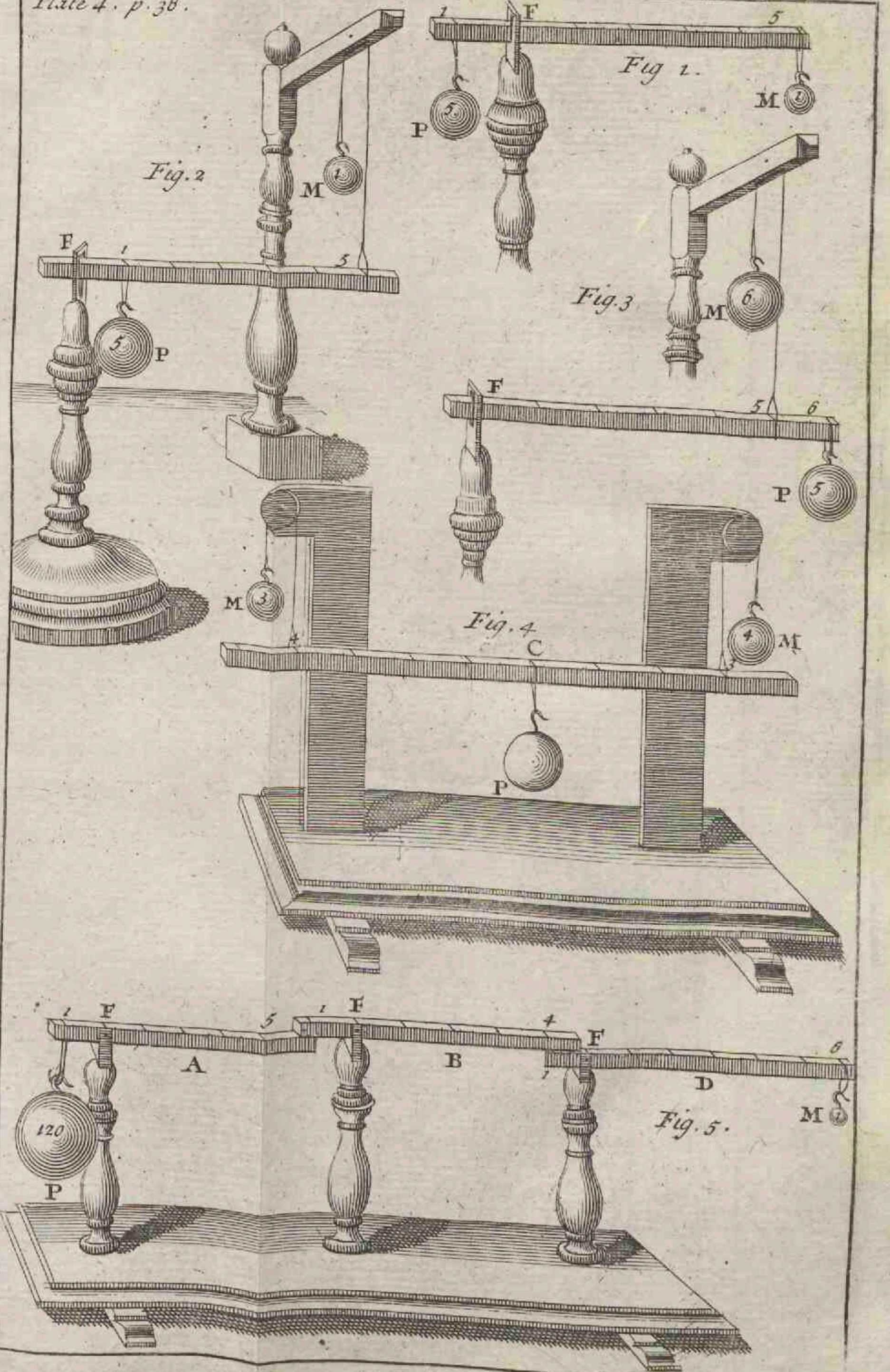
DEFINITION.

112 We call Axis in Peritrochio, a *Wheel which turns together with its Axis.* (*Plate* V. *Fig.* 5.)

The Power in this Machine is applied to the Circumference of the Wheel, by whose Motion a Rope that is tied to the Weight, is wound about the Axis by which the Weight is raised.

Let

Plate 4. p. 38.



Let *ab* be the Wheel, (*Plate* V. *Fig.* 6.) *de* the Axis, *p* the Weight to be raiſed, *m* the Power; as the Wheel is moved by the Power, the Points *b* and *d* deſcribe ſimilar Arcs, which are the Ways of the Power and the Weight, and are to each other, as *cb* to *cd*, that is, as the Diameter of the Wheel to the Diameter of the Axis; whence the following Rule is deduced.

113 *The Power has the greater Force, the greater the Wheel is, and its Action increaſes in the ſame Ratio as the Wheel's Diameter. The Weight reſiſts ſo much the leſs as the Diameter of the Axis is leſs, and its Reſiſtance is diminiſhed in the ſame Ratio as the Diameter of the Axis. And that there may be an Æquilibrum between the Weight and the Power, it is always requiſite that the Diameter of the Wheel be, to the Diameter of the Axis, in an inverſe Ratio of the Power to the Weight.** *70

It is to be obſerved, that you muſt add the Diameter of the Rope to that of the Axis.

Experiment 1.] This Rule is variouſly confirm'd (*Plate* V. *Fig.* 5.) by help of the Machine here repreſented. When the Axis is the twelfth Part of the Diameter of the Wheel, half a Pound ſuſtains ſix Pounds; and ſo on.

The Power alſo may be applied to an Handle or Spoke; as at D, and then the Diſtance of the Point to which it is applied, reckon'd from the Center, muſt be look'd upon as the Wheel's Semidiameter.

The Wheels, that have Teeth, work in the ſame Manner as this Machine; they being, in reſpect of the Axis in Peritrochio, what the compound Lever is, in reſpect of the ſimple Lever.

If the Axis of the Wheel has Teeth alſo, it ſerves to move a Wheel, whoſe Circumference has Teeth; and this Axis of the laſt Wheel may 114 again communicate Motion to a third Wheel, and ſo on. In that Caſe, *that the Power may ſuſtain the Weight, its Ratio to the Weight is compounded of the Ratio of the Diameter of the Axis of the laſt Wheel, to the Diameter of the firſt, and the Ratio of the Revolutions of the laſt Wheel, to the Revolutions of the firſt in the ſame Time.*

The Demonſtration of which Rule is alſo deduced from the Compariſon of the Ways run thro' by the Weight and the Power. (*Plate* V. *Fig.* 7.)

Experiment 2.] Let the Power repreſented by the Weight M be applied to the Wheel A B, and the Weight P to the Axis of the Wheel F G, the Diameter of that Axis is the eighth Part of the Diameter of the Wheel A B, and this Wheel goes round five Times, whilſt the Wheel F G goes round once: Therefore the Ratio of the Power to the Weight is compounded of the Ratio's of 1 to 8; and 1 to 5: Therefore it is the Ratio of 1 to 40; half a Pound in this Caſe ſuſtaining 20 Pounds.

CHAP. XIII.

Of the PULLEY.

IN ſeveral Caſes, where the Axis in Peritrochio cannot conveniently be applied, Pullies muſt be made uſe of to raiſe Weights; a Machine made by combining ſeveral of them, lies in a little Compaſs, and is eaſily carried about.

* 82 What a Pulley is, has been already explain'd.*

115 If the Weight be fix'd to the Pulley, ſo that it may be drawn up along with it, each End of the drawing or running Rope ſuſtains half the Weight. *Therefore when one End is fix'd, either to a Hook, or any other Way, the moving Force or Power applied to the other End, if it be equal to half the Weight, will keep the Weight in Æquilibrio.*

Expe-

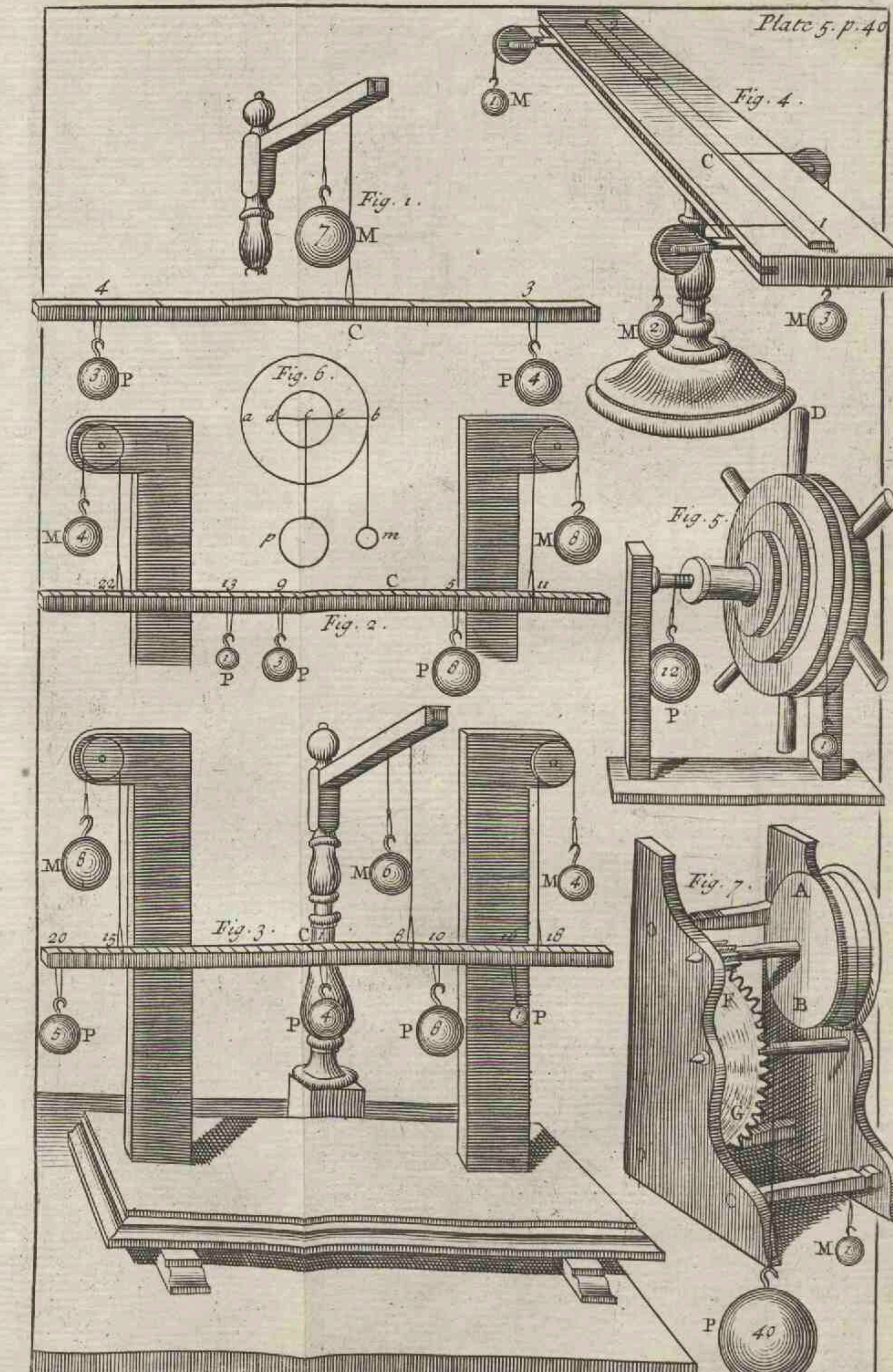


Plate 5. p. 40
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.

Experiment 1.] Make faſt the Weight P of two Pounds to a Pulley (*Fig.* 1. *Plate* VI.) yet ſo that the Wheel or Sheave may not be hindered from turning round; let one Part of the Rope *ef* be tied to a Hook; and the other End *cd* go round the fix'd Pulley, to change the Direction, * the * 83 Weight M of one Pound, fix'd to this End of the Rope, will ſuſtain the Weight P.

Several Sheaves may be joined in any Manner, 116 and the Weight be fix'd to them; then if one End of the Rope be fix'd, and the Rope goes round all thoſe Sheaves, and as many other fix'd ones as is neceſſary, a great Weight may be raiſed by a ſmall Power: In that Caſe *the greater the Number of Sheaves fix'd in a moveable Pulley or of moveable Wheels,* (for the fix'd ones do not change the Action of the Power *) *ſo much may the Power* * 83 *be leſs, which ſuſtains the Weight; and a Power which is to the Weight, as the Number One to twice the Number of the Sheaves, will ſuſtain the Weight.*

The Reaſon is, that the Number laſt mentioned is the Number of the Ropes that ſuſtain the Weight, and the Power is applied only to one Rope.

N. B. *The Workmen in* England *call a* Block, *the Box or Piece of Wood that has one or ſeveral Wheels in it; and thoſe Wheels,* Sheaves *or* Sheevers.

Experiment 2.] Hang the Weight P of 6 Pounds to the Piece AB (*Plate* VI. *Fig.* 2.) in which three Sheaves turn freely round. Let one End of the Rope be faſten'd to an Hook, and let the Rope go round thoſe three Sheaves, and three other fix'd ones: One Pound, fix'd to the other End of the Rope, will make an Æquilibrium.

Experiment 3, and 4.] The different Make of the Pullies, or the different Way of joining the Sheaves

Sheaves together, makes no Alteration; the laſt Sort is not very convenient for raiſing Weights, and therefore Workmen make uſe of unequal Sheaves, joined together in the manner repreſented in *Fig.* 3.; for the different Bigneſs of the Sheaves makes no Alteration. Oftentimes all the Sheaves move round the ſame Axis, as in the 4th *Fig.* and ſo the Pullies lie in the leaſt Room. Now in both theſe Caſes the Experiments anſwer as before.

When the End of the Rope, which in the foregoing Experiments was fix'd, is joined to the Weight, or to the moveable Wheels, then the Ratio of the Power to the Weight is no longer, as one to twice the Number of the Sheaves joined to the Weight; but this double Number muſt be increaſed by 1; and then, where two Sheaves are joined to the Weight, the Ratio will be as 1 to 5; for there are juſt ſo many Ropes, which ſuſtain the Weight. See (*Plate* VI. *Fig.* 5.)

117 *Experiment* 5.] If ſeveral moveable Pullies with one Wheel in each, and each having its own particular Rope, be diſpoſed in the manner repreſented in *Plate* VII. *Fig.* 1. the Action of the Power will be very much increaſed; for every Wheel doubles it, and therefore it is four Times greater with two Wheels, and eight Times greater with three, and ſo on.

The Rule above-mentioned, (namely, that the Spaces gone thro' by the Power and Weight, when they balance each other, are to one another inverſely as the Power to the Weight,) may be applied in all the Caſes above-mentioned.

Here we always ſuppoſe the Ropes parallel; we ſhall hereafter ſhew what Difference is made by the Obliquity of the Ropes.

CHAP.

CHAP XIV.

Of the WEDGE and SCREW.

FRom what has been ſaid, it plainly appears, how great a Weight may be ſuſtained, or even raiſed, by a little Power; but thoſe are not the only Ways of producing the ſame Effect. Mechanicks are not confined only to thoſe Methods; the Actions of Powers may be increaſed in all Caſes: A very good Inſtance of it appears in the *Wedge*, which is contrived for cleaving Wood, and alſo uſeful in ſeveral other Caſes.

DEFINITION I.

A Wedge *is a Priſm of a ſmall Height, whoſe Ba-* 118
ſes are æquicrural Triangles, as A, *Plate* VI. *Fig.* 6.

DEFINITION II.

The Height of the Triangle is the Height of the Wedge; as *d b*.

DEFINITION III.

The Baſe of the Triangle is alſo called the Baſe of the Wedge; as *c e*.

DEFINITION IV.

The Edge of the Wedge *is a Right Line, which joins the Vertices of the Triangles*; as *b f*.

The Edge of the Wedge is applied for cleaving Wood; and the Power is the Blow of a Hammer or Mallet, which drives the Wedge into the Wood.

When the whole Wedge, is driven in, the Space, gone thro' by means of the Blow or Blows, is the Height of the Wedge, which therefore may be look'd upon as the Space gone through by the Power; and the Space, which the Wood goes thro'

thro' as it yields on each Side, is half the Base of
119 the Wedge. Whence it follows, *That the Power is to the Resistance of the Wood* (when its Action is equal to it) *as the half Base of the Wedge is to its Height.*

What is here said, of the Resistance of the Wood, may be applied to all the other Uses of the Wedge.

120 The two wooden Rules A A, A A, are kept up in a parallel and horizontal Situation by the Feet B B, B B. (*Plate* VI. *Fig.* 7.)

The Brass Rulers C C, C C, are fix'd to them on the Inside.

Between them are moved the two Barrels, or Wooden Cylinders E E, which turn upon small Steel Axes that come out behind the Rulers, and have a small Return at their Ends, or the Bases are bigger than the Cylinders; each return is a little Convex on the Outside, that the Friction against the Rulers C C, C C, may be the less. In the Middle of each Ruler A A, there are two Pullies *d*, *d*, which almost touch one another, and whose upper Part is even with the Top of the Rulers C C.

The Rope, which in its Middle carries the Weight P, goes round the Pullies *d*, *d*, and each End of it is fixed to the Axis of one Cylinder E, by means of a small Plate that has a Hole through which the Axis goes. The other Weight P hangs in the same Manner upon such another Rope.

Therefore the Cylinders E E must be carried towards one another in an horizontal Motion (their Axes remaining parallel) by the Weight P, if they are equal.

Let there be a Wedge made of two wooden Planes F F, which make any Angle at Pleasure by Help of the Screw *g g*.

Experiment.] The Cylinders E E are separated by letting down the Wedge F F between them, which is drawn down by the Weight M, and you have an Æquilibrium, when the Weight M, together with the Weight of the Wedge, is to the Weights P, P, as the half Base of the Wedge to its Height.

The Force with which the Cylinders are carried towards one another, and which must be overcome to separate them, is here instead of the Resistance of the Wood; the Force with which the Wedge is driven or drawn between the Cylinders, that is the Weight of the Wedge, together with the Weight M, is here taken for a Blow with a Mallet; and so the foregoing Rule is reduced to Experiment, and confirmed by it.

The *Screw* has a great Affinity with the *Wedge*. It consists of two Parts.

DEFINITION V.

The first, which is called the Male Screw, or 121
Outside Screw, is *a Cylinder cut in, in a Helical Form*, as A B (*Plate* VI. *Fig.* 8.)

The second, which is called the Female Screw, or Inside Screw, and sometimes the Nut, and is different according to the different Uses of the Machine of which it is made a Part, is *a solid Body that contains an hollow Cylinder, whose Concave Surface is cut in the same Manner as the Male Screw, so that the Prominent Part of the one, may fit the Cavity of the other; as* D E.

N. B. The Prominent Helical Part is called the *Thread of the Screw.*

These two Parts are to move one within another, when the Screw is applied to Use. It serves chiefly to press together such Bodies as must be joined and firmly united; for in this Machine the

the ſmalleſt Power may produce a very great Preſſure. The Screw may alſo be uſed for raiſing Weights. In every Revolution of it, one Part remaining at reſt, the other is thruſt out as far as the Interval between two Threads. The Power which moves the Screw is applied to an Handle or *Hand-Spike*; and then *the Power*, which acts as ſtrongly as the Reſiſtance, *is to the Reſiſtance as the* ſaid *Diſtance between two Threads to the Periphery of a Circle, run thro' by that Point of the Handle to which the Power is applied.* For the Way gone through by that Point or Plane, wherewith the Reſiſtance is overcome, has the ſame Ratio to the Way of the Power.

Here we muſt obſerve, that when the Power balances the Weight in any Machine whatever, where no Friction is ſuppoſed; that, by increaſing the Power ever ſo little, it will over-balance the Weight. But when there is any Friction, that Friction muſt alſo be overcome by the Power; and how much muſt be added to it, to produce that Effect, cannot be determined mathematically. In the Machine laſt mentioned, this Friction is very ſenſible, and alſo of a great Uſe; for by it the Machine is kept in its Poſition, and cannot (either by the Action of the Bodies that are preſſed, or the Gravity of the Weights) receive a contrary Motion, ſo as to be puſhed back to its firſt Poſition.

CHAP. XV.

Of Compound Engines.

111 WE have already ſhewn, how a Machine
†114 may be compounded of ſeveral Levers†, or ſeveral Wheels; and that in ſuch Machines *the Power is to the Reſiſtance* (when it counterba-

 lances

lances it) *in a Ratio compounded of all the Ratio's, which the Powers in each simple Machine would have to the Resistance, if they were separately applied.* This Rule also obtains in all other Machines.

Not only simple Machines of the same Kind may be joined; but one may compound a Machine of several other Machines in different Manner: This will be plain enough by two Examples.

Experiment 1.] Join the running Rope of the Pullies to the Axle of the Wheel (*Plate* VI. *Fig.* 5.) and apply the Power to the Wheel: Now, as in this Case, the Action of the Power becomes five times greater by help of these Pullies, and the Diameter of the Axis is but the third Part of the Diameter of the Wheel; the Ratio of the Power to the Weight is compounded of the Ratio's of 1 to * 5, and 1 to † 3; it is therefore as 1 to 15; and therefore one Pound M sustains the Weight P of 15 Pounds. 122

*116 †113

The Axis in Peritrochio may be moved by a Screw: For this Purpose the Wheel must have Teeth, and those Teeth must stand askew, or be inclined, as you may see in the Wheel A, (*Fig.* 9.) which is carried round by the Screw B C. Such a Screw is called an *endless Screw*, and very much increases the Action of the Power; for there are so many Revolutions of the Screw, or of the Handle of it, required to turn the Wheel once about as the Wheel has Teeth. And if another Wheel with Teeth be added to the first, the Action of the Power will still be much more increased. 123

Experiment 2.] The Machine of *Plate* VI. *Fig.* 9. consists of an endless Screw, which is moved by the Handle D E. Here the Ratio of the Power

to the Weight when it balances it, is compounded of the Ratio of the Semidiameter of the Axis of the laſt Wheel F, to the Length of the Handle DE, and the Ratio of the Revolutions of that Wheel to the Revolutions of the Handle or Screw. The firſt Ratio in this Machine is the Ratio of 1 to 30; the ſecond is gathered from the Number of Teeth; the laſt Wheel F has in its Circumference 35 Teeth, and the Axis of the firſt Wheel 7; therefore the firſt Wheel goes round five times, while the ſecond Wheel goes round once: But this firſt contains 36 Teeth; therefore the Screw goes round ſo many times, for one
123 Turn of the Wheel. The Ratio compounded of theſe two is 1 to 180, which is the ſecond Ratio ſought; and the Ratio made up of that and the firſt (which is 1 to 30) is the Ratio of 1 to 5400, which would be the Ratio of the Power to the Weight, if there was no Friction; but as it is pretty great in all theſe Engines, the Power muſt be pretty much increaſed, to make it raiſe the Weight; tho' ſtill a very ſmall Power applied to it, will raiſe a prodigious Weight. The Handle ED may be twice or three times as long, or ſtill longer. which will double, or triple, or farther increaſe, the Action of the Power: And, in that Caſe, a ſmall Hair will overcome the Force of the ſtrongeſt Man.

A great number of other compound Machines may be made, whoſe Forces are in the ſame Manner determined, by Computation, by the Rule mentioned in the Beginning of this Chapter; or alſo by comparing the Way gone through by the Power with that gone through by the Weight, or any other Obſtacle; for their Ratio will be the inverſe Ratio of the Power, and the Weight or Reſiſtance.

The

Plate 6. p. 46.

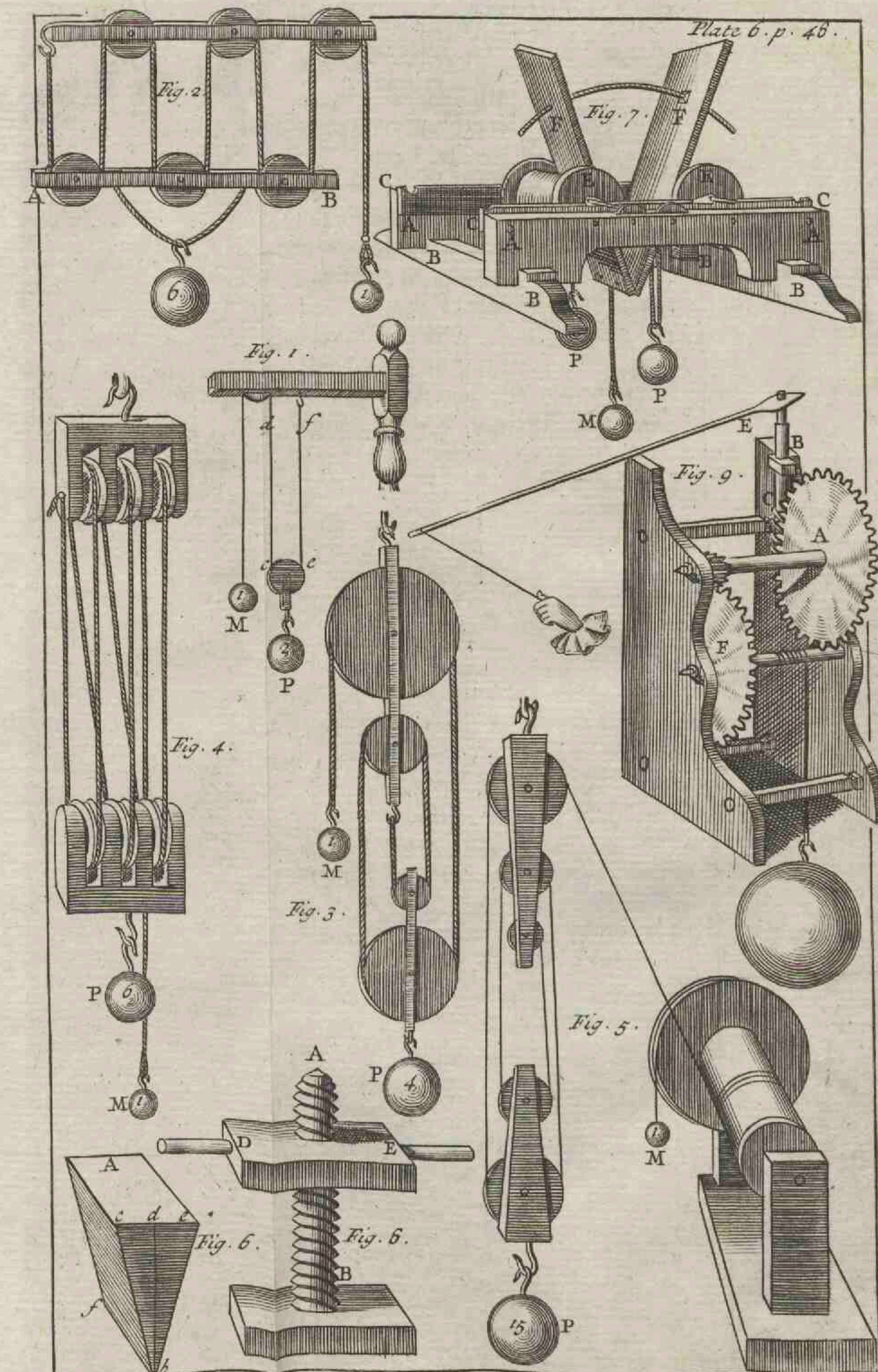


Fig. 2.
A
B
6
1
Fig. 7.
F
F
C
C
A
A
A
B
B
B
P
P
M
Fig. 1.
d
f
c
e
1
M
2
P
Fig. 4.
E
B
Fig. 9.
A
F
M
Fig. 3.
P
6
M
1
P
4
Fig. 5.
1
M
A
D
E
A
c d e
Fig. 6.
Fig. 6.
B
f
b
15
P

The Forces, which acting contrariwiſe balance one another, are always equal; if therefore the Intenſity of a Power be leſs than that of the Reſiſtance, it muſt run thro' a greater Space in the ſame Time; and that muſt always be in proportion as its Intenſity is leſs; for the Forces can differ in no other Reſpect, neither can we compenſate any other way for the Difference of Intenſity.

C H A P. XVI.

Of Sir Iſaac Newton*'s Laws of Nature.*

IN what we have ſaid of Machines, we have conſider'd the Actions of Powers and Weights acting continually againſt Obſtacles and other Reſiſtance; now we ſhall conſider Bodies left to themſelves and continuing in Motion, or freely falling: And here we muſt reaſon from Phænomena, (as one muſt do in all Natural Philoſophy) and from them deduce the Laws of Nature.

Sir *Iſaac Newton* has laid down three, by which we think that every thing that relates to Motion may be explained.

L A W I.

All Bodies continue in their State of Reſt, or Mo- 124
tion, uniformly in a Right Line, except ſo much as they
are forc'd to change that State by Forces impreſs'd.

We ſee that Bodies, by their Nature, are inactive, and incapable of moving themſelves; wherefore unleſs they be moved by ſome extrinſical Agent, they muſt neceſſarily remain for ever at reſt.

A Body alſo, being once in Motion, continues in Motion according to the ſame Direction, in the ſame Right Line, and with the ſame Velocity, as we ſee by daily Experience; for we never ſee any Change made in Motion, but from ſome Cauſe. But (ſince Motion is a continual Change of Place) how the Motion in the ſecond

Moment of Time ſhould flow from the Motion in the firſt, and what ſhould be the Cauſe of the Continuation of Motion, appears wholly unknown to me; but, as it is a certain Phænomenon, we muſt look upon it as a Law of Nature.

LAW II.

125 *The Change of Motion is always proportionable to the moving Force impreſs'd, and is always made according to the Right Line in which that Force is impreſs'd.*

If to a Body that is already in Motion, another Force be ſuperadded to move it in the ſame Direction, the Motion becomes quicker, and that in
58 proportion to the new Force impreſs'd.

When a new Force impreſs'd is contrary to the Body's Motion, the Retardation follows the Proportion of the Impreſſion; ſo that a Force which is double or triple, *&c.* produces a double or triple Retardation. And generally all Forces produce Changes in Motion, according to their Directions and Quantities; other Actions of Forces would imply a Contradiction: This will appear more clearly by ſuch Experiments made upon oblique Forces, as we ſhall mention in ſome of the following Chapters.

LAW III.

126 *Action is always equal, and contrary to Re-action; that is, the Actions of two Bodies upon each other are always equal, and in contrary Directions.*

Which Way ſoever one Body acts upon another, we ſee that Body always ſuffers an equal and contrary Re-action. If I preſs a Stone with my Finger, my Finger is equally preſs'd by the Stone. A Horſe that draws a Cart forward, is as much drawn backward by the Cart; for the Geers or Traces are equally ſtretched both Ways.

When a Body ſtrikes againſt another, whatever the Stroke be, both ſuffer it equally; but the Impreſſions are contrary. This is clearly confirmed by the Experiments of the Congreſs of Bodies.

The Loadſtone draws Iron, and is equally drawn by Iron.

Experiment.] Suſpend the Loadſtone M, (*Plate* VII. *Fig.* 2.) in ſuch manner that it may eaſily be moved; then, bringing a Piece of Iron within a ſmall Diſtance of it, the Loadſtone will come to the Iron: And if you pull back the Iron, before the Stone be come to it, the Stone will follow the Iron; juſt as the Iron goes towards the Stone and follows it, when the Iron is ſuſpended and moveable, and the Loadſtone brought to the Iron.

When a Man ſits in a Boat, and by a Rope pulls towards him another Boat, juſt as big and as much laden, both Boats will be equally moved, and meet in the Middle of the Diſtance of the Places in which they were at firſt. If one Boat be greater than the other, or more laden, the Velocities in each will be different, when they have different Quantities of Matter; but the Quantities of Motion on both Sides will be equal, abſtracting from the Reſiſtance of the Water.

And this Law takes place generally in all the Actions of Bodies upon one another.

CHAP. XVII.

Of the Acceleration and Retardation of heavy Bodies.

DEFINITION I.

AN accelerated Motion *is that Motion, whoſe Velocity becomes greater every Moment.* 127

Definition II.

128 A retarded Motion *is that, whose Velocity is di-
minished every Moment.*

The Force of Gravity acts continually upon all
Bodies, in proportion to their Quantity of Mat-
79 ter. When a Body falls freely, the Impression
made upon it the first Moment is not destroyed
in the second Moment; therefore there is super-
added to it the Impression made in the second
129 Moment, and so on. *The Motion* then *of a Body
that falls freely, is accelerated*, and that *equally in
equal Times*; because the Force of Gravity acts
75 every Moment in the same Manner, and there-
fore communicates an equal Velocity to Bodies in
equal Times. Whence *that Celerity, which is ac-
130 quired in the Fall, is always as the Time in which
the Body has fallen.* For Example: The Velocity
acquired in a certain Time will be double, if
the Time be double; and triple, if the Time be
triple, &c.

Let that Time be expressed by the Line AB
(*Plate* VII. *Fig.* 3.) and let the Beginning of the
Time be A. In the Triangle ABE, the Lines
1 *f*, 2 *g*, 3 *h*, which, being parallel to the Base,
are drawn through the Points, 1, 2, 3, are to
one another as their Distances from A, A1, A2,
A3; that is, as the Times which are expressed
by those Distances, and express the Velocities
of a Body falling freely after those Times. If,
instead of Mathematical Lines, others be taken
with a very small Breadth equal to each of them,
the Proportion will not be changed thereby;
and those small Surfaces will in the same manner
denote the above-said Velocities. In the least
Time the Velocity may be looked upon as equa-
ble, and therefore the Space gone through in
53 that Time is proportionable to the Velocity. In
each

each of those small Surfaces above-mentioned, if the Breadth of the Surface be called the Time, the Surface itself will be the Space gone thro'. The whole Time A B consists of those very small Times; and the Area of the Triangle A B E, of the Sum of all those very little Surfaces, answering to those small Parts of Times: Therefore that Area expresses the Space gone through in the Time A B. After the same Manner the Area of the Triangle A 1 *f* represents the Space gone thro' in the Time A 1; those Triangles are similar, and their Areas are to one another as the Squares of the Sides A B, A 1: That is, *the Spaces, gone through from the Beginning of the Fall, are* 131
to one another, as the Squares of the Times during which the Body fell.

This is confirmed by Experiments made on the following Machine.

The Balance A B (*Plate* VII. *Fig.* 4.) which 132
has but one Scale, is exactly in Æquilibrio; when a Weight is put into the Scale, an Iron, made in the Form of a Gnomon, keeps fast the Brachium A, and the Balance is retained in a horizontal Position.

At *f* there is a thin Spring *f g* fixed to the Gnomon, and which, when extended, reaches to *i*, where the End *g* is retained by help of the little Plate *i*, which is made fast to the Brachium A. Now by this Means the least Motion of the Balance becomes sensible; because then the Spring *f g*, being free, flies out, and returns to the Figure *f g*.

At the End of the Brachium B, there is a Hole, thro' which the String fastened to the Hook D passes freely, that String is kept in a vertical Situation by hanging on the Weight N.

The Weight M has a Hole thro' it for the above-mentioned String to pass freely thro'; in making Experiments, the Weight M is raiſed up along the String, and, when you let it go, it falls upon the ſame Point of the Brachium B.

Experiment.] Put the one Pound Weight P into the Scale; then the Body M, falling from the Height of three Inches, will move the Balance. When P is a two Pound Weight, let M fall from twelve Inches, and the Balance will be moved. If you lay on three Pounds in the Scale, the Body M muſt be let fall from a Height of 27 Inches, to move the Balance and raiſe P. And in all theſe Caſes, if the Height from which M falls be taken but a little leſs, the Balance, with the Weight in the Scale, will not be moved.

In this Experiment, the Weight which is laid upon the Scale, and raiſed by the Blow of the falling Body, is proportionable to the Stroke; the Quantity of Motion in the Body follows the Proportion of the Stroke: And that Quantity (becauſe we make uſe of the ſame Body) is propor-
63 tionable to the Celerity; and, laſtly, the Celeri-
130 ty here is proportioned to the Time of the Fall: Therefore the Weights above-mentioned follow the ſame Proportion of the Time. The Weights here are as the natural Numbers 1, 2, 3, and therefore the Times are in that Proportion: But the Spaces gone thro' in thoſe Times are as 3, 12, 27, or as 1, 4, 9, which Numbers are the Squares of the others.

133 Having divided the Time AB (*Plate* VII. *Fig.* 3.) into the equal Parts A 1, 1 2, 2 3, 3 B, thro' the Diviſions draw Lines parallel to the Baſe; *the Spaces gone thro'* in thoſe Parts of Time, that is, *in the firſt*, *ſecond*, *and third Moment*, &c. *ſuppoſing the Moments equal*, *are to one another* as the Areas A 1 *f*, 1 *f g* 2, 2 *g h* 3, 3 *h* EB; which Areas, as appears by the Figure, are to one another *as the odd Numbers* 1, 3, 5, 7.

If

If the Body, after it has fallen, during the Time AB, ſhould be no more accelerated, but with the Celerity BE, acquired by that Fall, ſhould uniformly continue its Motion, during the equal Time BC, the Space gone thro' by that Motion is expreſs'd by the Area BEDC, which is double the Triangle ABE. And therefore,

A Body falling freely from any Height, with that 134
Velocity which it has acquired by that Fall, will in a Time equal to the Time of the Fall (by an equable Motion) run thro' a Space double the ſaid Height.

Which Propoſition we ſhall alſo confirm by an Experiment.* * 211

The Motion of a Body thrown upwards is retarded in the ſame Manner, as the Motion of a falling Body is accelerated by the ſecond Law*: In this Caſe the Force of Gravity con- * 125
ſpires with the Motion acquired; and in that it acts contrary to it. But, as the Force of Gravity is equal every Moment, *the Motion of a Body thrown* 135
up is equally diminiſhed or retarded in equal Times.

The ſame Force of Gravity generates Motion in the falling Body, and deſtroys it in the riſing Body; therefore the ſame Forces are generated and deſtroyed in equal Times. *A Body thrown up* 136
riſes till it has loſt all its Motion; and ſo *goes up during the ſame Time, that a Body falling could have acquired a Velocity equal to the Velocity with which the Body is thrown up.*

If a Body be thrown up with the ſame Velocity that it would acquire in falling down the Line BC (*Plate* VII. *Fig.* 5.) it would aſcend in a Time equal to the Time of the Fall*, (and with an equable Motion) ſo as to * 136
come up the Height CA, the double of BC*; but * 134
as in the ſame Time, by the Force of Gravity, the Body goes thro' a Space equal to the Space AB, or BC: as theſe two Motions obtain here at

the ſame Time, and act contrariwiſe, the leſſer muſt be ſubſtracted from the greater; therefore the Body, after the End of the Aſcent, will be at
137 B. Whence it follows, that *a Body thrown up will riſe to the ſame Height from which falling, it*
138 *would acquire the Velocity with which it is thrown up.* And therefore, *the Heights, which Bodies thrown up with different Velocities can riſe to, are to one ano-*
* 131 *ther as the Squares of thoſe Velocities.**
130

CHAP. XVIII.

Of the Deſcent of heavy Bodies upon inclined Planes.

DEFINITION I.

139 WE *call an* inclined Plane *that which makes an oblique Angle with the Horizon.*

CB in *Plate* VII. *Fig.* 6. repreſent a Line parallel to the Horizon; AB makes with it the oblique Angle ABC, and repreſents an inclined Plane; and the Perpendicular AC is let fall from A, the upper Part of the Plane, to the Horizon.

DEFINITION II.

140 *The Length* AB *is called* the Length of the Plane.

DEFINITION III.

141 *The Line* AC *is called* the Height of the Plane.

Let two equal Bodies deſcend by the Force of Gravity from the Point A, the one along the Line AB, and the other along the Line AC; when they are come to the Points B and C, they have deſcended equally, that is, they will be got each equally near to the Earth's Center: Therefore the Forces with which they are impelled, as they are directed towards the Earth's Center, are

are equal; but the *Intensities* of equal Forces are reciprocally as the Spaces gone through*; and * 70
therefore here the Intensity of the Force, by which the Body is impelled along an inclined Plane, is to the Intensity of it, by which it is directly impelled towards the Center of the Earth, as AC to AB. Therefore, *A Body laid upon an inclined Plane, loses Part of its Gravity; and the Weight required to sustain it, is to the Weight of the Body,* 142
as the Height of the Plane to its Length.

The Plane NOQL (*Plate* VII. *Fig.* 7.) is placed 143
in an horizontal Situation; the Plane ABIH moves upon Hinges; and may be fixed at any Height, by help of the Screw V, and Quadrant *t*.

The wooden Ruler EF has a Pulley C fastened at one end, and revolves about the other; the Head D, about whose Center this Ruler moves, may be fixed (in any Place of the Slit *ps*) to the Plane NOQL, by a Screw under the Plane.

M is a wooden Cylinder, whose Axis is of Steel, and whose Bases somewhat exceed the Cylinder; so that, as it turns round along the Plane ABIH, the Bases only touch the Plane.

The Cylinder is sustained by a String that goes over the Pulley G; which String is fixed to a thin Brass Ruler, bent in such Manner, that the Axes of the Cylinder go thro' its Ends, and turn in them.

In making Experiments, the Pulley is so plac'd by the inclining Ruler EF, and moving the Head D along the Slit *rs*; that the String by which the Cylinder is sustained, is parallel to the inclined Plane ABIH.

Experiment 1.] Let the Plane ABIH be inclined in any Manner, the Weight of the Body M has the same Ratio to the Weight P, as the Length of the Plane AB to its Height AC; and the

the Body M, in what Part ſoever of the inclined Plane it be ſet, will be ſuſtained upon it by the Weight P.

As the Force, by which a Body is made to deſcend along an inclined Plane, ariſes from Gravity, it is of the ſame Nature with it; and therefore that Force every Moment, and in all Parts of the
* 75 Plane, is equal*: For the ſame Reaſon *the Motion*
144 *of a Body, freely running down upon an inclined Plane*, is of the ſame Nature with the Motion of a Body freely falling; and what has been ſaid of the one, may alſo be affirmed of the other. It *is* therefore *a Motion equably accelerated in equal Times*; * and
* 129 therefore the Propoſitions of Numb. 130, 131, 133, 134, 135, 136, 137 and 138, may be here applied, if we ſuppoſe a Motion upon an inclined
145 Plane, inſtead of a direct Aſcent or Deſcent.

The Forces *by which two Bodies deſcend, one of which falls freely, and the other runs down an inclined Plane, if the ſaid Bodies begin to fall at the ſame Time, are* always to one another *in the ſame*
* 129 *Ratio as in the Beginning of the Fall**; therefore
144 the Effect of thoſe Forces, that is, *the Spaces gone thro' in the ſame time, are in the* ſame *Ratio*; name-
* 142 ly, that *of the Length of the Plane to its Height**.

In the Plane AB (*Plate* VII. *Fig.* 8.) the Space gone through by a Body, whilſt another falls freely down the Height of the Plane AC, is determined by drawing CG perpendicular to AB: for then the Length of the Plane AB is to its Height AC, as AC to AG. If a Circle be deſcribed with the Diameter AC, the Point G will be in the Periphery of the Circle; becauſe an Angle in a Semicircle, as AGC, is always a Right Angle; and therefore a Point taken, as G, in any Inclination of the Plane, will always be in the Periphery of the ſaid Circle: Whence it follows, that all the

Chords

Chords, as A G, are gone thro' by Bodies running along them, in the Time that a Body, falling freely, would run down the Diameter A C; and therefore the Time of the Falls thro' those Chords are equal. Thro' the Point C there can be drawn no Chord, as H C, but what a Chord, as A *g*, may be drawn parallel to it, (that is, equally in- 147
clined) and equal; therefore in a Semicircle, as A H C, *whether a Body falls freely along the Diameter* A C, *or whether it falls down along any Chord, as* H C, *it will in the same Time come to the lowest Point of the Semicircle.*

The Time of the Fall along the whole Plane A B may be compared with the Time of the Descent along the Height A C; which for that Time is equal to the Time of the Fall along A G; and so the Squares of those Times are to one another, as A B to A G.* But A B is to A C, as A C *145
to A G; therefore the Squares of the Lines A B 131
and A C are to one another, as A B to A G; and therefore *those Lines* A B *and* A C *are to one another* 148
as the Times of the Fall along A B *and* A G, *or* A C; *that is, the Times in that Case are, as the Spaces gone through.*

In the same Case, the Velocities, at the End of 149
the Descent, are equal; for after equal Times, when the Bodies are at G and C, the Velocities are in the same Ratio as in the Beginning of the Fall*, *129
that is, as the Forces by which the Bodies are 144
impell'd, or as A C to A B.* When the Bo- *142
dy descends from G to B, the Velocity increases as the Time; and the Velocity at G is to the Velocity at B, as A C to A B:* Therefore the *148
Velocities at B and C have the same Ratio to the Velocity at G, and so are equal. Hence it appears, that *a Body acquires the same Velocity,* 150
in falling from a certain Height, whether it falls directly down, or along an inclined Plane; and since the

the Angle of Inclination causes no Alteration, *a Body may run down several Planes differently inclined, and even along a Curve,* (which may be consider'd as made up of an innumerable Number of Planes differently inclined) *and the Celerity acquired will always be the same, when the Height is equal.*

Experiment 2.] In this Experiment it is to be observ'd, that a Body hanging by a Thread, and describing a Curve by its Fall, falls in the same manner, as if it was to run down such a Curve cut hollow in a solid Body without any Friction.

Let the Body P, (*Plate* VII. *Fig.* 9.) suspended by a Thread, fall from the Height A C, in the Curve B C, and in the Curve D C, and in the Curve E F G C, made up of Parts of different Circles, and in each Case it will, with the same Force, strike against the Body Q, which is at rest; and always drives it to the same Height.

151 *A Body that has acquired any Celerity in falling down along any Surface, whether Plane or Curve, will rise up to the same Height along another similar Surface, with the same Velocity, in the same Time.*

152 *A Body will, with the same Celerity that it has acquired in falling down from a certain Height, rise up to the same Height in any Curve whatever.*

Experiment 3.] Let the Body P, (*Plate* VIII. *Fig.* 1.) hanging by a Thread, fall from the Height A C, along any Curve B C; with the Celerity which it has thereby acquired, it will ascend to the same Height on the other Side, in the Curves C D or C E, or C H G F.

CHAP.

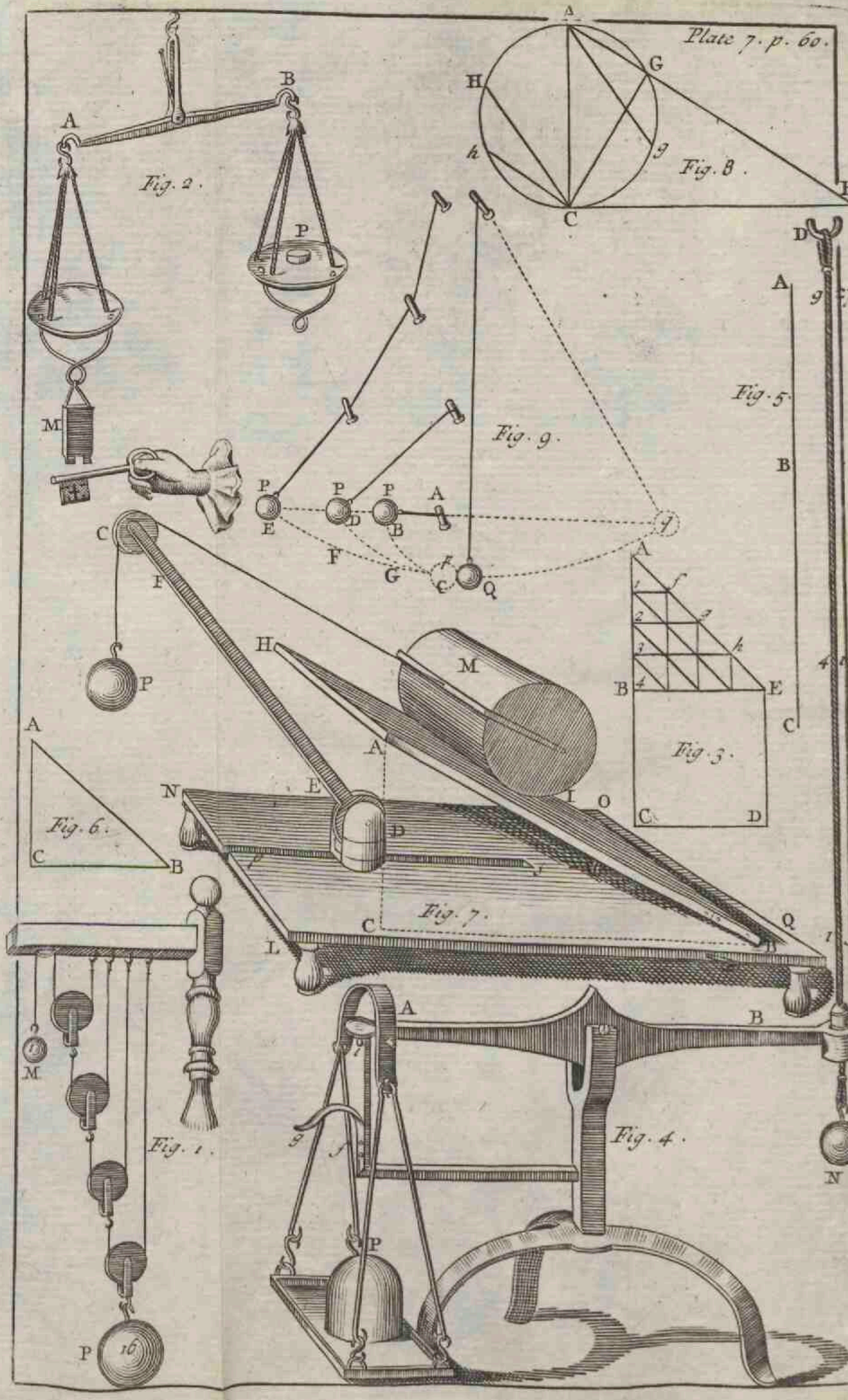


Plate 7. p. 60.
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.

CHAP. XIX.
Of the Oscillation or Vibration of Pendulums.

Definition I.

A *Heavy Body, hanging by a small Thread and moveable with the Thread about the Point to which the Thread is fix'd, is call'd* a Pendulum. 153

The Motion of a Pendulum is an oscillatory or vibratory Motion. When the Weight, the Thread being extended, is raised upon one Side, it descends by its Gravity, and, with the Celerity that it has acquired, rises up to the same Height on the other Side; * and then it returns by its Gravity, * 151
and so continues in its Vibrations.

We here suppose the Motion about the Point of Suspension to be perfectly free, and that there is no Resistance of Air, which is very small in great Pendulums.

The Body P (*Plate* VIII. *Fig.* 2.) does, in its 154
Motion, describe the Arc PBF; *if*, instead of that Motion, *a Body* should descend along the Chord PB, and again ascend along the Chord BF, and so *should perform its Vibration in Chords*; the Descent would be made in that Time in which the Body by its Fall would go thro' the Length of the Diameter AB; * that is, twice the Length * 147
of the Pendulum. In an equal Time, it ascends along the Chord BF;* therefore *in the Times of* * 151
one whole Vibration, the Body in falling might run through four Diameters; * that is, *eight Times the* * 131
Length of the Pendulum. And as the Descent and Ascent in any Chord is performed in the same Time, all the Vibrations in Chords, whether great or small, are likewise performed in the same Time. In small Vibrations, the Arcs of a Circle do not sensibly

ſenſibly differ from the Chords; *and the Vibrations*
155 *of the ſame Pendulum, tho' unequal, are performed in the ſame Time, as far as our Senſes can diſtinguiſh.*

Experiment 1. *Plate* VIII. *Fig.* 4.] If the two equal Pendulums, C P and *cp*, are let fall from the Points P and *p* in the ſame Moment of Time, they will at the ſame Time come to B and *b*, and then to F and *f*; and ſo they will continue their Motion, in the Arcs P B F and *pbf*, always in the ſame Time.

Here it is to be obſerved, that tho' the Propoſition 155 be true in all Pendulums, the Demonſtration given is only to be applied to ſhort Pendulums; in the longer Sort the Time of the Deſcent along a Chord differs ſenſibly enough from its Deſcent along an Arc; but in ſmall Arcs the Differences are equal, tho' the Chords be of different Lengths.

Let the Circle F B (*Plate* VIII. *Fig.* 3.) roll along the Line A D, till the Point B comes to A in the ſame Line; by ſuch a Motion the Point B deſcribes a Portion of the Curve B P A. Such another Curve B D may be deſcribed in the ſame Manner, and the whole Curve A B D is call'd *a Cycloid.* Let it be divided into two equal Parts at B, and let the Parts B A and B D be ſo diſpoſed as to have the Points A and D fall in together at C; and let the Point B coincide with the Points A and D in the Line A D. Let two Plates of Metal be bent according to theſe Curves, ſo that the Thred of a Pendulum ſuſpended at C, may on either Side apply itſelf to thoſe Plates, and fall in with their Curves as the Pendulum vibrates. Now if the Length of the Pendulum be C B, the Body P in its Vibrations will deſcribe the Cycloid A B D.

It

It is a Property of this Curve, that in whatever Point of it the Body P be placed, the Force, with which it is carried by its Weight along the Curve, is proportional to the Part of the Curve which is between that Point and the lowest Point of the Curve B. Whence it follows, that if two Pendulums, as CP, be let fall in the same Moment from different Heights, the Velocities, with which they begin to fall, are to one another, as the Spaces to be run through before they come to B: If therefore they should be acted upon by those Forces alone with a Motion not accelerated, they would come to B at the same Moment of Time; * after the same Manner by the Forces which are acquired, the second Moment, they also come to B at the same Time: The same may be said in Relation to the following Moments; and the half Vibrations made up of all the Forces together, however unequal they are, as also the whole Vibrations are performed in the same Time. 156

* 53

It is moreover demonstrated by Geometricians, that *the Time of each Vibration is to the Time of a vertical Fall, along the half Length of a Pendulum, as the Perpihery of a Circle to its Diameter.* In this Curve the lower Part coincides with a small Arc of a Circle, as to Sense: And this is the true Reason, why in a Circle the Times of small Vibrations (however unequal those Vibrations be) are equal; and therefore also the Duration of those Vibrations has the above-mentioned Ratio to the Time of a vertical Fall. 157

158 The Durations of the *Vibrations of unequal Pendulums* may be compared together. When the Arcs are similar, the Deviations, in Respect of the Chords, are also similar, and the Times of the Vibrations in the Arcs are, as the Times of the Vibrations along the Chords; but they are, as the Times of the Descent

Deſcent along Lengths, eight Times greater than
* 154 the Length of Pendulums; * and ſo *the Squares of the Durations are* as thoſe eightfold Lengths,
* 131 * or *as the Lengths of the Pendulums.*

Experiment 2.] Two Pendulums CP, *cp*, (*Plate* VIII. *Fig.* 5.) whoſe Lengths are as 4 to 1, are let fall at the ſame Time from the Points P and *p*, ſo that in their Vibrations they deſcribe ſimilar Arcs; the longer Pendulum vibrates once, whilſt the ſhorteſt vibrates twice; and ſo the Squares of the Durations of the Vibrations are as 4 to 1, namely, as the Lengths of the Pendulums.

When the Vibrations are ſmall, this Ratio alſo holds, tho' the Pendulums ſhould not vibrate in
* 155 ſimilar Arcs.*
159 *The Velocities of Pendulums in the loweſt Point, when the Vibrations are unequal, are to one another, as the Subtenſes of thoſe Arcs, which the Body deſcribes in its Deſcent.* So the Velocity of the Body P, (*Plate* VIII. *Fig.* 2.) falling in the Arc PB, is to its Velocity when it falls along D B, as the Chord PB to the Chord DB: For if you draw in a Circle the Lines P*f*, D*d* parallel to the Horizon, the Squares of the ſaid Chords are to one another, as the Lines *f* B, *d*B. The Squares of the ſaid Velocities are alſo, as thoſe Lines *f*B, *d*B;
* 150 therefore the Velocities are as the Chords.*
131 Concerning all that has hitherto been ſaid of
130 Pendulums, it is to be obſerved, that it is no
160 Matter how big the Weight of the Pendulum is, or whether the Weights of two Pendulums be different in Magnitude or different Sorts of Bodies; ſince Gravity is proportioned to the Quantity of Matter in all Bodies, all Bodies in the ſame Circumſtances are moved by Gravity with

with the ſame Velocity. Which is alſo confirm'd by the following Experiment.

Experiment 3.] Take two equal or unequal Balls, the one of Lead, and the other of Ivory, hang them up by Threads, that they may make Pendula of equal Lengths; let them vibrate, and their equal Vibrations (or even all their unequal ones, provided they be ſmall Vibrations) are perform'd in the ſame Time.

Oftentimes inſtead of a Thread, a ſmall, but 161
ſtiff, Iron-Rod is made uſe of, and ſometimes alſo two or more Weights are fix'd to it, and it is called a *Compound Pendulum*; in that Caſe the Rules above-mentioned are not applicable; but thoſe Pendulums are reduced to ſimple ones, by determining in them ſuch a Point, that, if all the Weights were united in it, the Vibrations would be of the ſame Duration as thoſe of the compound Pendulum. This Point is called the *Center of Oſcillation.*

The Center of *Percuſſion* in a compound Pen- 162
dulum is a Point, in which the whole Force of the Pendulum is as it were collected; ſo that if that Point ſtrikes againſt an Obſtacle, the Blow will be greater than if any other Point of the Pendulum ſhould ſtrike againſt it.

In a Vacuum, or a Medium that does not reſiſt, theſe two Centers coincide. They alſo coincide in the Air, as to Senſe, by reaſon of the ſmall Reſiſtance.

A Body of any Figure may be ſuſpended, and vibrate about a Point, or rather an Axis; and in ſuch a Body one may alſo determine the Center of Oſcillation.

When a Right Line, ſuch as is an Iron Wire, vi- 163
brates about one End, the Center of Oſcillation is

distant from the Point of Suspension two third Parts of the Length of the Wire.

Experiment 4.] The flat Iron AB (*Plate* VIII. *Fig.* 6.] must be so hung up, as to vibrate about the End A; let the simple Pendulum CP, whose Length is equal to two third Parts of AB, be suffered to descend at the same Time as the Iron; and the Vibrations of the Pendulum and the Iron will be perform'd at the same Time.

The Vibrations of Pendulums, as we have said,
* 155 tho' unequal, are perform'd in the same Time,* and this Property of Pendulums is of great Use in Clocks, to which an equable Motion is communicated by fixing on a Pendulum.

By carrying Clocks to different Places, it has appear'd that the Force of Gravity is not equal in all Parts of the Earth, because the Vibrations of the same Pendulum, in divers Countries, have been found unequal, in respect to Time; and that Difference of Gravity is measured by Pendulums.

164 *Let there be two Pendulums, whose Lengths are to one another, as the Forces of Gravity by which they are actuated*; if they run out into similar Arcs, in correspondent Points, the Force will always have the same Ratio to one another, and indeed the Ratio of the Spaces to be gone thro', (because similar Arcs are as the Lengths of Pendulums,) which therefore will be run thro' in equal
* 53 Times, *that is, *the Vibrations will be perform'd in the same Time.*

If they be reduced to the same Length by changing one Pendulum, the Square of the Time of the Vibration of the Pendulum, that is changed, is to the Square of the Time of Vibration before the Change (that is, to the Square of the Time of the Vibration of the Pendulum that is

Plate 8. p. 66.

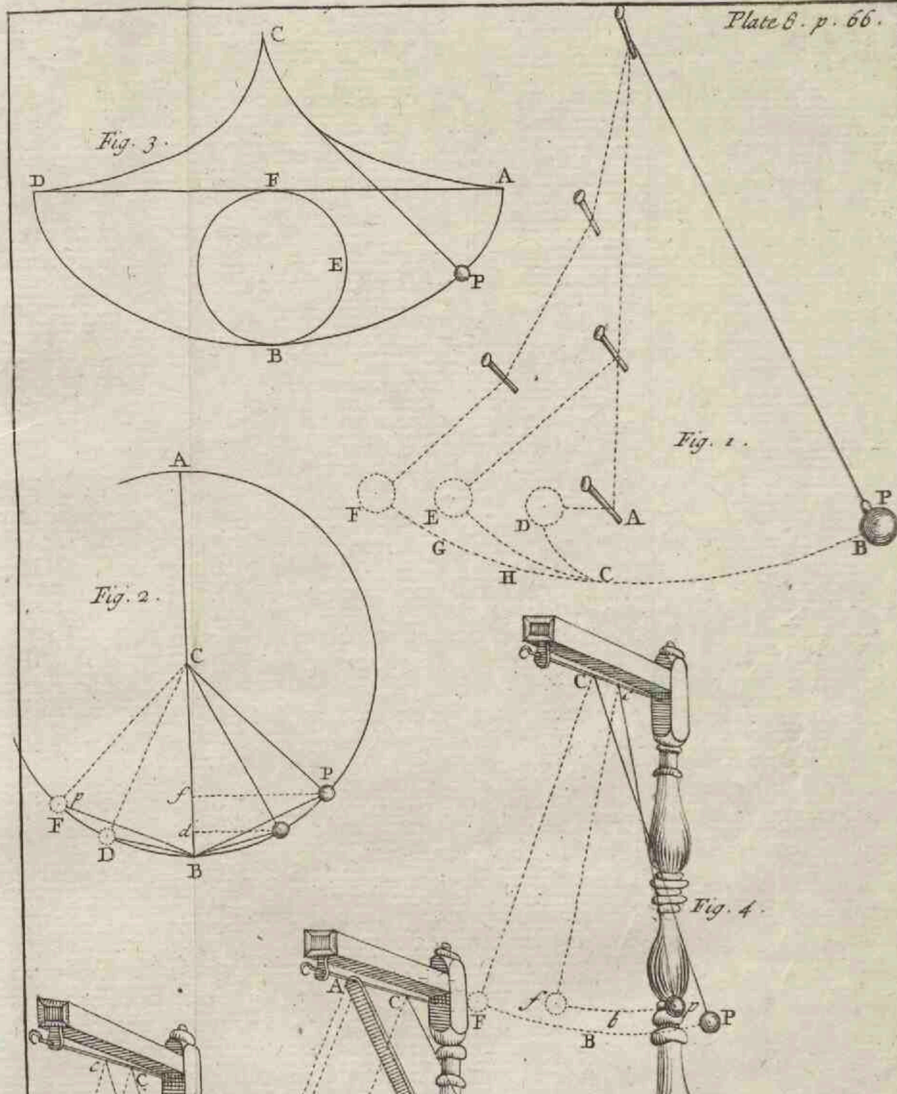


Fig. 3.
C
D
F
A
E
P
B
Fig. 1.
P
F
E
D
A
G
H
C
B
Fig. 2.
A
C
P
f
d
F
D
B
Fig. 4.
C
F
f
b
p
P
B
Fig. 6.
A
C
P
B
Fig. 5.
c
C
f
b
p
P
F
B

is not changed) as the Length of the Pendulum after the Change to its first Length: * which * 158
Lengths are to one another, as the Force of Gravity in the Pendulum that is not changed, to the Force of Gravity in the Pendulum that is changed.
And therefore the Squares of the Times of the Vibra- 165
tions *in equal Pendulums* are to one another, inversely, as *the Forces of Gravity with which the Pendula are acted upon:* which therefore *are to one another*, directly, *as the Squares of the Vibrations perform'd in the same Time.*

But whence this Difference of Gravity arises, shall be explain'd hereafter, when we speak of the Figure of the Earth.

CHAP. XX.

Of Percussion, and the Communication of Motion.

EVery Body that is at Rest, and hinder'd by no Obstacle, may be push'd forward by any other Body in Motion; and, when once it is put in Motion, it will continue in it, till it is hinder'd
by some external Cause.* That Cause is some- * 124
times a Stroke of another Body against it, or a Stroke which itself gives another Body; or lastly a Stroke of both meeting.

The Laws to be observed in that Percussion, are here to be explain'd.

All Bodies, here taken notice of, are supposed spherical; because the Laws of Motion ought to be examin'd in the most simple Case.

DEFINITION I.

A Body is said to *impinge* directly against ano- 166
ther, or two Bodies to strike or impinge against one another, *when the Direction of the Motion, or Motions, (if both are moved) goes thro' the Centers of both Bodies.*

Definition II.

167 *In all other Cafes the* Stroke *is ſaid to be* Oblique. When elaſtic Bodies impinge againſt one another, the Parts that are ſtruck yield inwards, and, by the Reſtitution of the Parts, the Bodies repel one another, and are ſeparated from one another.

168 *In Bodies that are perfectly ſoft, or perfectly hard,* there is no ſuch Action; and therefore, *in a direct Stroke they are not ſeparated after the Blow,* becauſe
169 *after their meeting, as well as before, they are moved in the ſame Line;* for nothing happens that can change the Direction.

I ſhall in this Chapter ſpeak of the Percuſſion of Bodies that are not elaſtic, and here, as alſo in the whole following Chapter, I ſhall ſpeak of direct Percuſſion; and confirm the whole by Experiments made with the following Machine.

170 ABC is a vertical Plane of Wood almoſt triangular, about 4 Foot and a half high, and 3 Foot wide at the Bottom. *Plate* IX.

In the upper Part there is a Slit *st* quite thro' it, which is horizontal, along which two ſquare Pins (and ſometimes more) are moved; theſe Pins, having a Shank that goes through the Plane, may be made faſt in any Part of the Slit by Screws which take the Shanks behind the Plane or flat Board, as may be ſeen from the Figure of the Pins at V.

A little ſquare Pipe of Iron X ſlips upon each Pin, and may be faſten'd to it by a little Screw *e*, in the upper Part of any Place of the whole Length of the Pin. Theſe little Pipes have Hooks in the under Parts, thro' which ſmall Threads or Fiddle-ſtrings run, and ſuſtain ſuch Balls as P and Q. Thoſe Strings go round the wooden Keys *l*, *l*, by turning which, the Balls are rais'd or lower'd.

The

The Pin, from which any Ball hangs, is fix'd to ſuch a Part of the Slit *st*, that its Center is diſtant from the Line A D (which divides the Machine into two Parts vertically) juſt one Semidiameter of the Ball; and that is to be done for all the Balls by means of Marks in the Surface of the Board.

The little Pipe and Hook, from which the Ball hangs, is fix'd to ſuch a Part of the Pin, that the Thread hangs but a little farther from the Surface of the Board than a Semidiameter of the Ball: There are Diviſions in the Pins, to determine the Place of the Iron Pipes upon the Pins, according to the Bigneſs of the Balls.

When you uſe two Balls, the Line A D ſeparates them, and in that Caſe (as alſo when ſeveral at once are made uſe of) if they are of different Bigneſs, the great Ball always determines the Diſtance of the little Ball from the Board; and the little Pipes are fix'd at ſuch Diviſions of the Pins, that the Centers of all the Balls may be equally diſtant from the Board. The Keys *l* bring all thoſe Centers to the ſame Heights; which is to be obſerv'd in all the Experiments.

There are two Braſs Rulers E G, E G, which ſlide horizontally in the Board, whoſe Surface is hollow'd to receive them, ſo that their Surface may lie even with it. Behind each Ruler there is a Slit in the Board of about 5 Inches, to tranſmit a Screw coming from the Backſide of the Ruler, which is fix'd behind a Nut in any Part of the Slit. In making Experiments, the End G of each Ruler is diſtant from the Line A D, one Semidiameter of the Ball, which hangs on the ſame Side.

Theſe Rulers are ſo divided as to ſhew equal Angles, run thro' by the Threads which carry the Balls.

To meaſure thoſe Angles in making the Experiments, there are four Indices, two great ones M M, and two leſs N N.

Theſe Indices, ſliding in a Groove, are moved along the Slits *or*, *or*, and are faſten'd behind the Board, where you pleaſe, in the ſame Line by Screws. The longer Indices reach to the Edge of the Board, tho' the Slits want about 3 Inches of it.

The ſeparated Figure M repreſents the greater Indices, in which *ab* is a Plate, which ſlides in the Groove of the Board; *cd* is the Index, perpendicular to that Plate, and about 3 Inches long.

The other ſeparated Figure N repreſents one of the leſſer Indices, whoſe Length is equal to the Semidiameter of the ſmaller Balls, which are applied to the Machine, and whoſe Diameter may be about 1 Inch and a Half: Theſe Indices are put among the great ones, becauſe they don't hinder the Motion of the Balls: Sometimes the two ſmall ones are put in the ſame Slit, when three Angles are to be meaſured on one Side.

In that Caſe the Ball Q is raiſed up, or riſes after its Fall towards the Side of the Board B. That the Index may be placed right for meaſuring that Angle, the end G of the Ruler E G, which is on the Side B, muſt be joined with the End G of the other Ruler, placed as above-mentioned.

The three Iron Screws F F F ſerve to ſet the Machine or Board truly vertical, ſo as to bring the Line A D perpendicular to the Horizon; which may be eaſily done by hanging on any one of the Balls, and putting on one of the great Indices; ſo that the Thread, cutting any Mark on the Index, may hang parallel to the Line A D.

For

For making Experiments on Bodies that have no Elaſticity, you muſt uſe Balls of ſoft Clay, made in the wooden Mould L.

This Mould conſiſts of five Parts, four of which may be ſeen at H, H, H, H; theſe being join'd, contain a ſpherical Cavity of an Inch and a Half Diameter; with a Hole in the lower Part: there is a Screw on the Outſide, by which they are preſs'd together by means of the Ring I, that has a Screw on the Inſide.

L repreſents all the Parts join'd together; there is a Hole in *v*, which has a Communication with the Inſide of the Mould: thro' this Hole muſt go a Thread, which lies irregularly in the Clay, almoſt thro' it. Before you put the Clay into the Mould, you muſt anoint the Inſide of it with Oil; then, when all the Parts have been join'd and preſs'd together by ſcrewing on the Ring, take them aſunder again, and you will find a ſmooth and round Clay Ball, to the Thread of which you may faſten another Thread, and immediately hang it upon the Machine.

The Experiments, relating to elaſtick Bodies, are made with Ivory Balls. You muſt have ſix ſmall ones, of an Inch and half Diameter. Beſides thoſe, one of double the Weight, another of three times the Weight, and a fourth of four times the Weight.

In the 11th Experiment of the following Chapter, the ſix equal Balls above-mentioned are hung on to the Machine at the ſame Time, ſo as to touch one another. And this is done (ſee *Fig.* Z) by means of the Plate *mn*, which is fix'd to the Machine by help of the Screws *q q*, which go thro' the Slit *st*. This Plate contains four Pins, *p*, *p*, *p*, *p*, in whoſe Ends are Holes, thro' which the Threads paſs that carry the Balls. The Threads are brought to a proper Length, and ſtaid by the

Keys *l*, *l*, *l*, *l*. The two other Balls hang from the two Pins V, already deſcribed.

To make the 13th Experiment of the next Chapter, there muſt be three ſuch Pins as V.

In this Machine the Percuſſion of the Balls, in the loweſt Part of it, is always direct; and the Balls (whether you let them go from different Heights, the ſame Way or contrary Ways) will
* 155 always come to the Bottom at the ſame Time; * and ſo in that Caſe the Percuſſion is always direct; the Celerities at the Bottom are mark'd by the
* 159 Diviſions of the Rulers EG, EG; * for in Arcs no greater than ſuch as the Balls deſcribe in this Machine, the Ratio between the Arcs and Chords does not ſenſibly differ. The Heights, from which the Balls are let fall, determine the Celerities before the Stroke; and the Heights, to which the Balls riſe, their Celerities after the Stroke.

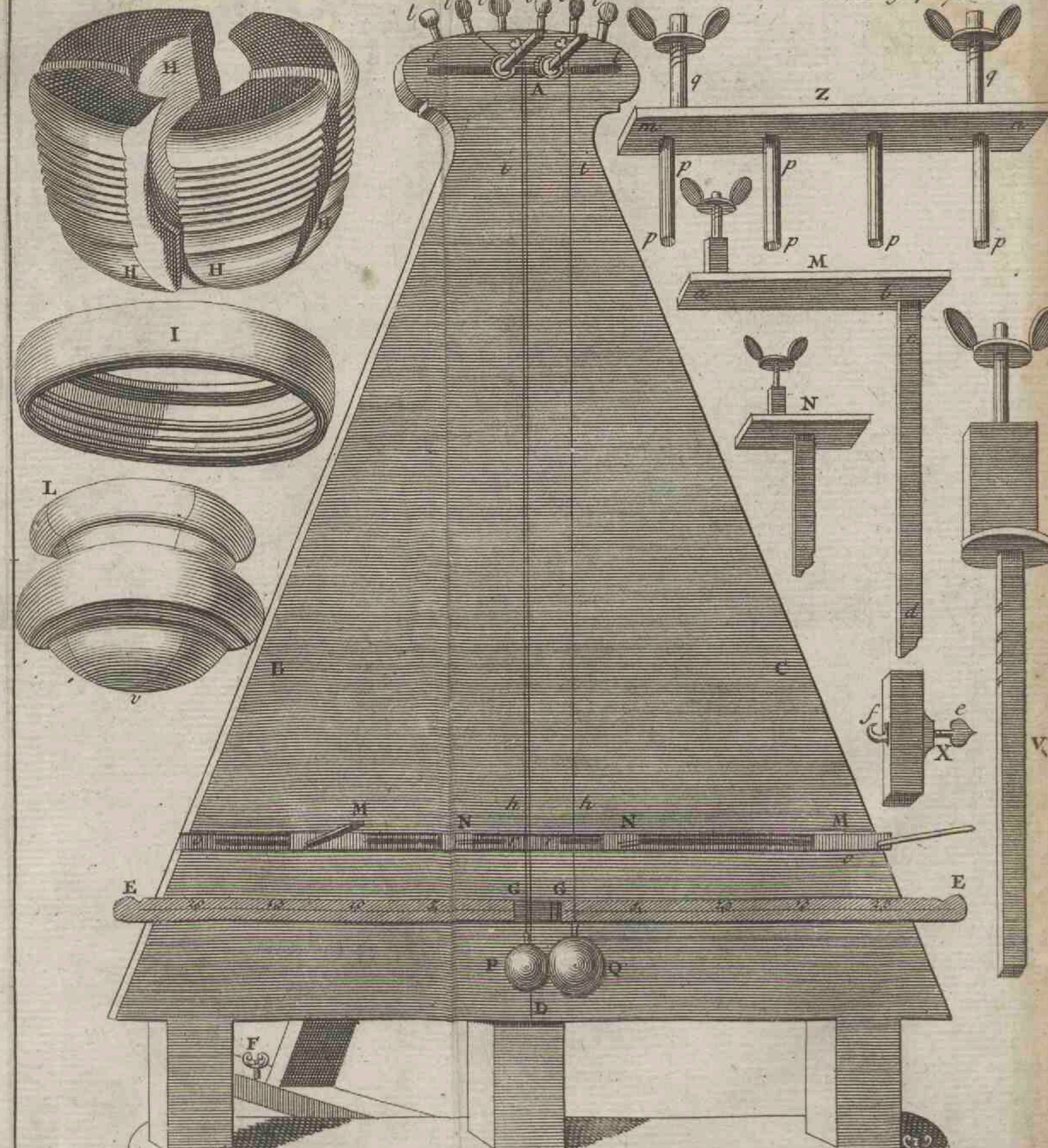
All that relates to the Percuſſion of Bodies not elaſtic, may be referr'd to the four following Caſes.

171 Caſe 1.] *If one Body ſtrikes againſt another Body that is at reſt, both together will continue their Mo-*
* 168 *tion in the ſame Direction as the firſt Motion*; * *and*
169 *the Quantity of Motion, in the two Bodies, will be the ſame after the Stroke, as in the ſingle one before it.*

For the Action of the Body in Motion, upon the other, communicates to it all the Motion that it acquires; now the Re-action of this laſt in the firſt retards its Motion; and as Action
* 126 and Re-action are equal, * therefore the Quantity of Motion, acquir'd by one Body, is equal to the Quantity of Motion loſt by the other; and ſo the Quantity of Motion is not changed by the Stroke.

This

Plate 9. p. 72.



This Quantity of Motion is found by multiplying the Mass of the first Body by its Velocity; * and dividing that Quantity by the Mass of both Bodies, you will have the Velocity after the Stroke. * 64

For Example, take two equal Bodies, in each of which the Quantity of Matter may be express'd by One; let the Velocity of the moving Body be Ten, the Quantity of Motion will also be Ten, which must be divided by Two, the Mass of both Bodies, and Five, the Quotient of the Division, will be the Celerity of the Bodies after the Stroke.

Experiment 1.] Take the two soft Clay Balls P and Q, and hang them upon the Machine of *Numb.* 170. See *Plate* X. *Fig.* 1.

Let fall the Ball P from the Height answerable to the tenth Division of the Ruler EG, so that it may strike against the Ball Q, which is at rest; after the Stroke they will both move together, and rise up on the other Side to the fifth Division of the other Ruler EG: The rest of the Experiments in this Chapter are made with the same Sort of Balls.

Case 2.] *If one Body strikes another that moves the same Way, but slower, they will both continue their Motion in the same Direction as before; and the Quantity of Motion, after the Stroke, will be the same as before.* 172

The Reason of this Proposition is the same as that of the foregoing.

In this Case the Celerity of the Bodies, after the Stroke, is determined by multiplying each Body by its Celerity, the Products of which Multiplications will give the Quantities of Motion in each Body; * by collecting them into one * 64

Sum, you have the Quantity of the whole Motion; which if you divide by the Maſs of both Bodies, the Quotient will be the Celerity required.

Experiment 2.] Take the equal Bodies P and Q (*Plate* X. *Fig.* 2.) and let them go towards the ſame Side, P with the Velocity 10, and Q with the Velocity 6; as the Maſs of each Body is 1, the
* 63 Quantity of Motion in both together will be 16;* which if you divide by 2, the Maſs of both Bodies, the Quotient will be 8; and the Experiment will ſhew the Velocity to be anſwerable to this.

173 Caſe 3.] *When two Bodies, with equal Quantities of Motion, are carried towards contrary Sides, the whole Motion will be deſtroyed by their meeting, and the Bodies will be at reſt.*

* 168 The Bodies are not ſeparated after the Stroke, * and the Line in which they move cannot be
*169 changed; * but that they may continue to move in the ſame Line, it is required that one Motion ſhould overcome the other, which implies a Con-
* 60 tradiction.*

Experiment 3. *Plate* X. *Fig.* 3.] Let two equal Bodies P and Q fall from contrary Sides with equal Velocities, and as ſoon as they meet they will be at reſt.

174 Caſe 4.] *Two Bodies moved with different Velocities contrariwiſe, after having ſtruck one another, will both together continue their Motion in the ſame Direction, towards that Side where there is moſt Motion; and the Quantity of Motion, after their meeting, is equal to their Difference of Motion before the Stroke.*

The

The greatest Motion overpowers; therefore
the Bodies must be carried together the Way that
the Motion is directed;* and a Body, which *168
has a less Quantity of Motion, is carried in the 169
same Line (but in a contrary Direction) as before the Stroke; for this is required, that by the Action of one Body, the whole Motion of the other be destroyed, which cannot bê done, unless that Body by the Re-action loses an equal Quantity of Motion; there remains therefore only the Difference of the Motions.

Multiplying the Mass of each Body by its Celerity, we have the Quantities of Motion; the least of which must be substracted from the greater to have the Differenee of the Motions; which Difference, if it be divided by the Mass of both Bodies, will give the Celerity after the meeting of the Bodies.

Experiment 4. *Plate* 10. *Fig.* 4.] Let the Body Q be moved with the Celerity 14, and an equal Body P in a contrary Direction with the Celerity 6; after meeting, the Body Q continues its Motion, and carries along with it the Body P with the Celerity 4.

Because of the Equality of the Bodies, the
Quantities of Motion will also be 14 and 6;* *63
and their Difference is 8; which Number being divided by 2, the Mass of both Bodies, the Quotient 4 will be the Celerity after the Stroke.

DEFINITION III.

We call Relative Celerity, *that with which one* 175
Body is carried towards another, or with which two Bodies are separated; in Motions directed the same Way, it is the Difference of the Celerities of the Bodies; and, in contrary Motions, it is the Sum of the Celerities.

In

176 *In the Congreſs of Bodies, the Stroke is proportional to that Relative Celerity.* For the Force of Bodies, ſtriking againſt each other, is increaſed or diminiſhed, according to the Celerity with which two Bodies come towards one another.

CHAP. XXI.

Of the Congreſs of Elaſtic Bodies.

AN Elaſtic Body, whoſe Figure is changed by any Force, will, when the Action of that Force ceaſes, by its Elaſticity or Spring, re-
44 turn to its firſt Figure.

DEFINITION.

177 *A Body has* perfect Elaſticity, *when it returns to its firſt Figure, with the ſame Force with which it was preſs'd in.*

178 In that Caſe, *the Stroke, ariſing from the Reſtitution of the Spring, is equal to the Stroke by which the Figure of the Body was alter'd.*

In this Chapter we ſuppoſe this Sort of Elaſticity, tho' we know no Bodies perfectly elaſtic: in different Bodies, the Force by which the Parts return to their former Figure is very unequal, for which Reaſon we can give general Rules only, concerning perfect Elaſticity; the nearer Bodies approach this Elaſticity, the more exactly will their Motion agree with theſe Rules.

The Experiments, that we ſhall mention in this Chapter, are to be made with the ſame Ma-
*170 chine * that the Experiments of the laſt Chapter were made with; and here we are to uſe Ivory Balls, ſuch as are mention'd in the Deſcription of the Machine; for the Want of perfect Elaſticity, and the Reſiſtance of the Air, do not make a ſenſible Error in the Experiments; which alſo, when

when Neceſſity requires, may be corrected by determining the Difference ariſing from it.

Which Way ſoever two Bodies ſtrike againſt each other, the mutual Actions of the one againſt the other are always equal.* By that Action *126
the Parts of Bodies are puſh'd inwards, and that with equal Force in both Bodies; by their Elaſticity alſo they return with equal Force to the firſt Figure. The Action of Bodies upon each other, from their Reſtitution by their Spring, is equal to the firſt Action from the Stroke;* *178
whence it follows, that *the Action of Bodies upon* 179
each other is double in elaſtic ones; that is, double in reſpect of each Body conſider'd ſingly, becauſe of the Equality of the Action in each. *The Change* therefore, *which in that Caſe is produced in the Motion of each Body by the Stroke, is double that which the Stroke would by the ſame Motion produce in Bodies that have no Elaſticity*; and as, in reſpect of theſe Bodies, the Change (both in reſpect to the Quantity of Motion, and in reſpect to the Celerity) is determined in the foregoing Chapter; we may alſo determine what the Change will be in thoſe, that is, in elaſtic Bodies: In which the following *Rules* are to be obſerv'd.

RULE I.

When Bodies that are not elaſtic ſtrike againſt 180
each other, if one Body acquires a certain Quantity of Motion, it would require twice as much, if the Bodies were elaſtic; and this double Quantity is to be added to the firſt Motion, in order to determine the Motion after the Stroke.

RULE II.

When two Bodies that are not elaſtic ſtrike againſt 181
each other, if one Body loſes a certain Quantity of Motion, it would loſe twice as much, if the Bodies were elaſtic;

elastic; and that double Quantity must be substracted from the first Motion, in order to determine the Motion after the Stroke.

What is said of Motion must also be understood of Velocity; because in the same Body the Mo-
63 tion is proportionable to the Velocity.

Experiment 1. *Plate* X. *Fig.* 5.] Let the Body P, whose Mass is 2, and Celerity 9, strike against Q, a Body at rest, whose Mass is 1; after the Stroke, Q moves with the Velocity 12, and P continues its Motion with the Velocity 3; which agrees with the former Rules: For if the Bodies were not elastic, the Celerities of both after meet-
171 ing would be 6; and so the Body Q would ac-
*180 quire 6 Degrees of Velocity, and by *Rule* I.* there-
fore it must acquire 12 Degrees; the Body P losing
*181 3 Degrees of Velocity, by *Rule* II.* must lose 6, which if you take from 9, the former Velocity, there remain 3 Degrees of Velocity.

Experiment 2. *Plate* X. *Fig.* 6.] If the Body P, whose Mass is 2, and Velocity 8, strikes the Body Q, whose Mass is 1, and which is carried the same Way with the Velocity 5; after the Stroke, the Body Q moves with the Velocity 9, and P with the Velocity 6; which again might have been determined by the foregoing Rules.

If the Bodies had not been elastic, both would have moved after the Stroke with the Celerity
172 7: the Body Q would acquire 2 Degrees of Celerity, which, by *Rule* I. must be doubled, and added to 5, the first Celerity, which gives us 9: The Body P lost one Degree of Velocity, and, by *Rule* II. it must lose 2; therefore it has 6 left.

RULE

RULE III.

When a Body loses its whole Motion, and acquires a Motion the contrary Way, those two Motions must be collected into one Sum, in order to have the Motion that is lost. 182

When the Quantity, which is to be substracted by Rule II. *exceeds the Quantity of Motion before the Stroke, from which it must be substracted, that whole Quantity of Motion is destroyed, and what remains,* (that is, its Difference from that which it should have been substracted from) *gives the Motion the contrary Way.*

Experiment 3. *Plate* X. *Fig.* 7.] Let the Body P strike with the Velocity 12 against another Body Q, which is three times as heavy, and at rest, and it will return with the Velocity 6. In this Case, Bodies not elastic would move with the Celerity 3; therefore the Body P would have lost 9 Degrees of Velocity, but by *Rule* II. * it must *181 lose 18; which if you substract from the former Velocity 12, you have 6 Degrees the contrary Way, by *Rule* III.* In this Manner may be de- *182 termined, by the following *Experiments*, what is laid down in the *Rules*.

Experiment 4. *Plate* X. *Fig.* 8.] Let the Body P be carried with the Velocity 19, the same Way as Q, that is, three times as heavy, and move with the Velocity 3; after the Stroke the Body returns with the Velocity 5.

Experiment 5. *Plate* X. *Fig.* 9.] Let the two Bodies P and Q come towards one another with equal Quantities of Motion; after the Stroke both will return with the same Celerities with which they came upon each other.

Expe-

Experiment 6. *Plate* X. *Fig*. 10.] Let P with the Velocity 5, and Q of triple the Weight, with the Velocity 11, move in contrary Directions; after the Stroke, Q continues its Motion with the Celerity 3, and P returns with the Celerity 19.

Experiment 7. Plate X. *Fig*. 11.] Let the same Bodies P and Q be carried in contrary Directions, P with the Celerity 16, and Q with the Celerity 8; both will be reflected after the Stroke, P with 20, and Q with 4 Degrees of Velocity.

All the Cases of the Percussions of elastic Bodies may be determined by the Rules above-mention'd; the following remarkable Observations are also deduced from them.

183 *When the Bodies are equal, and move the same Way, they continue their Motions, interchanging their Velocities; if they move contrariwise, then they are reflected from each other, likewise interchanging their Velocities.*

Case 1. *Plate* XI. *Fig*. 1.] Let the Bodies move the same Way, and let A B be the Velocity of one Body, and B C the Velocity of the other; the Velocities here are as the Quantities of Motion.* Let the Line A C be divided into two equal Parts at D, and let D *b* be equal to D B; A D or D C gives the Celerity of each Body after the Stroke, if they are not elastic; *so the Celerity B C is increased by the Quantity D B, but as it must be doubly increased because of the Elasticity, * it will become *b* C: The Celerity A B, in Bodies not elastic, is diminished by the Quantity, D B; but it must be diminished by double that Quantity, for the Reason above-mentioned;* and therefore it will now become A *b*.

*63 *172 *180 *181

There-

Plate 10. p. 80

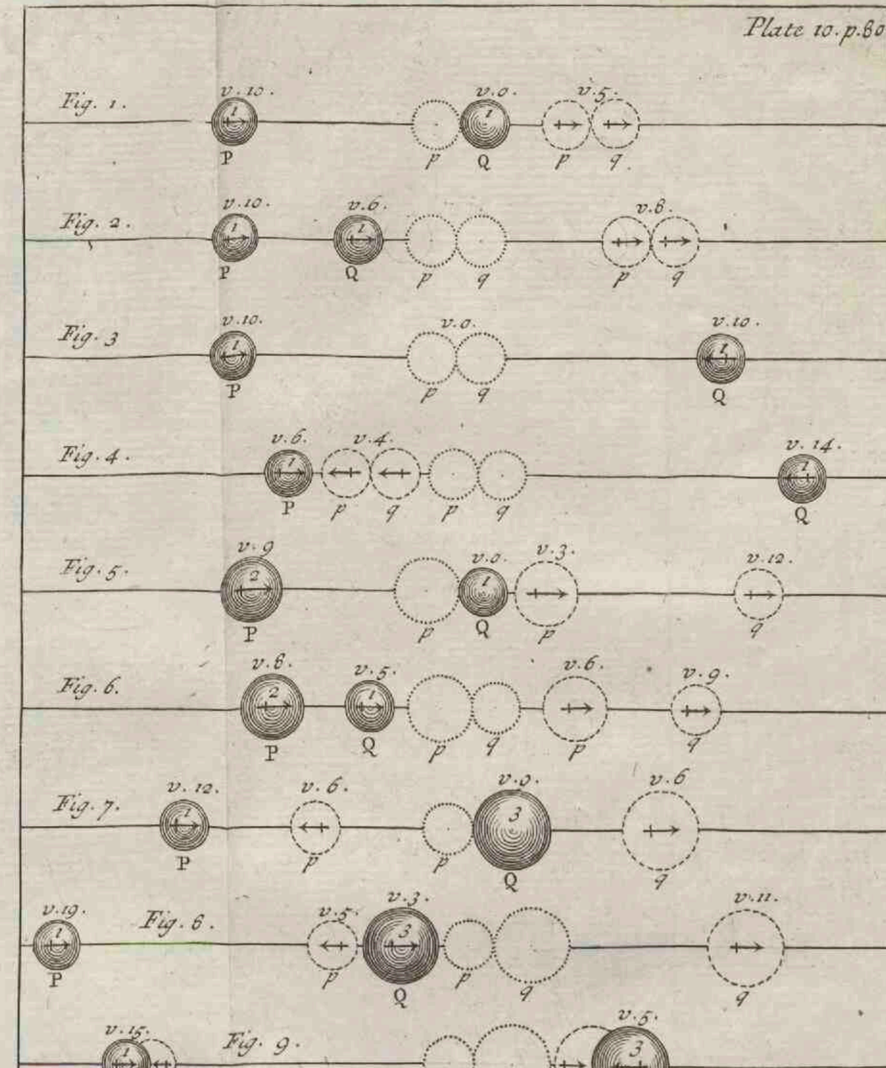


Fig. 1.
v. 10. v. 0. v. 5.
P p Q p q
Fig. 2.
v. 10. v. 6. v. 8.
P Q p q p q
Fig. 3
v. 10. v. 0. v. 10.
P p q Q
Fig. 4.
v. 6. v. 4. v. 14.
P p q p q Q
Fig. 5.
v. 9 v. 0. v. 3. v. 12.
P p Q p q
Fig. 6.
v. 8. v. 5. v. 6. v. 9.
P Q p q p q
Fig. 7.
v. 12. v. 6. v. 0. v. 6
P p p Q q
Fig. 6.
v. 19. v. 5. v. 3. v. 11.
P p Q p q q
Fig. 9.
v. 15. v. 5.
P p p q q Q
Fig. 10.
v. 19. v. 5. v. 3. v. 11.
p P q p q Q
Fig. 11.
v. 20. v. 16. v. 4. v. 8.
p P p q q Q

Therefore the Velocities A B and B C are changed into the Velocities A*b* and *b* C; but A B and *b* C, as also B C and A*b*, are equal to one another.

Experiment 8.] Let two equal Bodies P and Q, the first with the Velocity 10, and the other with the Velocity 5, be carried the same Way; they will continue their Motion after the Stroke, interchanging their Velocities: Which also agrees with the Computation by the foregoing Rules.

Case 2. *Plate* XI. *Fig.* 2.] Let A B be the Celerity of one Body, C B the Celerity of the other; let the Difference A C be divided into two equal Parts at D, and let A*b* be equal to C B. When the Bodies are not elastic, the Velocity of each of them, after the Stroke towards the same Side, is A D; * therefore the first Body has lost the Velocity D B, and the other has lost the whole Velocity C B, and acquired D C the contrary Way; therefore the whole Quantity lost is also D B; * if this Quantity be doubled, it will be *b* B, the Quantity of Celerity lost by both Bodies; * the Difference of that Velocity with the Velocity of each Body, does in each Body give a Velocity the contrary Way; * that Difference for the Motion A B is *b* A, and for the Motion C B is C*b*; but C*b* and A B, as also *b* A and C B are equal to each other.

* 174 * 182 * 181 * 182

Experiment 9.] If the equal Bodies P, with the Celerity 10, and Q with the Celerity 5, are carried contrariwise, they will both be reflected after the Stroke, interchanging their Velocities.

184 *When an elastic Body strikes another equal to it that is at rest*, that the Velocities may be changed, *the percutient Body will be at rest after the Stroke,*

and the other will go on with all the Velocity of the Percutient: Which is confirmed by

Experiment 10.] Let the Body P, with the Velocity 10, ſtrike the Body Q which is at reſt; P will be at reſt after the Stroke, and Q will go forward with the Velocity 10. And this ſerves to explain the following

Experiment 11.] 1*ſt*, Let ſeveral equal Bodies P, Q, R, S, T, V, (*Plate* XI. *Fig.* 3.) be placed in the ſame Line, and touching one another; if the Body ſtrike againſt Q with any Velocity, after the Stroke, P, Q, R, S, and T, will remain at reſt, and V only be moved.

2*dly*, Let P and Q move with equal Velocities, ſo that Q may ſtrike againſt R; after the Stroke P, Q, R, and S, will be at reſt; but T and V will move forward together.

3*dly*, If Three are let go together, they will alſo ſtrike off Three.

4*thly* and laſtly, If P, Q, R and S be moved at once, ſo that S ſtrikes T, after the Stroke P and Q will be at reſt, and R, S, T, V will move together. In general, let the Numbers of Balls be what it will, how many ſoever move on before the Stroke, ſo many alſo will move off in the ſame Direction after the Stroke.

In the firſt Caſe, the Body P (*Plate* XI. *Fig.* 3.)
* 184 ſtrikes Q, and then is at reſt, * Q ſtrikes R, and is alſo at reſt after the Stroke; and ſo it happens to the others, till at laſt T ſtrikes V, which, having no Obſtacle to ſtop it, does alone continue in Motion.

In the ſecond Caſe, the Body Q (*Plate* XI. *Fig.* 4.) does in the ſame Manner drive forward the Body V; P immediately follows, and ſtrikes Q, which, on account of the firſt Stroke, was

at

at reſt, but now communicates its new Motion forward to T, (in the Manner above-mentioned) which is not able to ſtrike V, that is already in Motion; and as the Motions of P and Q are equally ſwift, and thoſe Bodies follow one another very cloſe, there is no ſenſible Time between thoſe two Communications of Motion; which is the Reaſon that the Bodies V and T are moved equally ſwift, and not ſeparated from each other.

The relative or reſpective Velocity, with which 185
two elaſtic Bodies whatever recede from each other after the Stroke, is the ſame as the reſpective Velocity with which the Bodies came againſt one another.

If the Bodies were not elaſtic, they would jointly continue their Motion; * and in that Caſe, * 168
by the Action of the Bodies upon each other, the whole reſpective Celerity, by which they come to one another, is deſtroyed; the Action from the Reſtitution of the Spring is equal and contrary,
* and therefore it muſt generate the ſame reſpe- * 178
ctive Celerity, with which they recede from each other. Let the Inequality of the Bodies be what it will, nothing is changed thereby, becauſe of the Equality of the firſt and ſecond Action upon each Body.* * 126

The Quantity of Motion towards the ſame Side, 186
or the ſame Way, is the ſame after as before the Stroke.

For Bodies that are not elaſtic, this Propoſition is proved in all Caſes; * for when the Mo- * 171
tions do not conſpire, the contrary Motion muſt 172
be ſubſtracted from the Motion one Way, in or- 173
der to determine the Motion that Way. By the 174
Reſtitution on account of the Elaſticity, equal Quantities are generated towards each Side, ** * 120
by which the Quantity of Motion towards one Side is not changed. Theſe two laſt Propoſitions

are fully confirmed by the above-mentioned Experiments.

187 *When a ſmall Body ſtrikes againſt another greater Body, which is at reſt, the great Body acquires a greater Quantity of Motion than the ſmall one had before the Stroke.*

The Quantity of Motion, acquired by the great Body, is double the Quantity which the little one would loſe, if the Bodies had no Elaſticity; but in that Caſe, the little Body would loſe more than
* 180 half its Motion*.
171

Experiment 12. *Plate* XI. *Fig.* 7.] Let the Body P, with the Velocity 15, (which Number alſo does here expreſs the Quantity of Motion) ſtrike the Body Q, which weighs four Times as much, and is at reſt; the Body Q acquires the Velocity
* 171 6; * that is, 24 Degrees of Motion. But the
150 Body P returns with 9 Degrees of Velocity; and ſo the Quantity of Motion, towards that Way
* 186 where P was firſt directed, continues to be 15.*

The Motion is more increaſed in the Body Q, by the Interpoſition of a Body of mean Bigneſs between the Bodies P and Q.

188 *Experiment* 13. *Plate* XI. *Fig.* 8.] Let there be the two Bodies P and Q before-mentioned, and between them the Body R double the Body P; if the Body P, with 18 Degrees of Velocity, comes upon R which is at reſt, it will communicate to it
* 171 12 Degrees of Velocity; * with which if this Body
180 ſtrikes upon Q that is at reſt, it will communicate to it the Celerity 8, that is, 32 Degrees of Motion; but in this Experiment, becauſe of the double Percuſſion, the Error ariſing from the Want of perfect Elaſticity, is more ſenſible than in the others, where there is but one Percuſſion, and

and ſo the Quantity of Motion, acquired by the Body Q, is about 30.

The greater the Number is of unequal Bodies, which are interpoſed between two Bodies, if the Maſſes always increaſe from the firſt to the laſt, ſo much the greater will the Quantity of Motion be in the greateſt; and it will be the greateſt of all (the Number of Bodies interpoſed remaining the ſame) when the Maſſes of all the Bodies increaſe in a Geometrical Progreſſion. 189

Tho' the Quantity of Motion, directed the ſame Way in the Congreſs of Bodies, whether elaſtic or not elaſtic, remains the ſame, the Quantity of the Motion itſelf does not always remain the ſame, but is often diminiſh'd, * and alſo often * 173
increaſed; * ſo that there is no Reaſon to ſay, that * 174 187
there is always the ſame Quantity of Motion in the World.

CHAP. XXII.

Of compound Motion, and oblique Percuſſion.

A Body in Motion may be acted upon by a new Force, and driven according to another Direction; in that Caſe the Change of Motion follows the Proportion and Direction of that Force: * And as the firſt Motion is not deſtroy'd * 125
by that Action, from theſe two Motions a third ariſes, according to a new Direction.

Let the Body P (*Plate* XI. *Fig.* 9.) *be driven* 190
by any Force, according to the Direction PC, *and at the ſame time let it be driven by another Force, according to the Direction* PB; *and let the Celerities, ariſing from thoſe Forces, be as thoſe Lines* PC, PB. In order to determine what will happen, let the Parallelogram PBAC be compleated, by drawing the Lines BA, CA, parallel to the Lines above-mention'd; let PA be the Diagonal of

that Parallelogram. Let the Body be ſuppoſed by the firſt Force, that is, with the Celerity P C, to deſcribe the Line P C, and let that whole Line be carried along in the Direction, and with the Celerity P B; when that Line is come to *b a*, the Body will be at *p*, ſo that P *b* will be to P B, as *b p* to *b a*, or A B, that is, it will be in the Diagonal P A, and ſo always. When the Line P C is at B A, the Body will be at A; therefore *from a Motion compounded of the two Motions above-mentioned, there ariſes a Motion along the Diagonal* P A, *whoſe Celerity is proportional to the Length of*
*53 *the Diagonal*; * for the Diagonal will be run thro' in the ſame Time by the Body P with a compound Motion, as the Line P B or P C would have been gone thro' by the ſame Body, by only one Motion; that is, acted upon by one of the Forces.

To confirm this Propoſition experimentally, we muſt make uſe of the following Machine.

191 It conſiſts of two Boards, or wooden Planes, C D E, C D E, (*Plate* XII. *Fig.* 1.) of the Figure of a right-angled Triangle, whoſe Side C D is in Length about 3 Foot and a half, and the Side D E about 1 Foot and a half; theſe Boards are fixed ſo as to move in a vertical Situation about the Hinges A and B.

The Experiments upon this Machine are made with Ivory Balls of an Inch and a half Diameter. The Planes are ſo joined, that if you conceive two other Planes to run parallel to them, at the Diſtance of a little more than a Semidiameter of the Balls; the Lines, in which theſe imaginary Planes interſect, ſhall be the Axis of their Circumvolution: Which is brought to paſs by the Contrivance of the Hinges, (*Fig.* 2.) whoſe Pares *b*, *b*, are let into the Wood to be the firmer. In the Center of the upper Hinge A, there

there is a ſmall Cylinder *a*, (*Fig.* 2.) in whoſe Caſe there is a Hole, which meets another in the Side, thro' which the Thread *i b* is to run; at one End of this Thread a Ball, as P, hangs, and the other End is joined to the Key *l*. By Help of the Screws F, F, F, F, F, this Machine is ſet perpendicular; ſo as to have the Thread *b i* hang in the Axis of the Machine.

At *m*, *m*, there are two Pins fix'd to the two Planes, from which Pins the Balls Q, Q, hang, at ſuch a Diſtance from the Planes, that they may almoſt touch them; ſo that if you ſuppoſe a Line to paſs thro' the Centers of the Balls P and Q, it ſhall be parallel to the Plane on that Side: Beſides, it is required, that, when thoſe Balls hang at the ſame Height, they ſhall touch one another.

The Threads which are tied to the Balls go thro' the Holes in the ſaid Pins, and are fix'd to the Keys *l l*, ſo that the Balls may be raiſed or let down eaſily, and have all their Centers brought to the ſame Horizontal Plane. There is a Braſs Ruler, or graduated Limb R, bent up in the Arc of the Circle, ſo as to have the Ball P riſe along it in its Motion; and this Limb turns one of its Ends on a Center which is in the Axis of the Machine. This Piece of Braſs ſerves to ſhew to what a Height the Ball P aſcends.

Each Ball Q, when it ſwings, moves along the Plane to which it is applied; and the Height, from which it is made to fall, is ſhewn by an Index fixed to the Plane; to which End there are four Holes in each Plane containing equal Angles, in reſpect to the Motion of the Threads.

When the Ball Q is let fall from any Height, it ſtrikes upon the Ball P, and drives it to the ſame Height in the ſame Direction.* *184

Experiment 1.] The Horizontal Section of this Machine is here represented, (*Plate* XII. *Fig.* 3. and 4.) the Body P may be driven by either of the Bodies Q, with any Direction and Velocity. If the Bodies Q and Q are let fall at the same time, the Body P has two Motions impress'd upon
155 it at the same time, and therefore runs in the Diagonal P *p* of the Parallelogram made in the
190 Manner above-mentioned, to express those two Motions, and runs up to an Height proportionable to the Length of that Diagonal.

The Experiment answers very exactly, whether the Balls Q and Q are let fall from the same Height, or from unequal Heights; and whatever the Angle be that is made by the two Planes, that is, by the Directions of the Motions, whether the Angle be right, acute, or obtuse.

192 *A Body moved any how in the Right Line* PA, (*Plate* XI. *Fig.* 10.) *may always be consider'd as acted upon by two Motions*; and that as many Ways as you please: For you may draw as many different Parallelograms as you please, as PB, AC, pbac, *pbac*, whose Diagonal is the Line abovementioned; and, in every one of them, if there be supposed two Forces acting in the Directions PB and PC, from which the Celerities which the Body would have are as the Sides PB and PC, a Motion will always be produced by the Action of them both at once, which will give a Celerity proportional to the Diagonal.

From this Resolution of Motion into two other Motions, may be determined the Motion of Bodies that strike one another obliquely.

Let Q (*Plate* XI. *Fig.* 11. and 12.) be at rest, and P with the Direction and Celerity PA, strike against it. When P is come to A, draw thro' the Centers of both Bodies the Line DB, and

and then PB perpendicular to it, and compleat the Parallelogram ABPC, the Motion along PA is resolved into two others along PB and PC, or BA, CA: By the Motion in the Direction CA, the Body P does not act upon the Body Q; the Action therefore arises solely from the Motion in the Direction along BA, that is, the Body P, *by the oblique Stroke along* PA *with the Celerity* PA, *acts upon the Body* Q, *in the same Man-* 193
ner, as if it should strike it directly along BA *with the Celerity* BA. And so the Motion of the Body Q from that Action, whether the Bodies be elastic or not, is determined from what has been said of direct Percussion.

The Motion of the Body P (*Plate* XI. *Fig.* 11. and 12.) after the Stroke, is deduced from the same Principle; the Motion along CA is not changed; therefore by that Motion, with an equal Celerity, the Body P is carried in the Direction AE. Now let AE be equal to CA; the Change in the Motion BA is determined, in respect of the Body P, in the same Manner as the Motion of Q, by the two foregoing Chapters; let the Celerity of that Motion be AD, in *Fig.* 11. when the Body goes forward, and in *Fig.* 12. when it returns back; from that Motion, and the Motion along AE, arises a compound Motion in the Diagonal A*p*, which, by its Situation and Length, denotes the Direction and Celerity of the Body P after the Stroke.* *190

When *Bodies* are *equal and elastic*, the whole 194
Motion along BA is destroyed by the Percussion,* and only the Motion along CA is left, and *148
the Body P is also carried in that Direction. In that Case, both the Bodies *do* always *fly from each other in Directions that are at Right Angles with one another, which Way soever the Body* P *comes upon the other Body.*

Expe-

Experiment 2. *Plate* XII. *Fig.* 5.] In the Machine defcribed *Numb.* 191. let the Ball Q and P hang; having fet the Planes at Right Angles; let the Body Q with any Direction, and from any Height, come down upon P, and ftrike againft it: after the Stroke the Bodies will follow the Directions of the Planes, and rife to Heights, which may be determined by what has been faid hitherto.

195 We may alfo the fame Way determine the Motion of two Bodies after the Stroke, when both Bodies are moved, which Way foever they come upon one another. The chief Cafes are reprefented in *Plate* XIII. and all of them are explained exactly the fame Way.

Plate XIII. *Fig.* 1, 2, 3, 4, 5, and 6.] Let the Body P be moved with the Direction and Celerity P A, and the Body Q, with the Direction and Celerity Q *a*; draw the Line B *b*, which goes through the Centers of both Bodies where they touch one another, and let C A and *c a* be drawn perpendicular to the Line above-mentioned, and let the Parallelograms P B A C and Q *b a c* be compleated. The Motion of P is refolved into two others, of which the Celerities and Directions are exprefs'd by C A, B A. The Motions, into which the Motion of Q is refolved, are exprefs'd by *c a*, *b a*; by the Motions along C A and *c a* the Bodies do not act upon one another; therefore thefe Motions are not changed, and after the Stroke are exprefs'd by A E and *a e*, which are equal to A C and *a c*; the Percuffion, from the Motions in the Lines B A *b a*, is direct, and determined in the foregoing Chapters: Let the Body P move towards D, and its Celerity be A D, and the Body Q move towards *d* with the Celerity *a d*. After the Stroke therefore, the Motion

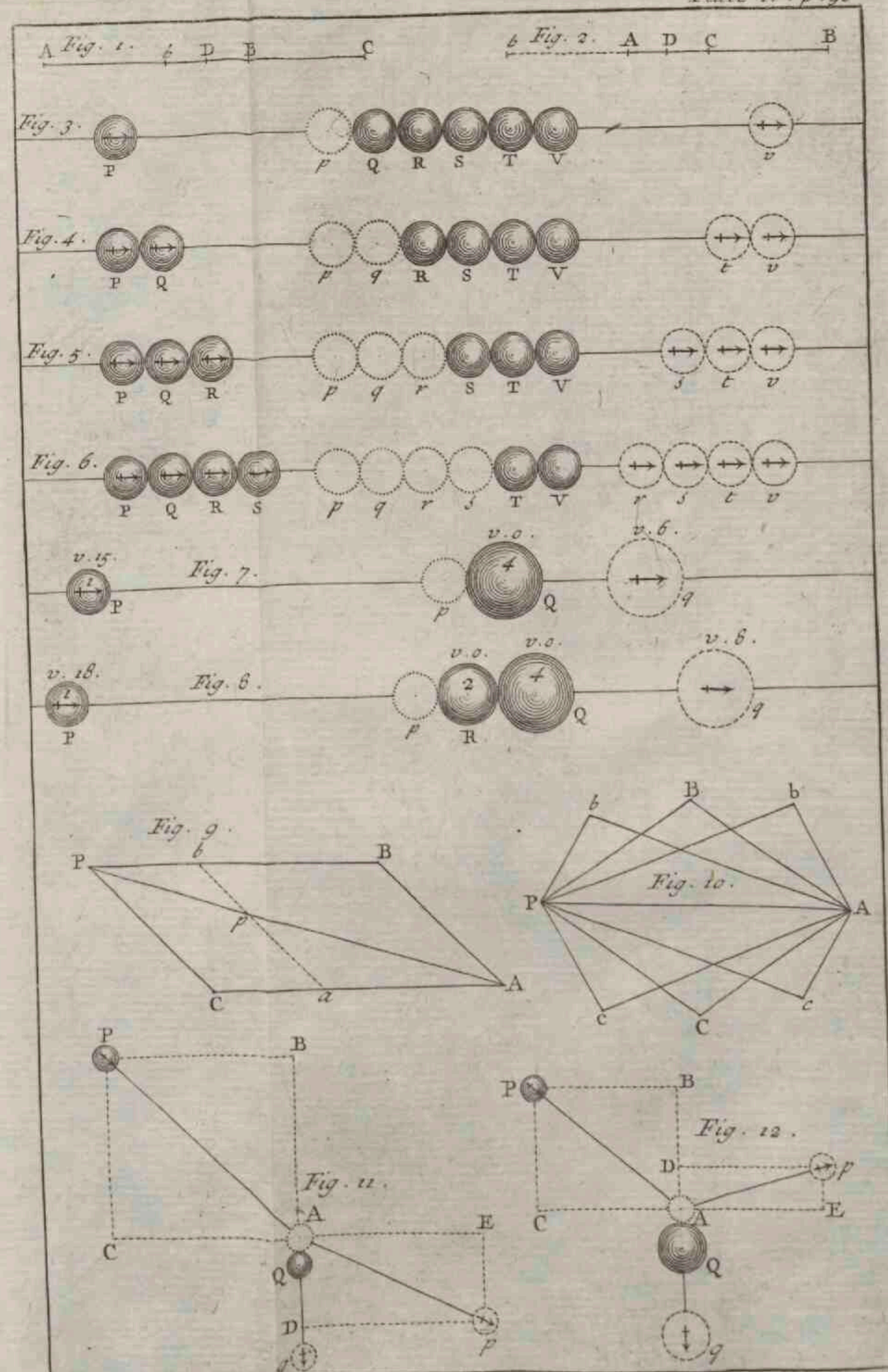


Fig. 1.
A b D B C
Fig. 2.
b A D C B
Fig. 3.
P p Q R S T V v
Fig. 4.
P Q p q R S T V t v
Fig. 5.
P Q R p q r S T V s t v
Fig. 6.
P Q R S p q r s T V r s t v
Fig. 7.
v. 15.
v. 0.
v. 6.
1
4
P p Q q
Fig. 8.
v. 18.
v. 0.
v. 0.
v. 6.
1
2
4
P p R Q q
Fig. 9.
P b B p C a A
Fig. 10.
b B b P A c C c
Fig. 11.
P B C A E Q D p q
Fig. 12.
P B D p C A E Q q

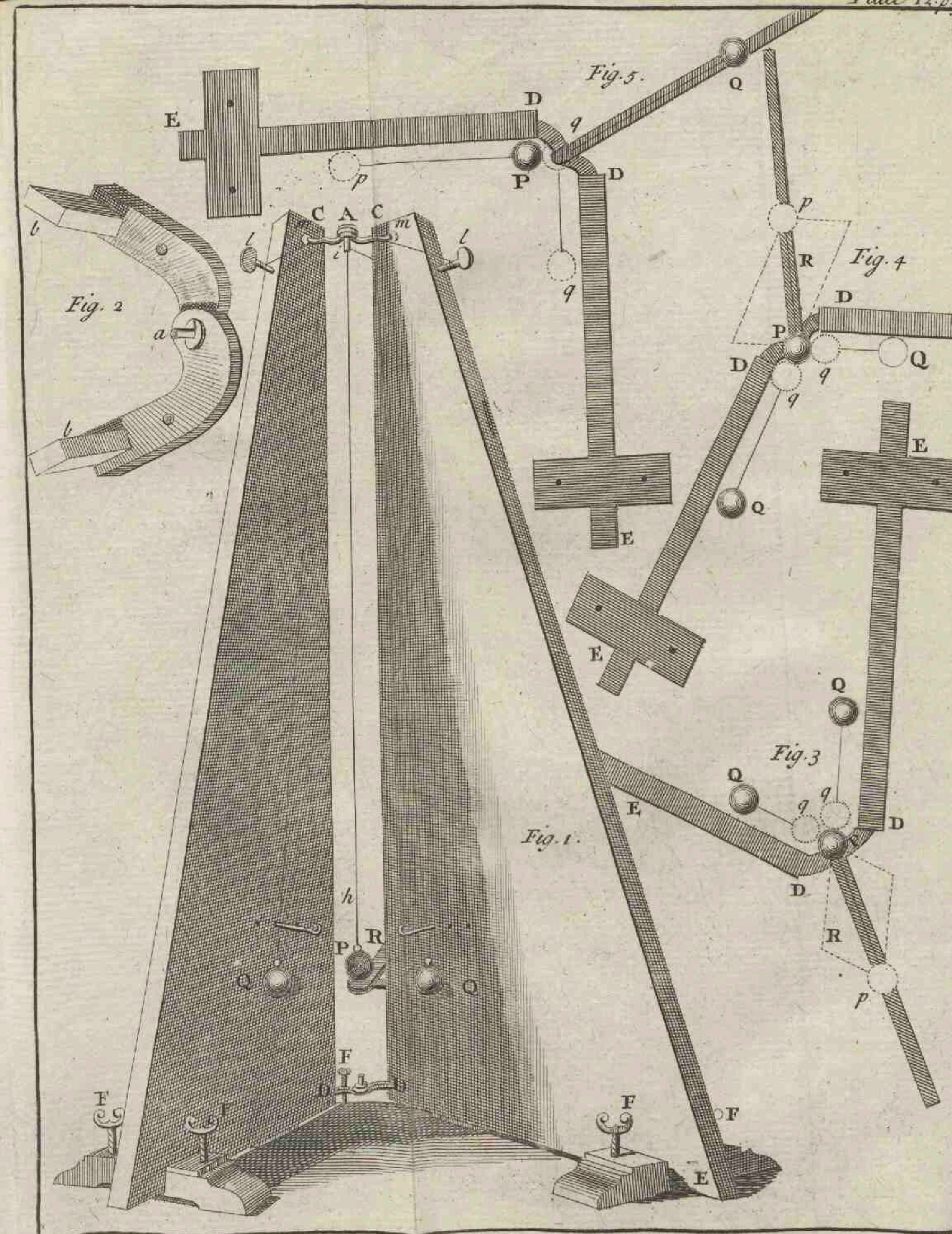


Fig. 1.
Fig. 2
Fig. 3
Fig. 4
Fig. 5.

Motion of the Body P is compounded of the Motions along AE and AD, and moved in the Diagonal A*p*. The Motion of the Body Q, after the Stroke, is compounded of the Motions along *ac* and *ad*, whence that Body is carried in the Diagonal *ae*; and the Lengths of thoſe Diagonals expreſs the Celerities of the Bodies after their Meeting. In the 1ſt, 2d, and 3d Figures the Bodies are ſuppoſed not elaſtic; and in the 4th, 5th, and 6th, the ſame Baſes are put, ſuppoſing the Bodies elaſtic. There are ſome Letters wanting in the firſt Figure, becauſe the Points, which are marked with thoſe Letters in the other Figures, do here coincide with other Points, and are not neceſſary for determining the Motions.

CHAP. XXIII.

Of Oblique Powers.

THE Body P (*Plate* XIII. *Fig.* 7.) being driven in the Directions PB and PC, with Celerities proportionable to thoſe Lines; from thence ariſes a Motion along P, the Diagonal of the Parallelogram PBAC, with a Celerity that is denoted by that Diagonal;* if there be a *190
third Force acting along the Line P*a*, ſo that the Celerity ariſing from it be PA; by that Action the Actions of both the ſaid Forces are deſtroyed, and the Body comes to reſt: If the aforeſaid Actions continue, the Body will continue at reſt; which happens when the Body is drawn towards C, B and *a*, with the ſaid Forces pulling by Threads. Whence it follows, That *a Body will* 196
be at reſt, which is drawn by three Powers, that are to one another, as the Sides of a Triangle made by Lines parallel to the Directions of the Powers.

2 This

197 This Propofition is confirmed experimentally, by the Machine reprefented in *Plate* XIII. *Fig*. 8. It confifts of a round Board of about 8 Inches Diameter, which is in a horizontal Pofition, and fuftain'd by a Foot: Round the Edge of it, within the Thicknefs of the Wood, is a Groove whereby Pullies are applied at Pleafure to any Part of the Circumference; for each Pulley has a Brafs Plate perpendicular to it, which fits into the Groove, when the Pulley is applied. See the Pulley with the Plate reprefented by F.

The Board above-mention'd is a little hollowed in, in the upper Part, fo as to receive a lefs orbicular Board DFA, whofe Thicknefs is about a Quarter of an Inch, and its Surface rifes a little above the firft Board; fo that a Thread that runs over any of the Pullies, being extended horizontally, may juft touch the faid Surface.

You muft have feveral of the leffer round Boards, for making different Experiments. They have Paper pafted upon them on both Sides, that the Lines (to be mentioned hereafter) may be the more eafily drawn upon them.

Experiment 1.] Let C be the Center of the fmall Board, and let there be drawn upon it the Triangle ABC, whofe Sides are to one another as 2, 3, and 4: Let CE be parallel to the Side AB of the Triangle, and let the Side AC be continued towards D.

Now if there be three Threads joined together at C, and ftretched over the Pullies faftened to the greater Board, fo as to be in the Lines CD, CE, and CB; if to the Thread CD you hang 4 Pounds, to CE 3; and laftly, but 2 to the Thread CF, the Threads will not be mov'd, and the Knot remains over C; but if it be mov'd out of that Point, it will not be at reft.

In

In this Proposition any two Powers are balanced by a third, that is, act but as one; which acts contrariwise in the Direction of that third. Therefore *the Actions of two Powers may be reduced* 198
to the Action of one.

So, when a Point is drawn by four Powers, there will be an Æquilibrium, if reducing two Powers to one, this new Power, with the other two remaining, be in the Position of *Numb.* 196; that is, if those remaining Powers being also reduced to one, the Power arising thence be equal with, and acts contrary to the new Power mentioned.

Experiment 2. *Plate* XIII. *Fig.* 9.] The Point 199
C is drawn by four Threads; towards B by the Weight of 1 Pound, towards F by 3 Pounds, towards E by 2 Pounds; and lastly, towards D by 4 Pounds; and this produces an Æquilibrium. Having drawn the Triangle CF*a*, or the Parallelogram CF*a*E, the abovesaid Powers, drawing in CF and CE, are reduced to one that acts in the Direction C*a*, with the Force of 4 Pounds; and then the three Powers, drawing in the Lines CB, CD, C*a*, give us the Case of *Numb.* 196: And therefore if the Powers, drawing along CB and CD, be reduced to one drawing along CA, it will act in the same Direction, but pull against the Power pulling in C*a*, and be equal to it.

What is here said of the four Powers, might be said of five or more; for of five, if two be reduced to one, we come to the last mentioned Case.

Experiment 3. *Plate* XIV. *Fig.* 1.] The Point 200
C is drawn by 5 Powers, pulling by the Threads CA, CB, CD, CE, and CF; the Powers are

are to one another as the Weight by which the Threads are drawn, and they have the ſame Proportion to one another as the Numbers that you ſee at the Pullies in the Figure, and you have an Æquilibrium.

The Powers, drawing in CB and CD, are reduced to one drawing in CG.

The Powers, drawing in CE and CF, may be reduced to one acting in CH; which brings us to the Caſe of *Numb.* 196. Laſtly, thoſe two new Powers, drawing in CH and CG, are reduced to one acting in C*a*, which are equal to the fifth drawing along CA, and pulls in the ſame Line, but contrariwiſe.

201 Beſides this, we deduced from the Propoſition mentioned *Numb.* 169, that the ſame Thing may be ſaid of *the Action of the Power*, which has been ſaid concerning Motion in the foregoing Chap-
*192 ter; * namely, that it *may be reſolved into the Actions of two other Powers*, and that in numberleſs Manners, becauſe Triangles of numberleſs Kinds may be made, tho' you keep one Side ſtill the ſame. Thus in all Engines we can reduce a Power, that acts obliquely, to a direct one; and can determine the Proportion between a direct and an oblique one: Which will appear by the following Examples, that are confirmed by Experiments.

202 *Experiment* 4. *Plate* XIV. *Fig.* 2 *and* 3.] To the Lever AB, whoſe Brachia are equal, apply at B the Weight P of two Pounds, and at A a power acting obliquely in the Direction AD, and which is repreſented by the Weight M. If you imagine a Line, as DE, parallel to the Lever in a horizontal poſition, and AE perpendicular to that Lever; and if AD be to AE as 3 to 2, and the Weight M be of three pounds, there will be an Æquilibrium.

The-

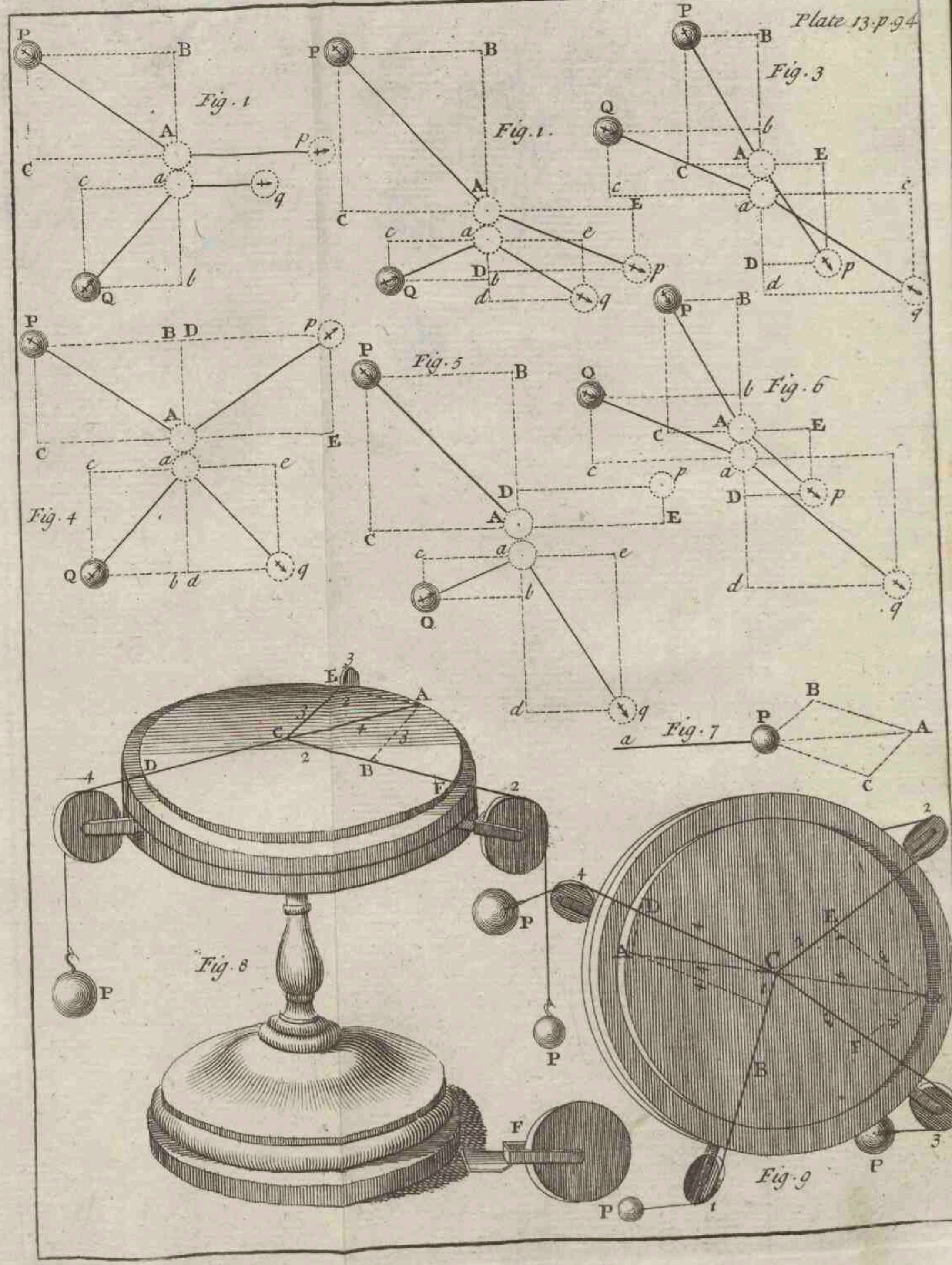


Plate 13. p. 94
Fig. 1
Fig. 1.
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Fig. 9

The Direction of the Motion of the Point A, by the Motion of the Lever, is perpendicular to the Lever, therefore it acts in the Line E A produced. As the Distance B A always remains the same, in the second Figure A is hindered from coming towards B, and as it were repelled in the Direction B A; in *Fig.* 3. the Point A is hindered from receding from B, and so A is as it were drawn towards B. Besides, the Point A is by M drawn towards D; therefore that Point is drawn by three Powers, whose Directions are parallel to the Sides of the Triangle A E D; which therefore, to produce an Æquilibrium, must be to one another as those Sides.

The Point A, by reason of the Equality of the Distances of the Points A and B from the Fulcrum, moving along E A, is drawn with the same Force as P descends, that is, with the Force of two Pounds; the Force therefore along A D must be of three Pounds, because the Sides A D and A E are to one another as 3 to 2. The Side D E expresses what the Fulcrum sustains by the Force with which the Point A in *Fig.* 2. is pushed towards B, or is drawn from it in *Fig.* 3.

The same also may be said of an oblique Power to the Axis in Peritrochio.

Experiment 5. *Plate* XIV. *Fig.* 4.] Let the Weight 203
P, fixed to a Pulley, be sustained by Powers applied on both Sides to the running Rope, but drawing obliquely in the Directions C A and C B; these Powers are equal to one another, because no Part of the Rope, that goes about the Pulley, can be at rest, unless it be equally drawn on both Sides. * The Weight P is as it were a *83
third Power, and so the Point C is drawn by three

three Powers; ſuppoſe the Line C E perpendicular to the Horizon, and the Line A E parallel to C B: If C E be to A E or A C (for theſe two Lines are equal, becauſe of the above-mentioned Equality of the Powers drawing along C B,
*196 C A; *) as 6 to 5, the Weight P of 6 Pounds will be ſuſtained by the Weight Q and Q of 5 Pounds each; the Reaſon of which is evident by *Prop.* 196.

If one End of the running Rope is faſtened to a Pin, the Weight P will be ſuſtained by only one of the Weights Q.

204 *Experiment* 6. *Plate* XIV. *Fig.* 5.] If the Weight P be not joined to the Pulley, but ſuſtained by the Rope C A and C B faſtened to it, it may be ſuſtained by two unequal Powers. Draw the Triangle C A E, as was done in the foregoing Experiments, and let A E be 11, and C A $12\frac{1}{2}$, and C E 12; you ſhall have an Æquilibrium, if the Weights Q and Q are to P, as the firſt Numbers to the laſt; the Reaſon of which Experiment is alſo evident from *Numb.* 196.

205 Here we are to obſerve, that from the given Inclinations of the Threads C A and C B to the Horizon, the Proportion of the Weight Q and Q to the Weight P may be determined by Trigonometrical Tables. If in the Triangle A C E you conceive a Line A*e* drawn thro' the Point A parallel to the Horizon, and that Line be taken for a Radius of a Circle, C A will be the Secant, and *e* C the Tangent of the Angle which C A makes with the Horizon, and A E will be the Secant, and *e* E the Tangent of the Angle of Inclination of the Thread C B to the Horizon: Whence it appears that the Weights Q, Q are proportional to the ſaid Secants, and that the Weight

Weight P follows the Proportion of the Sum of the aforesaid Tangents.

On the Machine with which these last Experiments are made, (the Make of which the Figure alone sufficiently expresses, especially if it be compared with the 4th *Fig.* of *Plate* IV.) draw Lines along which the Threads that go over the Pullies may be stretched; in the Middle of the Lines write down the Numbers, which express the Secants of the Angles which those Lines make with the Horizon; and, at the Ends of the Lines, write down the Numbers expressing the Tangents of these Angles.

Now in every Case where there is an Æquilibrium, the Weights Q and Q are as the Numbers in the Middle of the Lines along which the Threads are stretched; and the Weight P as the Sum of the Numbers at the Ends of those two Lines.

Experiment 7. *Plate* XIV. *Fig.* 6.] For this Ex- 206
periment we must make use of the Machine of *Numb.* 143. *Plate* VII. *Fig.* 7. The Body M, being laid upon an inclined Plane AB, is sustained by a Power drawing along MS; let MR be a Line perpendicular to the Horizon, and ASR perpendicular to the Surface of the Plane; in every Case where the Weight P is to the Weight of the Body M, as MS to MR, the Body will be at rest.

The Body M by its own Weight is drawn in the Directions RM, by the inclined Plane it is sustained in a Direction perpendicular to the Plane, and so that Experiment is reduced to the Proposition of *Numb.* 196.

Experiment 8. *Plate* XIV. *Fig.* 7.] The Brachia 207
of the Lever ACB are equal, and form such an

Angle, that if AC be continued towards D, and BD be drawn perpendicular to CD, DC ſhall be the Half of BC or CA. At A hang one Pound *p*, and at B the two Pound Weight P; then ſetting the Brachium CA in a horizontal Poſition,
* 105 you will have an Æquilibrium; * becauſe the Weight P hangs, as it would do upon a ſtreight Lever hanging at the Point D.

Change the Weights, and let the greater hang at A, and the leſſer be laid upon the Brachium BC at B; (*Plate* XIV. *Fig.* 8.) if by a vertical Plane you hinder this laſt Weight from falling off, you will again have an Æquilibrium.

The Brachia of the Lever are equal, and by the Motion of the Lever move equally; therefore, by the Force of the Weight P, the Weight *p* is as it were drawn towards E, in the Direction perpendicular to the Brachium BC; by the Action of the vertical Plane, that Weight is puſh'd horizontally; and at laſt is puſh'd vertically by the Force of Gravity. Therefore the Weight *p* is drawn by three Powers, which are to one another as the
* 196 Sides of the Triangle BED.* Therefore the Force tending towards the Earth, (that is, the Weight *p* to the Force drawing towards E, namely, the Weight P) is as BD to BE, or DC to CB or CA; that is, as 1 to 2. Which is alſo the Ratio between the Weights *p* and P. And here therefore the Reaſon of the Experiment is deduced from the often mentioned Propoſition of *Numb.* 196; to which all other Caſes imaginable, relating to oblique Powers, alſo belong.

CHAP.

CHAP. XXIV.

Of the Projection of heavy Bodies.

A Body, moved by two Impressions, has a 208
Motion compounded of both; * *if a Body* * 190
be projected, or thrown in the Line AB, (*Plate* XV. *Fig.* 1.) in the Time in which it could run the Length AB, it is by the Force of Gravity carried towards the Center of the Earth the Length BF, and so, by a Motion compounded of both, it is moved in AF; and by that Motion the second Moment it would run through FC, equal to AF, if that second Moment it was not by the Force of Gravity carried in CG, so that the Motion in the second Moment is in FG: After the same Manner, the Motion in the third Moment is in GH, and the fourth Moment in HI; but as Gravity acts continually, those Moments of Time are to be look'd upon as infinitely small, and so you will every where have a Motion compounded in different Directions; that is, an Inflection of Direction in the Body's Motion; in that Case therefore *it will move in a Curve Line.*

This Motion of a projected Body, or Projectile, 209
may be consider'd more simply in all Projections which we make; because all Lines, which tend towards the Center of the Earth, may be look'd upon as parallel, and the Direction from that Motion is always the same; when the projectile Motion *is made up of two Motions, the first equable in the Line of the Projection, and the second accelerated towards the Earth.** * 129

Let a Body be projected in the Line AD, parallel to the Horizon; in equal Times, by that

 Motion

Motion, it will run thro' the equal Spaces AB, BC, CD: By Gravity it will, in a Motion perpendicular to the Horizon, be carried in the Direction BF, CG, or DH, which here are supposed parallel; this Motion is accelerated, and therefore if after the first Moment the Body be at the Point F, after the second it will be at G, after the third at H; so that if you call BF
* 131 one, CG will be four, and DH nine. * The Body will run in a Curve, which goes through all the Points that may be determined in the same Manner as F, G, H, and that Line is called a *Parabola*.

The Machine made use of, for proving this Proposition experimentally, is made of three Parts, as may be seen in *Plate* XV. *Fig.* 3. A*h* is 6 Inches high, DE is exactly of the same Height: The Length *h*H is of 12 Inches, supposing the Point H to be distant 1 Inch from the End of the Cavity in which it is taken.

Let EA be hollowed circularly; or in any other Curve; and let this hollow Channel be overlaid with a Plate of very smooth Tin or Brass, that a Brass Ball may freely roll down it; but Care must be taken that the lower Part of the Curve, at A, shall have a horizontal Direction, that the Ball may quit it in that Direction.

A*h* must be divided into 9 equal Parts, of which A*f* is 1, and A*g* contains 4.

When, to this first Part of the Machine, you add the second B, (*Plate* XV. *Fig.* 4.) it reaches to *g*, and *g*G is 8 Inches long: If upon this you lay the third Part C, this last reaches to *f*, and *f*F is of 4 Inches.

The Diameter of the Ball P, which in making the Experiments is to be let fall along the Curve EA, is of about half an Inch; neither must a less Ball,

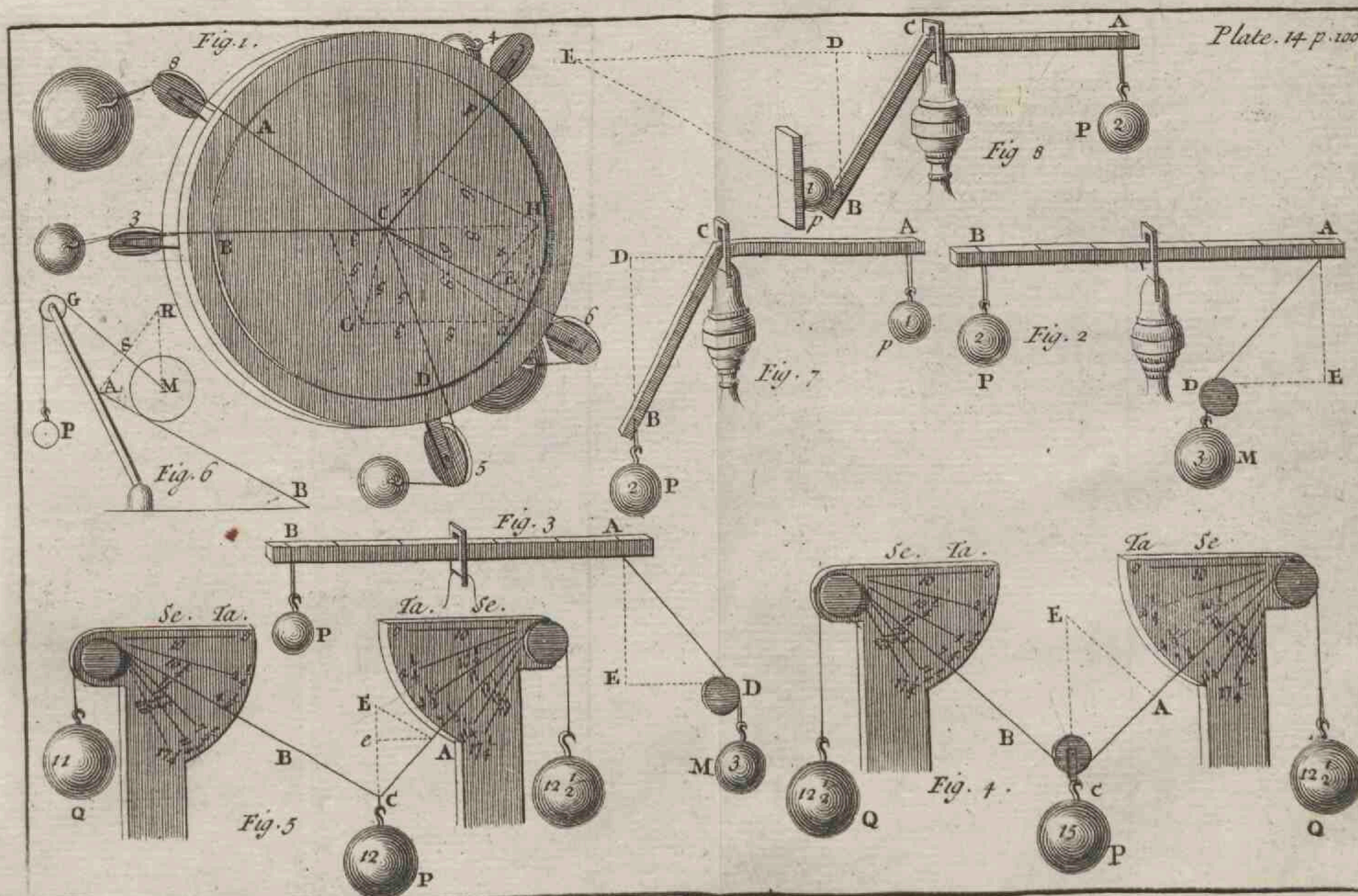


Plate. 14 p. 100
Fig. 1.
Fig. 2
Fig. 3
Fig. 4.
Fig. 5
Fig. 6
Fig. 7
Fig. 8
Se. Ta.
Ta. Se.

Ball, or a bigger Machine than what is here mentioned, be made uſe of; for the leſs the Bodies are, and the ſwifter their Motion, the more in Proportion is the Motion retarded by the Air's Reſiſtance; as ſhall be ſhewn in its Place.

When the Ball P is let fall from E, running down the Curve EA, it acquires ſuch a Degree of Velocity, as appears to be always the ſame in ſeveral Trials; and with that Velocity and horizontal Direction, it continues its Motion.

Experiment.] Having joined together the three Parts of the Machine, as in *Fig*.4. let go the Ball P from E, and it will ſtrike the Point F. Take away the leaſt Part C, and let the Ball come down as before, and it will ſtrike G. Laſtly, take away the Part B, and the Ball, deſcending as before, will ſtrike againſt H.

If you ſtick on a Piece of ſoft Clay upon H and G, the Point of the Stroke will be exactly mark'd; this will not do ſo well in the Point F, becauſe of the great Obliquity of the Motion there; but, by repeating the Experiment, the Point F will be well enough determined by Sight only.

The Propoſition of *Numb*. 134 may be experi- 211
mentally confirmed by this Machine; for, as we have already ſaid, the Ball running down EA will ſtrike the Point H.

Coming down EA, it acquires a Celerity which it could have acquired in falling in ED; * with * 150
that Celerity it is horizontally projected from the Point A, and it moves that Length *b*H equably according to that Direction, whilſt by its Fall it goes thro' A*b* equal to ED; but *b*H is double the Length A*b* or ED.

212 What has been ſaid of the Curve, deſcribed by a Body projected horizontally, may alſo be applied to any Projection.

Let the Body be projected in the Direction A E, (*Plate* XV. *Fig.* 5.) and let AB, BC, CD, DE, be equal; the Body will go through the Curve AFGHI, ſo that BF, CG, DH, EI, will be to one another, as 1, 4, 9, and 16; in which Caſe the Curve is alſo called a *Parabola*.

DEFINITION.

213 Let AI be drawn horizontal, and the Curve above-mentioned will cut it it I; AI is called the Amplitude of the Projection.

The Motions of Bodies projected with the ſame Celerity, but different Directions, may be compared together.

Let AL be the Height, to which a Body, thrown up with a determinate Degree of Celerity, may riſe: Let the Body with the ſame Celerity be thrown along AB, cutting in B the Semicircle deſcribed in the Diameter AL; let AB expreſs that Celerity, and MB be parallel to the Horizon. The Motion in AB may be reſolved
192 into two others; the firſt along MB, a horizontal, and the ſecond along AM, a vertical Line; and it is only by that ſecond Impreſſion that the Body aſcends: The Height therefore, to which the Body aſcends in that Caſe, is to the Height to which it would aſcend with the Celerity AB, as
138 the Square of AM to the Square of AB; that is, as AM to AL; but this is the Height to which the Body aſcends with the Celerity of the Projection; therefore alſo AM is the greateſt Height to which the Body comes in that Projection. In the Time of the Aſcent in AM, the Body might by

by an uniform Motion, with the ſame Celerity with which it moves in A M, in a horizontal Motion go thro' twice the Length of the Line MB; and as the Time of the Fall is equal to the Time of the Aſcent,* the Amplitude AB is four times the Length of the ſame Line M B. Now this Demonſtration will ſerve, whatever the Inclination of the Direction of AB is. Whence we deduce, *136

1. *That the Amplitude is the greateſt, with the ſame Celerity, when the Angle of the Projection is a Half Right Angle.* For then the Line *mb*, being a Radius of the Semicircle, is the greateſt of all. 214

2. *Except this Caſe, there are always two Inclinations, that give the ſame Amplitude*; for if thro' B, B*b* be drawn parallel to A L, cutting the Semicircle at *b*, and *m b* parallel to the Horizon, this Line will be equal to M B; therefore the Amplitude of the Projection, in the Direction A *b*, will alſo be A I. In the ſecond Part of the following Book, all this will be confirm'd by Experiments.

If the Celerity be changed, and the Body projected in the ſame Direction, the Amplitude is changed, in the ſame Ratio as the Diameter A L; that is, *the Amplitudes, the Direction remaining the ſame, are as* the Heights to which Bodies, with the ſame Celerities, being thrown up, may aſcend; and therefore they are as *the Squares of the Celerities.* 215

CHAP. XXV.

Of Central Forces.

124 A Body in Motion continues its Motion in a Right Line, and does not recede from it, unleſs a new Impulſe acts upon it; after ſuch an Impulſe the Motion is compound, and ſo from the two there ariſes a third Motion in a
190 Right Line alſo. If therefore a Body is moved in a Curve, it receives a new Impulſe every Moment; for a Curve cannot be reduced to Right Lines, unleſs you conceive it divided into Parts infinitely ſmall. We have an Example of that Mo-
208 tion in the Projection of heavy Bodies; and another in all Motions round a Point as a Center.

216 *If a Body, that is continually driven towards a Center, be projected in a Line that does not go thro' that Center, it will deſcribe a Curve; and, in all the Points of it, it endeavours to recede from that Curve, according to the Direction* of a Curvature; that is, *of a Tangent to the Curve*; ſo that if the Force driving towards the Center ſhould immediately ceaſe to act, the Body would continue its Motion in a Right Line along the Tangent.

A Stone whirl'd round in a Sling deſcribes a Curve, becauſe the Sling does every Moment, as it were, draw it back towards the Hand; but, if you let the Stone go, it will fly out in the Tangent of the Curve.

Definition I.

218 *The Force with which a Body in the Caſe abovementioned endeavours to fly from the Center*, ſuch as the Force by which the Sling in Motion is ſtretch'd, *is call'd* a Centrifugal Force.

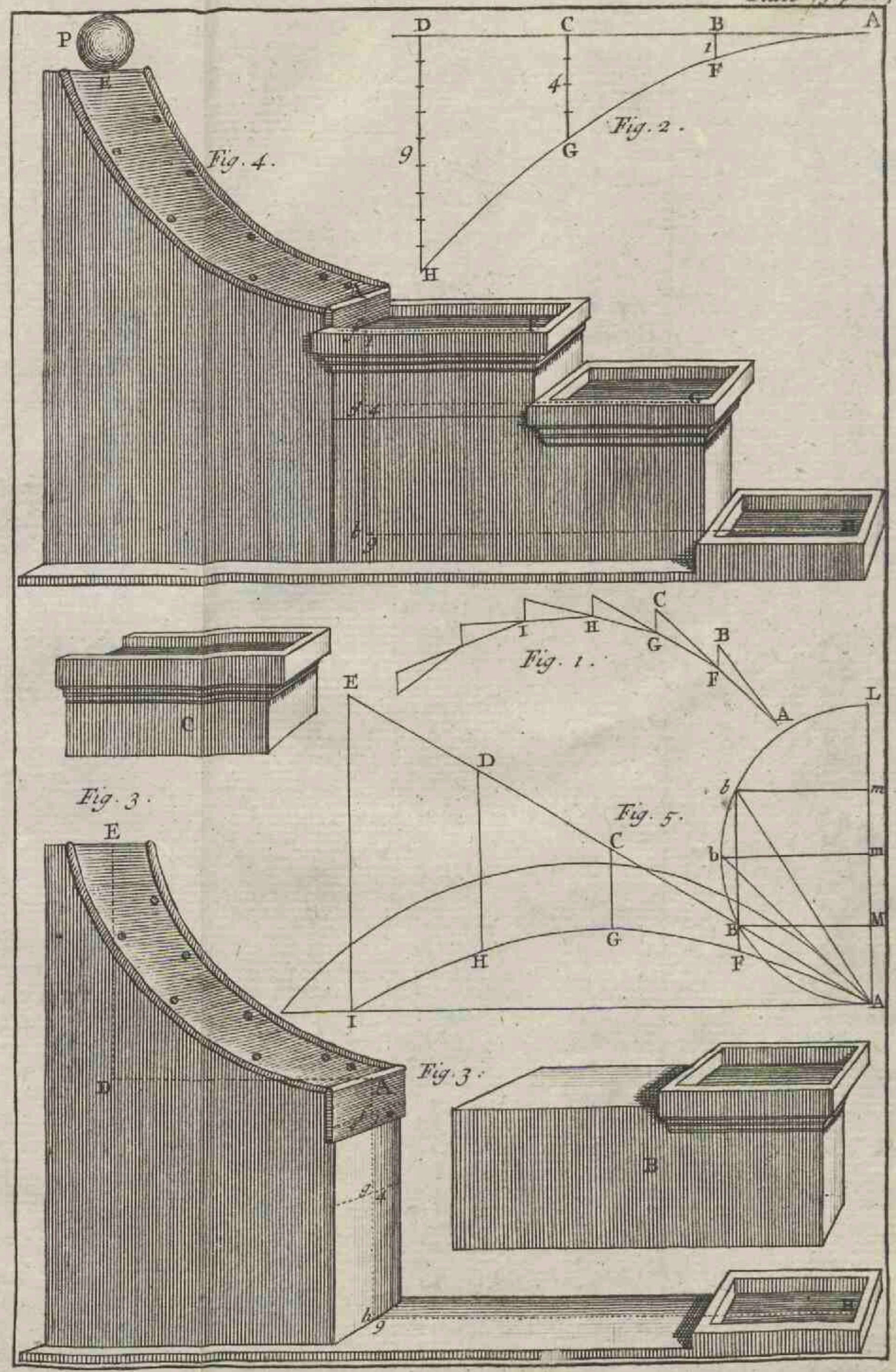


Fig. 4.
Fig. 2.
Fig. 1.
Fig. 3.
Fig. 5.
Fig. 3.
D C B A
F G H
9 4 1
P E
E D C B A
I H G F
L m M b

Definition II.

But *the Force, by which a Body is drawn or impell'd towards that Center, is call'd* a centripetal Force. 219

Definition III.

Theſe Forces are by a common Name call'd *Central Forces.* 220

In all Caſes, the centrifugal and centripetal Forces are equal to one another; for they act in contrary Directions, and deſtroy one another. 221

The whirl'd Sling is equally ſtretch'd both Ways,* and the Stone endeavours to recede from the Hand with as much Force as it is drawn towards it. *126

Central Forces are of great Uſe in Natural Philoſophy; for all the Planets move in Orbits, and moſt of them, if not all, turn upon their Axes.

I ſhall chuſe out the chief Propoſitions relating to theſe Forces, and confirm them by Experiments; but firſt we muſt deſcribe the Machines with which theſe Experiments are performed.

Plate XVI. *Fig.* 1. and 2.] A is a round Board or Table of 2 Feet and a half Diameter; whoſe vertical Section is ſeen in *Fig.* 2. in which *a a* repreſents the Section of the Table itſelf, and *g h* the Section of its turn'd Foot, which is joined perpendicularly to its Center; this Foot or Supporter of it conſiſts of two Pieces ſeparated at D, which are fix'd together by four ſmall Irons, whoſe Ends are rivetted to Rings of the ſame Metal. 222

The upper Part of the Foot has a Groove round it at *c c*; and has a cylindric Hole thro' it at *f g* of three Quarters of an Inch Bore.

The

The Frame of the whole Machine, repreſented at C, is very ſolid; and one Side receives the Foot of the Table A, which paſſes freely thro' an Hole in the upper Part of the Frame C; to which is firmly joined the wooden Collar F of *Fig.* 3. which fits in the ſaid upper Part of the Frame.

The Table with its Foot bears upon the croſs Piece ED, which has a Plate of Iron to receive the Braſs Center *h*. This tranſverſe Piece is fix'd but juſt above the Feet of the Frame, that, when the Table-Foot is let down upon it, the Groove *cc* may be but juſt above the wooden Collar; to the Top of which are ſcrewed down two Iron Plates R R (*Fig.* 3.) by four Screws, ſuch as *ss*. In this Poſition the Table will very freely move horizontally about its Center; and, that it may move the more eaſily, there is ſlipped on upon the Foot cloſe to the Table (where it is not round, but ſix or eight Square) a ſmall Wheel or Pulley, whoſe Section you ſee in *bb*, and which is joined to the Table by means of the Screws *el*, *el*. There muſt be three other ſuch Wheels whoſe Circumferences, taking them at the Bottom of their Grooves, are to one another as one, two, three; the leaſt of all the Wheels is of about 5 Inches Diameter.

Another Table B, made juſt like the firſt, is to be whirl'd about round its Center in the oppoſite Part of the Frame C. Tho' there is a ſmall Difference between them; for in this the lower Part of the Foot has an Hole thro' it as well as the upper, (ſee *i h Fig.* 4.) yet it is turn'd freely, having fix'd to its Bottom a Braſs Plate with a Hole in it to receive the little Pipe of another Plate M, whoſe vertical Section is ſeen at L, and which is fixed to ED, the other croſs Piece of the Frame, at the Place *m*. This croſs Piece

muſt

muſt be bored thro' to anſwer to the Hole of the Plate, in ſuch Manner that a Thread may go from the Top of the Table quite thro' the whole Foot and the Piece E D. Such a Wheel as *b b* is made faſt to the Table immediately under it, and is in Bigneſs juſt equal to the leaſt of thoſe which are made to take off and on, belonging to the Table A.

The two Tables A and B may be whirl'd, very ſwiftly, either ſeparately, or both together, by help of the great vertical Wheel Q: For performing of which, you muſt make uſe of the Machine of *Fig.* 5. which is a wooden Plane, or ſtrong flat Board, to which is perpendicularly fix'd a Parallelopiped, in whoſe upper Surface are vertically fixed the two Pullies *v v*, at the Ends, and ſideways at one End there is another Pulley, as *t*, which is horizontal. The Surface, when the Machine is applied to the Frame, is in the ſame Plane with the ſmall Wheels of the Tables.

If B alone is to be whirl'd, the Piece of *Fig.* 5. is to be fix'd to the Frame C, by help of two Screws going thro' ſuch Holes as *x* in the lower Part of the Piece, which muſt be ſo fixed, in reſpect to the little Wheel of the Table, as is repreſented in *Fig.* 6. where *b* repreſents that little Wheel: The Rope goes round the great Wheel Q, and from its lower Part goes from *d* towards *v*, goes round the Wheel *b*, and againſt the Pulley *t*, towards *c*, and ſo comes back to the upper Part of the Wheel Q.

The ſeventh *Figure* repreſents the Poſition of the Machine or Piece of *Fig.* 5. when both Tables are to be whirl'd round at once. A Sight of the Figure ſhews the Way of the Rope, which from *v* goes down to *d*, and ſo to the lower Part of the great Wheel.

Beſides

Beſides this, in ſeveral Experiments you muſt make uſe of long Boxes or Troughs I F, I F, which are laid upon the Tables, and fix'd to them with Screws; the Center of every one of theſe Boxes lies juſt over the Center of the Table where there is a Hole equal to the Hole *g*, *f*, (*Fig.* 2.) and exactly anſwering to it, in which Hole of the Box is thruſt a wooden Cylinder N, (*Fig.* 2.) as may be ſeen at G: thro' the Middle of this Cylinder goes a little Glaſs Tube of about a Quarter of an Inch Bore, whoſe Ends are thicken'd at the Flame of a Lamp, ſo as to make the Hole ſomething leſs and ſmooth; that a Thread or ſmall String may run up and down thro' it, without any ſenſible Friction.

One of the Troughs holds a Ball, tied to a Thread which goes thro' the above-mentioned Tube, and is alſo faſtened by a Screw to the Weight O, (*Fig.* 7.) which lies in D the Separation of the Foot. This Weight is laid upon the lower Half of the Foot, from whence it is raiſed up, as the Ball recedes from the Center of the Trough.

This Weight is a round Plate of Lead, and of about 2 Inches Diameter; it has a Braſs Cylinder fix'd to its Center, whoſe upper Part, in order to receive the String, is cleft into two Parts, which are drawn together by means of a Screw: this Plate of Lead with its Cylinder weighs half a Pound; and there muſt be two ſuch Weights.

There are ſeveral other Weights, ſome of half, ſome of a quarter of a Pound, repreſented by P, (*Fig.* 7.) which are to be laid upon the ſaid Weight O; that one may at Pleaſure vary the Weight to be raiſed by the Ball.

223 *When a Body, laid upon a Plane, does in the ſame Time, and about the ſame Center, revolve with that Plane,*

Plane, and so describe a Circle; if the centripetal Force, by which the Body is every Moment drawn or impelled towards the Center, should cease to act, and the Plane should continue to move with the same Celerity; the Body will begin to recede from the Center (in Respect of the Plane) in a Line which passes thro' the Center.

Experiment 1.] Take a Ball which is tied to a Thread, the other End of the Thread being fastened to the Center of one of the Tubes A or B, and lay it on the Table, which must be whirl'd singly so long, till the Ball is carried round with it; here the Ball is at rest, in respect of the Table, and in that Situation it is retained only by the String fastened to the Center; therefore it suffers no Impression in that Plane, except that by which the String is stretched, that is, whose Direction passes thro' the Center of the round Table; and so, if it be left to itself, it cannot the first Moment move in any other Direction in that Plane.

When a Body moves about a Center, if in its Motion it comes nearer to the Center, its Motion is accelerated; but on the contrary retarded, if it recedes from the Center. 224

Experiment 2.] Let the Trough F I, through whose Center the Cylinder G, with its Glass Tube, is fixed into the Center of the Table B, be fastened to the said Table.

Let the Ball L, tied to a Thread, be laid in the Trough, and the Thread put thro' the Tube above-mentioned, as also thro' the whole Foot of the Table, and the cross Piece at Bottom that sustains the Foot, and then with your Hand hold the End of the Thread.

Let the Table be turned round, and you will obſerve, that, during that Motion, the Ball will apply itſelf to one Side of the Trough, and is carried round ſo as to move with the ſame Velocity as the Trough. Let the Thread be pull'd, ſo as to bring the Ball nearer the Center, and it will immediately ſtrike the oppoſite Side of the Trough, becauſe it moves faſter than the Trough. If you bring your Hand nearer to the Foot of the Table, ſo as to give more String, the Ball recedes from the Center, and ſtrikes the firſt Side of the Trough, as moving more ſlowly than the Trough.

This Acceleration when a Body approaches nearer to the Center, and Retardation when it recedes from it, is determined by Geometricians: If a Body, for Example, which is driven towards the Center C (*Plate* XVI. *Fig.* 11.) be moved in the Curve AE, it will move faſter at E and ſlower at A: Draw the Lines AC, BC, and EC, DC, ſo that the Areas ABC and DEC may be equal to one another, the Parts AB and DE of the Curve are deſcribed in equal Times by
225 the Body; and therefore, *a Body that is retained in a Curve, by a Force tending towards a Center,* is ſaid *to deſcribe round that Center Areas proportionable to the Times.*

226 The inverſe Propoſition is alſo demonſtrated, namely, *That a Body which is moved in any Curve in a Plane, and deſcribes about any Point Areas proportionable to the Times, is turned out of the right Line, and urged by a Force tending to that Point.*

227 *The greater the Quantity of Matter in any Body is, the greater is its centrifugal Force;* which ariſes from a greater Quantity of Motion.

228 If Liquors of different Denſities be included in a determinate Space, ſo that the heavier cannot recede from the Center, unleſs the lighter come

come towards it; and they be ſo diſpoſed, that by their Weight the heavy Fluid comes to the Center; upon moving the Whole about that Center, the light Fluid will come towards the Center, and the heavy one fly from the Center.

If a Solid be included with a Liquid in a determinate Space, the ſame may be ſaid, as was ſaid of the two Liquids: If it be lighter than the Liquid, it will come towards the Center; if heavier, it will recede from it. All which ariſes from the great Centrifugal Force in the heavier Body.

Experiment 3. *Plate* XVI. *Fig.* 8.] Take four Glaſs Tubes of about one Inch Diameter each, and a Foot long, and having hermetically ſealed them, let them be firmly tied to an inclined Plane. In the firſt, you muſt have Quickſilver and Water; in the ſecond, Oil of Tartar *per deliquium*, and Spirit of Wine; and in the third Water with a Leaden Bullet; and laſtly, in the fourth, Water with a Piece of Cork; and all of them muſt be about half empty.

This inclined Plane muſt be faſtened to the whirling Table A or B (*Plate* XVI. *Fig.* 1.) ſo that the lower Part of the Plane may come almoſt to the Center of the Table, by means of two Screws, one of which goes thro' *x* (*Fig.* 8.) Let the Table be whirled round, and immediately the lower Part of the Tubes will remain empty, and the heavier Bodies will go to that Part of the Tube which is fartheſt from the Center; the Cork deſcends and ſtrikes to the lower Part of the Water, whilſt the Leaden Bullet goes to the Top of the Tube.

Central Forces not only differ on account of the Quantity of Matter, but the Diſtance does 229
alſo cauſe a Change, and likewiſe the Celerity with which the Body is moved round: There is

nothing

nothing elſe that can make any Difference in theſe Forces: and theſe are all the Things to be conſidered, when we compare them together.

Definition IV.

230 The Periodical Time *is the Time, in which a Body going round a Center performs one whole Revolution*; that is, if it deſcribes a Curve that returns into itſelf, the Time elapſed between its Departure from, and Return to a Point: If the Curve does not return into itſelf, inſtead of a Point we muſt take a Line paſſing thro' its Center.

231 The Periodical Time depends upon the Celerity of the Body; and therefore, in comparing central Forces, it muſt be taken for the Velocity.

When the Periodical Times are equal, and the Diſtances from the Center are alſo equal; the central Forces are as the Quantities of Matter in the revolving Bodies.

Experiment 4. *Plate* XVI. *Fig.* 1.] Of the three Wheels or Pullies *bb*, mentioned in the Deſcription of the Machines, apply the leaſt t , the Table A; ſo that if the two Tables A and B be whirled at the ſame Time by the Motion of the Wheel Q, they may run round in equal Times; to each of them fix the long Troughs IF, IF; and the Cylinders GG, that contain Glaſs Tubes, muſt be thruſt thro' the Center Holes of the Troughs quite into the Feet of the Tables.

Put a Ball L of half a Pound into the Trough of the Table B, and a Ball L of one Pound in the Trough of the Table A: Threads tied to the Balls go thro' the little Tubes GG, and are faſtened to Weights placed in the Separations or Hollows of the Feet of the Tables, in ſuch Manner, that the Diſtances of the Balls from the Center, when the Threads are ſtretched and the Weights not raiſed,

raised may be equal; now if the Weight in the Separation or Hollow of the Foot of A be one Pound, that in the Separation of B must be half a Pound; or if this last should be one Pound, the other must be two Pounds.

Let the Wheel Q be turn'd round faster and faster, till by the centrifugal Force of the Balls the Weights above-mentioned be raised, and both Weights will be lifted up precisely at the same Time; therefore Weights, that are as the Bodies, will, *cæteris paribus*, be overcome by the centrifugal Force.

When the Quantities of Matter in the revolving Bodies are equal, and the periodical Times also equal, the Forces are at the Distances from the Center. 232

Experiment 5. *Plate* XVI. *Fig.* 1.] This Experiment is made in the same Manner as the foregoing; instead of a Ball of half a Pound, put in the Trough of the Table B a Ball equal to the other that is of one Pound. Let the Distances from the Center be taken in any Proportion; if the Weights joined to the Balls in the same Proportion, and the Wheel Q be moved faster and faster, you will see the two Weights rise exactly at the same Time. As, for Example, if the Distance of the Ball upon A be of 12 Inches, and the Weight joined to it of 1 Pound and a half; and the Distance of the other Ball of 8 Inches, and the Weight joined to it of 1 Pound, the Experiment will succeed.

When the periodical Times are equal, but the Distances and the Quantities of Matter in the revolving Bodies differ, the central Forces are in a Ratio compounded of the Quantities of Matter and the Distances; which follows from the two last Propositions. To determine that compound Ratio, the Quantity of Matter in each Body must be multi- 233 plied by its Diſtance from the Center, and the Products will be to each other in the Ratio aforeſaid.

Experiment 6.] If in the laſt Experiment the Ball upon B be changed, and you place a Ball of half a Pound at 8 Inches from the Center, and you alſo change the Weight joined to it, and half a Pound be uſed inſtead of a Pound; the Experiment will alſo then ſucceed, and the Weights will begin to riſe at the ſame time. If you multiply the half Pound Ball by its Diſtance of 8 Inches, the Product is 4, and multiplying the 1 Pound Ball by 12 Inches, its Diſtance from the Center, the Product is 12; which Products are to one another as 1 to 3; that is, as the Weight of half a Pound to that of 1 Pound and a half, which are in this Experiment both lifted up at the ſame Moment.

The Differences of central Forces ariſing from the different Diſtances from the Center, and the different Quantities of Matter may compenſate
234 one another; for *ſuppoſing the Quantities of Matter in the revolving Bodies to be in an inverſe Ratio of the Diſtances from the Center, the central Forces will be equal*; as much as one Force is greater in reſpect of the Quantity of Matter, ſo much does the other exceed it by reaſon of its greater Diſtance.

Experiment 7.] Let a Ball of half a Pound be placed at the Diſtance of 14 Inches, and a Ball of one Pound at the Diſtance of 7 Inches; every thing elſe being as in the foregoing Experiment; if the Weights in the Foot or Spindle of each Table be alike, they will riſe at the ſame Moment.

235 There is a Caſe of this Propoſition, *when two Bodies joined by a Thread revolve about their common*

 Center

Center of Gravity. For the Diſtances from that Center are in an inverſe Ratio of the Weights of the Bodies,* and therefore the central Forces are equal. By the Force by which one Body endeavours to recede from the Center, the other is drawn towards it; and by reaſon of the Equality of the Forces, *they retain one another, and continue their Motion*; if they revolve about any other Point, they do not continue their Motion; and the Body whoſe centrifugal Force overpowers, recedes from the Center, and carries the other Body along with it. *95,90

Experiment 6. Plate XVI. *Fig.* 10.] Let two unequal Bodies P and Q be joined by a Thread, in which you muſt mark the Point C, which is the common Center of Gravity of thoſe Bodies, when the Thread is ſtretch'd.

In this Experiment you muſt uſe but one Table, and fix upon it a long Trough that reaches beyond the Diameter of the Table both Ways, and whoſe middle Point is over the Center of the Table. In this Trough you muſt place the Bodies above-mentioned, and the Thread that joins them being ſtretched, the Point C muſt be put in the Middle of the Trough. When the Table is whirl'd round, the Bodies are carried round with it, and remain at reſt in it. If the Point C be removed from the Middle of the Box, upon whirling the Table, both Bodies will be carried to that End of the Box which the Point C was placed neareſt to.

The Difference of the central Forces is alſo determined from the Difference of the periodical Time.

When the Quantities of Matter in the Bodies whirled round, and the Diſtances from the Center are equal, the central Forces are in an inverſe Ratio of 236 *the Squares of the periodical Times:* That is, directly as the Squares of the Revolution made in the same Time.

Experiment 9. Plate XVI. *Fig.* 1.] To the Table A apply a Wheel or Pulley, such as *b b*, (*Fig.* 2.) whose Circumference is double the Circumference of the Wheel which is fix'd to the Table B; so that when the two Tables are whirl'd both together, B shall go round twice for A once; that is, the periodical Time of that shall be double the periodical Time of this.

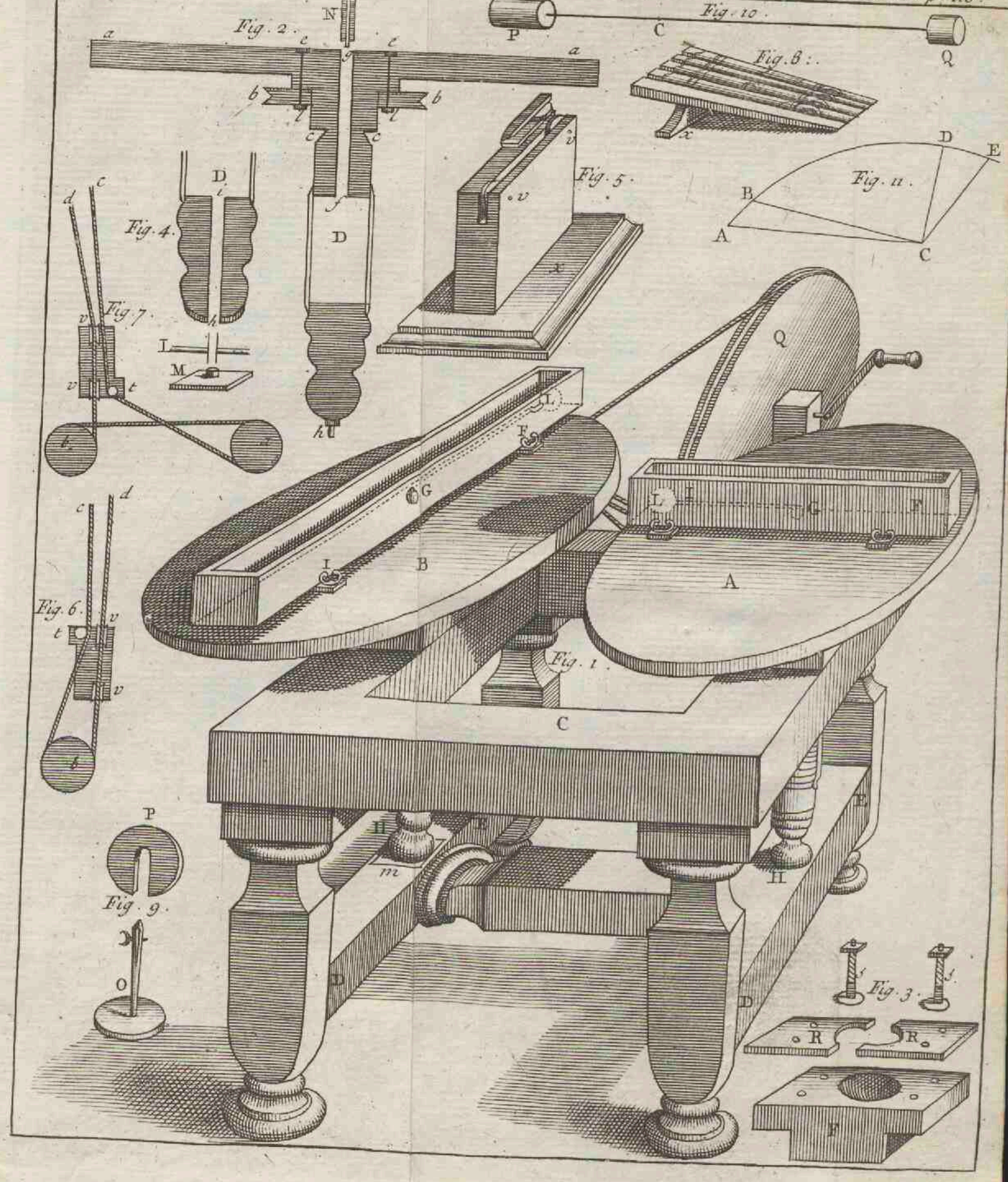
In each Trough I L, I L, lay a Ball of one Pound at equal Distances from the Center. The Ball laid on the Table B must be tied to two Pounds in the Foot, and the other Ball upon A is joined by its Thread to half a Pound in the Foot. Upon whirling the Tables, both Weights will rise at the same Time: Which Weights are here as 1 to 4, the periodical Times being as 2 to 1, whose Squares are reciprocally as 1 to 4.

237 *However the central Forces differ from one another*, they may, according to what has been said, be compared to one another; for *they are always in a Ratio compounded of the Ratio of the Quanti-*
*231 *ties of Matter in the revolving Bodies, * and the*
*232 *Ratio of the Distances from the Center, * and also an inverse Ratio of the Squares of the periodical*
*236 *Times.** If you multiply the Quantity of Matter in each Body by its Distance from the Center, and divide the Product by the Square of the periodical Time, the Quotients of the Division will be to one another in the said compound Retio, that is, as the central Forces.

Experiment 10.] Every thing being prepared as in the former Experiment, set a Ball of half a Pound, at the Distance of 8 Inches from the

 Center

Plate 16. p. 116.



Center of the Table B, and let it by the Thread be joined to one Pound in the Foot; let another Ball of two Pound be placed at the Distance of 12 Inches from the Center of the Table A, and joined with a Weight of 3 Quarters of a Pound; whirl the Tables, and the Weights will be raised just at the same Time.

Here the Bodies are as $\frac{1}{2}$ to 1; the Distances as 8 to 12; the Squares of the periodical Times at 1 to 4; multiplying one Half by 8, and dividing the Product by 1, the Quotient of the Division is 4; multiplying 1 by 12, and dividing the Product by 4, the Quotient is 3. Therefore the central Forces are to one another as 4 to 3, which Ratio also the Weights in the Feet have to one another.

When the Quantities of Matter are equal, the Distances themselves must be divided by the Squares of the periodical Times, to determine the Proportion of the central Forces. 238

In that Case, *if the Squares of the periodical Times be to one another as the Cubes of the Distances,* 239 the Quotient of the Divisions, as well as *the central Forces, will be in an inverse Ratio of the Squares of the Distances.*

Experiment 11.] Let the periodical Times of A and B be as 1 to 2, in the same Manner as in the two last Experiments. Take two equal Balls, and let the Distance from the Center on B be 10 Inches, and the other Ball's Distance from the Center be of 16 Inches: To the Thread of the first, fasten one Pound and a Quarter, and to the Thread of the other fasten half a Pound in the Hollow of the Foot A; whirling the Tables, the Weights will rise the same Moment.

In that Experiment the central Forces are as 5 to 2, which you also find by Calculation.* *238

This Ratio differs very little from the inverſe Ratio of the Squares of the Diſtances, which are to one another as 200 to 512; the Cubes of the Diſtances are alſo almoſt as the Squares of the periodical Times: Theſe Squares are as 1 to 4, and thoſe Cubes as 125 to 512, which Ratios do not much differ. If you take other Numbers, theſe Ratios will be exactly the ſame, and the Experiment will ſucceed in the ſame Manner; but it is not eaſy in the Experiment to vary the periodical Times or the Weights in what Ratio you pleaſe.

240 When the Force, by which a Body is carried towards a Point, is not every where the ſame, but is either increaſed or diminiſhed in Proportion to the Diſtance from the Center, ſeveral Curves will thence ariſe in a certain Proportion.

241 If the Force decreaſes in an inverſe Ratio of the Squares of the Diſtances from that Point, the Body will deſcribe an *Ellipſis*, which is an oval Curve, in which there are two Points called the *Foci*, and the Point towards which the Force is directed falls into one of them: So that in every Revolution the Body once approaches to, and once recedes from it. The Circle alſo belongs to that Sort of Curves, and ſo in that Caſe the Body may alſo deſcribe a Circle: The Body may alſo (by ſuppoſing a greater Celerity in it) deſcribe the two remaining Conic Sections, *viz.* the *Parabola*, or *Hyperbola*, Curves, which do not return into themſelves.

242 On the contrary, if the Force increaſes with the Diſtance, and that in the Ratio of the Diſtance itſelf, the Body will again deſcribe an *Ellipſe*; but the Point, to which the Force is directed, is the Center of the Ellipſe, and the Body in each Revolution will twice approach to, and again, twice recede from that Point. In this Caſe

Case also a Body may move in a Circle, for the Reason above-mentioned.

Experiment 12.] Hang up a leaden Ball with a long Thread; if the Ball be drawn back from its Point of rest, it is always carried towards it by its Gravity, and from equal Sides with equal Force, if the Distance be equal. The Ball in its Motion describes an Arc of a Circle, which Way soever it falls when you let it go: If those Arcs are not very great, they coincide with a *Cycloid*, and the Force, with which the Ball in any Point is carried towards the lowest Point, is as its Distance from that Point;
* therefore here the Force increases in the Ratio of *156
the Distance.

Let the Ball be pull'd back from the lowest Point, and projected obliquely; then it will describe an oval Curve about that Point, which (when the Ball does not run out to a great Distance) will hardly differ at all from an Ellipse, because of the Proportion of the Forces, and because in that Case the Ball does sensibly move in the same Place.

The Center of the Ellipse is the Point in which the Ball is at rest when it is not projected; and in every Revolution the Ball does twice approach to, and twice recede from it. If the Ball hangs over a Table so as almost to touch it when it is at rest, and the Point, over which it is, be mark'd upon the Table, the Experiment will become more sensible if you draw an Oval upon the Table with Chalk, by following the Body with your Hand.

If the Proportion (mentioned *Numb.* 241, and 243
242.) of the Forces by which a Point is driven towards a Center, be a little changed, the Body will no longer describe an Ellipse; but such a Curve as may be reduced to an Ellipse, by sup-

poſing the Plane in which the Body moves agitated by ſome Motion, which therefore will make the Ellipſis moveable.

Experiment 13.] Every Thing being as in the former Experiment, let the Ball be ſo thrown that it may run out to a greater Diſtance; and then it will deſcribe a Curve which may be referr'd to the moveable Oval: it will indeed twice in every Revolution come towards the Center, and twice recede from it; but the Place of the Points in which it is leaſt, or moſt diſtant, is changed every Revolution, and theſe Points are always carried the ſame Way, their Motion conſpiring with the Motion of the Ball.

CHAP. XXVI.

Of the Laws of Elaſticity.

* 44 WE have already ſhewn what *Elaſticity* is, and whence it ariſes; * and what is its Effect in the Congreſs of Bodies, whether they ſtrike one another directly or obliquely; what remains is to examine the *Laws* of *Elaſticity* itſelf, which we ſhall do from Phænomena.

All Bodies, in which we obſerve *Elaſticity*, conſiſt of ſmall Threads or Filaments, or at leaſt may be conceived as conſiſting of ſuch Threads; and it may be ſuppoſed that thoſe Threads laid together make up the Body; therefore that we may examine *Elaſticity* in the Caſe which is the leaſt complex, we muſt conſider Strings of muſical Inſtruments, and ſuch as are of Metal; for Catgut-Strings have a ſpiral Twiſt, and cannot be conſider'd in the ſame Manner as thoſe Fibres of which Bodies are form'd.

244 *The Elaſticity of Fibres conſiſts in this, that they can be extended, and taking away the Force by which they*

they are lengthen'd, they will return to the Length which they had at first.

Fibres have no Elasticity, *unless they are extended* 245
with a certain Force; as it apppears in Strings that have their Ends fix'd without being stretch'd; for if you remove them a little from their Position, they do not return to it: but what the Degree of Tension is, which gives Beginning to *Elasticity*, is not yet determined by Experiments.

When a Fibre is extended with too much Force, it 246
loses its Elasticity; and this Degree of Tension is also unknown; this we know, that the Degree of Tension in Fibres, which constitutes Elasticity, is confined to certain Limits.

Hence appears the Difference of Bodies that are 244
elastic, and such as are not so; why a Body loses its Elasticity, and how a Body destitute of Elasticity acquires that Property. A Plate of Metal, by repeated Blows of a Hammer, becomes elastic, and being heated, does again lose that Virtue.

Between the Limits of Tension, that terminate Elasticity, there is a different Force required for different Degrees of Tension, in or to stretch Chords to certain Lengths: What this Proportion is, must be determined by Experiments, which must be made with Chords of Metal, as was said before. But as these Wires are scarce sensibly lengthen'd, the Proportions of the Lengthening cannot be directly measured; therefore they must be measured by another Method.

Let AB (*Plate* XVII. *Fig.* 1.) be a small Wire stretch'd horizontally with a certain Force, whose Ends are fix'd at A and B: Let it be bent by a Weight hanging in the Middle of it, so that it may come to the Position ACB.

DEFI-

DEFINITION.

248 *The Line* C*c drawn from the middle Point of a String or Chord after its Inflexion, to the middle Point of the same when it was in its natural State, is call'd* the Sagitta (*Arrow*) of the Chord.

Let *ce* be an Arc of a Circle described about the Center B, with the Radius B*c*. Half the Chord or Wire by the Inflexion was stretch'd the Length C*e*, which Quantity has a certain Relation to the Sagitta C*c*.

The Weight also, by which theString is stretch'd, has a certain Relation to the Force with which the Fibre is lengthen'd, that is drawn along BC: and so in several Experiments by comparing the Sagitta C*c*, and the Weights with which the Chords are inflected, the Proportions of the Lengthenings are determined; as will be shewn in the following Experiments.

249 The Machine for performing them is a vertical Board, about 3 Foot long, and 1 Foot high. See *Plate* XVII. *Fig.* 2.

The Rulers of Wood *mn*, *mn*, are fix'd to the Board like a Moulding, and carry two Prisms H, H, made like a Wedge, which slide along upon the Rulers, and are fix'd any where upon them by means of Screws, which hold them behind the Board, their Shanks being moved backwards and forwards, by means of a Slit in the Board.

Between A and B there are equal Divisions reckon'd from the Middle on either Side, in order to determine the Places where to fix the Prisms.

At O there is a Groove, to hold the Pulley T in the Side of the Board; which Pulley is represented in *Plate* XIII. *Fig.* 8.

The Wire, with which the Experiments are made, is fix'd at one End of the Ruler *mn*, and as at the other End goes over the Pulley T, the Weight P ſtretching it, and the Priſms, H, H, ſuſtaining it in Points, which are equally diſtant from the Middle of the Machine.

There is a Braſs Plate *de* let into the Middle of the Board, and mark'd with very ſmall Diviſions, along which moves another Braſs Plate or Index *fg*, which hangs upon the Wire, having a Hole thro' which it runs: This Index has a Scale hanging from it, which, together with the Index *fg*, weighs juſt an Ounce. The Length of the Wire is determined in each Experiment by the Diſtance of the Edge of the Priſms H, H; for in the ſmall Inflexions made by hanging on Weights in C, concerning which alone Experiments are made, the String is not moved upon the Priſms, nor is the Weight P raiſed up, but only the Part AB is extended by theſe Inflections.

In the Inflections of the String, the Sagittæ are meaſured by the Diviſions on the Plate *ed*; for the End *g* of the Index *gc* does always deſcend equally with the Point C in every Inflection.

Experiment 1.] Let P be a two Pound Weight, 250
and let the Wire be inflected at C with the Weight of an Ounce, that is, with the Weight of the Scale and Index *fg*; and obſerve the Diviſion of the Plate *ed*, to which the End *g* of the Index *fg* deſcends. Change the Weight P to 4 Pounds, and alſo double the Weight by which the String is inflected, that *g* may deſcend to the ſame Diviſion, and this Weight will be two Ounces: Three Ounces will give the ſame Inflexions, when the Weight P is of ſix Pounds.

From this Experiment it follows, that *the* 251
Weight, by which a Fibre is increaſed a certain Length by its ſtretching, is in the different degree of Tenſion, as the Tenſion itſelf; if, for Example, there be

three

three Fibres of the ſame Kind, Length and Thickneſs, whoſe Tenſions are as 1, 2 and 3; any Weights in the ſame Proportion will equally ſtretch thoſe Fibres.

252 *The leaſt Lengthenings of the ſame Fibres are to one another nearly as the Forces by which the Fibres are lengthened.* As for Example, let a Fibre be ſtretched with the Weight of 100 Ounces, if it be ſeparately lengthen'd with the Weights of 1 Ounce, 2 Ounces, and 3 Ounces, the Lengthenings will be nearly as 1, 2, and 3; that is, each Ounce ſuperadded does equally lengthen the Fibre: For the Tenſions by the Weights of 100, 101, and 102 Ounces, by which the Fibre is ſtretch'd in each Caſe, when an Ounce is ſuperadded, do not ſenſibly differ from each other.

The Property of Fibres may be applied to their Inflexion, and is of great Uſe. Let the Wire AB (*Plate* XVII. *Fig.* 3.) be ſo inflected, as to acquire the Poſitions A*c*B, A*c*B, and ACB, yet ſo that in the greateſt Inflexion the Sagitta may not be ¼ Inch long, ſuppoſing the Wire 2 Feet and a Half; In thoſe Caſes the Lengthenings of the String are very ſmall, therefore they are in the Ratio of the Forces that
* 252 produce them, * and they ſerve to expreſs them; let cD expreſs the Force by which a String is ſtretch'd when it is not inflected, and with the Center B deſcribe the Circle D*d*; the Lines *dc*, *dc*, *d*C, which are longer than *c*D by the Quantity by which the Fibre was lengthened in every Caſe, expreſs the whole Forces, by which the Fibre is ſtretched in every Caſe. But here the Arc D*d* is hardly of one Degree, and D is always far enough diſtant from the Point c, wherefore D*d* may be looked upon as a Right Line parallel to cC, and the Lines *cd*, *cd*, C*d* have the ſame Ratio to the Lines *c*B, *c*B, CB. Therefore

fore the Point C is always drawn towards B*c* and A, by Forces proportionable to the Line CB or CA, and the Force by which the Wire is inflected, whose Direction is along *c*C, is as the double Sagitta, * or as the Sagitta itself. *There-* * 203
fore in all the least Inflexions of a Chord, Musical 253
String or Wire, the Sagitta is increased and diminished in the same Ratio as the Force with which the Chord is inflected.

Experiment 2. *Plate* XVII. *Fig.* 2.] Let the Wire AB, stretched by any Weight, be inflected by the Weight of 1, 2, and 3 Ounces; the Descents of the Point *g*, that is, the Sagittæ themselves are to one another as 1, 2, and 3.

In Chords of the same Kind, Thickness, and which 254
are equally stretched, but of different Lengths; the Lengthenings, which are produced by superadding equal Weights, are to one another as the Lengths of the Chords. This is plain, because the Chord is equally stretched in all its Parts; therefore the Lengthening of a whole Chord is double the Lengthening of half of it, or of a Chord of half the Length.

As to the Inflexion of those Chords, let AB, *ab*, (*Plate* XVII. *Fig.* 4.) be Chords of the same Kind and Thickness, but of different Length, equally stretched, and so inflected, that ACB shall be the Position of the first, and *adb* that of the last; and let the Triangles BC*c*, and *b*D*d* be similar: *c*B is to D*b*, that is, the Lengths of the Chords are as CB to *db*; therefore the Chords are lengthened in Proportion to their first Length, and consequently they are drawn by equal Porce in the Directions *bd*, *ad*, BC, AC: * But by * 254
the Likeness of the Triangles above-mentioned, 255
the Forces also acting along *c*C and D*d* are equal to one another*, and *the Sagittæ c*C, D*d*, *are as* * 203
the

the Lengths of the Chords; which does therefore, *cæteris paribus*, obtain *in unequal and inflected Chords.*

Experiment 3. *Plate* XVII. *Fig.* 2.] Let the Chord AB be ſtretched by any Weight, having fixed the Priſms H, H, at the ſixth Diviſion on each Side: Now let it be inflected with any Weight, ſo that the *Sagitta* may be equal to ſix Diviſions of the Plate *ed.* Let the Priſms be brought to and fixed at the fourth Diviſion on each Side, and the *Sagitta* will be equal to four Diviſions of the Plate; and ſo on for any Poſition of the Priſms.

One may compare together Fibres of the ſame Kind, but different Thickneſs; they may be looked upon as made up of ſeveral very fine Fibres of the ſame Thickneſs, whoſe Number in the abovementioned Fibres muſt be taken in a Ratio of the Solidity of thoſe Fibres, that is, as the Squares of the Diameters, or as the Weight of the Fibres when their Lengths are equal. Therefore theſe Fibres will be equally ſtretched by Forces that are in the ſame Ratio of the Squares of the Diameters; which Ratio alſo is required between the Forces by which the Chords are inflected, that the *Sagittæ* may be equal in the given Fibres. But by diminiſhing the Force by which the Fibre is ſtretched in the ſame Ratio as the Force by which
* 250 it is inflected, the *Sagitta* is not changed*. There-
256 fore, *if the Forces by which the Fibres are ſtretched be equal, and they are inflected by equal Forces, even in that Caſe alſo the Sagittæ will be equal, however different the Thickneſs be.*

Experiment 4. *Plate* XVII. *Fig.* 2.] Take any Chords of the ſame Kind, and unequal Thickneſs;

neſs; and let them be ſeparately applied to the Machine, leaving the Priſms HH in the ſame Place; if they be ſtretched by the ſame Weight P, and alſo be inflected by the ſame Weight L, the Sagittæ will be equal.

Let the Chord AB (*Plate* XVII. *Fig.* 1.) *ſtretched any how, be* ſo *inflected* as to acquire the Figure ACB, *then left to it ſelf*, and by its Elaſticity it will return to its firſt Figure, and in that Caſe the Motion of the Point C is accelerated; for when the Chord is let go from the Poſition ACB, the Point C is moved with the Force that is able to retain it in that Poſition. This Motion is not deſtroyed, but there is ſuperadded to it, in all the Points of the Sagitta, the Force by which the Point C might be retained in them. The Celerity is the greateſt of all at *c*, and by that Celerity the Point C is carried farther, and then returning, *it will perform ſeveral Vibrations*, in which the Point C runs out but ſhort Spaces; for which Cauſe the Force, by which the Point C is acted upon in all Diſtances from *c*, is as the Diſtance * in each Point. Therefore the Motion agrees with the Motion of a Body vibrating in a Cycloid, *and how unequal ſoever the Vibrations are, they are performed in the ſame Time.* * 257

* 253

* 155

If there be two equal and ſimilar Chords, but unequally ſtretched, unequal Forces are required to inflect them equally; therefore the Vibrations are performed in unequal Times. One may compare their Motions with the Motions of the Pendulums which vibrate in Cycloids *, and deſcribing ſimilar Cycloids by different Forces; which Forces are inverſly as the Squares of the Times of the Vibrations. * In Chords therefore likewiſe *the Squares of the Times of the Vibrations are to one another inverſly*, as the Forces by which they are equally 258

* 257

* 165

equally inflected; which are *as the Weights by which*
* 250 *the Chords are ſtretched.**

259 *When the Chords are ſimilar, equally ſtretched, but of different Lengths*, their Motion muſt be compared with that of Pendulums by another Method; for as the Times of the Vibrations are to be conſider'd, the Celerities alſo, with which the Chords are moved, muſt alſo be conſidered: And in the Chords ACB, *adb* (*Plate* XVII. *Fig.* 4.) whoſe *Sagittæ* are equal, and in which the Points C and *d* may be conſidered as deſcribing ſimilar Cycloids, the Celerities, with which thoſe Points are moved in correſpondent Points, are to each other in an inverſe Ratio of the Squares of the Times of
* 165 the Vibrations.* In Pendulums and equal Chords, the Forces are taken for the Celerities; becauſe in
77,63 thoſe Caſes they are in the ſame Ratio.

Let the Chords AC, *ab*, be divided into very ſmall Parts, but each into an equal Number of Parts; the Ways to be run thro' by correſpondent Parts, ſuppoſing the Sagittæ equal, will be equal, and theſe ſmall Parts will perform ſimilar Vibrations; but the Particles of Matter in the correſpondent Particles are as the whole Chords: That therefore their Celerities may be determined in correſpondent Points, the Forces with which the Chords are inflected, when the Sagittæ are equal, muſt be divided by the Quantity of Matter in the Chords, as it follows from *Numb.* 64. It is therefore plain, that thoſe Celerities are to one another directly as the Weights by which the Chords are inflected, and inverſly as the Quantities of Matter in thoſe Chords, that is, inverſly as their Lengths: But thoſe Weights are alſo in an inverſe
* 255 Ratio of the Lengths of the Chords;* therefore
253 the Celerities are in an inverſe duplicate Ratio of thoſe Lengths, that is, inverſly as the Squares of the Lengths;

Length; and then, as was said before, the Squares of the Times of the Vibrations will also be in that inverse *Ratio. The Lengths* therefore *of the Chords will be as the Times of the Vibrations.*

One may, in the same Manner, compare *the* 260
Times of the Vibrations of Chords of different Thickness, supposing the Chords equal, and stretched with equal Weights; the Quantities of Matter *are as* the Squares of the Diameters; therefore to determine the Celerities of the correspondent Points, the Weights, by which the Chords are inflected, are to be divided by those Squares, when the *Sagittæ* are equal; * the Celerities therefore are * 256
inversly as the Squares of the Diameters, and therefore *the Diameters* are as the Times of the Vibrations.

Any Chords of the same Kind being given, the 261
Durations of the Vibrations may be compared together; for they *are in a Ratio compounded of the inverse Ratio of the square Roots of the Weights, by which the Chords are stretched,* * *and the Ratio of* * 258
the Lengths of the Chords, * *and the Ratio of the* * 259
Diameters. * If you multiply the Diameters by * 260
the Lengths, and divide the Product by the square Root of the Weight that stretches the Chord, and go through the same Operation for several Chords; the Quotients of the Division will be to one another as the Times of the Vibrations.

Elastic Plates may be considered as a Congeries, 262
or Bundle of Chords; when the Plate is inflected, some Fibres are lengthened, and there are unequal Lengthenings in several Points of the Plate; now the Curve, which is formed by the inflected Plate, may be discovered from what has been said concerning Chords.

By comparing together *the Inflexions of the* 263
same Plate, they *are proportional to the Forces by*

which the Plate is bent. Let AB (*Plate* XVII. *Fig.* 5.) be an elaſtic Plate or Spring, whoſe End A is fixed, and let it be inflected by two Forces,
264 ſo as to be brought into the Poſition *ab* and *ab*; if the one be doubled, the other, *bb* and *b*B, will be equal; and therefore in the Vibrations the Motion of the Spring is accelerated in the ſame
* 253 Manner as the Motion of a Chord *, and the Mo-
* 156 tion of a Pendulum in a Cycloid *; *and the Vibrations of this Plate are performed in the ſame Time.*

Experiment 5. *Plate* XVII. *Fig.* 6. The Spring A is made up of ſeveral elaſtic Plates, and put into the Box B, and there moves on each Side; between the Rulers *cd*, *cd*, two Strings are fixed to the upper Part of the Spring, and run through the Hole *e*, *e*, in the Bottom of the Box. If you hang half a Pound upon the Threads, it will deſcend half an Inch; add another half Pound, and it will deſcend half an Inch more; and ſo on, 'till the Spring can be compreſſed no farther.

Each ſmall Plate is bent in Proportion to the Weight; and the Motion of the Weight, on account of all the Inflexions together, follows the ſame Proportion. The Experiment is made with ſeveral Plates joined together; becauſe in various Inflexions the Direction of the Action of the Weight on the Plates is not ſenſibly changed.

265 What has been ſaid of the Inflexion of Plates, may be applied to the curve Plate or Spring ACB (*Plate* XVII. *Fig* 7.) If it be preſſed by two Weights, ſo as to acquire the Poſition *acb*, *aeb*, and the Weights are to each other as 1 to
* 263 2, the Diſtances *cc* and *c*C will be equal *: Therefore the bending in of the Spring, or Spaces gone

gone through by the Point C, are as the Weights with which the Plate is pressed. Which may also be applied to the bending in of several Plates joined together.

The Ball ACB (*Plate* XVII. *Fig.* 8.) being made of an elastic Substance, may be considered as consisting of several Plates; and the Introcessions (or Yieldings inward) of the Point C will be proportionable to the Forces with which the Body is compressed. 266

Let the Point C of the Ball ACBE (*Plate* XVII. *Fig.* 9.) strike several times against any Plane, and let that Point go inwards to *d*, *d*, and D; the Strokes will be to each other as the Lines C*d*, Cd, and CD. At the first Stroke the Part *abc* becomes flat, the second Stroke *acb* is flattened, and the third ACB: As here we always consider the least Arcs, the Arcs (that is, the Diameters of the plane Surfaces made by the Strokes,) are to one another sensibly as the Chords C*a*, Ca, and CA; therefore the Surfaces are as the Squares of those Chords; in which Ratio also, from the Nature of the Circle, are the Lines C*d*, Cd, and CD, which are to each other as the Strokes. Therefore *in elastic Spheres, the Planes made by the Strokes follow the Proportion of the Strokes.*

Experiment 6.] Take a flat Piece of blue Marble made fast in a horizontal Position, and a little wet, so as to make the Colour the more intense; if you let an Ivory Ball fall upon this Plane, that Part of the Ball, which by being made flat applies itself to the Stone, leaves a very round Spot in the Surface of it: Let the Ball fall from the Height of 9 Inches, and the Spot be E: then let it fall from the Height of 3 Feet which is the Quadruple of the other, and the 268

Spot will be F; laſtly, let it fall from the Height of 6 Feet and 9 Inches, which is nine times the firſt, and the Spot will be G. In that Experiment, the Strokes of the Body againſt the Stone
131 are to each other as 1, 2, and 3: In which
130 Ratio alſo are the Spots E, F and G; for if you draw the right-angled Triangles D A B, D B C, in which the Sides D A, A B, B C, are equal to one another, and to the Diameter of the Spot E, the Line B D will be exactly equal to the Diameter of the Spot F, and the Line C D to the Diameter of the Spot G.

END *of the* FIRST BOOK.

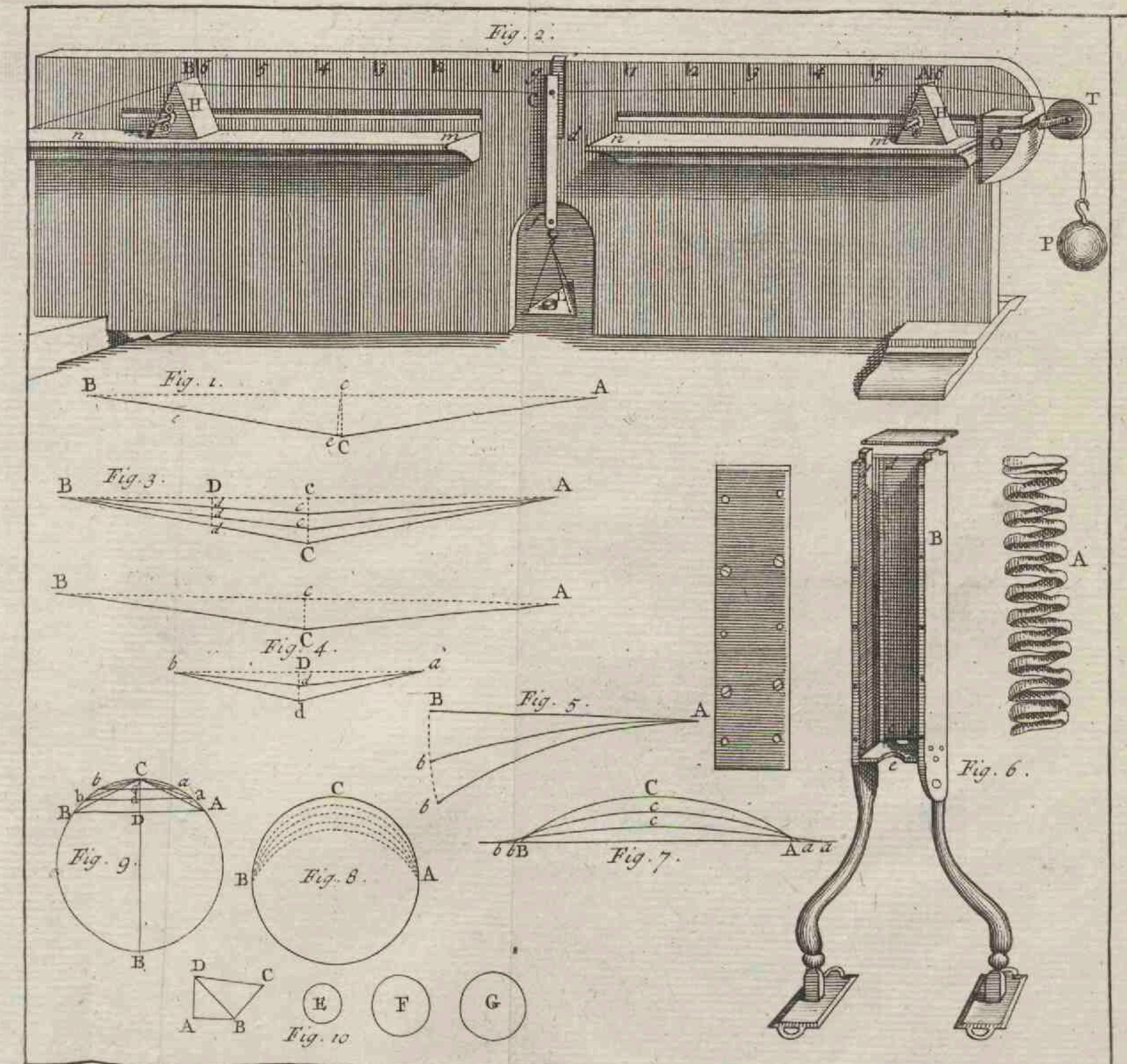


Fig. 2.
B
H
n
m
d
f
A
O
T
P
Fig. 1.
c
C
Fig. 3.
D
Fig. 4.
a
b
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Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 9.
Fig. 10
E
F
G

MATHEMATICAL ELEMENTS OF *Natural Philosophy*, CONFIRMED BY EXPERIMENTS.

BOOK II.

PART I. *Of the Gravity, Pressure, and Resistance of* FLUIDS.

CHAP. I.

Of the Gravity of the Parts of Fluids, and its Effect in the Fluids themselves.

A FLUID is a Body whose Parts yield to any Force impressed, and by yielding are very easily moved one amongst another.* Whence it follows, * 30
that *Fluidity arises from this, That the* 269
Parts do not strongly cohere, and that the Motion is not hinder'd by any Inequality in the Surface of the Parts, as it happens in Powders.

But the Particles, of which Fluids conſiſt, are of the ſame Nature with the Particles of other Bodies, and have the ſame Properties; for Liquids are often converted into Solids, when there is a more ſtrong Coheſion of them, as in Ice. On the contrary, melted Metals give us an Inſtance of a Solid changed into a Fluid.

270 *Fluids agree in this with Solid Bodies*, viz. *That they conſiſt of heavy Particles, and have their Gravity proportionable to their Quantity of Matter, in any Poſition of the Parts.* If in the Liquid itſelf that Gravity be not ſenſible, it is owing to this, that the lower Parts ſuſtain the upper, and hinder them from deſcending: But it does not follow from thence, that the Gravity is taken away; becauſe a Liquid contained in a Veſſel will preſs down the End of a Balance, which carries the Veſſel, in Proportion to its Quantity. The following Experiment will alſo ſhew, that the Gravity is preſerved in any Part of the Liquid.

271 In this, as well as in other hydroſtratical Experiments relating to the Gravity of Fluids, we uſe a very exact Pair of Scales, differing from common Scales only in this, that each Scale has a Hook V V under it, (*Plate* XVIII. *Fig.* 1.) for ſuſpending ſuch Bodies as are to be immerſed in Liquids.

272 The Balance itſelf hangs by a Line which goes round two Pullies T T, and is faſtened to a Weight P, (*Plate* XVIII. *Fig.* 1.] that, by moving the Weight, the Balance may be conveniently raiſed and depreſſed, and ſuſpended at any Height.

Experiment 1.] Immerſe in Water the Phial D cloſe ſhut, and hanging by a Horſe-hair, nnd balance it with the Weight in the oppoſite Scale; then, without taking the Phial out of the Water, open it, and let it be filled with Water, and you will

will find, that the Water in the Phial will bring down the End of the Balance, although it has no Communication with the external Water: If you restore the *Æquilibrium*, by putting more Weight into the opposite Scale, the Phial will remain suspended in any Part of the Water.

From this Gravity it follows, that *the Surface of* 272
a Fluid contained in a Vessel, to keep it from flowing out, if it be not pressed from above, or if it be equally pressed (for that makes no Alteration) *will become plain*, or flat, *and parallel to the Horizon.* For, as the Particles yield to any Force impressed, they will be moved by Gravity, 'till at last none of them can descend any lower.

The lower Parts sustain the upper, and *are pres-* 273
sed by them; *and this Pressure is in Proportion to the incumbent Matter, that is, to the Height of the Liquid above the Particle that is pressed*; but, as the upper Surface of the Liquid is parallel to the Horizon,* all the Points of any Surface, which you * 272
may conceive within the Liquid parallel to the Horizon, are equally press'd.

If therefore *in a Part of such a Surface there is* 274
a lesser Pressure than in the other Parts, the Liquid, which yields to any Impression *there*, will be mov'd, that is, *will ascend*, *'till the Pressure becomes equal.*

Experiment 2. *Plate* XVIII. *Fig.* 2.] Take a Glass Tube C open at both Ends, and stopping one End with your Finger, immerse the other in Water; when the Tube is full of Air, the Water will rise in it but to a very small Height: If you take away your Finger, that the Air that is compressed may go out, the *imaginary Surface* (as Mr. *Boyle* used to call it) that you conceive in the Water, just at the Bottom of the Tube, and parallel to the Horizon, is less pressed just against the Hole of the Tube, ſo that the Water will riſe up into the Tube till it comes up to the ſame Height with the external Water.

275 *The Preſſure upon the lower Parts, which ariſes from the Gravity of the ſuperincumbent Liquid, exerts itſelf every Way, and every Way equally.*

Which follows from the Nature of a Liquid, for its Parts yield to any Impreſſion, and are eaſily moved; therefore no Drop will remain in its Place, if, whilſt it is preſſed by a ſuperincumbent Liquid, it is not equally preſſed on every Side: But it cannot be moved on account of the neighbouring Drops, which are preſſed in the ſame Manner, and with the ſame Force, by the ſuperincumbent Liquid; and therefore the firſt or loweſt Drop is at reſt, and equally preſſed on all Sides, that is, in all Directions.

Experiment 3. *Plate* XVIII. *Fig.* 2.] Let the Glaſs Tubes A, B, D, be immerſed in Water, in the ſame Manner as in the laſt Experiment; and, upon taking away the Finger, the Water will riſe in all the Tubes to the ſame Height as in the Tube C: In C the Preſſure is directed upwards, in B downwards, in A ſidewiſe, and in D obliquely; yet the Preſſure is equal in each. If you pour in a greater Quantity of Liquid into the Veſſel, it will alſo riſe equally in each Tube.

276 Hence it follows, that *all the Particles of Liquids* are preſſed equally on all Sides, and therefore *are at reſt*; and that they do not continually move among themſelves, as ſeveral have ſuppoſed.

277 *In Tubes that have a Communication, whether equal or unequal, whether ſtrait or oblique, a Fluid riſes to the ſame Height*; that is, all the upper Surfaces are in the ſame horizontal Plane; which is eaſily deduced from what has been ſaid.

Plate XVIII. *Fig.* 3.] Let A be a Vessel, and B a vertical Tube, and D an inclined Tube; they must communicate by Means of the Tube C, E: Let there be a Liquid poured into them, and let *f g h* be a Surface parallel to the Horizon. If the Heights *f i* and *g l* be unequal, the Water will ascend where that Difference is least.* For the * 274
same Reason, unless the Pressures at *g* and *h* be equal, the Water will not be at rest; but they are equal when *l* and *n* are in the same horizontal Plane: For since the Pressure arises from the Gravity of the Parts, which tends towards the Center of the Earth, the Height of the pressing Liquid must be measured according to that Direction, that is, it will be *h m*; but the Obliquity of the Column *h n* causes no Change, because at the same Depth the Pressure every Way is equal.* * 275

Experiment 4. *Plate* XVIII. *Fig.* 4.] Pour Water 278
into the Machine represented by *Fig.* 3. and after any Agitation it will not rest, unless all the Surfaces be in the same horizontal Plane. The Glass Vessel A is joined to the Glass Tubes B and D, by help of the Brass Tube C E.

All Liquids are not equally heavy, that is, have not the same Quantity of Matter in the same Space; but what has been said will agree to every Liquid by itself.

When Liquids of different Gravities are contained 279
in the same Vessel, the heaviest lies at the lowest Place, and is pressed by the lighter, and that in Proportion to the Height of the lighter.

Experiment 5. *Plate* XVIII. *Fig.* 5.] Take Water tinged with some Colour, and pour it into the Glass Vessel A to the Height of *b c*; immerge into it the Glass Tube *d* E; the Water will

* 274 will riſe in it to the Height *bc.** Now pour in Oil of Turpentine, which is a Liquid lighter than Water, and immediately the Water will riſe in the Tube; and ſo much the higher as the Oil is poured in, to a greater Height: Yet the Water in the Tube does not riſe to the ſame Height as the Oil in the Veſſel; becauſe, ſince Water is heavier, there is not required the ſame Height of Water as there would be required of Oil to produce the ſame Preſſure.

If you have a Mind to try this Experiment with Mercury and Water, you will find a greater Difference in their Heights, by reaſon of their greater Difference of Gravity.

Experiment 6. *Plate* XVIII. *Fig.* 6.] Let the End of a Tube be immerſed in Water, and pour Oil into it. The Water in the Tube is depreſs'd as far as *d*; yet the Height of the Oil *de* is greater than the Height of the Water in the Veſſel. If the Tube be immerſed deeper, the Water will run into it in greater Quantity; if you raiſe it up, the Water will again go out at *a*, and the Water in the Tube *de* follow it, if it be raiſed to ſuch a Height, that the Preſſure of the Oil may overcome the Preſſure of the Water in the lower Part of the Tube.

CHAP. II.

Of the Actions of Liquids againſt the Bottoms and Sides of the Veſſels that contain them.

280 THE *Bottom and Sides of a Veſſel, which contain a Liquid, are preſſed by the Parts of the Liquid which immediately touch them*; and be-
* 126 cauſe Re-action is equal to Action,* thoſe Parts all ſuſtain an equal Preſſure. But, as the Preſſion of Liquids

Liquids is equal every Way, the Bottom and Sides are preſs'd as much as the neighbouring Parts of the Liquids; therefore *this Action increaſes, in Proportion to the Height of the Liquid,* * * 275
and is every Way equal at the ſame Depth, de- 273
pending altogether upon the Height, and not at all upon the Quantity of the Liquid. Therefore, when the Height of the Liquid, and Bigneſs of the Bottom remain the ſame, the Action upon the Bottom is always equal, however the Shape of the Body be changed. In every Caſe the Preſſure, ſuſtained by the Bottom, is equal to the Weight of a Column of Water, whoſe Baſe is from the Bottom itſelf, and the Height of the vertical Diſtance of the upper Surface of the Water from the Bottom itſelf.

Plate XVIII. *Fig.* 7 and 8.] Take the hollow 281
Cylinder A, open at both Ends, and finely poliſhed within, whoſe Diameter and Height alſo are about three Inches and an half; the Ring E is faſtened to it by a Screw, ſo as it may be ſuſtained by a Trevet.

Let the Cylinder have a moveable braſs Bottom F, with which the braſs Ring G, having a Screw in the Inſide, is joined: This Ring retains and fixes a Leather Ring, broader than the Bottom, all round by half an Inch; this Leather covers the external Surface of the braſs Ring when the Bottom is thruſt into the Cylinder, and it hinders the Water from going out when it is moved up and down. This Leather muſt be ſoaked in Oil, and after a few Days it muſt be taken out and ſoaked as long in Water; after which Preparation the Leather muſt be well anointed with Oil and Water, and moved ſeveral times up and down the Cylinder, and left in it in that Condition two or three Days. When you are going to uſe the Machine

chine, you muſt anoint the Leather again with Oil and Water, then the Bottom will move eaſily, and hold Water. The Leather muſt be neither too thick nor too thin, which muſt be left to the Judgment of the Workman.

The Bottom has in its Middle a ſmall Braſs Cylinder *h i* faſtened to it, by which the Motion of the Bottom is directed, for this Cylinder goes thro the Hole *m* in the Plate *b*, which is laid upon the larger Cylinder *A*, and let into it by a Cut in the Edge. In the upper Surface of the Cylinder *h i* there is a Cavity which contains a Screw, by which the Bottom is joined to the Braſs Wire *n p*, which is carried through the Tube D, that the Bottom may be faſtened to the Brachium of a Balance by the help of this Wire.

Let the Cylinder A have the Cover C laid upon it; and, to hinder the Water from running out, the Mouth of the Cylinder muſt be cover'd with a Leather Ring, which is ſtrongly preſſed by the Screw which joins the Leather Cover to the Cylinder. To the Cover and Cylinder itſelf may be added a Handle, that the Cylinder may the more eaſily be ſhut and open'd. The Cover has a Hole in the Middle, and the hollow Cylinder *l*, which has a Screw on the Outſide, is faſten'd to it, that the Tube *d* may be joined to the Machine with a Leather upon the Screw, to hinder the Water from coming out.

Experiment 1. *Plate* XIX. *Fig.* 1.] Having joined together all the Parts of the Machine in the Manner juſt mentioned, hang upon one End of the Beam of a Balance the Braſs Wire which is fixed to the moveable Bottom, ſo that the Beam may be exactly horizontal when the Bottom is two Inches diſtant from the Cover; then put

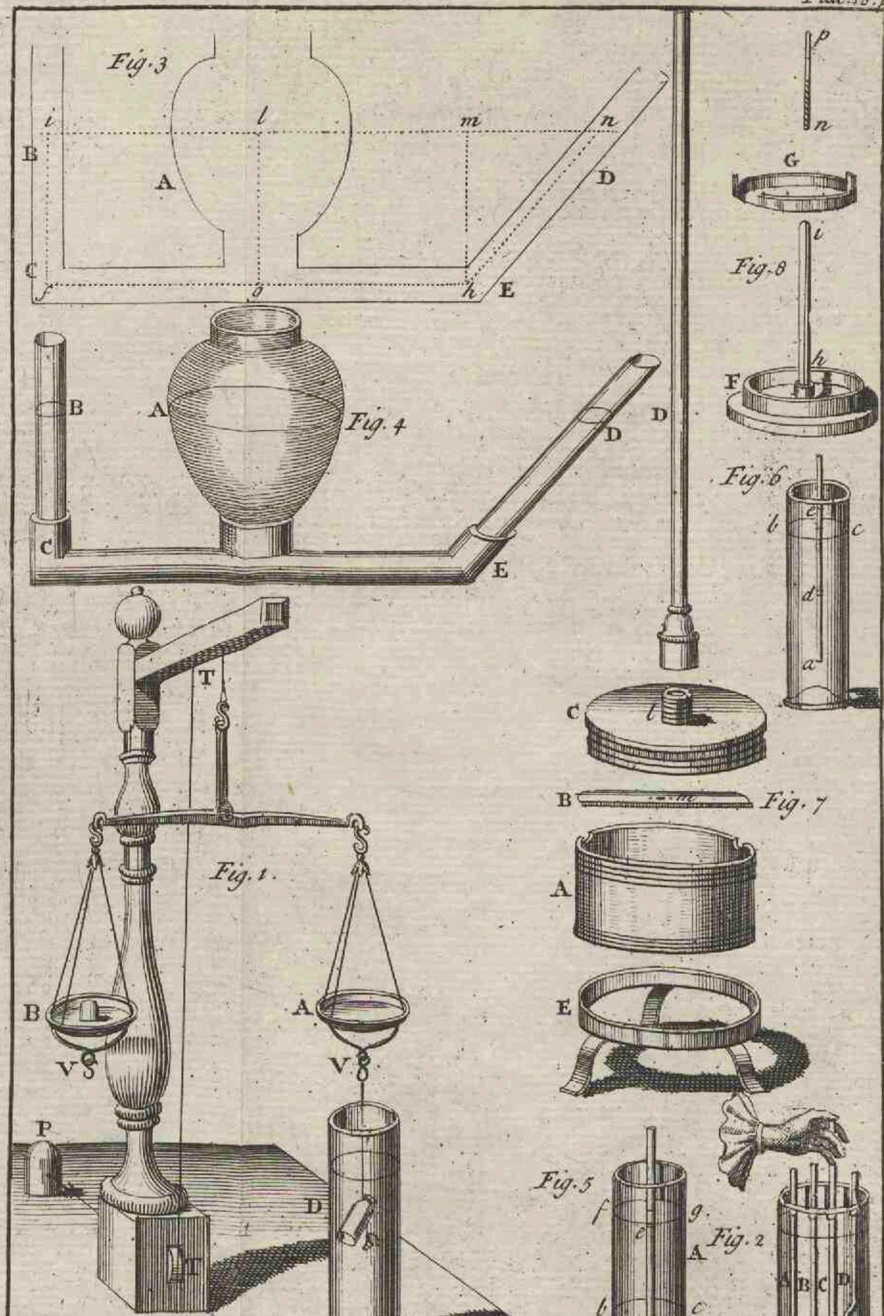
Fig. 3
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Fig. 4
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Fig. 7
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Fig. 5
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Fig. 2
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d
A B C D

put such a Weight in the opposite Scale as will make an *Æquilibrium* with the Weight of the Bottom only. Let the Tube be one Foot long; and, the Beam of the Balance being placed horizontal, pour Water into the Tube D, so that it may rise up to its upper End; another Weight of 4½ Pounds, being put into the upper Scale, will make an *Æquilibrium* with the Water; and, if you diminish or increase this Weight, the Bottom will move upwards or downwards. But you must observe, that, in altering the Weight, you must put in, or take out a pretty considerable Weight; for Example, half a Pound, because of the Friction of the Bottom.

The Diameter of the Bottom is almost 3½ Inches, and the Height of the Top of the Water, in this Experiment, is 14 Inches; the Weight of a Pillar of Water of that Height, whose Base is equal to the Bottom, is 4½ Pounds; and just so much does the Water press against the Bottom; tho' there be but a small Quantity of Water in the Machine.

Since only the Motion of the Bottom is to be observed, the Machine is to be fix'd down, lest it should be wholly raised; which is done by laying such Weights upon it as are represented by PP, *Plate* XX. *Fig.* 1.

Experiment 2. *Plate* XIX. *Fig.* 2.] Having taken away the Cover and the Tube, join the Cylinder A to the Vessel DE, which has at the Bottom a Ring with a Screw. Into this Machine pour Water upon the Bottom as high as in the foregoing Experiment; the rest of the Experiment is made in the same Manner as the former, and the Success is the same; for the Pressure is not changed, tho' you alter the Figure of the Vessel

Veſſel and the Quantity of the Water, provided you keep the Water to the ſame Height.

Experiment 3. *Plate* XIX. *Fig.* 3.] Hang the Cylindrick Veſſel A to the End of a Balance, which Veſſel muſt be filled in part by a wooden Cylinder D*e*, which Cylinder is fixed any how to the Piece of Wood BC, and neither touches the Sides nor the Bottom of the aforeſaid Veſſel; if you pour Water into this Veſſel to any Height, and make an Æquilibrium by putting Weights in the oppoſite Scale, that Weight will be the Weight of the whole Water which would be contained in the Veſſel, the Cylinder being taken away, ſuppoſing it filled to the ſame Height as in the Experiment. And ſo a ſmall Quantity of Water, whoſe upper Surface is raiſed, ſo as the Preſſure againſt the Bottom be increaſed, will ſuſtain a great Weight.

It will viſibly appear, that the lateral is equal to the vertical Preſſure, making uſe of the following Machine.

282 *Plate* XIX. *Fig.* 4.] The Veſſel DB is a Parallelopiped of Wood about a Foot and a half high; in the Side towards the Bottom there is a Hole in which there is a Braſs Ring containing a Screw, that the Cylinder A, mentioned in the firſt and ſecond Experiments, may be ſcrewed to it. Here you muſt take away the Trevet which ſuſtained the Cylinder in thoſe Experiments, and was fixed to the lower Ring by Screws. Now the Motion of the Bottom, in the Cylinder is horizontal. Two croſs Pieces of Wood are joined to the Sides of this Machine, one of which is ſeen in GH; along them the Ruler CC is moved horizontally, which is wider in the Middle towards F, that by its Motion the Bottom of the Cylinder may be thruſt inwards, which the

the Ruler presses a little below the Center. At CC Ropes, as CE, are fasten'd to this Ruler, which are stretch'd along the Pieces, as GH, and going over Pullies at the Extremities of the said Pieces, as T, have Weights joined to them, as P.

Experiment 4.] Pour Water into the Vessel BD, so that the Surface of the Water may be higher by 14 Inches than the Ruler CC; let the Weights, as P, be of 2 Pounds and a Quarter each; so that both taken together shall amount to 4 Pounds and a Half, the Pressure of the Water will sustain that Weight, and the Bottom in that Case will be moved with the same Ease towards either Part.

The following Experiment proves that the Force, with which Water presses upwards, is equal to that with which it presses downwards and sideways.

Experiment 5. *Plate* XX. *Fig.* 1.] In the Middle of the upper Surface of the Block or Foot B there is a Cylinder of about 2 Inches Diameter, on which you must put the moveable Bottom of the Cylinder A, so often mentioned; so that, the Bottom remaining fixed, the Cylinder may be moved. The Cylinder must have its cover on, and to it the Tube D, 3 Feet and a Half long, must be fastened; pour in Water, by which, the Bottom remaining fix'd, the Machine will be raised; put the Weights PPP, which all together weigh 9 Pounds, upon the Cover, and they, with the Weight of the whole Machine, will be sustained by the Water in the Tube; but the Weight of the Machine is more than 3 Pounds and a Half.

The Force, which acts against the Cover, is equal to the Weight of a Pillar of Water, whose whofe Bafe is the Cover, excepting the Hole to which the Tube is fix'd, and whofe Height is the Height of the Water-Tube above the inward Surface of the Cover; which agrees with this Experiment.

If you apply the fame Tube to a greater Machine, the Action againft the Cover will increafe in the fame *Ratio* as the Cover; fo that a prodigious Weight may be fuftained, and even raifed by a fmall Quantity of Water.

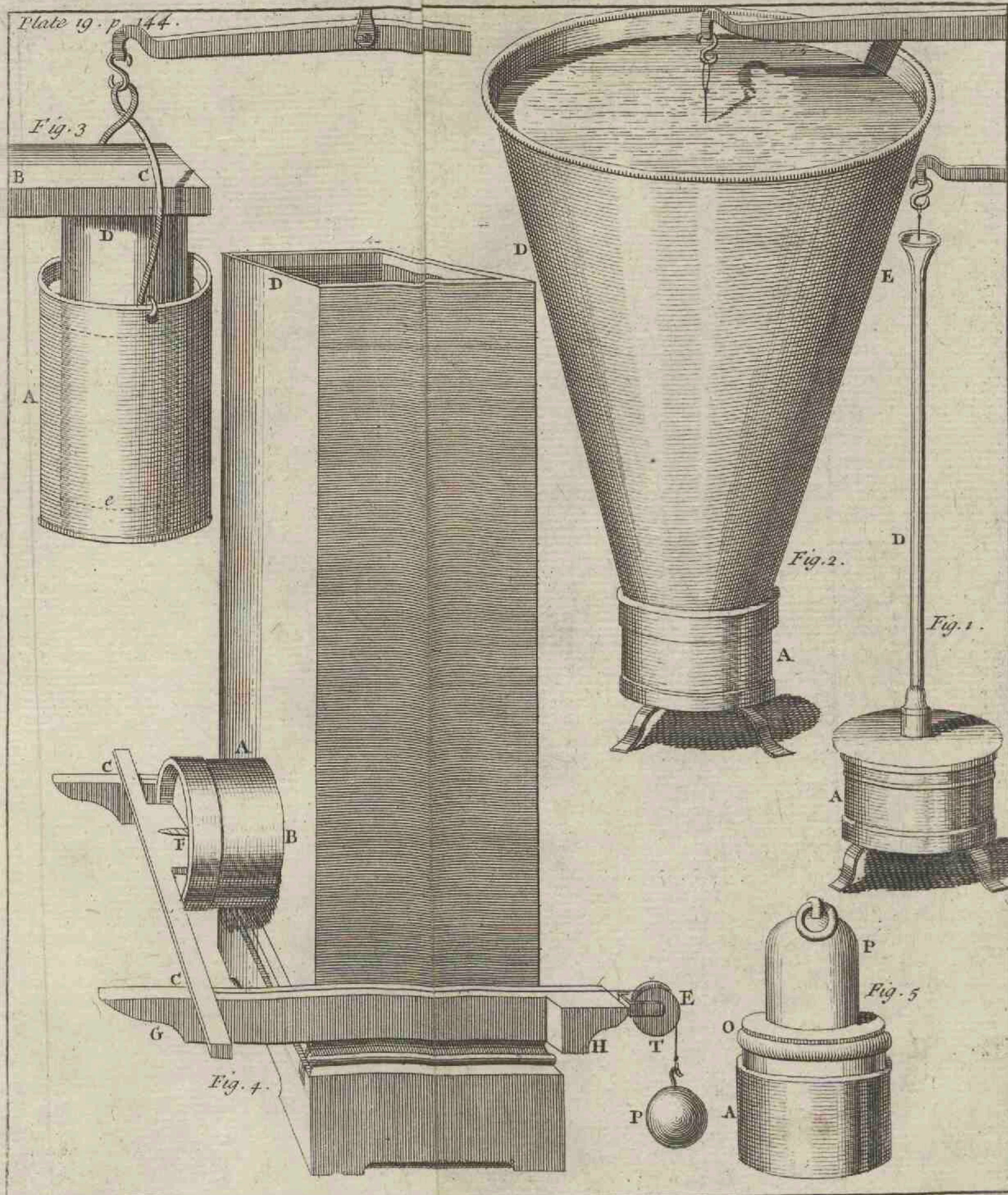
283 *Plate* XX. *Fig.* 2.] Take two round Boards AB, AB, of 15 Inches Diameter, and join them together with a Piece of Leather, fo that they may make a Cylindrick Veffel fomething like a Pair of Bellows, fo that it may contain Water.

There is a Hole *l* in the upper Board, to which is fix'd a brafs Cylinder that has a Screw, whereby the Tube D is fix'd to it, which is as long as the Tube ufed in the former Experiment.

Experiment 6.] Pour Water into this Bellows thro' the Tube, and the Water in the Tube will fuftain the Weights P, P, P, P, P, P, all which together weigh more than 250 Pounds. Thefe Weights will even be raifed by continuing to pour Water into the Tube.

Though thefe are Paradoxes, they follow from the Nature of Liquidity; every Drop which is at reft, endeavours to recede every Way with
*275 equal Force; *if therefore it be preffed on one Side, it endeavours to recede that Way with the fame Force, becaufe Action and Re-action are equal, and with that very Force itfelf will prefs every Way. In the firft Experiment, the Water which touches the Bottom, and correfponds with the

Plate 19. p. 144.



the Tube, ſuſtains the Weight of the Column of *Water* contain'd in the Tube, and reaching quite to the Bottom, and preſſes the Bottom with ſuch a Force, that it acts with the ſame Force upon the *Water* next to it; and ſince that *Water* cannot flow out againſt the Bottom, the *Water* next to that is alſo preſs'd with the ſame Force. The ſame may be ſaid of the *Water* next to that; and ſo in all Parts of the Bottom there is a Preſſure equal to that which lies under the *Water* in the Tube; and therefore the Bottom in this Caſe is as much preſs'd as if a Pillar of *Water*, of the ſame Height as the *Water* in the Tube, and of a Baſe equal to the Bottom, ſhould lie upon it.

The fifth and ſixth Experiments are illuſtrated by the ſame Reaſoning.

In the ſecond Experiment, ſuppoſe that the Cylinder A ſhould be continued, ſo as to reach up to the Surface of the *Water*; by this Means the external *Water* would be ſeparated from the *Water* contain'd in this Cylinder, and then no *Water* but this interior *Water* would preſs upon the Bottom, and the Bottom would ſuſtain it all. The *Water* in the Cylinder preſſes againſt the Sides of the Cylinder, and the external *Water* preſſes upon the external Surface of the Cylinder, and the outward Surface is preſs'd exactly in the ſame manner as the inward, and the Preſſures againſt oppoſite Points are preciſely equal; ſo that if the Surface was taken away, theſe Preſſures would deſtroy one another; therefore it is no matter, whether there be ſuch a Surface or not, ſo that taking it away (that is, taking away the Continuation of the Cylinder) the Action againſt the Bottom is no way alter'd.

The third Experiment is alſo illuſtrated by what has been ſaid; for the Weight placed in the Balance is not only ſuſtain'd by the *Water* in the Veſſel, but alſo by the Action of the inferior Surface in the Cylinder D*e* againſt the *Water*.

Tho' all that has been ſaid depends upon the Gravity of Liquids, their Actions muſt be diſtinguiſh'd from their Gravity, which laſt is always proportionable to the Quantity of Matter*.

* 270

CHAP. III.

Of Solids immerſed in Liquids.

WE have often ſaid, that the different Gravity of Bodies, whether Solids or Liquids, ariſes from this, that they contain a greater or leſs Quantity of Matter in an equal Space.

Definition I.

284 *The Quantity of Matter in a Body being conſider'd in relation to its Bulk*, that is, in relation to the Space poſſeſs'd by it, *is call'd* the Denſity of the Body.

A Body is ſaid to have double, or triple, *&c.* the Denſity of another Body, when, ſuppoſing their Bulks equal, it contains a double, or triple, *&c.* Quantity of Matter.

Definition II.

285 *A Body is* ſaid to be *Homogeneous, when it is every where of the ſame Denſity.*

Definition III.

286 Heterogeneous, *when the Denſity is unequal in different Parts of the Body.*

Definition IV.

287 *The Gravity of a Body, conſidered with relation to its Bulk, is called* the ſpecifick Gravity of a Body.

The ſpecifick Gravity is ſaid to be double, when under the ſame Bulk the Weight is double.

Therefore *the ſpecifick Gravities and Denſities of Bodies, in homogeneous Bodies, are in the ſame Ratio; and they are to one another as the Weights of equal Bodies, in reſpect to their Bulk.* 288

If homogeneous Bodies are of the ſame Weight, their Bulks will be ſo much leſs as their Denſities are greater, and under the ſame Weight the Bulk is diminiſhed in the ſame Ratio in which the Denſity is increaſed; therefore in that Caſe the Bulks are *inverſly as the Denſities.* 289

When a Solid is immerſed in a Liquid, it is preſs'd by the Liquid on all Sides, and that Preſſure increaſes in Proportion to the Height of the Liquid above the Solid; as it follows from what has been ſaid in the foregoing Chapter; and which may alſo be proved by a direct Experiment.

Experiment 1. *Plate* XX. *Fig.* 3.] Tie a Leather Bag S to the End of a Glaſs Tube B *m*, and fill it with Mercury; you may alſo make uſe of a Bladder; let this Bag be immerſed in *Water*, but ſo, that the End B of the Tube may be above the *Water*; by the Preſſure of the *Water* againſt the Surface of the Bag, the Mercury in the Tube will riſe to *m*; and the Aſcent of the Mercury follows the Proportion of the Height of the *Water* above the Bag.

When a Body is immerſed in a Liquid to a great Depth, the Preſſure againſt the upper Part differs very little from the Preſſure againſt the under Part; whence *Bodies very deeply immerſed, are, as it were, equally preſs'd on all Sides*; which Preſſure may be ſuſtained by ſoft Bodies, without any Change of Figure, and by very brittle Bodies, without their breaking.

Experiment 2. *Plate* XIX. *Fig.* 5.] Take a Piece of soft Wax of an irregular Figure, with an Egg, and inclose it in a Bladder full of Water, and the Bladder being exactly shut must be put into a brass Box A; let it be covered with a wooden Cover O, so that it may be sustained by the Bladder, lay on a Weight P of 70 or 80 Pounds, and the Egg will not be broken, nor the Figure of the Wax any way changed.

292 *A Body specifically heavier than a Liquid, being immersed in a Liquid in any Depth, will descend.* The inferior Part of the Body presses the Surface of a Liquid which it touches, and this Pressure is equal to the Weight of a Column made up of the Body itself and the superincumbent Liquid, and with this Force the Body is carried downwards. The Weight of a like Column, but which consist wholly of a Liquid, is the Force by which
* 290 the Body is press'd upwards by the Liquid.* But
275 when the Solid is supposed specifically lighter than the Liquid, this Force is less than that, and therefore is overcome by it.

293 It is proved by the same Reasoning, *That a Solid specifically lighter than a Liquid, and immersed into it, must ascend to the highest Surface of the Liquid.*

294 But *suppose a Solid of the same specifick Gravity with the Liquid, it will* neither ascend nor descend, but *remain suspended in the Liquid at any Height, and the Liquid will sustain the whole Body*; in which Case, by reason of the Equality of the specifick Gravities, the Liquid sustains a Weight equal to the Weight of the Quantity of the Liquid, which would fill the Space taken up by the Body. But a Liquid acts in the same manner upon all equal Solids immersed to the same Depth, and will sustain

sustain them equally; therefore they all lose Part of their Gravity.

Definition V.

A Weight, which keeps a Body immersed in a Liquid, is called its respective Gravity. 295

And *this respective Gravity is the Excess of the specifick Gravity of a Solid above the specifick Gravity of a Liquid*; for since *a Solid immersed in a Liquid* loses that Part of its Weight which is sustained by the Liquid, it *loses the Weight of the Quantity of the Liquid, which could fill the Space taken up by the Body.* 296 297

Experiment 3. *Plate* XXI. *Fig.* 1.] Hang the hollow Brass Cylinder E to the Balance above-mentioned; * hang the solid Cylinder C of the same Metal by a Horse-hair to a Hook fix'd to the Bottom, which, if it be put into the other Cylinder E, will exactly fill it; so that E, when it is full of Water, will contain such a Quantity of Water as will fill the Place taken up by C; put a Weight in the opposite Scale to make an Æquilibrium; let the Balance descend, that the Cylinder C may be immersed into the Water contained in the Vessel D; by that Means the Æquilibrium is destroyed, because C is partly sustained by the Water; but is restored, if E be fill'd with Water.

* 271

Hence it follows, that *all equal Solids, but of different specifick Gravity, when they are immersed into the same Liquid, they lose equal Parts of their Weight.* The last mentioned Experiment will succeed in the same Manner with a Cylinder of any other Metal, and by pouring in the same Quantity of Water, that is, so much as will fill the Vessel E, the Æquilibrium will always be restored. 298

299 Moreover, from what has been ſaid it follows, that *however the Denſities of equal Bodies differ among themſelves, if they be immerſed in the ſame Liquids, the Weight which they loſe is in the Ratio of their Bulks*, for the Spaces which they take up in a Liquid are in the ſame Ratio.

Therefore Bodies of the ſame Weight, but of different Denſities, loſe an equal Part of their Weight, when they are immerſed in the ſame Liquid, becauſe of the Inequality of their Bulks.

Experiment 4. *Plate* XX. *Fig.* 4.] Let two Pieces of Metal of the ſame Weight, the one of Gold, and the other of Lead, *g*, *g*, be ſuſpended with the Hook V V of the Balance above-mentioned
* 271 with Horſe-hair, * and you will have an Æquilibrium; let the Balance deſcend, and the Bodies *g*, *g*, be immerged in the *Water* contained in the Veſſel F F, and the Æquilibrium will be deſtroyed. When a Solid, ſpecifically heavier than a Liquid, is ſuſpended in a Liquid, the Liquid acts on every Side againſt that Solid, in Propor-
* 290 tion to its Weight, * and the Solid re-acts equally againſt it; therefore thoſe Actions are the ſame as if the Space taken up by the Solid were fill'd with the Liquid; therefore *it is no Matter, in reſpect of the Gravity of the Liquid, whether a Solid, ſpecifically heavier than the Liquid, be ſuſpended in it, or a Quantity of the Liquid be poured in, which takes up a Space equal to the Solid.*

300 *Experiment* 5. *Plate* XXI. *Fig.* 2.] Take the Veſſel A containing *Water*, hang it to one End of the Balance, immerge into it the Braſs Cylinder C, which is ſuſtained by a Horſe-hair, leſt it ſhould touch the Bottom of the Veſſel, putting a Weight into the oppoſite Scale, and you will have an Æquilibrium; take the Cylinder Court of

of the Water, the Æquilibrium will be destroyed: and it will be restored again by pouring in Water as much as can be contained in the hollow Cylinder E, which will be exactly filled by the following Cylinder C.

By comparing together the *Numb.* 297 and 300, as also the third and fifth *Experiments*, which confirm them, it appears, that a *Liquid acquires the Weight which the immersed Solid loses.* The Force of Gravity is always proportionable to the Quantity of Matter, and is not changed by the Immersion of a Solid into a Liquid; wherefore the Sum of the Weight of the Solid, and of the Liquid, do not differ before and after the Immersion. 301

Experiment 6. Plate XX. *Fig.* 5.] Hang the Solid C to the Balance, and make an Æquilibrium, by putting into the opposite Scale B the Weights P and *p*, of which *p* equal to the Weight which the Body C loses in Water. Take the Vessel E, which contains Water, and is suspended to the Balance EF, and, putting a Weight into the opposite Scale, make an Æquilibrium here also; let the Balance descend with the Body C, that it may be immersed in the Water contain'd in D, by this Means you will destroy the Æquilibrium in both Balances, which will be restored by taking out of the Scale B the Weight *p*, and putting it into the Scale of the Brachium F.

A Body specifically heavier than a Liquid, and 302
which descends in it, is carried downwards with a greater Force than it is pressed upwards, as has been explained before*; the Difference of which * 292
Forces is the respective Gravity of the Body.

The first Force in part consists of the Weight of the Liquid incumbent over the Body, and the Body may be immersed to such a Depth, that

that Weight ſhall be equal to the above-mentioned ſpecifick Gravity: If in that Caſe you take away
303 the ſuperincumbent Liquid, the Body will be ſuſtained by the Preſſure of the Liquid under it. If the Body immerſed to a greater Depth, and the Liquid be alſo hindered from paſſing upon the upper Surface of the Body (becauſe the Preſſure by which a Body is puſhed up, increaſes as the Depth
* 290 to which it is immerſed) * the Body then will be carried upwards with greater Force than downwards by Gravity; wherefore, if it could move freely, it would aſcend.

Experiment 7. *Plate* XXI. *Fig.* 3.] To the Cylinder C, which is open at both Ends, apply at Bottom the Plate of Lead F, a Quarter of an Inch thick; if it fits ſo exactly to the Cylinder as to let no Water ſlip by, and the Plate be held up by a Thread faſtened to the Hook V in the Center of the Plate, until it be immerſed with the Cylinders to the Depth of about 3 Inches, the Lead will be ſuſtained by the Water, as appears by letting go the Thread. If you immerge it to a greater Depth, it will ſtick cloſer to the Cylinder; but if to a leſs, it will fall off.

If this Experiment was made with a Plate of Gold, it ought to be immerſed to a greater Depth; for the ſpecifick Gravity of Gold is to the ſpecifick Gravity of Water as 19 to 1; and therefore its reſpective Gravity is to that of Water as 18 to
* 296 1.* Therefore to have a Pillar of Water equal in reſpective Gravity to the Plate of Gold, that Pillar muſt be above 18 times its Height; and therefore the Height of the Water, above the upper Surface of the Plate of Gold, muſt be at leaſt equal to as many times its Thickneſs.

Experiment

Experiment 8. *Plate* XXI. *Fig.* 3.] Take a Cylinder A with a moveable Bottom, that has also a Cover with the Tube D joined to it, as was before described; * immerge it in Water, and when it * 381
comes to be a Foot under Water, the Bottom will rise, although it weighs a Pound and a Quarter, and has P a Pound Weight screwed to it at Bottom.

If the same Solid be immersed into Liquids of 304
different Density, it will lose different Parts of its Weight: * And therefore, when two Bodies of * 297
the same Density and Weight are immersed in Liquors of different Density, they will lose their Æquilibrium.

Experiment 8.] *Plate* XXI. *Fig.* 4.) Take two flat Pieces *g g*, of the same Metal and equal, and hang them upon the Hooks V V of the Scales A and B; then by the Descent of the Balance immerge them in the Liquids contained in the Vessels F F, the one in Water, the other in Oil of Turpentine, and the Æquilibrium will be destroyed, the Piece which was immersed in Oil becoming lighter.

A Solid lighter than a Liquid, and immersed in it, 305
ascends and remains at the upper Part of the Liquid, * so as to be immersed only in Part; but the * 293
greater is its specifick Gravity, the more it descends, and the Body *will not be at rest till the immersed Part takes up such a Space in the Liquid, that the Bulk of the Liquid, which would fill that Space, shall weigh as much as the whole Body.* For in another Case the Solid does not act with the same Force against the neighbouring Parts of the Liquid, as the Liquid would act, if it should take up the Place of the Body; therefore in this Case alone the Liquid and the Body can be at rest. * * 276

It

It follows from this Propoſition, that *the immerſed Parts of the Bodies, ſwimming on the Surface of the ſame Liquor, are to one another, as the Weights of the Bodies.* Therefore if, by ſuperadding a Weight, the Gravity of the Body is changed, the immerſed Part is increaſed in the ſame Proportion, and *the Parts, which deſcend into the Liquid by laying on of different Weights, are to one another as thoſe Weights.*

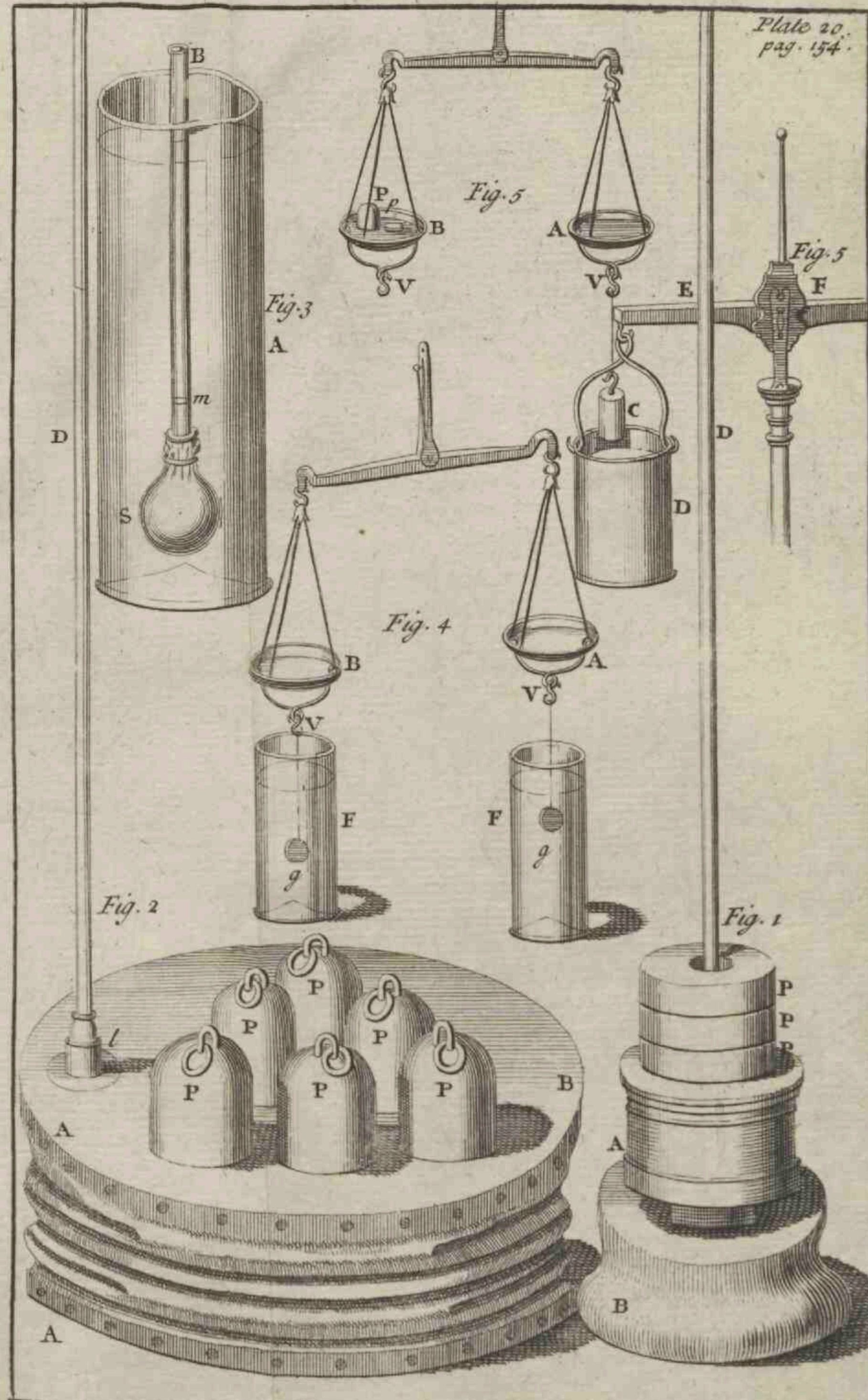
Experiment 10. *Plate* XXII. *Fig.* 1.] Take a Veſſel A containing Water in it; let C be a hollow Cylinder of any Metal; lay upon it the Weight *p*, that it may deſcend into the Water with its Part *bd*; adding the Weight of one Pound, meaſure how far it will deſcend; then adding another equal Weight, you will find that it will deſcend equally every time.

308 In *Numb.* 302 and 303, confirmed by the *Experiments* 7 and 8, it appeared how a Body, ſpecifically heavier than a Liquid, may be made to ſwim; by the ſame Method a Body, ſpecifically lighter than a Liquid, may be retained at the Bottom: In that Caſe the Preſſure of the ſuper-incumbent Water is taken off; but here you muſt take off the Preſſure of the inferior Water whereby the Body is puſhed upwards.

Experiment 11. *Plate* XXII. *Fig.* 2.] Upon the Foot D, which is fixed at the Bottom of the Veſſel A, there is a Braſs Plate *bc* exactly flat and poliſhed; there is another Braſs Plate *bc*, like the former, faſtened to a large Piece of Cork E, ſo that together with the Cork it ſhall make up a Body ſpecifically lighter than Water: Lay this Plate upon the other, ſo that they may fit, and keep the Cork down with a Stick while you pour in Water; leaving the Cork, it will not aſcend until, by

moving

Plate 20.
pag. 154.



moving it out of its Place, the Plates are ſepara-
ted, ſo that the Water may exert its Preſſure a-
gainſt the Plate joined with the Cork, and puſh it
upwards together with the Cork.

CHAP. IV.

Of the Manner of comparing the Denſities of Liquids.

SInce the Denſity of Bodies is in Proportion to 309
their Gravity, by comparing the Weights of
equal Bodies we diſcover their Denſities. * *If* * 288
therefore *any Veſſel be exactly filled with a Liquid,
and that Liquid be weighed; and if you make the
ſame Experiment with other Liquids, their Weights
will be as their Denſities.* But, as this Method is
liable to ſeveral Difficulties in Practice, I ſhall not
ſpend any Time in explaining it here.

When the Preſſures of two Liquids are equal, the 310
Quantities of Matter, in Columns that have equal
Baſes, do not differ; * wherefore the Bulks, that * 273
is, *the Heights* of the Columns, *are inverſly as the
Denſities*; * whence may be deduced the Method * 289
of comparing them together.

Experiment 1. *Plate* XXII. *Fig.* 3.] Pour Mercury into a curve Tube A, ſo as to fill the lower Part of the Tube from *b* to *c*; pour in Water in one Leg from *b* to *c*; in the other Leg pour in Oil of Turpentine, till both the Surfaces of the Mercury *bc* be in the ſame horizontal Line, and the Height of the Oil be *cd*: Theſe Heights will be as 87 to 100, which is the inverſe Ratio that the Denſity of the Water has to the Denſity of Oil of Turpentine; and therefore theſe Denſities are to each other as 100 to 87.

The Mercury is poured in, left the Liquids ſhould be mix'd in the Bottom of the Tube.

The Denſities of Liquids are alſo compared together, by immerging a Solid into them; for if *a*
311 *Solid, lighter than the Liquids to be compared together, be immerſed ſucceſſively into different Liquids, the immerſed Parts will be inverſely as the Denſities of the Liquids*; for, becauſe the ſame Solid is made uſe of, the Portions of the different Liquors, which in every Caſe would fill the Space taken up by the
305 immerſed Part, are of the ſame Weight; therefore the Bulks of thoſe Portions, that is, the immerſed Parts themſelves, are inverſely as the Den-
289 ſities.

312 *Plate* XXII. *Fig.* 4.] Take the Glaſs A, which is a hollow Ball that has a Tube divided into equal Parts; at the Bottom of the great Ball there is a ſmall one, Part of which is fill'd with Mercury, or very ſmall Shot, whoſe Weight ſerves to make the Tube deſcend vertically in Liquids, and ſtand in that Poſition: Care muſt be taken not to have too much Weight in the little Ball, for the whole Glaſs muſt be lighter than the Liquids to be compared together. The *Hydrometer*, (for ſo it is called) deſcends to different Depths in different Liquids; and thoſe Denſities, as we have already ſaid, are inverſely as the Parts immerſed, which therefore are to be compared together. Tie a Thread to the Hydrometer, and weigh it together with the Thread; the Weight (if it be like mine) will be 573 Grains; if put into Water, it will deſcend to *b*; therefore a Bulk of Water, equal to the immerſed Part of the Hydrometer, weighs
305 573 Grains, and may be expreſſed by that Number. Faſten the Thread above-mentioned to the Hook V of the Scale A of the Balance repreſented in *Plate* XVIII. *Fig.* 1. the Hydrometer remains

remains immerſed; put 20 Grains in the Scale B, and let the Weight P be moved gently to raiſe the Balance, (by which the Tube will be drawn a little way out of the *Water*) till there be an Æquilibrium, and the Surface of the *Water* then will be given with the Point *d*; the *Water* ſuſtains the Weight of the whole Machine, except 20 Grains; that is, it ſuſtains 553 Grains; and the Weight of the ſame Bulk of *Water*, which is now immerſed, weighs juſt ſo many Grains, and is expreſſed by that Number; wherefore one may call the Bulk of the Parts *db* of the Tube 20; if the Space *db* be divided into 10 equal Parts, and you continue the Diviſions upwards beyond *b*, and downwards below *d*, each Diviſion may be called 2; and by obſerving the Diviſion to which the Inſtrument deſcends in a Liquid, you will have the Bulk of the immerſed Part; ſo, if the whole Tube ſtands out above the *Water*, the immerſed Bulk will be 549; if it riſes to the upper Diviſions, the immerſed Bulk will be 579; and the Denſities of the Liquids, in which this happens, will be inverſly as thoſe Numbers, that is, as 579 to 549, and only the intermediate Denſities may be compar'd by this Inſtrument; if the Ball was leſs in Proportion to the Tube, it would ſerve for comparing together Liquids whoſe Denſities differ more than this. When ſeveral Liquids are compared together, the Numbers which expreſs the Bulk of the immerſed Parts are the Denominators of Fractions, which have 1 for their Numerator; and theſe Fractions expreſs the Ratio of the Denſities; for they are to one another inverſly as the Denominators.

Experiment 2.] Let the Denſities of Waters, containing different Quantities of Salts, be to be com-

compared, the Hydrometer deſcends in the one to the Diviſion a; if it be immerſed in another, it only deſcends to the Diviſion c, their Denſities will be to one another as $\frac{1}{379}$ to $\frac{1}{585}$, as may be eaſily deduced from what has been ſaid.

This Method is alſo liable to ſeveral Difficulties beſides this, that it is difficult to compare together Liquors very different in Denſity by the ſame Hydrometer.

813 The beſt Method of all is, to make uſe of a Solid heavier than the Liquids. *When the ſame*
314 *Body is immerſed in different Liquids, the Weights, which it loſes in the Liquids, are to each other as*
*297 *the Denſities of thoſe Liquids.* * Here you muſt
288 uſe a hydroſtatical Balance, * and beſides a ſolid
*271 Piece of Glaſs, as C, which may hang to one of the Scales by a Horſe-hair, *Plate* XXI. *Fig.* 5, and 6, you muſt have a Weight, as P, which æquiponderates with the Glaſs C, when it is immerſed in Water, as is repreſented in *Fig.* 4. The Difference between the Weight P, and the Weight of the Glaſs C, when it is taken out of the *Water*, is the Weight which the Body has loſt, when weigh'd in *Water*: This muſt be obſerved, that it may ſerve in all the Experiments; in our Balance it weighs 722 Grains. Suſpend the Body in any other Liquid, unleſs it be of the ſame Denſity as *Water*, the Æquilibrium will not be preſerved: Let it be reſtored by putting Grain Weights in either of the Scales; if they be put into the Balance A, add them to the above-mentioned Difference of 722 Grains; if the Weights be put into B, ſubſtract them from that Number; and by that Means in each of thoſe Caſes, as it appears, the Weight loſt by a Body is determined, that is, the Weight which expreſſes the Denſity of the Liquid.

Experi-

Experiment 3.] Let the Weight C, which hangs from the Scale A, be immersed in Oil of Turpentine, whilst the Weight P hangs from the Scale B; put 94 Grains in the Scale B, and you will have an *Æquilibrium*. Then immerge the same Weight in Milk, that the Balance may return to an *Æquilibrium*, a Weight of 22 Grains must be put in the Scale A. Substracting the first Number from 722, and adding the second to it, you will have 628 and 744, expressing the Densities of the above-mentioned Liquids; whilst 722 shews the Density of the Water itself.

CHAP. V.

Of the Hydrostatical Comparison of Solids.

IN all homogeneous and equal Bodies, the Densities are as the Weights;* in unequal Bodies *288
of the same Weight, the Densities are inversly as the Bulks;* if therefore both the Bulks and Weghts *289
differ, the Ratio of the Densities is compounded of the direct Ratio of the Weights, and the inverse Ratio of the Bulks; and therefore, *dividing the Weights by the Bulks, you have the Densities*; 315
that is, you will have Numbers that are to each other as those Densities.

The Weight of all Bodies may be compared by 316
Means of the Balance; the Bulks are found by *immerging Bodies in the same Liquid*; for *the Weights, which they lose, are as the Bulks.** *299

Plate XXI. *Fig*. 7.] Here also the hydrostati- 317
cal Balance is to be used,* as likewise the Glass Vessel D, in which the Bodies to be compared are to be placed; you must also have such a Weight as is represented by P in *Fig*. 6. that is, equal to the Weight

Weight of D; and laſtly, the Weight *p* (*Fig.* 8.) equal to the Weight which the Glaſs D loſes, when it hangs in the *Water*.

With a Horſe-hair you muſt faſten the Glaſs D in the Place of the Body C (*Fig.* 4.) in the Scale A, and hanging the Weight P in the Scale B, you will have an Æquilibrium. The Body, whoſe Denſity is required, is placed in the Glaſs D (as we have ſaid before) and weighed, the Veſſel and Body being immerſed in *Water*; putting the Weight *p* into the Scale B, the Æquilibrium is reſtored in reſpect to the Glaſs D; you muſt add as much Weight beſides as is required to make an Æquilibrium, and that will be the Weight loſt by the Body weighed; by this Weight therefore you muſt divide the Weight of the Body itſelf, to have
315 the Denſity.

Experiment 1.] A Piece of Gold, weighing 137 Grains, loſt in *Water* $7\frac{1}{4}$ Grains. A Piece of Silver, weighing 248 Grains, loſt in *Water* 24 Grains. Therefore their Denſities are as $18\frac{26}{29}$ to $10\frac{1}{2}$, that is, nearly as 11 to 6. By faſtening a Body, whoſe Denſity is required, and is heavier than a Liquid, to a Body lighter than the Liquid, the Denſity is alſo deſcovered.

318 *Plate* XXII. *Fig.* 5.] Take the Machine A, like the Machine deſcribed in the preceding Chap-
312 ter, and let it have fixed to its Bottom the Ring DE, and to its Top the Ring FG; then the Ball will, by its own Weight, be in part immerg'd in *Water*. This Machine cannot be apply'd to Uſe, unleſs you know by ſome other Method, how much of its Weight any Body loſes in *Water*; therefore we lay down as known, that $109\frac{7}{10}$ Grains of Lead weigh in *Water* but 100 Grains; therefore lay juſt as many Grains upon the Ring

 DE,

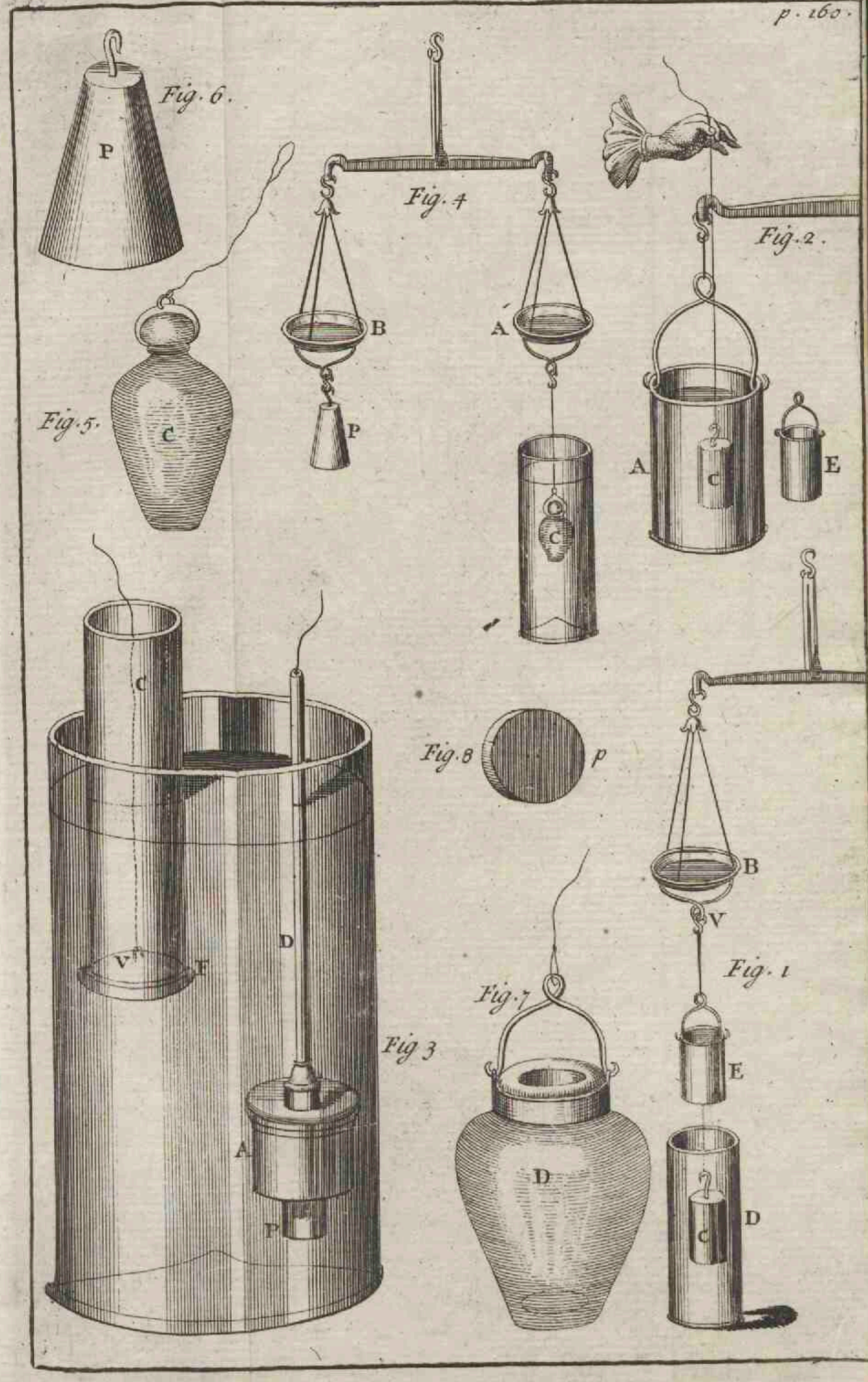


Fig. 6.
P
Fig. 4
Fig. 2.
Fig. 5.
C
B
P
A
C
A
C
E
C
V
F
D
Fig. 8
p
Fig 3
A
P
B
V
Fig. 1
Fig. 7
D
E
D
C

D E as will make the Machine, when immerg'd in Water, descend to *a*; lay what Number of Grains you please upon the Ring F G, for Example, Eight, and the Machine descends to *c*; the Space *a c* must be divided into Eight Parts, and the Divisions must be continued upwards and downwards; if you make *a* the hundredth Division, *c* will be the hundred and eighth Division, and the lowest of all in this Figure will be the 97th. 'Tis plain, that if the Proposition of *Num.* 307 be compar'd with the aforesaid Preparation, the Division, to which the Machine descends in Water, shews the Weight of the Grains which press down the Machine; therefore laying a Body upon the Ring D E, its Weight in Water will be determined; by substracting this Weight from its Weight out of the Water, you will have the Weight lost in the Water; by which if the Weight out of the Water be divided, the Density is discover'd, as has been said in the Beginning of this Chapter.

Experiment 2.] Lay a Piece of Brass, weighing, for Example, 100 Grains, on the Ring D E by which the Ball of the Machine is not immerg'd; lay any Weight, for Example, of 17 Grains, on the Ring F G, and the Machine descends to *b*, that is, to the 105th Division; which proves, that the Machine is press'd down by so many Grains; from this Number of Grains substract the 17 last mentioned, the remaining 88 are the Weight of the Piece of Brass in Water, which therefore loses 12 Grains. If again, the Weight 100 Grains be divided by the 12 Grains, you have $8\frac{1}{3}$, expressing the Density of the Brass. The Densities of any other Bodies may be found after the same Manner.

This Method has several Difficulties. The foregoing is the best of all.

CHAP. VI.

Of the Reſiſtance of Fluids.

319 ALL *Bodies moved in Fluids ſuffer a Reſiſtance, which ariſes from two Cauſes. The firſt is the Coheſion of the Parts of the Liquid.* A Body in its Motion, ſeparating the Parts of a Liquid, muſt overcome the Force with which thoſe Parts cohere, and thereby its Motion is retarded.
12 *The ſecond is the Inertia,* or Inactivity *of Matter*, that belongs to all Bodies, which is the Reaſon, that a certain Force is required to remove the Particles from their Places, in order to let the Body paſs. The Body acts upon the Parts to remove them, and they diminiſh its Motion by Re-action.

320 The *Retardation from* the firſt Cauſe, that is, *the Coheſion of Parts*, is always the ſame in the ſame Space, the ſame Body remaining, be the Velocity of the Body what it will. The ſame Coheſion is to be overcome in every Caſe; therefore this Reſiſtance increaſes as the Space run through, in which Ratio the Velocity alſo in-
* 53 creaſes; * therefore it *is as the Velocity* itſelf.

321 *The Reſiſtance ariſing from the* Inertia, *or Inactivity of Matter, when the ſame Body moves through different Liquids with the ſame Velocities*, follows the Proportion of the Matter to be removed in the ſame Time, which *is as the Denſity of the Liquid.*

322 *When the ſame Body moves thro' the ſame Liquid with different Velocities, this Reſiſtance increaſes* in Proportion to the Number of Particles ſtruck in an equal Time, which Number is as the Space run through in that Time, that is, as the Velocity. But this Reſiſtance does farther increaſe in Proportion to the Force with which the Body runs againſt every Part; which Force is alſo as the

the Velocity of the Body. And therefore, if the Velocity is triple, the Resistance is triple from a triple Number of Parts to be removed out of their Places. It is also triple from a Blow three times stronger against every Particle; therefore the whole Resistance is ninefold, that is, *as the Square of the Velocity*.

A Body therefore moved in a Liquid is resisted 323
partly in a Ratio of the Velocity, and partly in a duplicate Ratio of it. *The Resistance from the Cohesion of Parts in Liquids, except glutinous ones, is not very sensible* in respect of the other Resistance; which as it increases in a Ratio of the Square of the Velocities, * but the first in a Ratio of the * 222
Velocity itself: * By how much the Velocity in- * 220
creases, by so much more do these Resistances differ amongst themselves; wherefore, *in swifter* 324
Motions the Resistance alone is to be considered, which is as the Square of the Velocity.

I shall not now treat of tenacious or glutinous Liquids, nor of slow Motions, in which the Resistance, arising from the Cohesion of the Parts, must be considered.

If a Liquid be included in a Vessel of a prismati- 325
cal Figure, *and there be moved along in it with equal Velocity*, and a Direction parallel to the Sides of the Prism, *two Bodies, the one spherical and the other cylindric, so that the Diameter of the Base of this last be equal to the Diameter of the Sphere, and the Cylinder be moved in the Direction of its Axis, these Bodies will suffer the same Resistance.* To demonstrate this, suppose the Bodies at rest, and that the Liquid moves in the Vessel with the same Velocity that the Bodies had; by this the relative Motion of the Bodies and the Liquid is not changed, therefore the Actions of the Bodies on the Liquid, and of the Liquid on the Bodies, are not changed. The Retardation which the Liquor

ſuffers in paſſing by the Body, ariſes only from this, That in that Place it is reduced to a narrower Space, but the Capacity of the Veſſel is equally diminiſhed by each Body; therefore each Body produces an equal Retardation. And becauſe Action and Re-action are equal to one another, the Liquids act equally upon each Body; wherefore alſo each Body is equally retarded, when the Bodies are moved, and the Liquid is at reſt.

326 *This Demonſtration* will alſo obtain, tho' the Veſſel be ſuppoſed much bigger; and it will do in an infinite Liquid compreſſed; therefore it *may be referred to Bodies deeply immerſed.* Here we ſpeak of a continuous Liquid, and whoſe Parts cannot be reduced into a leſſer Space by Preſſure; otherwiſe there will be an Accumulation before the Body, and a Relaxation behind; and ſo much the more, the more blunt the Body
327 is; which alſo cauſes a greater Irregularity in the Motion of the Liquid, and a greater Retardation in the Motion of the Body.

When a Body is moved in any Liquid along the Surface, the Liquid is raiſed before the Body, and depreſſed behind; and theſe Elevations and Depreſſions are greater, *the more blunt the Body is*, and *by that Means it is more retarded*; for there is alſo a greater Irregularity in the Motion of the
328 Liquid in this Caſe, which ſtill more increaſes the Retardation of the Body. *This is alſo true, if the Body be not immerſed deep*; yet in that Caſe the Irregularity of the Motion of the Liquid is the chief Cauſe of the Retardation.

Therefore, to take away theſe Irregularities, we muſt conſider Bodies as deeply immerſed, and give Rules relating to them; by which the Retardations in ſeveral Caſes may be compared together. We ſuppoſe the Bodies ſpherical, tho' the Demonſtrations will ſerve for all ſimilar Bodies moved in the ſame manner.

Here

Here you must observe, that *the Resistanc, is to* 329
bedistinguished from the Retardation; the Resistance produces the Retardation. *When we speak of the same Body, the one may be taken for the other,* because they are in the same Proportion; but, supposing the Bodies different, the same Resistance often generates different Retardations. *From the* 330
Resistance arises a Motion contrary to the Motion of the Body; the Retardation is the Celerity, and the Resistance itself is the Quantity of Motion.

Let the Bodies be equal, but of different Densi- 331
ties, and moved thro' the same Liquid with equal Velocity, the Liquid acts in the same Manner upon both; therefore they suffer the same Resistance, but different *Retardations*; and they *are to one another* as the Celerities, which may be generated by the same Forces in the Bodies proposed; * that is, they are inversly as the Quantities * 330
of Matter in those Bodies, * or *inversly as the* * 60
Densities.

Now, *supposing Bodies of the same Density, but* 331
unequal, moved equally fast thro' the same Fluid, the Resistances increase according to their Superfices, that is, as the Squares of their Diameters; the Quantities are increased in Proportion to the Cubes of the Diameter; the Resistances are the Quantities of Motion, the Retardations are the Celerities arising from them; * dividing the * 330
Quantities of Motion by the Quantities of Matter, you will have the Celerities; * therefore the * 64
Retardations are directly as the Squares of the Diameters, and inversly as the Cubes of the Diameters, that is, *inversly, as the Diameters themselves.*

If the Bodies are equal, move equally swift, 332
and are of the same Density, but are moved thro' different Liquids, their Retardations are as the * 339
Densities of those Liquids. * 321

334 *When Bodies, equally dense and equal, are carried thro' the same Liquid with different Velocities, the*
* 324 *Retardations are as the Squares of the Velocities.* *
329 From what has been said, *the Retardations of*
335 *any Motions* may be compared together, for they
* 334 *are first, as the Squares of the Velocities*; * *secondly, as the Densities of the Liquids thro' which*
* 333 *the Bodies are moved; thirdly, inversly, as the Di-*
* 332 *ameters of those Bodies; lastly, inversly, as the*
* 331 *Densities of the Bodies themselves.* *

The Numbers in the Ratio, compounded of those Ratio's, express the Proportion of the Retardations. Multiplying the Square of the Velocity by the Density of the Liquid, and dividing the Product by the Product of the Diameter of the Body multiplied into its Density, and working thus for several Motions, the Quotients of the Divisions will still have the same compound Ratio to one another.

These Retardations may also be compared together, by comparing the Resistance with the
336 Gravity. It is demonstrated, that *the Resistance of a Cylinder, which moves in the Direction of its Axis* (to which the Resistance of a Sphere of the
* 325 same Diameter is equal *) *is equal to the Weight of a Cylinder made of that Liquid, thro' which the Body is moved, having its Base equal to the Body's Base, and its Height equal to half the Height, from which a Body falling* in Vacuo *may acquire the Velocity with which the said Cylinder is moved thro' the Liquid.* From the given Celerity of the Body moved, the Height of the Liquid Cylinder is found, as also the Weight of it from the known specifick Gravity of the Liquid and Diameter of the Body. Let a Ball, for Example, of 3 Inches Diameter be moved in Water with that Celerity with which it would go thro' 16 Foot in a Second: From what has been said of falling Bodies

Bodies and Pendulums, * as alſo by Experiments * 157
made on Pendulums, it has been found that this 158
is the Celerity which a Body acquires in falling 134
from a Height of 4 Foot; therefore the Weight of a Cylinder of Water, of 3 Inches Diameter, and 2 Foot high, that is, a Weight of about 6 Pounds and 3 Ounces, is equal to the Reſiſtance of the aforeſaid Ball.

Let the Reſiſtance ſo diſcovered *be divided by* 337
the Weight of the Body, which determines its Quantity of Matter, and *you will have the Retardation.** * 79
By which Rule the Proportion of the ſeveral Retardations is diſcovered, * and found to be the * 330
ſame as is given by the foregoing Rule. 335

Having conſidered the Retardations of direct Motions, we paſs on to the Motion of Pendulums.

The Arc deſcribed by a Pendulum oſcillating *in* 338
Vacuo, with a Celerity that it has acquired by deſcending, is equal to the Arc which is deſcribed by the Deſcent; * the ſame does not happen in a * 151
Liquid, and there is a greater Difference between thoſe Arcs, the greater the Reſiſtance is; that is, if you ſpeak of the ſame Liquid and Pendulum, the greater the Arc is which is deſcribed in the Deſcent.

Let the Reſiſtance of the Liquid be in Proportion 339
to the Velocity, and two Pendulums, entirely alike, oſcillating in a Cycloid, perform unequal Vibrations, and begin to fall the ſame Moment; they begin to move by Forces that are as the Arcs to be deſcribed;* * 159
if thoſe Impreſſions alone, which are made the firſt Moment, be conſidered, after a given Time, the Celerities will be in the ſame Ratio as in the Beginning; for the Retardations, which are as the Velocities themſelves, * cannot change their * 329
Proportions, for the Ratio between Quantities is not changed by the Addition and Subſtraction of the Quantities in the ſame Ratio. Therefore

in equal Times, however the Celerities of Bodies are changed in their Motion by the Refiftance, Spaces which are gone thro' are as the
* 53 Forces in the Beginning; * that is, as the Arcs to be defcribed by the Defcent; therefore after any Time the Bodies are in the correfpondent Points of thofe Arcs. But in thefe Points the Forces are
* 156 generated in the fame Ratio as in the Beginning,* and the Proportion of the Celerities, which is not varied by the Refiftance, fuffers no Change from the Gravity. In the Afcent, Gravity retards the Motion of the Body, but in correfpondent Points its Actions are in the fame Ratio as in Defcents. And therefore every where in correfpondent Points the Celerities are in the fame Ratio. But as in the fame Moments the Bodies are in thefe correfpondent Points, it follows that the Motion of both is deftroyed in the fame Moment, that is, *they finifh their Vibrations in the fame Time.* The Spaces, run thro' in the Time of one Vibration, are as the Forces by which they are run thro';
340 that is, *the Arcs of the whole Vibrations are as the Arcs defcribed by the Defcent*, whofe Double are the Arcs to be defcribed *in Vacuo. The De-*
341 *fects of the Arcs to be defcribed in Liquids, from the Arcs to be defcribed* in Vacuo, are the Differences of Quantities in the fame Ratio, and *are as the Arcs defcribed by the Defcent.*

342 Since there is the fame Proportion between thofe different Arcs, it follows, that *the Celerities, in the correfpondent Points of the Arcs defcribed, are every where as the Arcs defcribed by the Defcent*; for thefe correfpondent Points are alfo the correfpondent Points of the Arcs to be defcribed *in Vacuo*, in which we have demonftrated that this Proportion holds.

343 Now *let the Refiftance increafe in the Duplicate Ratio of the Velocity, and let the Pendulum perform*

unequal

unequal Vibration, the greatest will last the longest, because the Resistance increases more than in the Case *Numb.* 239.

Yet *the Celerities, supposing the Arcs not very* 344
unequal in the correspondent Points of the Arcs described, are every where nearly in the same Ratio, and indeed *in the Ratio of the Arcs described by the Descent.* If the Resistance was in the Ratio of the Celerity, this Proportion would obtain; * but now it is disturbed by reason of * 342
a greater Resistance in a greater Vibration, by which the Motion in this is more diminished. But it is more accelerated by two Causes, 1st, this greater Vibration lasts longer, * and the * 343
Body stays longer in a certain Space, than in the correspondent Space in a less Vibration, and is accelerated during a longer Time. 2dly, the Defect of the Arc described here, from an Arc to be described *in Vacuo*, is greater in Proportion, in a greater Vibration, because in this the Resistance differs more from the Resistance in a less Vibration than in *Numb.* 241. Therefore the correspondent Points, keeping the same Proportion, are more distant from the lowest Point in the greater than in the lesser Arc, as long as the Body descends in it: therefore in Proportion it has a greater Acceleration, because the Force which acts continually on the Body, is as its Distance from the lower Point; * therefore there is a Compensation, * 156
and the Proportion above-mentioned is restored. In the Ascent of the Body, the Duration of the Retardation concurs with the Resistance to disturb that Proportion; but now the correspondent Points are less distant from the lowest Point in the greater Arc (the same Proportion continuing) than in the lesser, and the Gravity in Proportion produces a less Retardation; and there-

therefore now (the Proportion continuing) the Difference of the Diſtance of the correſpondent Points from the loweſt Point is increaſed, ſo that a Compenſation is given from this alone.

The Reſiſtances which are as the Squares of the Celerities, and therefore every where in correſpondent Points, as the Squares of the Arcs deſcribed by the Deſcent, in which Ratio alſo the
* 329 Retardations are; * but, as each of them keep the ſame Proportion in correſponding Points, the Sums of them all will be in the ſame Proportion; that is, the whole Retardations, which are the Defects of the Arcs deſcribed in the Liquid from the Arcs to be deſcribed *in Vacuo*; or, what is the ſame, *the Differences betwen the Arcs deſcribed in the*
345 *Deſcent and the next Aſcent.* Therefore theſe Differences, *if the Vibrations are not very unequal, are nearly as the Squares of the Arcs deſcribed by the Deſcent.* Which is alſo confirmed by Experiments in greater Vibrations; for in theſe the Proportion of Reſiſtance, which we treat of here, ob-
* 324 tains. *

346 Fill the wooden Veſſel ABFCD (*Plate* XXII. *Fig.* 6.) 3 Foot long, 1 Foot wide, and 1 Foot high, with *Water*; hang up the Pendulum V *p* by a Hook V hanging over the Middle of the Veſſel. This Pendulum is made of an Iron Wire 7 or 8 Foot long, and a Leaden Ball *p* of the Diameter of an Inch and a Half; when the Pendulum is at reſt, the Ball is diſtant 3 Inches from the Bottom of the Veſſel. At P there is a greater Ball of Lead, of 3 Inches Diameter, joined to the Iron Wire, that the Ball P may be the leſs retarded in Water.

A-croſs the Top of the Veſſel, upon the Brim of it, may be moved a Board about 5 Inches high, to which muſt be applied the divided Braſs Rulers EG, EG, and the Indices M, M, for measuring the Angles described by the Pendulum in the Descent, or Ascent, by the Method given *Numb.* 170. *Page* 70.

Experiment.] Let the Rulers E G, E G, be so disposed, that the Ends G, G, may be over-against the Pendulum when it is at rest, and in such Manner that between their Ends there may be a Distance equal to the Diameter of the Wire to which the Bodies P *p* are fixed. Let one Index be applied to the 16th Division of the Ruler, and another to the 14th Division of the other Ruler; let the Pendulum fall from that Division, and it will rise almost to this. If, instead of these Divisions, you take 20 and 16 $\frac{7}{8}$, the Experiment will succeed in the same Manner, as also when you apply the Indices to the Divisions 24 and 19 $\frac{1}{2}$. Take care that the Water be perfectly at rest.

In this Experiment the Arcs described in the Descent are to one another as 4, 5, and 6, whose Squares are 26, 35, 26; the Difference of those Arcs from the Arcs described in the Ascent, are 2, 3 $\frac{1}{8}$, 4 $\frac{1}{2}$; which Numbers are to one another as the aforesaid Squares, as appears by multiplying them by 8.

A Body freely descending in a Liquid is accelerat- 347
ed by the respective Gravity of the Body which continually acts upon it; *yet not equally*, as in a Vacuum, * the Resistance of the Liquid occasions * 129
a Retardation, that is, a Diminution of Acceleration, which Diminution increases with the Velocity of the Body. *For there is a certain Velocity* 348
which is the greatest that a Body can acquire by falling; for *if* its Velocity be such, that *the Resistance arising from it becomes equal to the respective Weight of the Body*, its Motion can be no longer accelerated; for the Motion which is continually generated

nerated by the reſpective Gravity, will be deſtroyed by the Reſiſtance, and the Body forced to go on equably: *The Body continually comes nearer and nearer to this greateſt Celerity, but can never attain to it.*

When the Denſities of a Liquid and a Body are given, you have the reſpective Weight of the Body; and by the knowing the Diameter of the Body, you may find out from what Height a Body falling *in Vacuo*, can acquire ſuch a Velocity, that the Reſiſtance in a Liquid ſhall be e-
* 336 qual to that reſpective Weight, * which will be that greateſt Velocity above-mentioned.

If the Body be a Sphere, it is known that a Sphere is equal to a Cylinder of the ſame Diameter, whoſe Height is two third Parts of that Di-
350 ameter; which Height is to be increaſed in the Ratio in which the reſpective Weight of the Body exceeds the Weight of the Liquid, in order to have *the Height of the Cylinder of the Liquid, whoſe Weight is equal to the reſpective Weight of the Body*; but, *if you double this Height, you will have a Height, from which a Body, falling* in Vacuo, *acquires ſuch a Velocity as generates a Re-*
* 336 *ſiſtance equal to this reſpective Weight,* * *and which therefore is the greateſt Velocity which a Body can*
* 348 *acquire, falling in a Liquid from an infinite Height.**

349 Lead is eleven times heavier than *Water*, wherefore its reſpective Weight is to the Weight of *Water* as 10 to 1; therefore a leaden Ball, as it appears from what has been ſaid, cannot acquire a greater Velocity, falling in *Water*, than it would acquire in falling *in Vacuo*, from an Height of $13\frac{1}{3}$ of its Diameters.

351 *A Body lighter than a Liquid, and aſcending in it by the Action of the Liquid, is moved exactly by the ſame Laws as a heavier Body falling in the Liquid.* Where-ever you place the Body, it is ſuſtained by the

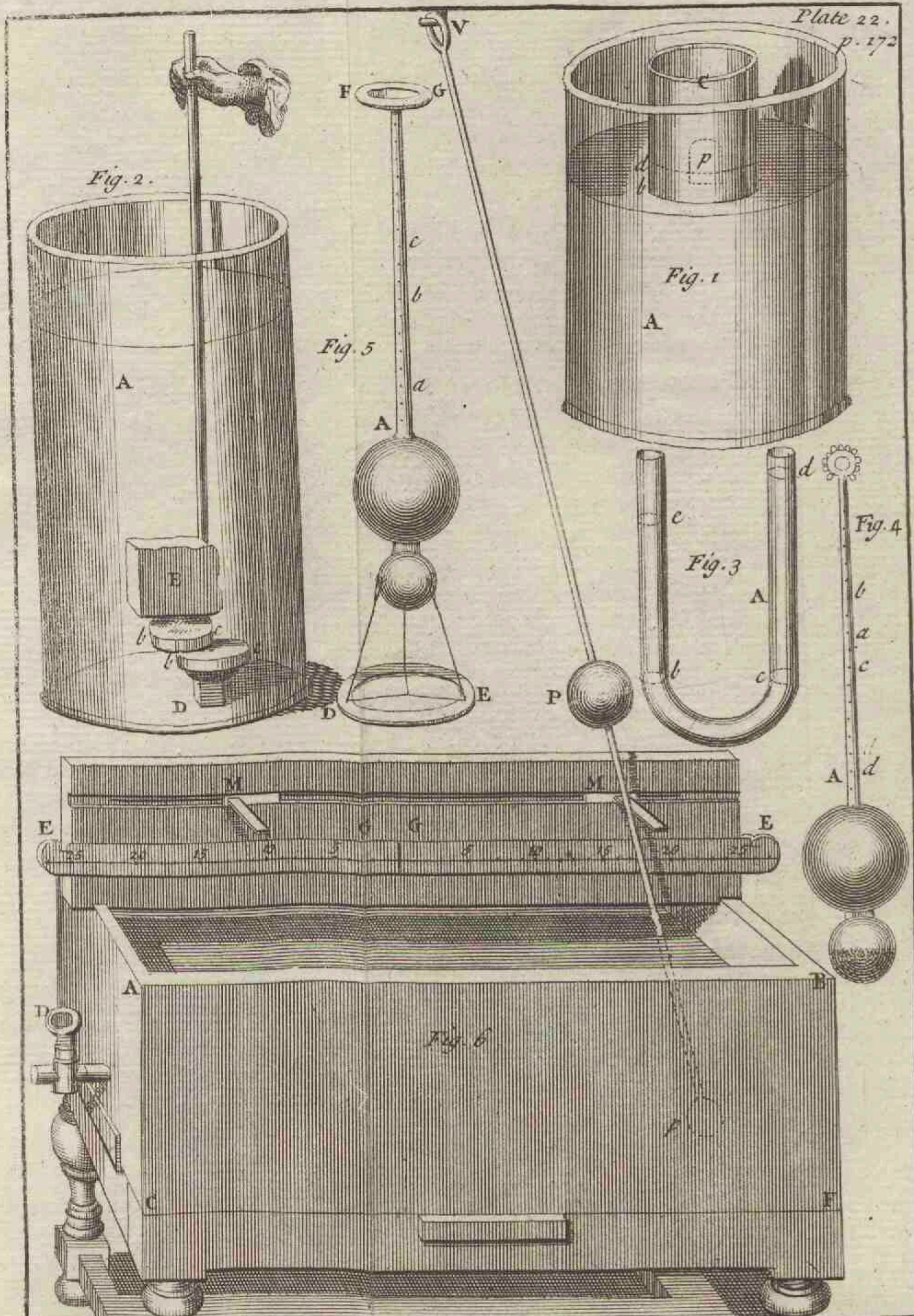


Plate 22.
p. 172
Fig. 1
Fig. 2.
Fig. 3
Fig. 4
Fig. 5
Fig. 6

the Liquid, and carried up with a Force equal to the Difference of the Weight of the Quantity of the Liquid, of the ſame Bulk as the Body, from the Weight of the Body, as appears by comparing *Numb.* 293 with *Numb.* 292; therefore you have the Force that continually acts equably upon the Body, by which, not only the Action of the Gravity of the Body is deſtroyed, ſo that it is not to be conſidered in this Caſe, but by which alſo the Body is carried upwards by a Motion equably accelerated, in the ſame Manner as a Body, heavier than a Liquid, deſcends by its reſpective Gravity; but the Equability of the Acceleration is deſtroyed in the ſame Manner by the Reſiſtance, in the Aſcent of a Body lighter than the Liquid, as it is deſtroyed in the Deſcent of a Body heavier than the Liquid.

When a Body, ſpecifically heavier than a Liquid, 352
is thrown up in it, it is retarded upon a double Account, on account of the Gravity of the Body, and on account of the Reſiſtance of the Liquid; therefore *a Body riſes to a leſs Height than* 353
it would riſe in Vacuo *with the ſame Celerity. But the Defects of the Height in a Liquid, from the Heights to which a Body would riſe* in Vacuo *with the ſame Celerities,* have greater Proportion to each other than the Heights themſelves; and, *in leſs Heights,* the Defects *are nearly as the Squares of the Heights* in Vacuo.

PART

Part II. *Of the Motion of Fluids.*

CHAP. VII.

Of the Celerity of a Fluid arising from the Pressure of the superincumbent Fluid.

AN inferior Fluid is pressed by a superior,
* 275 and that equally every Way; * because
Action is equal to Re-action, it endeavours to
recede every Way with equal Force; therefore *if
you take off the Pressure on one Side, the Liquid will
354 move towards that Side; and which Way soever the
Pressure be taken away, it will move with the same
Celerity*; which will be confirm'd by the Experiments to be mention'd in the following Chapter.

At the same Depth the Celerity is also every
where the same, by reason of the Equality of the
* 273 Pressure, * but when the Depth is changed, the
272 Celerity is also changed.

Yet the Velocity does not follow the same Proportion as the Depth; tho' the Pressure, from
which the Velocity arises, does increase in the
* 273 same Ratio as the Depth. * The Quantity of Motion, which is produced in the Liquid, is the Effect of the whole Pressure; and this Quantity
* 59 increases as the Pressure; * but the Ratio of the
Quantity of Motion is compounded of the Ratio
of the Velocity and the Quantity of the Matter
* 64 moved. * Here the Matter moved is the Water,
which goes out of the Hole, whose Quantity,
the Time remaining the same, increases with the
Celerity;

Celerity; it will be double, if the Celerity be doubled, in which Case the Quantity of Motion 355
is quadruple; that is, increased as *the Square of the Celerity*, which obtains in any Celerity: Therefore that Square *increases* as the Pressure; that is, *is the Height of the Liquid above the Hole from which the Water spouts.*

Plate XXIV. *Fig.* 1.] Fill with Water the Pa- 356
rallelopiped A B, which is 15 Inches long, and as wide, and 2 Foot high; it must be so placed that its Bottom may be raised about 8 Inches above the horizontal Bottom of a hollow Trough C D, whose Length is almost 4 Foot, and Breadth a Foot and a half, and Depth 5 or 6 Inches.

At E, near the Bottom of the Vessel A B, there is fix'd a Brass Tube horizontally, above half an Inch in Bore; the fore Part of it is shut by a Plate, in the Middle of which there is a Hole, whose Diameter is equal to $\frac{1}{12}$ Inch: that Hole is shut with a Cover that screws on upon the fore Part of the Tube.

The Celerities with which the Water flows out from E, when you have open'd the Hole, are compared together by Help of this Machine. Let it move, for Example, in the Line E L, and at L let it come to the Bottom of the Vessel C D; this Motion may be resolved into two Motions; the one horizontal along E I, in the Direction that the *Water* has in going out of the Hole, and the other vertical along I L; the first is equable, and the *Water*, with the Celerity with which it goes out, runs thro' the Space E I, in the same Time that in falling it runs thro' I L; * whatever the Celerity be, I L is not changed, because E I and the Bottom of the Vessel are horizontal, therefore the Time is not changed, in which several such Lines as E I may be run thro'

* 209

thro', and therefore they are as the Celerities with
* 53 which the Water goes out: * if you meaſure the Diſtance to which the Water ſpouts, you will have the Line E I.

Experiment 1.] Let there be Water in the Veſſel A B, up to the Height of five Inches above the Hole at E; let the Diſtance be meaſured to which the Water ſpouts; if more Water be poured in, to the Height of twenty Inches, it will ſpout to a double Diſtance. The Squares of the Diſtances are here as the Height of the Water, in which Ratio alſo are the Squares of the Celerities.

357 *The Velocity of a Liquid, at any Depth, is the ſame as that which a Body, falling from a Height equal to the Depth, would acquire*; for the Velocity of a Liquid increaſes, when the Depth of the Hole below the Surface of the Liquid increaſes, in the ſame Ratio as the Celerity of the falling Body increaſes, when the Space gone thro' by the Fall in-
* 355 creaſes *; and in the Beginning theſe Velocities
131 are equal; for in a Liquid the upper Parts, as
130 well as in a Body at the Beginning of the Fall, endeavour to deſcend by Gravity only.

Experiment 2. *Plate* XXIV. *Fig.* 1.] This is performed with the ſame Machine as was uſed in the former Experiment; the Veſſel A B is filled with Water, and a Tube with a Hole, like the Tube E, is placed at F; ſo that the Height of the upper Surface of the Water, above the Bottom of the Veſſel C D, is divided by that Hole into two equal Parts; the Water from that Hole will ſpout to M, ſo that the horizontal Diſtance from the Point M to the Hole will be double the Height of the Hole, above the Bottom of the Veſſel C D; therefore the Water, by an equable

equable Motion, and with the Celerity with which it goes out, runs thro' double the Space of that Height, in the Time in which a Body can fall from F to the Bottom of the Vessel C D, and therefore it moves with the Celerity which a Body can acquire in falling from that Height *; but this * 134
Height is equal to the Height of the Surface of the Water above the Hole.

CHAP. VIII.

Of spouting Liquids.

A *Liquid, spouting vertically out of a Hole,* a- 358
rises up with that Celerity, with which it *would come up to the upper Surface of the Liquid, yet it never comes up to that Height* *; and that for * 357
several Causes. 1. The Celerity, by which the Li- * 137
quid ascends, is diminished every Moment, and the Column of the spouting Liquid consists of Parts, which are moved to different Heights by different Celerities; all the Parts of a Column, which is every where of the same Thickness, are necessarily moved by the same Celerity; the said Column every where will be broader every Moment, as the Celerity of the Liquid is diminished; which arises from the Impulse of the Liquid following, and which from the Nature of a Liquid yields to every Impression, and is easily moved every Way; by that Impression the Motion is retarded every where. 2. This Motion is also diminished by the Liquid, because, when it hath lost all its Motion, it hangs in the upper Part of the Column, and is sustained for a Moment by the Liquid that follows, before it flows off on the Sides, which retards the Liquid that follows it, and that Retardation is communicated to the whole Column. 3. By the Friction against the

Sides of the Hole, the Celerity of the Liquid is diminiſhed; which Friction is increaſed, when the Liquid is brought through Pipes and Cocks. 4 Laſtly, the Air's Reſiſtance ſtops the Motion of Liquids.

The firſt Cauſe above-mentioned of the Retardation cannot be corrected.

The ſecond is corrected by ſomewhat inclining the Direction of the Liquid, as is ſelf-evident;
359 and this is the Reaſon why *a Liquid riſes higher, if its Direction be a little inclined, than if it ſpouts vertically.*

Experiment 1. *Plate* XXIV. *Fig.* 1.] To the
* 356 Machine above-deſcribed, * by help of a Screw at N, join the Curve Tube N O, from which the Water, thro' a ſmall Hole, ſpouts up vertically by turning the Tube a little, which is eaſily done by reaſon of the Screw at N, the Direction of the ſpouting Water will be inclined, and it will aſcend higher. But by this Inclination the Beauty of a Jet is deſtroyed.

As to the third Cauſe of the Retardation, 'tis to be obſerved, that there is a greater Friction, in Proportion, in ſmall Holes, than in great ones; the Celerity being increaſed, the Friction alſo
360 is thereby increaſed; and therefore *the Holes are to be increaſed according to the Heights of the ſpouting Water.*

The Ends of the Pipes, from which the Water ſpouts, have commonly the Figure of a truncated Cone, as is repreſented at P, (*Plate* XXIV. *Fig.* 3.) in which End the Water ſuffers a great deal of Friction, and is moved irregularly, and ſpouts up with that Irregularity. This may be a-
361 mended by *covering the End of the Tube with a flat, ſmooth, and poliſhed Plate, fixed to it, which has a Hole in it*; for then *the Water ſpouts higher, and* becauſe it

it rises with a Motion intirely regular, it *is perfectly transparent*.

Experiment 2. *Plate* XXIV. *Fig.* 3.] Take the Tube above-mentioned P, as also the Cylinder Q, shut up at one End with a bored Plate; let those be screwed on, one after another, to the End O of the Tube N O (*Fig.* 1.) the Water remaining at the same Height in the Vessel A B, let it spout from the Tube P, and the Cylinder Q, and the Experiments will fully confirm what we have said.

The Pipes which bring the Water from a Reser- 362
voir, must be very wide, in Proportion to the spouting Hole, that the Water may move slowly in these Pipes, and have no sensible Friction.

For the same Reason *the Water-way*, or Paf- 363
sage, *of the Cocks must be very large.*

Experiment 3. *Plate* XXIV. *Fig.* 1.] To the Vessel A B, at the same Height as the Tube F, fix the Cock H, the Pipe of the Cock must be shut up in the same Manner as the Tube F, and bored with a Hole of the same Size; the Water-way of this Cock is a Quarter of an Inch. The Water which goes thro' this Cock is brought thro' a narrower Space than that which moves thro' the Tube F; therefore this last is more transparent, and spouts to a greater Distance.

The Resistance of the Air has a sensible Effect upon 364
the Motion of Liquids; (for itself may be reckoned amongst Liquids, as will be said in the Second Part;) therefore we may here apply what has been said of the Ascent of any Body in a Liquid; *and in small Heights, the Defects of the Heights from the Heights* in Vacuo are in the Ratio of the Squares of those Heights *; that is, abstract- * 353
ing from the other Causes of Retardation, they

are *in the Ratio of the Square of the Height of the
Liquid above the Hole.* Beſides this Reſiſtance
there is alſo another, not to be over-looked, which
is the Action of the Air againſt the ſpouting Li-
quid. It encloſes the whole Column of the ſpout-
ing Liquid, and reſiſts that Part of its Motion,
whereby it ſpreads itſelf ſide-wiſe, as it becomes
wider, and there is required a greater Force of the
Liquid that comes after, than if this Reſiſtance
was taken away; therefore the Air reſiſts by its
lateral Preſſure. The Reſiſtance from the Stroke
of the Liquid againſt the Air encreaſes with the
impingent Surface, that is, if the Celerities remain
the ſame, encreaſes with the Hole; in which Ra-
tio alſo, the Quantity of the Matter moved en-
creaſes, and upon this Account it is no matter of
what Bigneſs the Hole is.

The lateral Preſſure follows the Proportion of
the Surface of the Column; the Matter moved,
which (the Celerity being the ſame) is in the ſame
* 62 Ratio as the Quantity of Motion, * follows the
Proportion of the whole Column, that is, of the
Square of its Surface; and therefore, if the Hole
be encreaſed, the Quantity of Motion encreaſes
faſter than the Cauſe retarding it; and for that
365 Reaſon *in the greateſt Heights of ſpouting Liquids*,
that the lateral Preſſure (which exerts a greater
Action when it acts the longer) may be the bet-
ter overcome, *greater Holes are required*; which
we have alſo ſhewed before to be required in the
* 360 ſame Caſe from another Cauſe: * where, as well as
here, we ſuppoſed the greater Holes only neceſſary
for the greateſt Heights, though the Demonſtrati-
ons prove that theſe Holes, which are very neceſ-
ſary in the greateſt Heights, are in general to be
preferred to others.

Great Holes alſo hinder the Motion; for
then there is a greater Surface which is preſſed

upon by the highest Part of the Liquid, which has lost all its Motion, and hangs on a longer Time, before it runs off down the Sides. From these two contrary Effects joined together, *in all* 366
Heights there is a certain Measure of the Hole, thro' which the Liquid will rise to the greatest Height possible. Yet one cannot give Rules to determine the Diameter of the Hole, because the Bigness of the Pipes of Conduct and their Inflexions require it different, so that there may be a Variation to Infinity.

In respect to the greatest Heights, it is to be 367
observed, that *the Bigness of the Hole, and also the Height to which the Liquid can ascend, have their Limits,* which they cannot exceed. Not only the Liquid which is directly against the Hole runs out, but, that there may be a constant Supply, the neighbouring Liquid continually comes towards the Hole with an oblique Motion, and in going out it spouts with a compound Motion, and the Motion of the Liquid, spouting vertically, is disturbed; the greater the Hole is, the greater is the Disturbance arising from that Cause, and in spouting Waters the Holes should never exceed an Inch and a Quarter. When the Celerity of the Liquid is too great, it strikes against the Air with so much Force, that it is dispersed into Drops; in which Case, by diminishing the Celerity, the Height to which the Liquor spouts will be increased, and there is a Height which is the greatest to which a Liquor can ascend, which Height in spouting Water scarce exceeds 100 Feet.

Liquids, which spout obliquely, are not retarded 368
from so many Causes, nor so much, as those that spout vertically. The second Cause of Retardation, above-mentioned, * has no Place here, and the Effect of * 358
the first is less. As for the rest, one may apply here what has been said of Solids oblique pro-

 jected

369 jected in *Chap.* XXIV. *Book* I. from *Numb.* 212. quite to the End of the Chapter. And *a Liquid may be considered as an innumerable Quantity of Solids, following one another*, and running the same Way. In the Motion of the Liquid, the Way gone thro' may be perceived by our Senses, and what has been said of Solids, obliquely projected, may be reduced to Experiments by the Help of Liquids; for doing which we must make use of Quickfilver, because of the great Specifick Gravity of this Liquid in respect of others: But these Experiments are to be made by a particular Machine contrived as follows.

370 *Plate* XXIII. *Fig.* 1.] The wooden Trough A, B, C, D, E, F, H, is four Foot and an half broad, eight or ten Inches long, and six or seven Inches high; the Bottom is made of a Board hollowed in half an Inch, to contain the Mercury the better.

In the End H, of the Side E, F, H, you have a Board H I six Inches wide, and two Foot high, which has in it a Slit *o t*. By this means you may fix to any Height upon the Board the wooden Parallelopiped *s*, which has a Screw fixed in its hinder Part.

The second Figure represents this Parallelopiped at S: There is fastened to it a cylindric Vessel of Box Wood, which has a Groove round it to receive two Brass Plates, one of which may be seen at *f e*; their Ends are joined together by the Screw G, so as to make the Box Vessel immoveable, till it is loosened by unscrewing *g*, which will allow it to move about its Axis.

In the Bottom of this Vessel there is a cylindric Cavity *a b*, a Quarter of an Inch Diameter; this communicates with a like Cavity *b c*, which terminates in the Middle of the greater Cavity *c d*, whose

whose Diameter is above half an Inch, that it may receive the truncated Cone of Box H, which is joined to the Cylinder I L, *Fig.* 3.

The truncated Cone H exactly fills the Cavity *c d*, and is held fast in it by help of the Screw R, that goes thro' the Brass Plate Q O, but so that this truncated Cone may turn upon its Axis.

In this Cone, as well as in the Cylinder I L, there is a Cavity *h i l*, of the same Diameter as the Cavity *b c*, and answering to it. This Cylinder I L has a Glass Tube N cemented to it.

The Tube is a Foot and a half long, one End of which is seen at N M (*Fig.* 5.) which is cemented also to the Box Cylinder L I, which is hollowed at *l i h*, with a round Hole in the Form of a Gnomon, or the Carpenter's Square; at *b c* the Cavity is greater, to receive the truncated Cone C D, that exactly fills it, and is moveable about its Axis by the Help of the Handle E A.

The Cavity *h i* answers to the Cavity *d e*, which communicates with *f g*; this Part of the Box has driven upon it an Iron Ferril B Q, in which is drilled a very small Hole g, which, when the Parts of the Machine are joined together, communicates with the Cavity of the Box P (*Fig.* 2.)

To prevent the Tube from breaking the Ends L L of the Box Cylinders (*Fig.* 3. and 5.) together with the Tube, are joined close to a streight and stiff Piece of Wood *m n* (*Fig.* 1.) whose lower End *m* has an Iron Plate fixed to it (as may be seen in *Fig.* 6.) whose End L P B Q is bent in the Figure of a double Gnomon; when the End L (*Fig.* 5.) of the Box Cylinder is applied to the End of the Piece M N I of *Fig.* 5. it fits to I of *Fig.* 6. and the Screw Q, applied to *o*, presses the Cylinder

B D of *Fig.* 5. and joins it firmly to the Cylinder L l.

All the Parts of the Machine may be ſeen joined together at *Fig.* 1. Quickſilver being poured into the Veſſel *p*, ſpouts out of the Hole *g*, *Fig.* 5. When the Mercury is at the ſame Height in the Box, and you do not vary the Inclination of the Piece *n m*, the Mercury ſpouts with the ſame Celerity in any Direction; but the Inclination of the Direction may be varied by moving the Handle *e a* (E A, in *Fig.* 5.) the Angle that the Direction, in which the Mercury goes out of the Hole, makes with the Horizon, may be meaſured by Help of the Quadrant *q*, along with the Index *f h* is moveable, which by its Weight is always kept in a vertical Poſition. This Quadrant may be ſeen in *Fig.* 7. with its Index F H: It has two Rings behind, to receive the Handle E A, *Fig.* 5. When this Handle is vertical, the Index hangs againſt the 45th Degree, and the Direction of the Motion of the Mercury, which ſpouts out then, makes a half Right Angle with the Horizon.

In *Fig.* 1. the Jets of Mercury in their ſeveral Directions are repreſented: They become the more viſible by help of a wooden Plane G painted black, along which the Mercury in its Motion does almoſt ſlide: Upon this Plane muſt be drawn the Ways which a Body (according to what is ſaid in *Numb.* 212) runs thro', when it moves with the ſame Celerity according to Directions which make different Angles with the Horizon. Alſo the Semicircle A L of *Plate* XV. *Fig.* 5. muſt be drawn upon this Plane, tho' it could not be repreſented in this Figure.

There are ſeveral other ſuch Planes, in which the ſame Things are drawn, but ſo as to repreſent the Ways of Projectiles, &c. according to different Celerities.

This

This Plane ſtands upright near the Middle of the Trough, and bears againſt the Side E F H, ſo as to move backwards and forwards according to the Length of the Trough.

The Celerity of the ſpouting Mercury is varied, as you change the Inclination of the Piece *n m*; and, by lowering the Veſſel *p*, the Hole through which the Mercury ſpouts, is ſet to the Direction of the Lines drawn on the Plane.

The Mercury will ſtop its ſpouting, when the Cavity *a b* (*Fig.* 2.) is ſtopped with the Pin D E of *Fig.* 4.

Experiment 4. *Plate* XXIII. *Fig.* 1.] The Parts of this Machine being joined and fixed together, as in the Manner above deſcribed, incline the Piece *n m*, till the Height to which the Mercury is to ſpout, when it aſcends to a Direction almoſt vertical, is nearly equal to the Diameter of the Semicircle deſcribed on the Plane G. Let the Veſſel P be fixed at ſuch a Height, and the Plane G be ſo placed, that the Axis of the Circumvolution of the Cylinder B D (*Fig.* 5.) ſhall anſwer to the loweſt Point of the Semicircle above-mentioned. Which Way ſoever the Inclination of the Direction of the Jet (that is, of the Projection) be, its Amplitude will always be the Quadruple of the Line B M in the Semicircle A B L (*Plate* XV. *Fig.* 5.) There is indeed a ſmall Difference, which chiefly ariſes from the Reſiſtance of the Air, and muſt be obſerved in the following Experiments. 371

Experiment 5.] The Machine being diſpoſed as in the foregoing Experiment, if the Mercury ſpouts in two Directions, and the Inclination of one of them exceeds a half Right-Angle as much as the other is under it, the Mercury will cut the Horizontal Line which is drawn from the lower 372

Point

Point of the Semicircle on the Plane G, juſt in the ſame Place in both Caſes.

373 *Experiment* 6.] Every thing being diſpoſed as before, if the Way for any Direction of Motion be drawn on the Plane, and the Index *f g* agrees with that Diviſion of the Quadrant which denotes that Inclination, the Mercury in its Motion will follow the Line drawn to repreſent its Way. If you draw the Ways for ſeveral Angles, by the Motion of the Handle *a e*, you will bring the Mercury to ſpout in Jets that go along theſe very Lines.

374 *Experiment* 7.] Let there be another Plane as G, in which all the Lines above-mentioned are drawn from another Celerity of the Mercurial Jet, and the Experiments will ſucceed in the ſame Manner.

By the ſame Method, as we do by a Semicircle determine the Diſtance to which Bodies obliquely projected will fall, one may find the Diſtance to which the Liquor coming out of the Side of a Veſſel ſpouts, when the Veſſel is ſet upon a Horizontal Plane; which Diſtance is different according to the different Height of the Hole, the upper Surface of the Liquid remaining the ſame.

375 *Plate* XXIV. *Fig.* 4.] *Let A B be the Height of a Veſſel filled with a Liquid*; ſuppoſe this Height cut into two equal Parts at C; with the Center C and Diſtance E A deſcribe a Semicircle; *let there be a Hole at E*; laſtly, draw E D perpendicular to A B, and terminated in the Circumference of the Semicircle at D. *Let the Liquid ſpout from E to F in the Horizontal Plane, and the Diſtance B F will be double the Perpendicular E D.*

Which

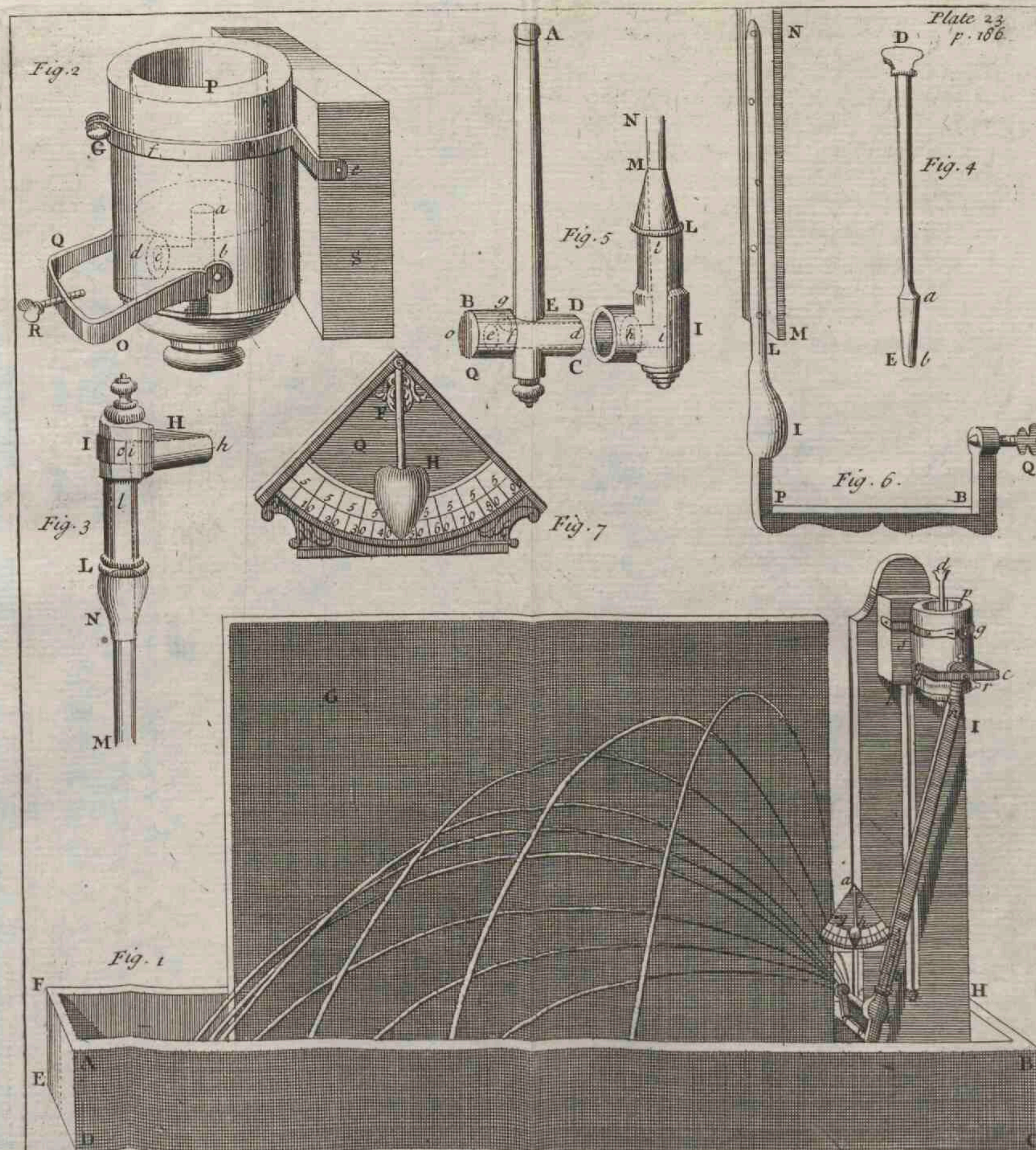


Plate 23
p. 186
Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6.
Fig. 7

Which will be demonstrated, if we consider, that the Liquid, with an equable Motion with the Celerity that it has coming out of the Hole, would (in the Time that a Body can fall from E to B) run thro' the Space B F *. * 209

In all Motion, the Time remaining the same,
the Space gone thro' is as the Celerity *; the * 53
Celerity remaining the same, it is as the Time: Therefore if you change the Time and the Celerity, the Space gone thro' will be in a Ratio compounded of the Celerity and the Time; and multiplying the Time by it, you will have the Space gone thro'; that is, if you apply this Operation to different Motions, you will have such Quantities as will exprefs the Proportion of the Spaces gone thro'. If you compute the Squares of the Celerities and the Time, you will have the Proportion of the Squares of the Spaces gone thro'. A E here expresses the Square of the Ce-
lerity; * E B the Square of the Time; * there- * 355
fore the Product of those Lines expresses the * 131
Square of the Space gone thro' E F. But that Product is the Square of the Line E D, which therefore, changing the Hole, increases and diminishes in the same Ratio as the Distance B F. Suppose the Hole in the Center C; B G, the Distance to which the Liquor spouts, is equal to
B A, * and it is double the Perpendicular, which * 357
from C may be drawn to A B in the Semicircle; 134
which therefore obtains in all Holes, and E D will be the Half of B F.

Hence it follows, that *a Liquid, spouting from* 376
a Hole in the Center C, *will go to the greatest Distance possible.*

Experiment 8. *Plate* XXIV. *Fig.* 1.] Here we must make use of the Machine, described in the
foregoing Chapter *. Let the Water spout from * 356

the Hole F, as in *Experiment* 2. *Chap.* VII. Let it ſpout at the ſame Time from E, and alſo from G, where there is a Tube like thoſe which are made faſt at F and E; the Hole G is leſs than F, but the Hole E is farther diſtant from the Surface of the Water, the Water comes from neither of them to the Diſtance to which it comes, when ſpouting from F.

Plate XXXIV. *Fig.* 4.] From what has been ſaid, it follows, That *the Water ſpouts to the ſame Diſtance from the Holes* E *e equally diſtant from the Center* C, becauſe in that Caſe the Perpendiculars E D and E *d* are equal.

377 *Experiment* 9. *Plate* XXIV. *Fig.* 1.] From F let there be drawn a horizontal Line which goes thro' H; if H G and H E are equal, the Water will go from each Hole G and E to L.

CHAP. IX.

Of a Liquid flowing out of Veſſels, and the Irregularities in that Motion.

378 THE Quantity of a Liquid, which *in a given Time* flows *from a given Hole*, increaſes in Proportion to the Velocity of the Liquid going out; this depends upon the Height of the Liquid above the Hole, and it is no matter to what
* 354 Part the Motion of the Liquor is directed; * therefore *the Squares of the Quantities flowing out are in the Ratio of the Heights of the Liquid above*
* 355 *the Holes**.

In the Time in which a Body falling freely goes through the Height of the Liquid above the Hole, a Column of the Liquid flows out
357 equal in Length to twice that Height; * the
* 134 Hole itſelf is the Baſe of the Column, and is given;

given; if the Height of the Liquid above the Hole is known, the whole Column is known; the Time also is easily determined by Experiments of Pendulums; * but having found what * 157 Quantity flows out in a known Time, one may know what Quantity will flow out in a given Time.

Here you must observe, that the Resistance of the Air, and the Friction of the Liquid against the Sides of the Hole, hinders the Motion of the Liquid, and that the Rule above-mentioned does not exactly obtain, and that there always flow out a less Quantity than what there is determined by it. Yet making Experiments with Water, it is plain, that the Quantities, which flow from the same Hole in equal Times, sensibly keep the Proportion of the Squares of the Heights of the Water above the Hole, in Heights not exceeding fifty Foot.

In Vessels which are not supplied by the flowing in of the Liquid, the Celerity of the Liquid flowing out is continually changed; to which Regard must be had, when you compare together the Times in which different Vessels are emptied.

Here we consider cylindric Vessels; and what is here said may be applied to any Vessels that are of the same Bigness from Top to Bottom; we suppose the Liquid to flow out from a Hole in the Bottom.

The Times in which cylindric Vessels of the same 379
Diameter and Height are emptied, the Liquid flowing from unequal Holes, are to each other inversly as those Holes.

If we suppose that these Vessels are divided into very small equal Parts, by Planes parallel to their Base; and that the Divisions of each Vessel do not differ from one another, when we consider the

the ſmalleſt Parts, one may conceive that the Celerity is not changed in the Evacuation of one Part. The Quantity of a Liquid which flows from a Hole, if the Celerity is not changed, increaſes with the Hole and with the Time; that is, in a Ratio compounded of the Time and of the Hole. The correſpondent Parts in the Veſſels are emptied with equal Celerities, and the aforeſaid compounded Ratio obtains here: The ſame Parts alſo, that is, the Quantities of the Liquid which flow out, are equal; wherefore the Difference of the Times is recompenſed by the Difference of the Holes; that is, the Times are in the ſame, but inverſe Ratio, as the Holes. Now as this happens in all the correſpondent Parts, it muſt alſo be referred to the Times of the whole Evacuations of the Veſſels.

380 *When the Veſſels are cylindric, unequal, and equally high, they are emptied thro' equal Holes, in Times that are to one another as the Baſes of the Cylinders.*

Let the Veſſels again be ſuppoſed to be divided into very ſmall Parts, and equal in Number in each Veſſel; the Liquid of the correſpondent Parts flows thro' equal Holes, and with equal Celerity: Therefore the Quantities that flow out are as the Times; and conſequently the correſpondent Parts themſelves are in that Ratio of the Times, which are as the Baſes of the Cylinders: But the Times of the whole Evacuations are as the Times in which the correſpondent Parts are evacuated.

381 Laſtly, *Let there be two cylindric Veſſels, whoſe Baſes are equal, but their Heights, for Example, as* 1 *to* 4, *and let them be evacuated thro' equal Holes.*

Let theſe alſo be conceived to be divided into very ſmall Parts, by Planes parallel to the Baſe; and let the Number in thoſe Parts be equal in each Veſſel; thoſe Parts will be to one another

as

as the Veſſels, that is, as 1 to 4. We can conſider every Part as evacuated by an equable Motion, becauſe the Parts are very ſmall; the Celerities in the correſpondent Parts are every where, as 1 to 2, *becauſe the Heights of thoſe Parts above the Baſes are as the Heights of the Veſſels, which are as the Squares of thoſe Numbers. Whence it follows, that the Times, in which correſpondent Parts are evacuated, are to one another, as 1 to 2; becauſe in twice the Time with a double Celerity, a quadruple Quantity is evacuated. But as the Times are in the ſame Ratio for each correſpondent Part, the Times in which the whole Veſſels are evacuated are alſo, as 1 to 2. If the Veſſels are, as 1 to 9, the Times will be, by a like Demonſtration, as 1 to 3; and generally the Times are as the Celerities in which correſpondent Parts are evacuatad, the Squares of whoſe Celerities are *as the Heights of the Veſſels*, * in which Ratio alſo are *the Squares of the Times.*

* 355
* 355

Experiment 1. *Plate* XXIV. *Fig.* 2.] Let there be three thin cylindric Veſſels of Metal A, C, B, having equal Diameters, and whoſe Heights are, as 1, 3, and 4; let each of them have a Lip in the Top to let the Water run out when it comes to a certain Height, which Lip muſt be reckoned the Top of the Veſſel; in the Bottoms of the Veſſels A and B, which are as 1 and 4, let there be equal Holes, and let them be filled with Water; let the Holes be opened in the ſame Moment; if the Water running out of B be received in the Veſſel C, it will be filled in the ſame Time that A is evacuated. C contains three Quarters of the Veſſel B; the Quarter which is left will alſo be evacuated in the ſame Time as the Veſſel A,

which

which is evident to Senſe; therefore A is emptied twice, whilſt B is emptied once.

382 *The Times in which any cylindric Veſſels are eva-*
*380 *cuated, are in a Ratio compounded of the Baſes,* *
*379 *of the inverſe Ratio of the Holes,* * *and of the ſquare*
*381 *Roots of the Heights.**

383 *The cylindric Veſſel may be ſo divided, that the Parts intercepted between the Diviſions ſhall be emptied in equal Times, which will happen, if the Diſtances of the Diviſions from the Baſe be as the Squares of the natural Numbers*; for the Times of the Evacuations of the Veſſels, whoſe Heights are
381 in that Proportion, are as the natural Numbers, and the Differences of the Times are equal.

The Time in which a cylindric Veſſel is emptied, is as the Celerity with which the Liquid be-
*381 gins to run out; * therefore the Celerity, while
355 the Liquid deſcends in the Veſſel, is diminiſhed in the ſame Ratio as the Time of the Evacuation of the Liquid remaining in the Veſſel, and *the*
384 *Motion of a Liquid running out of a cylindric Veſſel, is equally retarded in equal Times.*

385 *If thro' equal Holes a Liquid runs out of a Cylinder, and out of another Veſſel of the ſame Height (and in which the Liquid is always ſupplied ſo as to be kept at the ſame Height) in the Time in which the Cylinder is emptied, there runs out twice as much Water from the other Veſſel as from the Cylinder.* For, becauſe of the equal Height of the Veſſels, the Celerities in the Beginning are equal; the Celerity of the Liquid, which comes out of the Veſſel that is always kept full, is equable; the Celerity of the Liquid, which runs out of the Cylinder, is equably retard-
*384 ed. * Therefore, whilſt the Cylinder is emptying, there will flow twice as much Water out of this Veſſel as out of the Cylinder: For if two Bodies are driven with the ſame Celerity, and the firſt

goes

goes with an equable Motion, and the second with a Motion equally retarded, and they move till they have lost all that Motion, the first in that
Time will run double the Space of the second; * 135
* here the Liquor that runs out may be looked * 136
upon as the Space gone through, because the * 134
Holes are equal.

Besides the Irregularities from Friction, and the 386
Resistance of the Air, there are several others arising from the Cohesion of Parts, even in Liquors that are not glutinous. I shall here only speak of Water. We observe in Relation to it, that tho' it be driven by the same Force in any Direction,
* the Height of the Water above the Hole re- * 354
maining the same, yet it will descend the more swiftly in a vertical Direction; the Water in falling, is continually accelerated in its Motion, it coheres with the following, and accelerates that, and increases the Velocity of the Water flowing out of the Vessel.

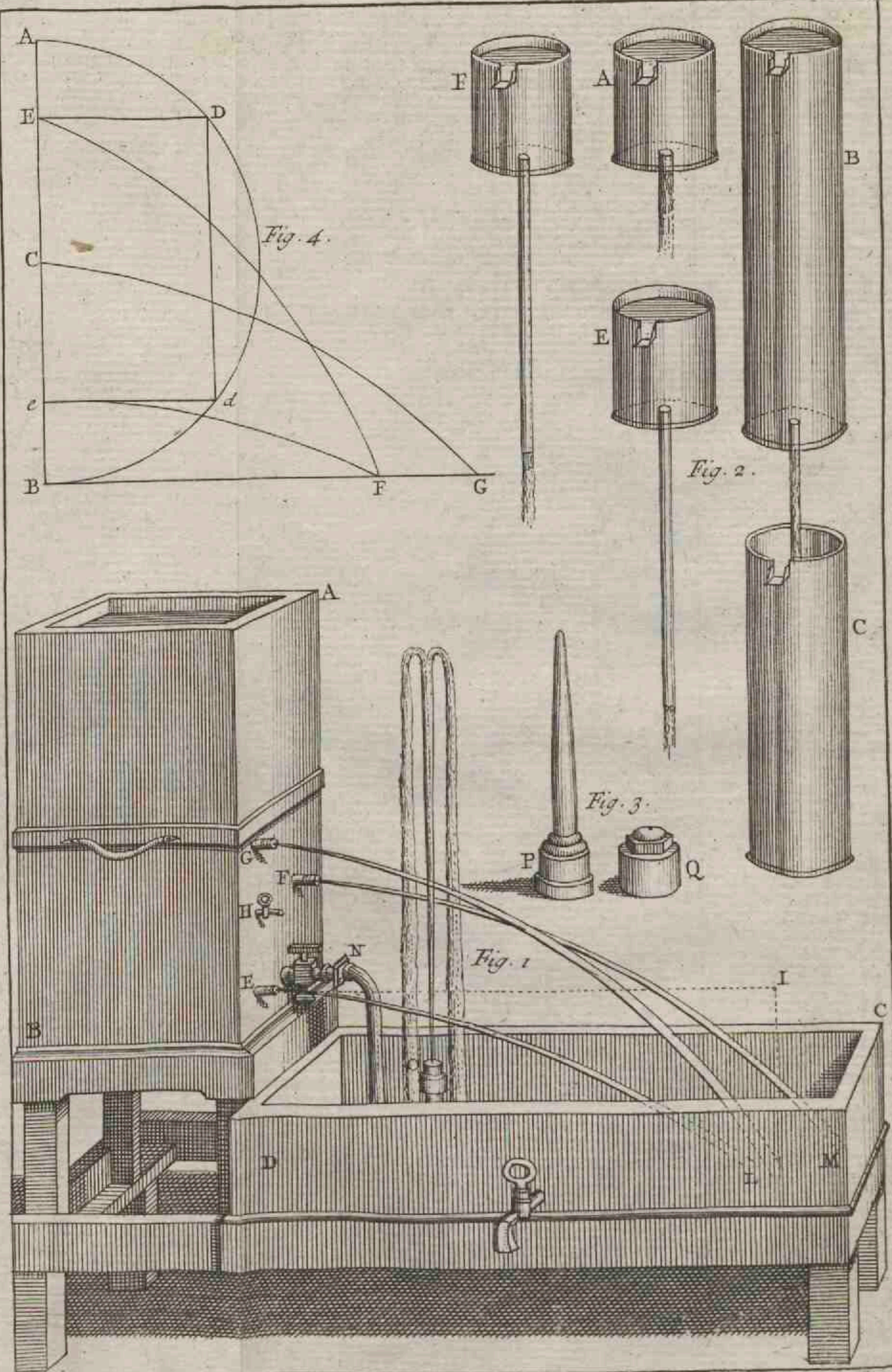
Plate XXIV. *Fig.* 2.] For this Reason *the Mo-* 387
tion out of a Vessel, that has a Tube fixed to its under Side, is also accelerated. Let E be such a Vessel equal and similar to the Vessel A, and which, together with the Tube, makes up the Height of the Vessel B; let the Tube have the Holes at both Ends, equal to the Holes at the Bottoms of the Vessels A and B, fill the Water in the Vessels A, E, and B. In the Beginning of the Motion, the Water flows from the Vessel E and B with equal Celerity, because the Heights of the Water above the Holes, from which the Water goes out, are equal; but the Celerity, in the Vessel E, is immediately diminished, because there cannot run a greater Quantity of Water out of the Tube than what comes in at the upper Hole of the Tube, into which Hole no more Water

can run in, than what can flow out at the Veſſel A. Since the Parts of the Water cohere, the Water, which runs out, accelerates that which runs into the Tube, and this laſt retards that which runs out; and ſo the Quantity of Water, which in a certain Time runs out of the Veſſel E, is a mean Quantity between the Quantities of Water that can run out at the ſame Time from the Veſſels A and B.

Experiment 2.] The Veſſels A E and B being made of ſome thin Metal, in the Proportions above-mentioned, fill with Water A and E; having opened the Holes at the ſame Inſtant of Time, the Water of the Surface at E will deſcend faſter than that at A: On the contrary, if you make uſe of the Veſſels E and B, it will deſcend faſter in the laſt than in the firſt.

388 *Let the upper Hole of the Tube, by which it communicates with the Veſſel, remain as before; and the lower Hole be opened wider; then a greater Quantity of Water will flow out*, and the Water which goes into the Tube will be more accelerated; this Hole may be made ſufficiently wider without altering the Length of the Tube, inſomuch that a greater Quantity of Water ſhall flow out from it than from the Veſſel B. In that Caſe thro' the upper Hole of the Tube, at a ſmall Depth below the Surface of the Water, there flows out a greater Quantity of Water, than from an equal Hole four Times the Depth. The ſame may be done by applying a longer Tube, without widening its lower Hole.

Experiment 3. *Plate* XXIV. *Fig.* 2.] Take the Veſſel F no Way different from the Veſſel E, but in having the lower Hole of its Tube bigger; take alſo the above-mentioned Veſſel B. The Diameters


A
E
D
Fig. 4.
C
e
d
B
F
G
F
A
B
E
Fig. 2.
C
A
Fig. 3.
P
Q
G
F
H
N
Fig. 1
E
I
C
B
D
L
M

Diameters of the Hole in the Bottom of this, and of the upper Hole of the Tube which is joined to F are of four Lines ($\frac{1}{42}$ of an Inch) the lower Hole of this Tube is of five Lines. Let the Veſſels be filled with Water; and let the Water begin to run out of both at the ſame Moment; the Surface of the Water in F will deſcend faſter than that of B. The Veſſel B is about 16 Inches high.

CHAP. X.

Of the Running of Rivers.

DEFINITION I.

THE *Water that runs by its own Gravity, in* 389
a Channel open above, as A E, *is called* a River (*Plate* XXV. *Fig.* 1.)

DEFINITION II.

A River is ſaid to remain in the ſame State, 390
or to be in a permanent State, *when it flows uniformly, ſo as to be always at the ſame Height in the ſame Place.*

DEFINITION III.

A Plane, which cutting a River is perpendicular 391
to the Bottom, as *p o n q*, *is called* the Section of a River.

When a River is terminated by flat Sides parallel to each other, and perpendicular to the Horizon, and the Bottom alſo is a Plane either horizontal or inclined, the Section of the River with theſe three Planes makes Right Angles, and is a Parallelogram.

In every River that is in a permanent State, the 392
ſame Quantity of Water flows in the ſame Time thro' every Section. For unleſs there be in every Place

as great a Supply of Water, as what runs from it, the River will not remain in the ſame State. And this Demonſtration will not hold good, whatever be the Irregularity of the Bed or Channel, from which, in another Reſpect, ſeveral Changes in the Motion of the River ariſe; as, for Example, a greater Friction in Proportion to the greater Inequality of the Channel.

The Irregularities in the Motion of a River may be infinitely varied, and Rules cannot be given to ſettle them: Therefore ſetting aſide all Irregularities, we muſt examine the Courſe of Rivers; for, unleſs the Laws of Motion be known in that Caſe, we have no certain Foundation for determining any Thing.

Therefore, we ſuppoſe the Water to run in a regular Channel, without any ſenſible Friction, and that the Channel is terminated with plane Sides, that are parallel to one another and vertical; and alſo that the Bottom is a Plane, and inclined to the Horizon.

Let A E be the Channel, into which the Water runs from a greater Receptacle or Head; and let the Water always remain in the ſame Height at the Head, ſo that the River may be in a permanent State. The Water deſcends along an inclined
* 144 Plane, and is accelerated; * whereby, becauſe
the ſame Quantity of Water flows through every
* 392 Section, * *The Height of the Water, as you re-*
393 *cede from the Head of the River, is continually*
diminiſhed, and the Surface of the Water will acquire the Figure *i q s*.

To determine the Velocity of the Water in different Places, let us ſuppoſe the Hollow of the Channel A D C B to be ſhut up with a Plane; if there be a Hole made in the Plane, the Water will ſpout the faſter through the Hole, as the Hole is more diſtant from the Surface of the

Water

Water bi; and the Water will have the ſame
Celerity that a Body, falling from the Surface of
the Water to the Depth of the Hole below it,
would aquire; * which ariſes from the Preſſure * 357
of the ſuperincumbent Water. There is the ſame
Preſſure, that is, the ſame moving Force, when
the Obſtacle at A C is taken away; then every
Particle of Water enters into the Channel, with
the Celerity that a Body would acquire in falling
from the Surface of the Water to the Depth of
that Particle. This Particle is moved along in
an inclined Plane in the Channel, with an acce-
lerated Motion; and that in the ſame Manner,
as if, in falling vertically, it had continued its Mo-
tion to the ſame Depth below the Surface of the
Water in the Head of the River.* So, if you * 150
draw the horizontal Line it, the Particle at r
will have the ſame Celerity as a Body falling the
Length i C, and running down Cr, can ac-
quire; which is the Celerity acquired by the
Body in falling down tr. Therefore the Cele-
rity of a Particle may be every where meaſured,
drawing from it a Perpendicular to the horizon-
tal Plane, which is conceived to run along the
Surface of the Water in the Head of the River;
and the Velocity, which a Body acquires in fal-
ling down that Perpendicular, will be the Cele-
rity of the Particle; which is greater, the longer
the Perpendicular is. From any Point, as r, draw
rs perpendicular to the Bottom of the River,
which will meaſure the Height or Depth of the
River. Since rs is inclined to the Horizon, if
from the ſeveral Points of that Line you draw
Perpendiculars to it, they will be the ſhorter,
the more diſtant they are from r, and the ſhort-
eſt of them all will be sv: Therefore the Cele-
rities of the Particles in the Line rs are ſo much
the leſs, the nearer they are to the Surface of the

394 River, *and the lower Water is moved faster than the upper Water.*

395 *But yet the Celerities of those Waters, as the River runs on, continually approach nearer and nearer to an Equality.* For the Squares of those Celerities are as *r t* to *s v*, the Difference of which Lines, as you recede from the Head of the River,
* 393 is continually lessened, because of the Height *r s*,* which is also continually diminished as the Lines themselves are lengthened. Now as this obtains in the Squares, it will much more obtain in the Celerities themselves, whose Difference therefore is diminished as they increase.

396 If the Inclination of the Bottom be changed at the Head of the River, so as to become *y z*, *and a greater Quantity of Water flows into the Channel, it will be higher every where in the River, but the Celerity of the Water is no where changed.* For this Celerity does not depend upon the Height of the Water in the River, but, as has been demonstrated, from the Distance of the moved Particle from the horizontal Plane of the Surface at the Head continued over the said Particle; which Distance is measured by the Perpendicular *r t* or *s v*; but these Lines are not changed by the Afflux of Water, provided that the Water remains at the same Height in the Bason or Head.

397 *Let the upper Part of the Channel be stopped up by an Obstacle, as* X, *which descends a little Way below the Surface of the Water*; the whole Water which comes cannot run through, therefore it must rise up: But the Celerity of the Water below this
* 396 Cataract is not increased; * and the Water that comes on is continually heaped up, so that at last it must rise so as to flow over the Obstacle or the Banks of the River. But *if the Banks be raised and the Obstacle be continued, the Height of the Water would rise above the Line i t*; but, before that,

that, the Celerity of the Water cannot be increaſed: In which Caſe the Height of all the Water in the Head will be increaſed; for as we ſuppoſe the River in a permanent State, there muſt continually be as great a Supply of Water to the Head, as there runs from it down the Channel; but, if leſs Water runs down, the Height muſt neceſſarily be increaſed in the Head, till the Celerity of the Water flowing under the Obſtacle be ſo much increaſed, that the ſame Quantity of Water ſhall run under the Obſtacle, as uſed to run in the open Channel before.

All theſe Things, as we have already ſaid, if we abſtract from all the Irregularities, are true; and, the leſs the Irregularities are, the more will the true Motions agree with what we have ſaid: concerning which, before we can make any Judgment, we muſt be able to compare the Velocities of Water by Experiments, and ſo determine the Velocities themſelves, as to know the Spaces gone through in a certain Time.

Plate XXV. *Fig.* 2.] Let ACB be a Quadrant divided into Degrees, * with a Thread in * 398
the Center, that has at the other End a Ball P hanging, which is heavier than the Water.

Let the Ball hang within the running Water, 399
whilſt you hold the Side CA of the Quadrant in a vertical Poſition; the Ball by the Motion of the Water will be ſo far ſuſtained, that the Thread PC will make the Angle PCA, with the Side CA, which will ſerve to determine the Celerity of the Water running againſt the Ball.

The Ball, being at reſt in the Water, is drawn by three Powers; by its Gravity it endeavours to deſcend vertically; by the Action of the Liquid it is carried in the Direction of the Motion of the Water; and laſtly, it is drawn by the Thread

along P C. Draw the Triangle E F G, in which
E F repreſents the vertical Line; let F G make
with that Line the Angle E F G, equal to the
Angle which the Direction of the Motion of the
River makes with the vertical Line; laſtly, let the
Angle G E F be equal to the Angle P C A. The
Sides of the Triangle E F G are parallel to the Di-
rections of the three Powers above-mentioned;
therefore the Powers are to one another, as thoſe
* 196 Sides *. If therefore E F expreſſes the reſpective
Gravity of the Ball, F G will expreſs the Action of
the Water on the Ball. If you make ſeveral Expe-
riments in different Places with the ſame Ball, you
muſt draw ſuch Triangles, the Side F remaining,
(which denotes the reſpective Gravity of the
Ball that never changes) the Sides that are as F G,
will have the ſame Proportion as the Actions
of the Water on the Ball. But theſe are as the
Squares of the Velocities of the Waters in the
* 324 Places in which the Experiments are made *; for
there is no Difference in reſpect of the Action of
the Water on the Ball, whether the Ball be mo-
ved and the Water at reſt, or, on the contrary,
the Water be moved and the Ball at reſt.

The Action of the Water againſt the Ball may
be compared with the Weight, for it is to the
reſpective Gravity of the Ball, as F G to E F.

400 But this Action is equal to the Reſiſtance
which a Body ſuffers, when it is moved through
quieſcent Water with the ſame Celerity with
which the flowing Water does now ſtrike againſt
the Body which is at reſt: By knowing the
Weight, which is equal to the Reſiſtance, we
know what Space could be run through in a gi-
ven Time, with the Celerity with which the
* 336 Body moves; * therefore we ſhall alſo here
know what Space the Water can go through in a
known Time, and ſo likewiſe what Quantity of
Water

Water flows in a given Time through a Place given in the Section of the River.

Here it is to be obſerved, that the Determination of the Velocity of the Water will not be exactly ſettled, if the Experiment he made towards the Surface of the Water, becauſe there the Action of the Water upon the Globe is irregular. * * 328

This Celerity may be determined by immer- 401
ging in Water a Body which is but a little lighter than Water, and which, ſwimming at the Surface, does not float ſo high above it, as to be affected by the Motion of the Wind; as the ſpecifick Gravities of the Water and the Body ſcarce differ at all, and this Body may be looked upon as wholly immerſed, it will move with the ſame Celerity as the Water; and you may, by Help of a Pendulum, meaſure the Time in which a Body runs through a certain Space that was meaſured before. When the Surface of the Water is agitated by the Wind, the Experiment will not ſucceed well, becauſe of the Motion of the Waves, which cauſe an Irregularity in the Motion of the Body.

CHAP. II.

Of the Motion of the Waves.

THE Surface of the ſtagnant Water is plain, 402
and parallel to the Horizon *; if it be- * 272
comes hollow at A (*Plate* XXV. *Fig.* 3.) upon any Account whatever, this Cavity is ſurrounded with the Elevation B B; this raiſed Water deſcends by its Gravity, and, with the Celerity acquired in deſcending, it forms a new Cavity, by which Motions the Water aſcends at the Sides of

of this Cavity, and fills the Cavity A, whilſt there is a new Elevation towards C; and, when this laſt is depreſſed, the Water riſes anew towards the ſame Part; whence there ariſes a Motion in the Surface of the Water, and a Cavity, which carries an Elevation before it, is moved from A towards C.

DEFINITION I.

403 *This Cavity, with the Elevation next to it, is called* a Wave.

DEFINITION II.

404 The Breadth of a Wave *is the Space taken up by a Wave in the Surface of the Water, and meaſured according to the Direction of the Wave's Motion.*

The Cavity, as A, is encompaſſed every Way with an Elevation, and the Motion above-mentioned expands itſelf every Way; therefore the
405 *Waves are moved circularly.*

Plate XXV. *Fig.* 4.] Let AB be an Obſtacle, againſt which the Wave, whoſe Beginning is at C, does run; we muſt examine, what Change the Wave ſuffers in any Point, as E, when it is come to the Obſtacle in that Point. In all Places through which the Wave runs, whilſt it goes forward its whole Breadth, the Wave is raiſed; then a Cavity is formed, which is again filled up, which Change while the Surface of the Water undergoes, its Particles go and come through a ſmall Space. The Direction of this Motion is along CE, and the Celerity may be repreſented by that Line; let this Motion be conceived to be reſolved in two other Motions along GE and DE, whoſe Celerities are reſpectively re-
* 192 preſented by thoſe Lines *. By the Motion along

long D E the Particles do not act againſt the Obſtacle, and after the Stroke continue the Motion in that Direction, with the ſame Celerity; and this Motion is here repreſented by E F, ſuppoſing E F and E D to be equal to one another, by the Motion along G E, the Particles ſtrike directly againſt the Obſtacle, and this Motion is deſtroyed; * for though theſe Particles are elaſtick, * 168
yet, as in the Motion of the Waves they run thro' but in a ſmall Space, going backward and forward, they are moved ſo ſlowly, that the Figure of the Particles cannot be changed by the Blow, and ſo they are ſubject to the Laws of Bodies perfectly hard. But there is a Reflection of the Particles from another Cauſe; the Water which cannot go forward beyond the Obſtacle, and is puſhed on by that which follows it, yields that Way where there is the leaſt Reſiſtance, that is, aſcends: And this Elevation greater than in other Places is cauſed by the Motion along G E, becauſe 'tis by that Motion alone that the Particles come againſt the Obſtacle. The Water, by its Deſcent, acquires the ſame Velocity with which it was raiſed; and the Particles of Water are repelled from the Obſtacle with the ſame Force in the Direction E G, as that with which they came againſt the Obſtacle. From this Motion and the Motion above-mentioned along E F, ariſes a Motion along E H, whoſe Celerity is expreſſed by the Line E H, which is equal to the Line C E; and by the Reflexion the Celerity of the Wave is not changed, but it returns along E H in the ſame Manner, as if, taking away the Obſtacle, it had moved along E *h*. If from the Point C, C D be drawn perpendicular to the Obſtacle, and then produced, ſo that D *c* ſhall be equal to C D, the Line H E continued will

go through *c*; and, as this Demonſtration holds
good in all the Points of the Obſtacle, it follows
406 that *the reflected Wave has the ſame Figure on that
Side of the Obſtacle, as it would have had beyond
the Line* A B, *if it had not run againſt the Obſta-*
407 *cle. If the Obſtacle be inclined to the Horizon,*
the Water riſes and deſcends upon it, and ſuffers
a Friction, whereby *the Reflexion of the Wave is
diſturbed, and often wholly deſtroyed.* This is the
Reaſon why very often the Banks of Rivers do
not reflect the Waves.

408 *When there is a Hole, as* I, *in an Obſtacle, as*
B L, the Part of the Wave, which goes through
the Hole, continues its Motion directly, and ex-
pands itſelf towards Q Q, and *there is a new
Wave formed, which moves in a Semicircle, whoſe
Center is the Hole.* For the raiſed Part of the Wave,
which firſt goes through the Hole, immediately
flows down a little at the Sides, and then by de-
ſcending, makes a Cavity, which is ſurrounded
with an Elevation on every Part beyond the Hole,
which moves every Way in the ſame Manner, as
was ſaid concerning the Generation of the firſt
* 402 Wave. *

409 In the ſame Manner a Wave, to which an Ob-
ſtacle, as A O, is oppoſed, continues to move
between O N; but expands itſelf towards R in
a Part of a Circle, whoſe Center is not very far
from O.

10 Hence we may eaſily deduce what muſt be the
Motion of a Wave behind an Obſtacle, as M N.

411 *Waves are often produced by the Motion of a tre-
mulous Body, which alſo expand themſelves circular-
ly*, tho' the Body goes and comes in a Right Line;
for the Water, which is raiſed by the Agitation,
deſcending, forms a Cavity, which is every where
ſurrounded with a Riſing.

Different

Different Waves do not disturb one another, when 412
they move according to different Directions. The Reason of which Effect is, that whatever Figure the Surface of the Water has acquired by the Motion of the Waves, there may in that be an Elevation and a Depression, as also such a Motion as is required in the Motion of a Wave.

Whoever has attentively considered the Motion of the Waves, will find that all these Things agree with Experiments.

To determine the Celerity of the Waves, ano- 413
ther Motion, analogous to their Motion, is to be examined. Let there be a Liquid *in the recurve cylindric Tube* E H (*Plate* XXV. *Fig.* 5.) and let the Liquid in the Leg E F be higher than in the other Leg by the Distance *l* E; which Difference is to be divided into equal Parts at *i*. The Liquid by its Gravity descends in the Leg E F, whilst it ascends equally in the Leg E H; and so, when the Surface of the Liquid is come to *i*, it is at the same Height in both Legs, and that is the only Position in which the Liquid can be at rest: But, by the Celerity acquired by descending, it continues its Motion, and ascends higher in the Tube G H, and in E F it is depressed quite to *l*, except so much as it is hindered by the Friction against the Sides of the Tube. The Lquid in the Tube G H, which is higher, also descends by its Gravity; and so the Liquid in the Tube rises and falls, till it has lost all its Motion by the Friction.

The Quantity of the Matter to be moved is the whole Liquid in the Tube; the moving Force is the Weight of the Pillar *l* E, whose Height is always double the Distance E *i*; which Distance therefore increases and diminishes in the same Ratio with the moving Force. But the Distance E *i* is the Space to be run through by the Liquid, that

that from the Position E H it may come to the Position of Rest; which Space therefore is always as the Force which continually acts upon the Liquid: But we have demonstrated that it is upon this Account, that all the Vibrations of a Pendulum, oscillating into a Cycloid, are perform-
* 156 ed in the same Time; and * therefore here also, *whatever be the Inequality of the Agitations, the Liquid always goes, or comes, in the same Time.*

414 *The Time in which a Liquid, thus agitated, ascends, or descends, is the Time in which a Pendulum vibrates, whose Length*, that is, the Distance between the Center of Oscillation and Suspension, *is equal to Half the Length of the Liquid in the Tube*, or to Half the Sum of the Lines E F, F G, G H. This Length is to be measured in the Axis of the Tube.

Plate XXV. *Fig.* 6.] Let such a Pendulum vibrate in a Cycloid, in the Manner explained above (*Page* 62 *and* 63.) Let the Pendulum B C and the Arc A D be of the same Length; for the Arc C A is equal to the Arc A D, and the Thread, by which the Pendulum is suspended, applies to it, when the Body suspended is at A; in that Point the Direction of the Curve is perpendicular to the Horizon, and the Body endeavours to descend with all its Weight along the Curve: But this Weight is to the Force acting upon the Body, when it is at P, as A D,
* 156 or P C to P D. * Now let the Liquid be in such a Position, that *i* E (*Fig.* 5.) be equal to P, D; the Weight of the whole Matter to be moved, that is, of the whole Liquid, is to the Weight *l* E (which is the Force acting upon the Liquid in that Position) as the Length of the Liquid in the Tube to the Line *l* E, in which Ratio also the Halves of those

Quantities are, that is, P C to P D (*Fig.* 6.) Therefore in the Pendulum the Weight of the Matter to be moved is to the Force acting upon it, at P, as in the Tube, the Weight of the Matter to be moved is to the Force acting upon it in the Poſition E H. Therefore the pendulous Body and the Liquid, in this Caſe, are acted upon by equal Forces, and this always obtains where the Spaces to be run through by the Liquid in Agitation, and by a Body in Vibration, are equal; therefore in this Caſe the Agitation and the Vibration are performed in the ſame
Time, and not only in this Caſe, but always. * * 413
But, as the ſmall Vibrations in a Circle do not differ from the Vibrations in a Cycloid, the Demonſtration will agree with them.

Experiment.] Take a cylindric recurve Tube as E F G H; let the Length of the Legs be one Foot, and the Bore of the Cylinder half an Inch; pour Mercury into this Tube, and having made a Pendulum, whoſe Length is equal to Half the Length of the Cylinder of Mercury in the Tube; if the Mercury be agitated in the Tube, it will aſcend and deſcend in the ſame Time as the Pendulum will go and come.

Plate XXV. *Fig.* 7.] To determine the Celerity of the Waves from what has been ſaid, we muſt conſider ſeveral equal Waves that follow one another immediately, as A, B, C, D, E, F, which move from A towards F, the Wave A runs its Breadth, when the Cavity A is come to C; which cannot be, unleſs the Water at C aſcends to the Height of the Top of the Waves, and again deſcends to the Depth C; in which Moment the Water is not agitated ſenſibly below the Line *b i*; therefore this Motion agrees with the Motion

tion in the Tube above-mentioned, and the Water ascends and descends; that is, the Wave goes through its Breadth, whilst a Pendulum of the Length of half B C performs two Oscillations,
* 414* or whilst a Pendulum of the Length B C D, that is four times as long as the first, performs
* 158 one Vibration *.

Therefore the Celerity of a Wave depends upon the Length of a Line B, C, D, which is greater, according as the Breadth of the Waves is greater, and as the Water descends deeper in the Motion of the Waves.

415 In the broadest Waves, which do not rise high, such a Line as B C D does not much differ from the Breadth of the Wave; and in that Case *a Wave runs through its Breadth, whilst a Pendulum, equal to that Breadth, oscillates once.* In every equable Motion, the Space gone through increases with the Time and the Celerity; wherefore multiplying the Time by the Celerity, you have the Space gone thro'; whence it follows, that the *Celerities*
416 *of the Waves are as the Square Roots of their Breadths*: For as the Times in which they go
* 158 through their Breadths are in that Ratio, * the
415 same Ratio is required in their Celerities, that the Products of the Times by their Celerities may be as the Breadth of the Waves, which are the Spaces gone through.

All these Things must be only looked upon as nearly true, because the Motion of the Waves differs something from the Motion in the Tube; which Error is in part taken off, because the Length of the Pendulum is measured along the inclined Lines B C and C D.

PART

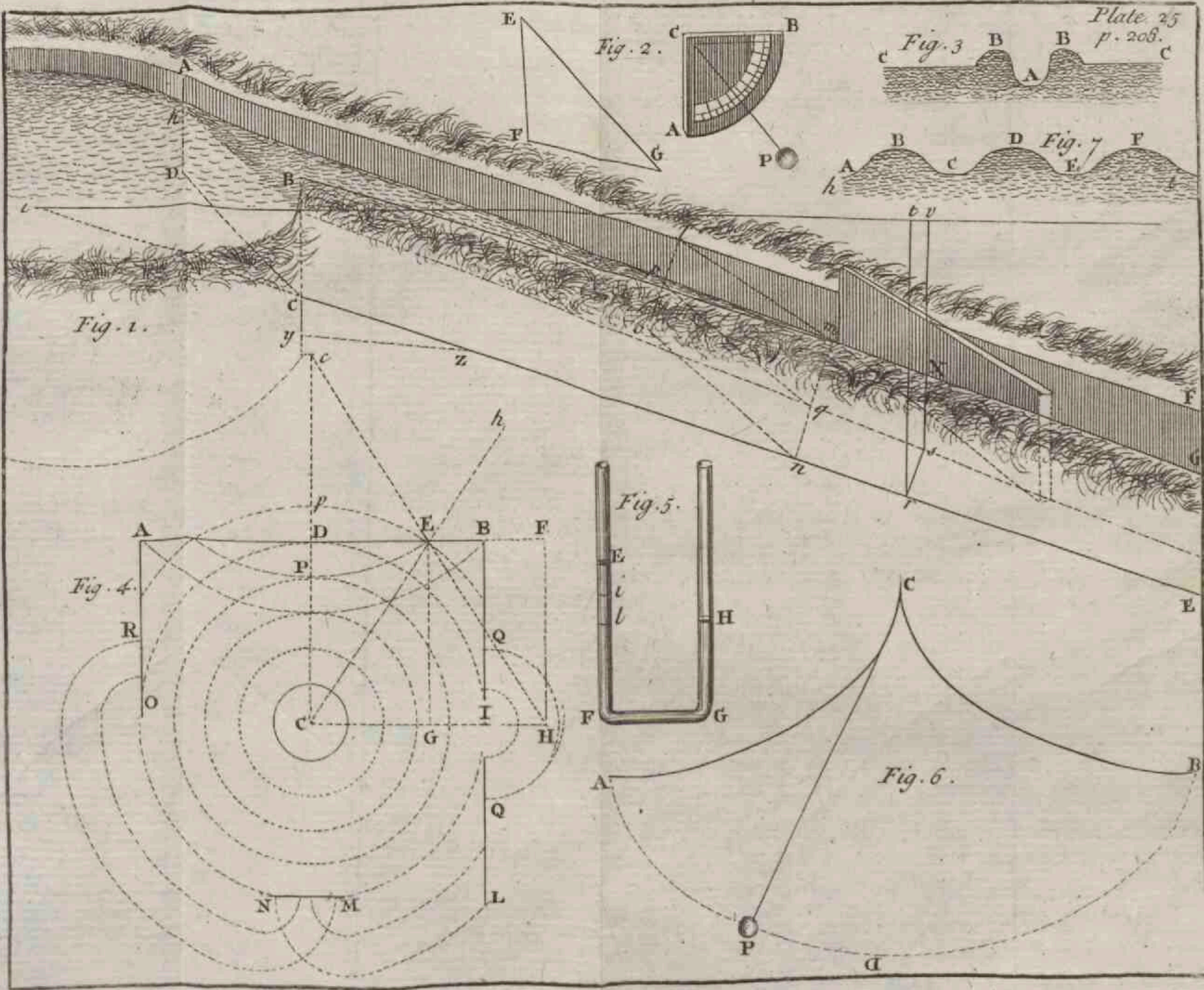


Plate 25
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Fig. 1.
Fig. 2.
Fig. 3
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7

PART III. *Of the Air, as an Elastic Fluid.*

CHAP. XII.

That Air has the Properties of Fluids.

WE have often spoken of the Air; and as we live in, and are always encompassed by it, we must have Regard to its Effect in several Experiments, as we have said in other Parts of this Treatise: But now we shall consider its Properties singly.

417 *The Air* is corporeal, heavy, its Parts yield to any Force impressed, and are very easily moved one amongst another; it presses in Proportion to its Height, and the Pressure every Way is equal: It is plain therefore, that it *ought to be reckoned amongst Fluids.*

DEFINITION I.

418 *All the Air which the Earth is encompassed with, considered together, is called* the Atmosphere of the Earth, *or simply*, the Atmosphere.

DEFINITION II.

419 *The Height of the Air above the Surface of the Earth, is called* the Height of the Atmosphere.

420 *That the Air is a Body*, appears from its excluding all other Bodies from the Place where it is. *

* 14

421 *That it yields to any Impression, and has its Parts easily moved,* is not doubted by any one.

422 *That it is heavy,* is proved by its pressing upon the Surface of other Fluids, and sustaining them in Tubes.

Experiment 1. *Plate* XXVI. *Fig.* 1.] Take a Glass Tube A B, about three Foot long, of about $\frac{1}{4}$ Inch Bore; if you stop up the End A, and let the Tube be filled with Mercury, and let the other End be immersed in a Vessel full of Mercury, the Mercury will be sustained at the Height of about 29 Inches. This is occasioned by the Pressure of the Air on the Surface of the Mercury in the Vessel, which cannot press equally in every Part of it, unless in the Tube where no Air is, there be a Column of Mercury, which presses
* 274 equally with outward Air*.

Experiment 2. *Plate* XXVI. *Fig.* 1.] That this Pressure may not be changed when the Tube is inclined, it is required that the Mercury should
* 277 keep the same perpendicular Height*: If therefore there be two Vessels containing Mercury, in which Tubes in the Manner above-mentioned are immersed, of which E D is inclined to the Horizon, the Mercury is sustained at the Heights *hf* and *ig*, so that *f* and *g* are in the same Horizontal Lines; supposing the Surfaces of the Mercury in the Vessels to lie in the same Plane.

Experiment 3. *Plate* XXVI. *Fig.* 2.] The same Pressure of Air sustains the Water of the Glass U, which is yet immerged in Water and filled with it, and then is pulled out all but the Orifice, which still remains immersed.

423 Water would be sustained in the same Manner, tho' the Height should be 32 Foot; for Quick-

silver

ſilver 14 times heavier than Water, and a Pillar of Water a little more than 32 Foot high preſſes equally with a Column of Mercury 29 Inches high, which Preſſure is equal to the Preſſure of the Atmoſphere.

That the Preſſure of the Air depends upon its 424
Height, may be eaſily deduced from what has been ſaid; but it is immediately proved by carrying the Tube with the Mercury above-mentioned to a higher Place; for, when you carry this Machine up a Hill, for 100 Foot that you riſe perpendicularly, the Mercury deſcends a Quarter of an Inch.

That Air preſſes equally every way, appears from 425
this, that the Preſſure is ſuſtained by ſoft Bodies without any Change of Figure, and brittle Bodies without their breaking, though this Preſſure be equal to the Preſſure of a Pillar of Mercury 29 Inches high, or a Height of Water of 32 Foot *; any Body may ſee that nothing can pre- * 423
ſerve theſe Bodies unchanged, but the equal Preſſure on all Parts; but it is plain that the Air does preſs in that Manner. * If you take away the Air * 291
on one Side, the Preſſure is ſenſible on the oppoſite Side.

Exper. 4. *Plate* XXVI. *Fig.* 3.] Hang a Glaſs Tube to one of the Scales of a Balance A B, which is ſhut at D, and 3 Foot long; fill this Tube with Mercury, and let the End E be immerſed in the Mercury that is contained in the Veſſel U. The Mercury by the Air's Preſſure is ſuſtained at the Height *f* in the Tube, and the upper Part of the Tube *f* D is left void of Air; to make an Æquilibrium, you muſt put into the oppoſite Scale a Weight equal to the Weight of the Tube and the Mercury contained in it. The Mercury in the Tube cannot preſs the Balance;

for its Action againſt the Sides of the Tube is horizontal; but the Air acts upon the upper Part of the Tube, and the Column that is ſuſtained by the Tube is æquiponderate with the Column of Mercury that is contained in the Tube: If letting the Mercury run out, you ſuffer the Air to come in, then nothing but the Tube weighs down the Scale; which proves, that the Action againſt the inferior Surface of the upper Part of the Tube deſtroys the Action on the exterior Surface, and that the Air preſſes upwards and downwards with the ſame Force.

By this Experiment alſo is confirmed what has been ſaid of the Air's Gravity.

CHAP. XIII.

Of the Air's Elaſticity, or Spring.

WE have ſhewn, the Air has the Properties of other Liquids; but beſides, it has another Property, which is, that it can take up a greater or leſſer Space, according as it is compreſſed with a different Force; and, as ſoon as that Force is diminiſhed, it expands itſelf.

426 By reaſon of the Analogy of this Effect with the Elaſticity of Bodies, this Property of the Air is called its *Elaſticity*.

427 That *the Air may be compreſſed*, appears from an
* 14 Experiment already mentioned. *

428 That *it may be dilated*, may appear from the following

Experiment 1. *Plate* XXVI. *Fig.* 4.] Take the Tube A B cloſe at the End A, and pour Mercury into it, ſo that there may be ſome Air left in the Tube, which, when in the State of the external

ternal Air, will take up the Space A *l*; if the End B of the Tube be immersed into Mercury in a Vessel, the Mercury in the Tube will descend to *g*, and there remain. The Height *i g* differs very much from the Height of the Mercury in the first Experiment of the foregoing Chapter, which does not arise from the Weight of the Air in the Tube; for its Weight is too little to produce any sensible Difference in the Height of the Mercury: The Expansion of the Air causes this Effect.

From this Experiment we deduce this Rule, 429
That *the Air dilates itself in such a Manner, that the Space taken up by it is always inversely as the Force by which it is compressed.*

The Force, by which the common or external Air is compressed, is the Weight of the whole Atmosphere, which is equal to a Pillar of Mercury of the Height *h f*, *Fig.* 1. therefore the compressing Force may be expressed by that Height; the Space taken up by the Air in the Tube, when it is compressed with such a Force, is A *l*.

But in the last Experiment, the Pressure of the Atmosphere exerts two Effects; it sustains the Pillar of the Mercury *i g*, and it reduces the Air in the Tube to the Space *g* A; if the Force, by which the Mercury is sustained at the Height *g i*, be substracted from the Pressure of the whole Atmosphere; that is, if the Height *g i* be taken from the Height *h f*, (*Fig.* 1.) there remains the Force by which the Air is compressed in the upper Part of the Tube; but this Difference of the Heights of the Mercury *h f*, and *g i*, is always to *hf*, as A*l* to A*g*, that is, their Forces are inversely as the Spaces.

This Rule also obtains in compressed Air.

Experiment 2. *Plate* XXVI. *Fig.* 5.] Take a curve Tube ABCD, open at A, and ſhut at D; let the Part BC be filled with Mercury, ſo that the Part CD may contain Air of the ſame State or Tenor as the external Air; therefore, the compreſſing Force is the Column of Mercury, whoſe Height is *b f*, *Fig.* 1. and by this Height muſt this Force be expreſſed, as in the foregoing Experiment; but the Space taken up by the Air is CD. Pour Mercury into the Tube AB, that it may riſe up to *g*, the Air will be reduced to the Space *e* D: Now the compreſſing Force acts as ſtrongly as a Column of Mercury of the Height *fg*, and alſo the Preſſure of the external Air upon the Surface *g* of the Mercury; this Force is expreſſed by the Sum of the Heights *fg* in this Figure, and *bf*, in *Fig.* 1. This Sum is always to *bf* (*Fig.* 1.) as CD to *c* D; and again the Forces are inverſely as the Spaces.

430 *The Elaſticity of the Air is as its Denſity*; for this laſt is inverſely as the Space taken up by the
* 489 Air *; therefore, as the Force compreſſed the
429 Air *, which is equal to that by which the Air endeavours to expand itſelf; but this Force is its Elaſticity.

Hence it follows, that the Air in which we live is reduced to the Denſity which it has near the Earth, by the Preſſure of the ſuperincumbent Air, and that it is more or leſs compreſſed, according to the greater or leſs Weight of the Atmoſphere; for which Reaſon alſo the Air is leſs denſe at the Top of a Mountain than a Valley, as being compreſſed by a leſs Weight.

How far this Property of expanding itſelf is extended, we do not certainly know; and it is very probable that it can be determined by no

Ex-

Experiments. Nevertheless, if you compare the following Experiments with the Experiment of the Air compressed in a Pump*, it will appear * 14 that the Air may take up twenty thousand Times more Space in one Case than in the other.

Experiment 3. *Plate* XXVI. *Fig*. 6.] Let the Glass A B, about fourteen Inches high, be exactly filled with Water; it has a brass Cap fixed to it at the End B, by which it is to be screwed to the Pump that is represented in *Plate* XXIX. *Fig*. 6. by drawing out the Piston of the Pump, the Water descends into it by its Gravity; and the Place in the upper Part of the Vessel is void both of Air and Water. The Air bubbles in the Water, which are now compressed, because the Air does not act upon the Surface of the Water, expand themselves, and rise up to the Surface of the Water; in that Motion the Bubbles are accelerated, so as not to be seen distinctly near the Surface, upon Account of their very swift Motion; they also grow bigger as they ascend, and if you compare the Diameter of a Bubble at B with its Diameter, when it is come almost up to the Surface of the Water, but so far from it as to be seen distinctly, its Diameter is at least four Times as great as before.

The upper Part of the Glass, as was said before, is entirely void of Air; for the small Quantity of Air, which is continually going out of the Water, is not to be taken notice of here; therefore the Air-bubbles near B, which is about a Foot below the Surface of the Water, are compressed only by the superincumbent Water; which Pressure is to the Pressure of the Atmosphere nearly as one to thirty-two; * in which Ratio * 423 also is the Space taken up by the Air, when it is compressed by the whole Atmosphere, to the

 Space

Space taken up in the Bubbles above-mention-
* 429 ed; * their Diameter in their Aſcent, as has been ſaid before, becomes quadruple; that is, the Bubble becomes 64 times bigger than it was; and ſo the Space taken up by the Air, in this laſt Caſe, is to the Space taken up by the Air, when compreſſed by the Atmoſphere, as 64 times 32 (that is 2048) to 1. The Air compreſſed by the Atmoſphere is reduced to a Space 10 times leſs in a Forcing Pump; and ſo the Denſity of the Air above-mentioned is to the Denſity of this Air, as 1 to 20480. Extracting the Cube Roots of theſe Numbers, we ſhall find that the Diſtances between the Center of the Particles, in theſe two Caſes, are, as 1 to 27.

Hence we conclude, that the Particles of Air are not of the ſame Nature with other elaſtic Bodies, for the ſingle Particles cannot expand themſelves every way into 27 times the Space, and ſo be increaſed 2000 times, preſerving their Surface free from every Inequality or Angle; for in every Expanſion or Compreſſion, the Parts are eaſily moved one amongſt another; but, as the
431 Air may be dilated much more than in this Experiment, it follows, that *the Air conſiſts of Particles which do not touch one another, and that repel each other.* We have ſhewn, that in ſeveral Caſes there are Particles endowed with ſuch a Proper-
*40 ty; * and it is plain enough, that it obtains here; but we are entirely ignorant of the Cauſe of this Force, and it muſt be looked upon as a Law of Nature, as is plain from what has been ſaid between *Numb.* 4 and *Numb.* 5.

432 *The Force, by which the Particles of the Air fly from each other, increaſes in the ſame Ratio as the Diſtance in which the Centers of the Particles are diminiſhed*; that is, that Force is inverſely as this Diſtance. To demonſtrate which, let us conſider two equal

equal Cubes A and B (*Plate* XXVI. *Fig.* 7.) containing unequal Quantities of Air; let the Diſtances between the Center of the Particles be as 2 to 1, the Numbers of the Particles will be in the ſame, but inverſe Ratio, in the Lines *d e* and *h i*; the Numbers of the Particles acting upon the Surfaces *d g* and *h m* are, as 1 to 4, namely, as the Squares of the Numbers of the Particles in equal Lines, and as the Cubes of thoſe Numbers, that is, as 1 to 8, ſo are the Quantities of Air contained in the Cubes; in which Ratio alſo are the Forces compreſſing the Air in the Cubes. * *429
The Forces acting upon the equal Surfaces *d g* and *h m* are as the Forces by which the Air is compreſſed; * they are alſo in a Ratio compound- *126
ed of the Numbers of the Particles acting, and the Action of the ſingle Particles; therefore, this compound Ratio is the Ratio of 1 to 8: The firſt of the compounding Ratio's, as has been ſaid, is that of 1 to 4; wherefore, neceſſarily the ſecond is that of 1 to 2, which is the inverſe Ratio of the Diſtances of the Particles. And this Demonſtration is general; for by 1 and 8 we expreſs any Cubes whatever; by 1 and 4, the Squares of the Cube Roots; and laſtly, by 1 and 2, the Roots of thoſe Cubes. This Demonſtration proves that the Action, which the Particles continually ſuffer from all Sides, is increaſed between the Ratio in which the Diſtance of the Centers of the Particles is diminiſhed, whether the Action is to be referred only to neighbouring Particles, or alſo to thoſe which are more diſtant. In the firſt Caſe the repellant Force itſelf, which every Particle is endowed with, is as the Action above-mentioned, that is, inverſely as the Diſtance between the Centers of the Particles.

In the ſecond Caſe, the repellent Force is equal at all Diſtances; for then the Action againſt each

each Particle depends upon their Number in the ſame Line, which Number is inverſely as the Diſtance between the Number of the Particles. Then alſo, ſuppoſing the Air of the ſame Denſity, the Elaſticity will be the greater, where the Quantity of the Air will be the greater; but, as this does not agree with Experiments, therefore, the firſt Cauſe muſt be true.

433 *The Effects of the Elaſticity of the Air are like thoſe of its Gravity*, and included Air acts by its Eaſticity, juſt as Air not included does by its Weight.

The Air which is loaded by the Weight of the whole Atmoſphere, preſſing every Way from the very Nature of Liquids, and the Force which it exerts, does no Way depend upon the Elaſticity, becauſe, whether you ſuppoſe Elaſticity, or not, that Force which ariſes from the Weight of the Atmoſphere, and is equal to it, can be no Way changed; but, as the Air is elaſtic, it is reduced to ſuch a Space by the Weight of the Atmoſphere, that the Elaſticity, which re-acts againſt the
*126 compreſſing Weight, is equal to that Weight *. But the Elaſticity increaſes and diminiſhes as the
*432 Diſtance of the Particles diminiſhes or increaſes *, and it is no Matter, whether the Air be retained in a certain Space by the Weight of the Atmoſphere, or any other Way; for in either Caſe it will endeavour to expand itſelf with the ſame Force, and preſs every Way. Therefore, if the Air near the Earth be included in any Veſſel, without altering its Denſity, the Preſſure of the included Air will be equal to the Weight of the whole Atmoſphere.

Experiment 4. *Plate* XXVII. *Fig* 3.] Take the Tube mentioned in the firſt Experiment of the laſt Chapter, immerge it in Mercury included in the

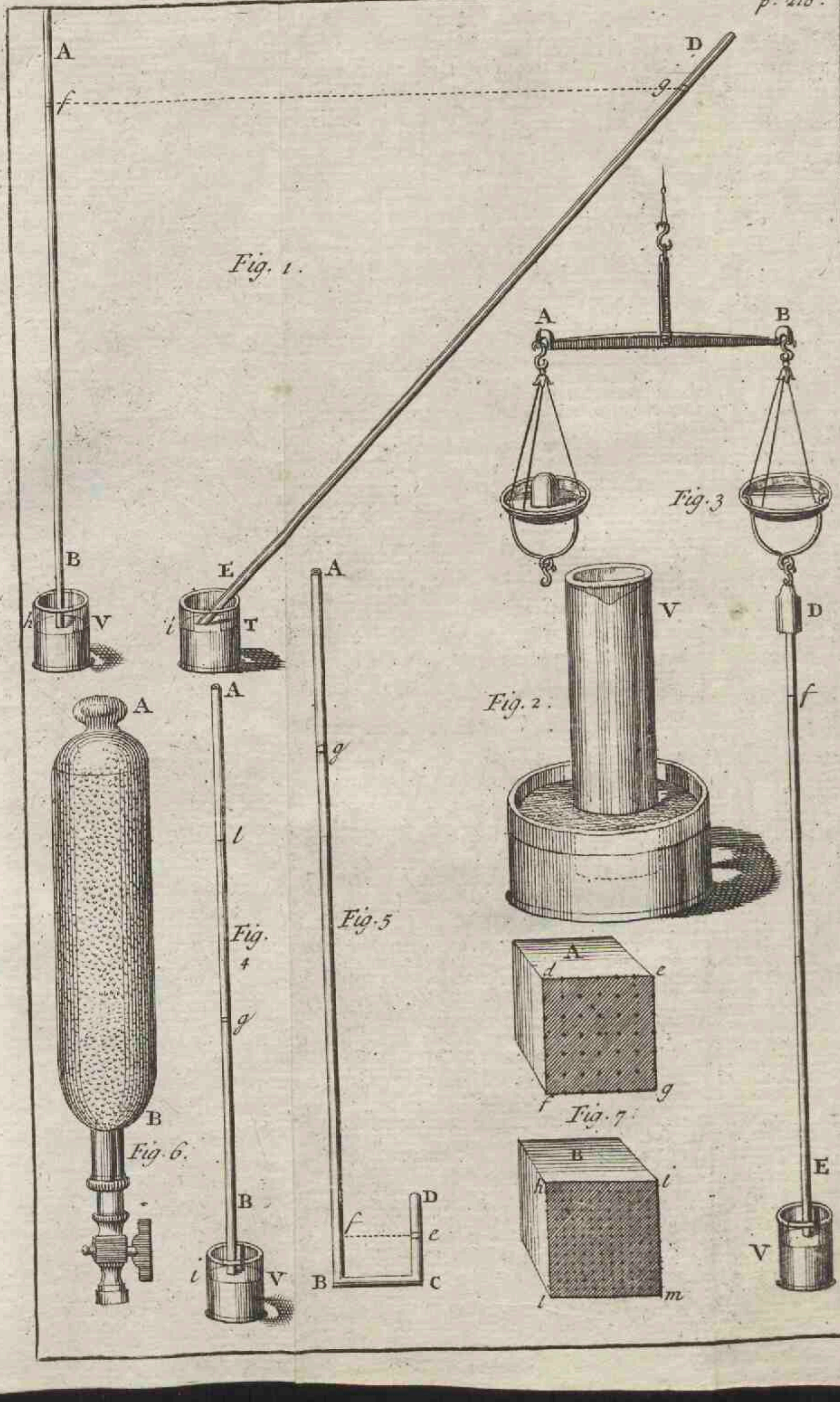
Fig. 1.
Fig. 2.
Fig. 3
Fig. 4
Fig. 5
Fig. 6.
Fig. 7.

the Glaſs DC, ſo that the Air, preſſing upon the Surface of the Mercury contained in the Veſſel U, may have no Communication with the external Air; the Mercury in the Tube is ſuſtained at the ſame Height by the Elaſticity of the Air, as it was ſuſtained in the open Air.

The Tenor of the Air continuing the ſame, what we have ſaid will always obtain; but this Tenor or Temper of the Air is not always the ſame; *the repellent Force of the Particles is often increaſed, or diminiſhed, though the Diſtance between their Centers is not changed:* I ſhall ſpeak of this Alteration in the following Book: *The Elaſticity increaſes by Heat, and diminiſhes by Cold.* 434

CHAP. XIV.

Of the Air-Pump.

THE Elaſticity of the Air is the Foundation of the Conſtitution of a Machine, by which the Air may be drawn out of any Veſſel. This Machine is called an Air-pump, which is made ſeveral Ways: The chief Part in all of them is a Barrel, or hollow Cylinder of Metal, bored ſmooth, and poliſhed in the Inſide; in this Barrel muſt move a Piſton, that fills its Bore ſo exactly as to let no Air ſlip by. This Piſton is thruſt down cloſe to the Bottom of the Barrel, and then raiſed up in ſuch a Manner as to exclude all the Air from the Cavity of the Cylinder or Barrel; if this Cavity communicates with any Veſſel, by means of a Pipe at the Bottom of the Barrel, the Air in the Veſſel will expand it ſelf, and Part of it will enter into the Barrel, ſo that the Air in the Barrel, and in the Veſſel, will have the ſame Denſity. Shut up the Communication between the Veſſel and Barrel, and letting the Air out of the 435

Barrel, apply the Piſton cloſe to the Bottom. If you raiſe the Piſton a ſecond time, and open the Communication between the Barrel and Veſſel above-mentioned, the Denſity of the Air in the Veſſel will again be diminiſhed; and repeating the Motion of the Piſton, the Air in the Veſſel will be reduced to the leaſt Denſity. Yet all the Air can never be exhauſted by this Method; for at every Stroke the Air does ſo expand itſelf, as to have the ſame Denſity in the Barrel as in the Veſſel; in which laſt therefore, there is always a little Air left.

436 All Air-Pumps have in common the Parts above deſcribed, but they differ in ſeveral other Things. *Firſt*, the Communication between the Receiver to be exhauſted, and the Cylinder or Barrel, is opened and ſhut different Ways. *Secondly*, there are different Ways of getting the Air out of the Cylinder or Barrel, when the Piſton is brought to the Bottom. *Thirdly*, the Piſtons differ in different Pumps. *Fourthly*, the Poſition of the Cylinder is not the ſame in all Pumps. *Fifthly*, there are different Contrivances for moving the Piſton.

There are often two Barrels, in one of which the Piſton is raiſed, when it is depreſſed in the other.

Our Pump is here repreſented in *Plate* XXVII. *Fig.* 1. the other Side of it is repreſented in *Plate* XXX. *Fig.* 2. I ſhall defer the particular Deſcription of it to another Place, and only here mention ſome Things in general. This Pump has two Braſs Barrels C, C, of 2 Inches Diameter, and about 5 Inches high.

In theſe Barrels the Piſtons move, one of which deſcends, while the other riſes, which Motion is communicated to them by the Wheel R, which is moved by the Handle M M, *Fig.* 2.

fixed

fixed to the Axis *a*. The angular Motion of the Wheel is the eighth Part of a Circle, by which in a leſs Wheel there is produced an angular Motion of 120 Degrees. This leſſer Wheel is fixed to a third Wheel, by means of which the Piſtons are immediately moved; they make a Stroke of 3 Inches and a half.

The Contrivance of the Piſton is much the ſame as in the Pumps which they uſe in *England*; tho' we think that we have made ours more perfect, by ſome Alterations in them.

The Glaſs is to be exhauſted, or ſet upon the round Plate L L; they communicate with the Barrels, by means of a Pipe, one End of which is at D, and which ſoldered to the lower Side of the Plate, the Continuation of this Tube is ſeen at E E; there are two Cocks in it, E, E, between the Cocks is fixed the Pipe *l*, *l*, which communicates with the Cylinders C, C.

When the Air is exhauſted, one of the Cocks above-mentioned ſerves to ſhut the Communication between the Receiver (ſo the Glaſſes are called from which the Air is to be pumped out) and the Barrels; the other Cock ſerves to let the Air in again, and to cut off the Communication with the mercurial Gage.

The mercurial Gage could not be conveniently 438
repreſented in this Figure; it ſerves to determine what Quantity of Air is drawn out of the Receiver, as alſo what Quantity of Air remains in it; it is likewiſe of uſe for meaſuring the ſolid Contents of the Receivers, which ought to be exactly known in ſeveral Experiments; our Gage differs from the common Gages in ſeveral Reſpects.

A little Cylinder, with a Screw upon it, is often ſcrewed into the Plate at D, for applying a Globe to be exhauſted to the Pump.

In

In the Middle of the Plate L L there is a Hole, which is ſhut up with a Screw; but ſometimes it ſerves for joining ſeveral Machines to the Plate.

439 By this Means alſo there is often applied to the Pump a cylindric Box, full of Leathers ſoaked in Wax, thro' the Center of which a braſs Wire paſſes, which may be moved by the Help of a Handle, ſo as to communicate Motion into a Place void of Air; the Box has a Cover, which enters into it with a Screw, for preſſing the Leathers together, and to prevent the entring in, or eſcaping out
440 of the Air; ſuch a Box, or Collar of Leathers, is often joined to the Cover which is laid over the Recipients, as may be ſeen in *Fig.* 2. *Plate* XXVIII. and in *Fig.* 2. *Plate* XXIII.

441 When the Receivers are laid upon the Plate L, L, or when the Receivers are ſtoped with Covers, or when the Screws are joined to the Machine, and in general, when the Air is to be hindered from running in, we make uſe of Wax, which is ſoftned by mixing as much Oil and Water to it as is found neceſſary.

CHAP. XV.

Several Experiments concerning the Air's Gravity, and its Spring.

* 422 WE have ſhewn that Air is heavy, * it may be weighed like other Bodies, and ſo its Denſity may be compared with that of other Bo-
* 288 dies.* If the Veſſel that contains the Air be weighed, when it is full of Air, and again, when the Air is exhauſted, the Difference between their Weights is the Weight of the Air; which Method has this Inconvenience, that ſuch a ſmall Difference of Weight cannot eaſily be diſcovered, when a Balance, tho' ever ſo nice, is loaden with a great Weight; therefore we muſt make uſe of the following Method. *Expe-*

Experiment 1. *Plate* XXVIII. *Fig.* 2.] Having 442
exhausted the Air out of the Glass Ball, whose solid Contents are 283 Inches, and having tied such a Weight to it, that it may be almost equal in specifick Gravity to Water, let it be immersed into the Water continued in the Vessel D E, and let it be fastned by a Thread to the Hook of the Scale of the Balance A B, above described; * raise the Balance till you make an Æquilibrium with a very small Weight; if by opening the Cock you let the Air into the Globe, a Weight I, of about 100 Grains, will be required in the opposite Scale to restore the Æquilibrium, sometimes more, or sometimes less, according to the different Tenor of the Air, which here near the Earth is varied according to the different Weight of the Atmosphere, and according to the Difference of Heat and Cold.

* 271

Bodies immersed in Liquids are sustained by them, and the more or less according to the greater or less Bulk of the Body, * and the Weight lost in that Case is determined from the known Density of the Liquid; * by the foregoing Experiment, therefore, it may be known, how much Bodies gravitate less in Air than in a Vacuum.

* 299

* 297

Hence also may be deduced, that *Bodies that* 443
are in Æquilibrio in the Air, if their Bulks are unequal, will lose their Æquilibrium in a Vacuum: Which is confirmed by the following Experiment.

Experiment 2. *Plate* XXVIII. *Fig.* 3.] In the Scales of the Balance *a b*, lay a Piece of Wax, *c*, and a Weight of Metal, *p*, and you will have an Æquilibrium. Hang up the Balance in a Glass Receiver, and, having exhausted the Air, the

Wax

Wax will preponderate, its Bulk being greater than the Bulk of the Body *p*, it muſt be more ſuſtained by the Air; and, therefore, when you let the Air into the Receiver again, the Æquilibrium is reſtored.

The Elaſticity or Spring of the Air, which has been proved in *Chap.* XIII. becomes more ſenſible by the following Experiment.

444 *Experiment* 3.] Tie up a Bladder very cloſe, with a ſmall Quantity of Air in it; put a Receiver over it, and pump out the Air, whereby the Preſſure upon the external Surface of the Bladder is diminiſhed, and immediately the Air included in the Bladder will expand itſelf, and ſwell it out. We have proved, that the Spring of the Air is equal to the Weight of the whole Atmoſphere; the following Experiment will make it viſible.

445 *Experiment* 4. *Plate* XXVIII. *Fig.* 1.] Take a Bladder tied up very cloſe, and not quite full of Air, and put it in a Braſs Box A, whoſe Diameter is three Inches and an half; ſo that the Cover, which is of Wood, and does not exactly fit the Box, may be ſuſtained by the Bladder; you muſt put the Lead Weights P, P, upon the Cover: They have a Hole in the Middle for a wooden Cylinder E, which is fixed to the Cover, to go thro': When you pump out the Air, the Bladder is ſwelled, as in the foregoing Experiment, and by that means the Weights are raiſed. You may uſe ſeveral Weights according to the Bigneſs of your Glaſs Receiver; and tho' they ſhould amount to 60 or 70 Pounds, they would be eaſily raiſed. The Gravity of the Air, its Preſſure that ariſes from the Gravity, as alſo its Elaſticity, produce very

very different Effects; some of which I shall select, and confirm by Experiments.

Experiment 5. *Plate* XXVII. *Fig.* 3.] To the Hole in the Middle of the under Side of the Air-Pump L L, screw on a small hollow brass Cylinder which has a Hole through it, and an open Glass Tube A B cemented to it at Bottom, whose lower End B must be immerged into Mercury. Let Mercury be sustained in the Tube *e g*, that is close at *e*, and void of Air in the Manner before-said. * Set the Vessel U with the Tube upon the Plate L L, and cover it with a tall Glass D C, so as to cut off all Communication between the external Air and the Vessel U, as also the Cavity of the Tube A B. The Air in this Tube does, by its Elasticity, hinder the Mercury from rising up in the Tube, by the Pressure of the external Air. The Air also, that is included in the Receiver D C, does by its Spring sustain the Mercury in the Tube *g e*. * Pump the Air out of D C; as the Density diminishes, the Elasticity does also decrease, * and the Force by which the Mercury is sustained in the Tube *g e* becomes less; therefore, the Mercury descends. At the same time the Pressure of the external Air overcomes the Resistance in the Tube A B, and the Mercury ascends in the Tube. The Diminution of the Spring in the Tube A B, and in the Vessel B C, is the same, and the Effect of the Diminution the same in both Cases; therefore, the Mercury descends as much in the Tube *e g* as it rises in the Tube A B, which agrees with the Experiment. By this Method the Mercury is raised up to *f*, while the Tube *g e* becomes almost wholly empty; when you let in the Air again, the Mercury rises in the Tube *g e*, as it is depressed in the Tube A B.

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*422 *433 *430

447 *Experiment* 6. *Plate* XXVIII. *Fig.* 4.] Take the little Pump or Syringe A, and its Piſton being thruſt cloſe to the Bottom, let the Tube which is joined to the Syringe be immerged in Water; when you raiſe up the Piſton, the Water will follow it, and fill up the Cavity between the Bottom of the Pump and the Piſton; which Effect ariſes from the Preſſure of the external Air.

For this Reaſon Water does not riſe *in vacuo*.

448 *Experiment* 7. *Plate* XXVIII. *Fig.* 5.] Join the Glaſs Tube *bc* to the Syringe A, which is ſcrewed to the Cover of a Glaſs Receiver, ſo that the End *c* of the Tube may deſcend below the Surface of the Water in the Veſſel U; thruſt down the Piſton to the Bottom of the Syringe, and let all the Air be pumped out of the Receiver. If then you pull up the Piſton, the Water will not riſe.

449 *Experiment* 8. *Plate* XXVIII. *Fig.* 6.] The Force, by which the Air preſſes upon Bodies, often breaks them, when the Preſſure is not equal every Way. Let the Braſs Cylinder A be covered with a flat Piece of Glaſs; when you pump the Air out of this Cylinder, the Plate of Glaſs will be broken into a great many little Pieces by the Preſſure of the external Air.

450 *Experiment* 9. *Plate* XXIX. *Fig.* 1.] Take a Syringe A of an Inch Diameter; puſh down the Piſton to the Bottom of it, and ſhut up the Hole at the Bottom of the Syringe, and hang on a Weight P of 10 Pounds to the lower End of the Syringe; if you hold the Handle B of the Piſton in your Hand, the Syringe will not deſcend; for it cannot deſcend, unleſs the Weight hanging at it

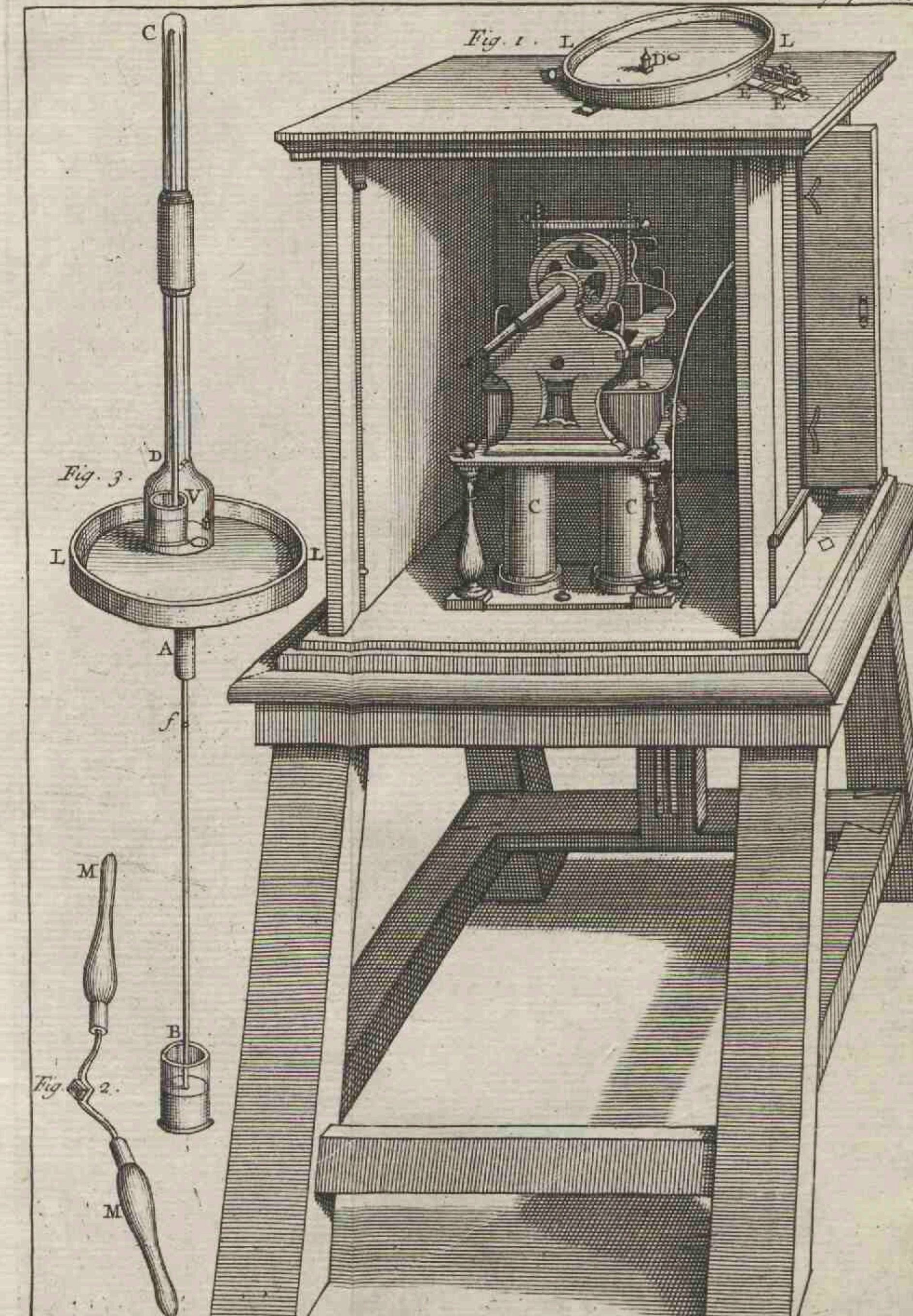
Fig. 1.
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Fig. 3.
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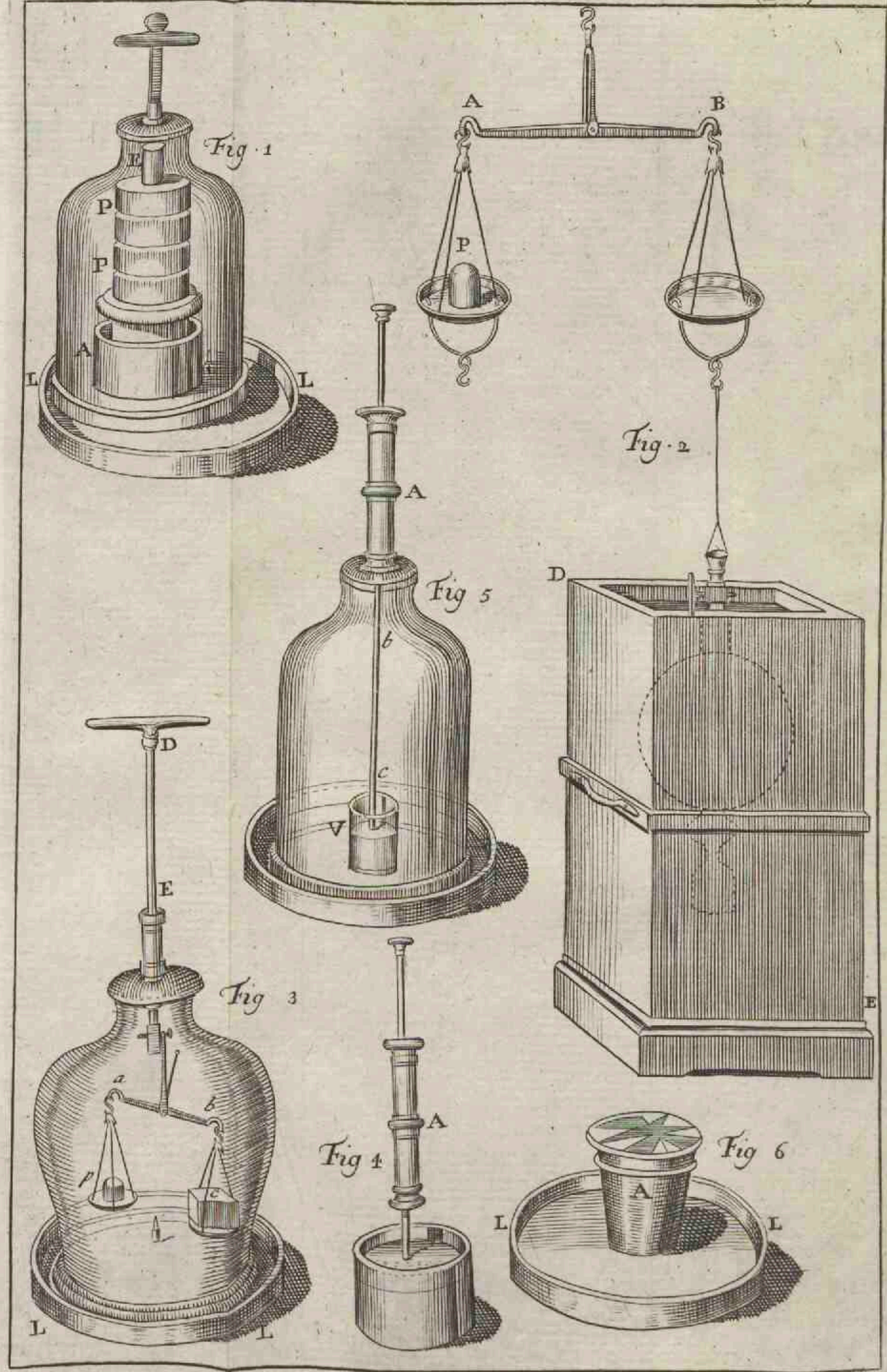

Fig. 1
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Fig. 2
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Fig 5
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Fig 3
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Fig 4
Fig 6
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it overcomes the Pressure of the Air and Friction of the Piston; but the Pressure of the Air alone does here exceed 10 Pounds.

Experiment 10. *Plate* XXIX. *Fig.* 2.] The Syringe descends by the Weight *p* alone in a Vacuum, which is but just sufficient to overcome the Friction of the Piston. 451

Experiment 11. *Plate* XXIX. *Fig.* 3.] We see a more sensible Effect of the Pressure of the Air, when the two Segments of a Sphere, H and I, are joined together. Let the Brim or Edge of each of them be well polished, so that they may fit together, and, when they are applied close, put a little Wax between, to exclude the Air. There is a Cock in the Segment H, by which the two Segments, when joined together, may be applied to the Air-Pump, and which must be shut, when you have exhausted the Air. The Segments are suspended by the Ring A, and, by the Help of the Ring Q, you may hang to them the Weights that are laid upon the great wooden Scale T. If the Diameter of the Segments be three Inches and an Half, a Weight of about 140 Pounds will be required to pull them asunder. 452

Experiment 12. *Plate* XXX. *Fig.* 1.] Let the Segments be joined together and exhausted, as in the former Experiment; if they be suspended in a Vacuum, with a little Weight P hanging on, which is just able to overcome the Cohesion of the Wax, they will be separated in this Experiment. There must be fastned to the Plate L L, the little brass Box, or Collar of Leathers, * thro' which a brass Wire, that has the Weight hanging to it, slips. Lest the Receiver should be broke by the Fall of the lower Segment, you must put un- 453

* 439

der it the hollow wooden Cylinder M, to let it fall into. In this Figure the Segments are fuſpended to the Cover of the Receiver which is to be exauſted; they may alſo be fuſpended from a Pillar faſtened to the Cylinder M. That the Hemiſpheres may not be ſeparated without Difficulty, it is not required that they ſhould be empty of Air; as great a Force will be required to ſeparate them as in the 11th Experiment; when having included them in a Veſſel, and applied them cloſe together (ſo as to leave them full of common Air, and ſhut the Cock, that the Air between them may not be changed;) the Air on the Outſide of them in the Veſſel, that contains them, is reduced to a double Denſity; which to confirm by an Experiment, we muſt firſt deſcribe the Machine with which we make Experiments in compreſſed Air.

454 *Plate* XXXI. *Fig.* 5.] A round braſs Plate N is laid upon a Board *a a*, about 15 Inches long, and 10 wide; the Diameter of this Plate is about 5 or 6 Inches, as you may ſee by its ſeparated Figure at N, in *Plate* XXIX. *Fig.* 4. and has fixed to it here a Cylinder P, which is not perforated, and goes through the Board *a a*. Upon this Plate you muſt put a Glaſs U U, about 10 Inches high, that is terminated at each End in a cylindric Form, and the cylindric Parts have braſs Rings, or Ferrels, upon them.

The Veſſel muſt have upon it the Cover D. The Pillars C S, C S, are faſtened to the Board *a a*, and go thro' the Wood *d e*, by which the Cover D is firmly joined to the Glaſs, as alſo the Glaſs to the Plate N, by the Force of the Screws *f f*. It is very neceſſary to preſs all theſe Parts cloſe together, having firſt ſpread Wax upon the upper and lower Edges of the Glaſs.

The

The Cover is represented in the separated Figure D, *Plate* XXIX. *Fig.* 4.] There is fixed to it the Collar of Leathers; * and, lest the Piece of Wood *de* should be applied to too small a Surface, the Cover is made in the Shape of a round open Box.

There is a perforated brass Wire C, which goes through the Collar of Leathers, to which a Cock B is joined.

Plate XXIX. *Fig.* 6.] In order to compress the 455
Air in this Vessel, screwing on the Syringe A B to the Cock B last mentioned, which Syringe has joined to it another Cock, in the Key of which, besides the usual Hole, there is another oblique Hole which goes to *f*, and by which when you shut the Communication between the Glass and the Syringe, the Syringe has a Communication with the common Air, and is filled with it, when you raise up the Piston. When you open the Communication between the Glass and the Syringe, by pushing down the Piston, you force the Air which was contained in the Syringe into the Glass; and, by often repeating this Operation, you at last bring it to the Density required.

Experiment 13. *Plate* XXIX. *Fig.* 4.] Now, to 456
separate these Segments or Cups in compressed Air, the Segment I is joined to the Plate N, by Means of the Pillar M L, which has Screws at M and L. The other Segment H, by Screws at F and E, is joined to the Wire C. The Segments must be applied to each other. As for the rest, you must observe what has been said in the Description of the Machine; and the Air must be compressed in the Vessel, so as to have twice the Density of that which is compressed only by the Atmosphere. At P the Ring Q

is joined to the Plate N, as alſo the Ring A to the Cock B. Invert the Machine, as in *Fig.* 5, and ſuſpend it by the Ring Q. The Scale T, on which the Weights are laid, hangs upon the Ring A; and, until the Weights, laid on, come to be about 140 Pounds, the Segments will not be ſeparated. Three Screws X, X, hinder the Scale T from deſcending too low in ſeparating the Segments.

457 *Experiment* 14. *Plate* XXX. *Fig.* 2.] Apply the Tube A B to the under Side of the Air-Pump Plate L L, which Tube has a Cock in its upper Part, and is joined to the ſmall Tube which ſtands above the Plate. Put on the Receiver R, which covers the prominent Tube. The End B of the Tube A B muſt be immerſed in the Water contained in the Veſſel U, and having exhauſted the Veſſel R, you muſt open the Cock; the Water will ſpout up into the Receiver with a great Force, for the ſame Reaſons as the Water is ſuſtained
* 425 at the Height of 32 Foot in a Pipe void of Air. *

458 *Experiment* 15. *Plate* XXX. *Fig.* 3.] The Air's Elaſticity produces the ſame Effect. Let there be a braſs Cylinder U exactly ſhut. There muſt be a Hole in the Bottom to pour in Water, which afterwards you ſhut up with a Screw. To the upper Part of the Veſſel there is ſoldered a Pipe, which goes down almoſt to the Bottom; and to the other End of it, that ſtands above the Veſſel, a Cock is joined (See *Fig.* 4.) This Veſſel is to be ſcrewed on to the lower Part of L L, the Air-Pump Plate, and from it a Pipe goes quite thro' the Plate, and ſtands up above it, which is covered by the Receiver R. After you have pumped out the Air, the Veſſel U being about two Thirds full of Water, when you open the Cock, the

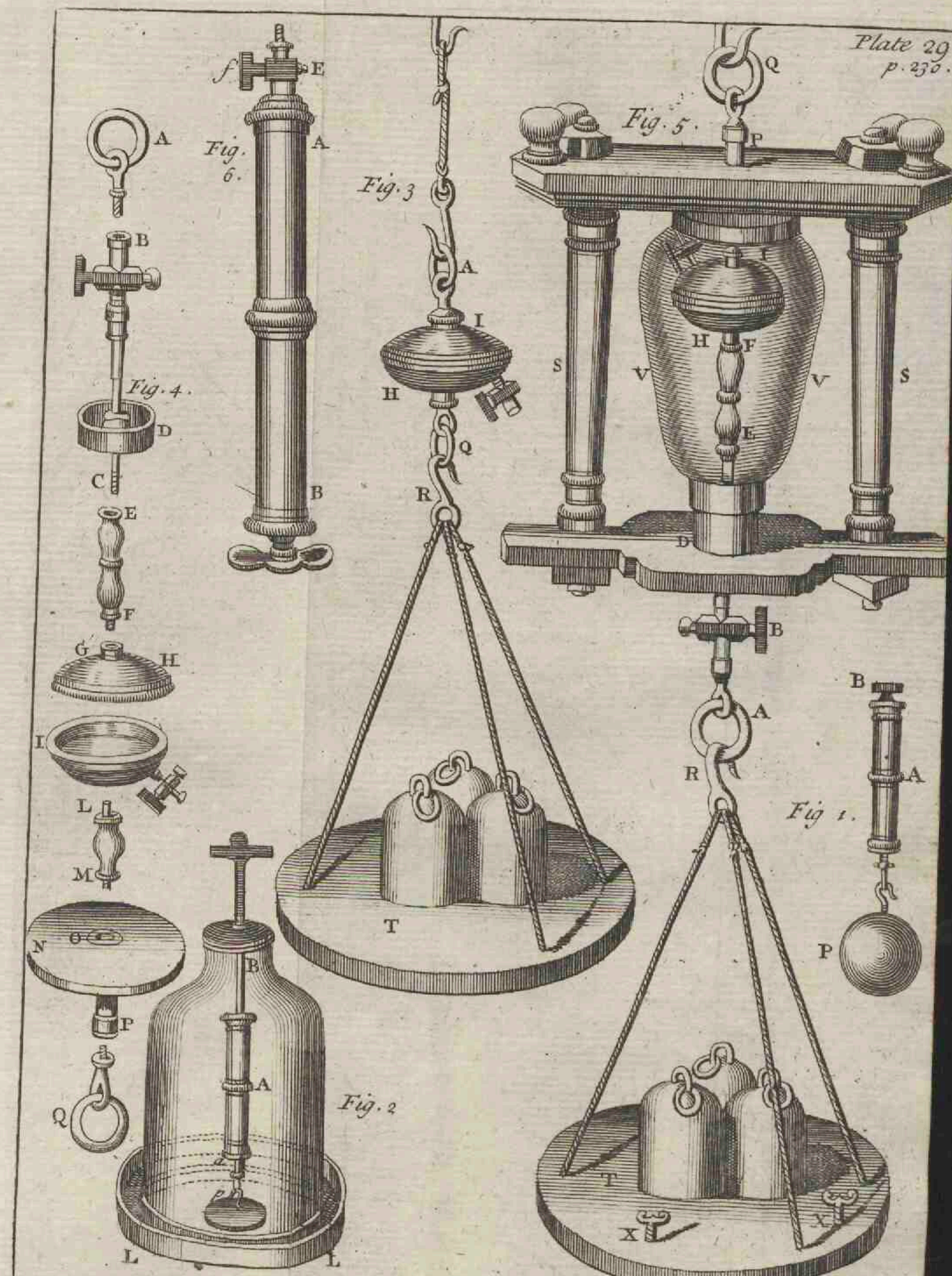
Plate 29
p.230.
Fig. 1.
Fig. 2
Fig. 3
Fig. 4.
Fig. 5.
Fig. 6.

the Water will violently ſpout up into the Receiver, by the Force of the Spring of the Air contained in the upper Part of the Veſſel U. Here the Air preſſes upon the Surface of the Water; when you open the Cock, the Preſſure in the Tube becomes leſs, therefore the Water muſt go into the Tube.

Experiment 16. *Plate* XXX. *Fig.* 4.] Even in the 459
open Air, the Water will violently ſpout out of the Veſſel U, if, having filled it two Thirds full of Water, the Air be compreſſed in the upper Part of it; which is done by help of the Syringe above-mentioned *. * 455

Experiment 17. *Plate* XXXI. *Fig.* 1.] Invert the 460
Glaſs R, and immerge it in the Water contained in U, the Air keeps out the Water at whatever Depth it be immerged; yet the deeper the Glaſs is put down, the leſs Space the Air is reduced into.

Upon this Principle are made the Machines in 461
which divers go down into the Sea. They are made like Bells, and deſcend by their own Gravity; the Water does not riſe up to the Diver in the Bell; freſh Air is ſent down continually by Bladders tied to a Rope, which he draws down to him; the Air, heated by his Reſpiration, riſes to the upper Part of the Bell, and is there driven out through a Cock, by the Preſſure of the Water, that puſhes up, and compreſſes the Air in the lower Part of the Bell; which Preſſure overcomes the Force with which the Water endeavours to deſcend thro' the Cock; for the Preſſure of Liquids is increaſed in Proportion to their Depths. * * 290

Experiment 18. *Plate* XXXI. *Fig.* 2.] Take lit- 462
tle Figures of Glaſs that are made hollow, of an Inch and half long, repreſenting Men, which may be had at the Glaſs-Blowers; theſe little

Images have a ſmall Hole in one of their Feet, and are lighter than Water. Immerge them into the Water contained in the Glaſs A B. This Glaſs is about a Foot, or 15 Inches high, and covered with a Bladder, which is tied faſt over the Top. A ſmall Quantity of Air is to be left between the Bladder and Surface of the Water. If the Veſſel be preſſed with the Finger, the Air above-mentioned is reduced to a leſs Space, and the Surface of the Water is more compreſſed; the Water which is more compreſſed, enters in the little Men through the Hole at their Feet, and compreſſes the Air in their Bodies more than it was. The little Images, becoming heavier, by this Means deſcend towards the Bottom of the Veſſel, and that faſter or ſlower, according to the Bigneſs of the Hole, and alſo according as the ſpecifick Gravity of the Images comes nearer to the ſpecifick Gravity of the Water. Taking away your Finger, the Air in the little Men, being leſs compreſſed, expands itſelf, and drives out the Water; ſo the Images riſe up again to the Surface of the Water.

463 *Experiment* 19. *Plate* XXXI. *Fig*. 3.] Animals cannot live without Air. If any Animal be included in the Receiver U, and the Air be drawn out, the Animal will immediately be in Convulſions, and will fall down dead, unleſs the Air be ſuddenly re-admitted. Some Animals will live in a Vacuum longer than others.

464 *Experiment* 20. *Plate* XXXI. *Fig*. 4.] Some Fiſhes alſo cannot live without Air; but in others you ſee no ſuch Change, but the Swelling of their Eyes. What Experiments you make upon Fiſhes, muſt be made in the Glaſs Receiver U, which is ſet upon the Plate of the Air-Pump, and to the Hole,

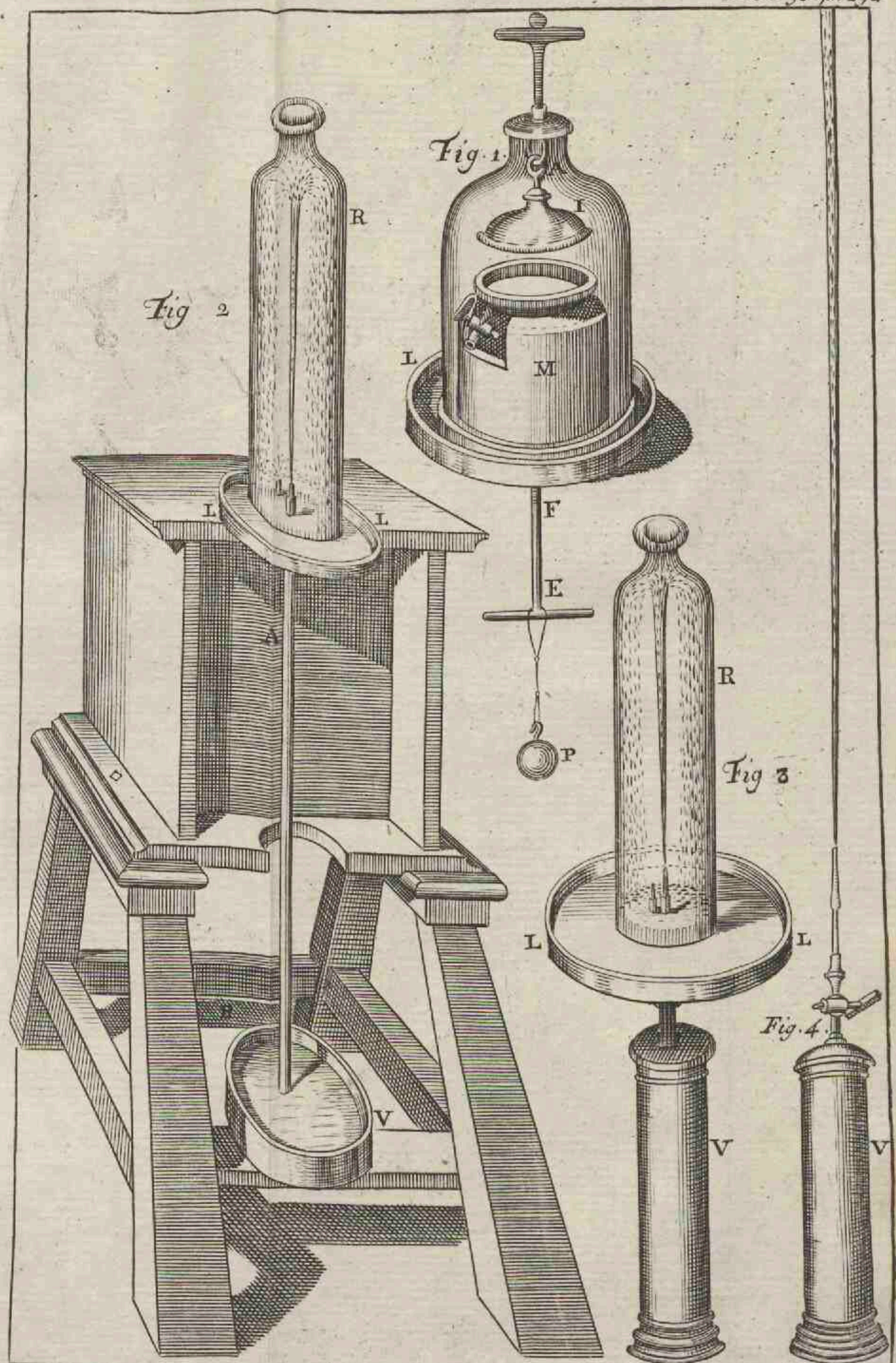
Fig. 1.
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E
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Fig 2
R
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L
A
V
Fig 3
R
L
L
V
Fig. 4.
V

Hole, through which the Air is drawn out, you muſt ſcrew on a Pipe, which comes up almoſt to the upper Part of the Glaſs U; pour in Water, and then put a Cover over the Glaſs Receiver U, and exhauſt the Air out of the upper Part of it. Having taken away the Preſſure of the Air from the Surface of the Water, the Air in the Fiſh's Body expands itſelf, by which Means the Fiſh, becoming lighter, cannot deſcend in the Water.

Experiment 21. *Plate* XXXI. *Fig.* 5.] Experi- 465
ments are made upon Animals in compreſſed Air, by Help of the Machine above deſcribed.* In *454
that Caſe Animals do not ſoon die, becauſe the Veſſels in the Body are not broken; yet if they continue long in that condenſed Air, it muſt be hurtful to them; nay, and in a greater Compreſſion of the Air (for which a Veſſel of Metal is required) they will die in a little Time.

Experiment 22.] Several Liquors contain Air. 466
If you put them under a Glaſs Receiver, and draw out the Air, then the Air contained in the Liquors will expand itſelf and go out. In that Caſe very often the Action of the Particles of the Liquid upon one another, is changed, and a Fermentation ariſes.

CHAP. XVI.

The Deſcription of ſeveral Machines, and the Explanation of their Effects.

[*Experiment* 1. *Plate* XXXII. *Fig.* 1.]

LET one End *a*, of the Curve Tube *a* S *b*, 467
be immerged in Water, whilſt the other End *b* deſcends below the Surface of the Water. If by ſucking, or any other Way, the Air be taken

out

out of this Tube, the Water will run through *b*. This Inſtrument is called a *Syphon*.

468 This Effect ariſes from the Preſſure of the Air, which drives on the Water in the Syphon, by its Preſſure upon the Surface of the Water in the Veſſel. The Air does alſo preſs againſt the Water that goes out of the Orifice *b*, and ſuſtains it. Theſe Preſſures are equal, and act contrariwiſe in the upper Part of the Syphon, with a Force equal to the Weight of the Atmoſphere, taking away the Weight of the Pillars of Water which are ſuſtained by the Preſſure. The Pillar of the Water in the Leg S *b*, is longer than the oppoſite Pillar of Water; therefore, the Preſſure of the Air is more diminiſhed on the Side *b* S, and the oppoſite Preſſure overcoming it, the Water flows towards *b*.

469 *Experiment* 2. *Plate* XXXI. *Fig*. 6.] The Syphon above-mentioned has this Inconveniency, that, if once it ceaſeth to work, the Water will not run again, unleſs the Air be drawn out of the Tube afreſh. This may be corrected by making a Syphon in the Figure *a* S *b*, whoſe Legs are equal, and turned up again: For if the Syphon be filled with, and one Leg be immerſed in Water, ſo that the Surface of the Water may be above the Orifice, then the Water will run out through the other Leg, for the Reaſon given in the Explication of the former Experiment. Since the Legs are returned upwards, the Syphon will not be emptied, when the running out of the Water ceaſes; and ſo the Syphon, being once filled, is always ready to work its Effect. The Water runs backward or forward through it, according as it is higher on one Side or the other.

Plate

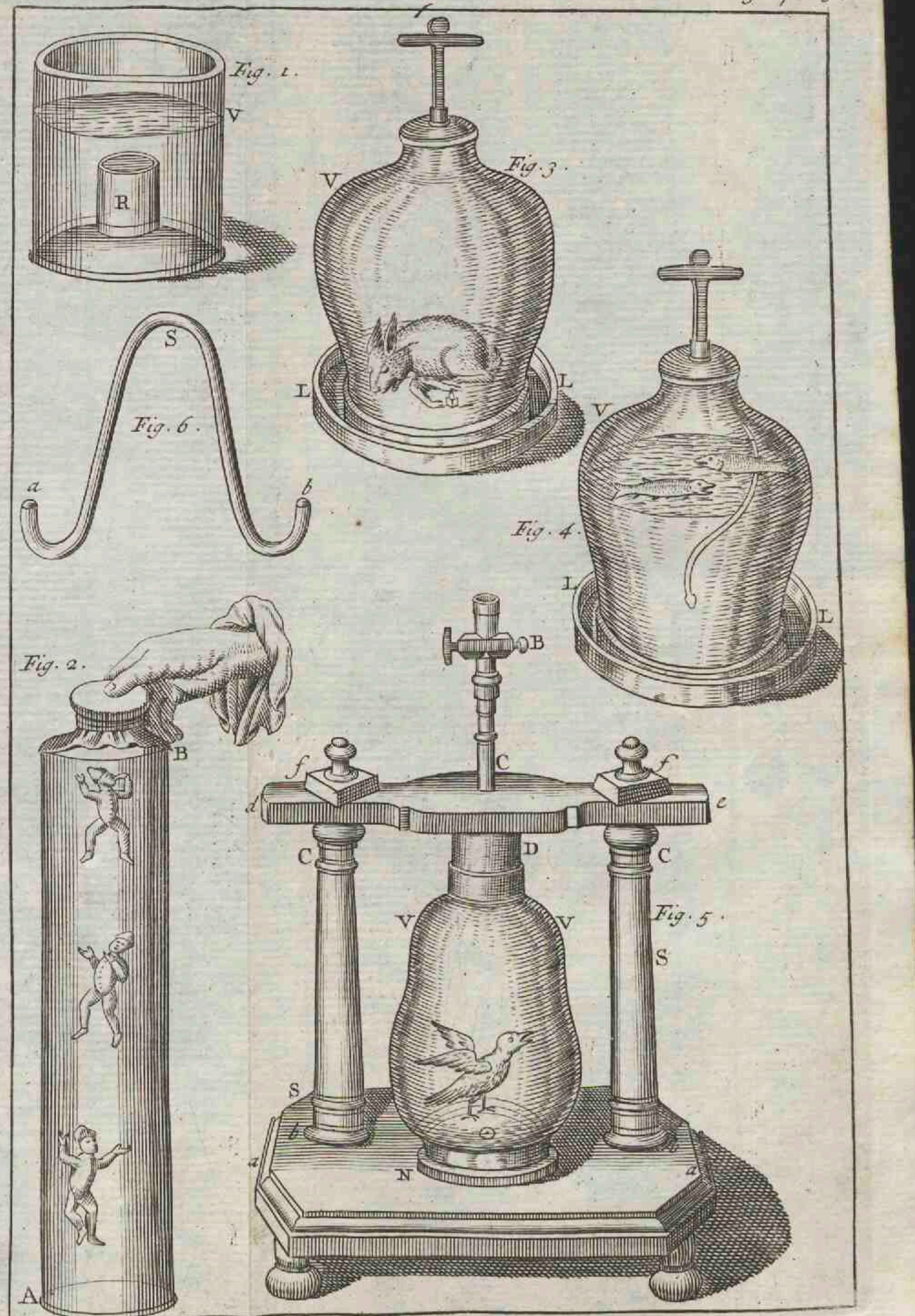
Fig. 1.
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Fig. 3.
V
L
L
Fig. 6.
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b
Fig. 4.
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L
L
Fig. 2.
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f
f
d
e
D
C
C
Fig. 5.
V
V
S
S
b
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N
a

Plate XXXII. *Fig.* 2.] Upon the ſame Principle as the foregoing Machines, is contrived the Syphon for raiſing Water into a Ciſtern. The Effect of this Syphon becomes viſible by the Help of a Machine made up of two hollow Glaſs Balls H and I, which are joined together by the Tube C D E, the Ball I communicates with the Water to be raiſed by means of the Tube A B, which comes up almoſt to the Top of the Ball; to the Ball H at the lower Part is joined the Tube F G, as long as the whole Tube A B. 470

The Ball H muſt be filled with Water through a Hole by a Funnel, and then the Hole muſt be ſhut up cloſe.

In ſuch Machines as are applied to Uſe, for raiſing Water out of a Reſervoir that contains it, the Water is brought away in the Veſſel H, and the Communication between the Veſſel and the Reſervoir is ſhut up with a Cock.

Experiment 3.] Opening the Cock G, the Water will run out that Way, and the Water will aſcend through the Tube A B up into the Veſſel I; which being filled, the Water is ſuffered to run away to the Place where you would have it; and, by repeating the Operation, the Elevation of the Water continues.

Opening the Cock G, the Air preſſes againſt the 471
Water going out of the Tube F G; the Air alſo preſſes upon the Water in the Reſervoir, and ſuſtains that which is in the Tube A B. Theſe Preſſures are equal, and if you take from them the Columns of Water which they ſuſtain, you will have the Forces by which they act upon the Air, contained in the upper Part of the Veſſels and the Tube CDE. The Pillar F G, becauſe there is ſuperadded to it the Height of the Water in the Veſſel H, does always

always overcome the Column in the Tube A B, as being longer; therefore the Preſſure at G is leſs diminiſhed than the other, and ſo overcome by it; and therefore the Water muſt riſe in the Tube A B, and deſcend down F G.

472 To render the Effect of common Pumps viſible, let there be a little Pump made of Glaſs in the following Manner; A B (*Plate* XXXII. *Fig.* 3.) muſt be a Cylinder of Glaſs, and about an Inch and a half Diameter. In the Bottom of it join a Tube of any Length, as C D. Let the upper Part of it be ſhut with a leaden Ball, ſo that the Water may not be able to deſcend out of the Cylinder, but may eaſily riſe into it, by raiſing up the Ball, which we make uſe of here inſtead of a Valve. The Piſton is moved in the Cylinder A B, which, being ſurrounded with Leather, exactly fills its Cavity: There is a Hole in the Piſton, which likewiſe is ſtopped with a Ball of Lead inſtead of a Valve; ſo that the Water may riſe, but not deſcend through the Piſton.

Experiment 4.] Puſh down the Piſton to the Bottom; pour Water upon it to hinder the Paſſage of the Air; if the End of the Tube C D be immerſed into Water, and the Piſton be raiſed, the
*447 Water will aſcend up into the Cylinder A B * from which it cannot deſcend; wherefore, it comes up through the Piſton, when it is puſhed down. If you raiſe the Piſton again, the Cylinder is again filled with other Water, and the firſt Water is raiſed up into the wooden Cylinder which is joined into the glaſs one, from which it runs out thro' the Tube G.

473 Since the Effects of all the Machines, deſcribed in this Chapter, depend upon the Preſſure of the Atmoſphere, the Water will not riſe in theſe Ma-
423 chines much higher than 32 Foot.

There

There are ſeveral little artificial Fountains, that are called the Fountains of *Hero*; I ſhall here give the Conſtruction of one of them.

Plate XXXII. *Fig*. 4.] Let there be two equal elliptical Veſſels A B and C D, exactly ſhut on all Sides, made of one Sort of Metal. 474

In each of them there is a Separation paſſing through the Center of the Ellipſe, which divides the whole Veſſel into two equal Parts.

The Separation *m n i*, in the Veſſel D C, is perpendicular to the Axis of the Ellipſe, the Separation *c f g h* of the other Veſſel muſt be inclined to that Axis.

There is a Brim raiſed round about the upper Part of the Veſſel A C B, to make a Baſon.

Four Tubes are joined to theſe Veſſels. The firſt *o p* goes through the Cavity B of the Veſſel A B, without having any Communication with it, and deſcends almoſt to the Bottom of the Cavity D; the ſecond *s t* is ſoldered to the upper Part of the Cavity D, and aſcends to the upper Part of the Cavity B, but not quite ſo high as to touch the upper Plate of it. The third *q r* reaches from the lower Part of the Cavity B, almoſt to the Bottom of the Cavity C; the 4th, *x u*, is made faſt to the upper Part of the Cavity C, and reaches almoſt to the upper Part of the Cavity A.

Laſtly, there is a Tube *z y*, which, going thro' the upper Plate, is ſoldered to it, and reaches down ſo deep in the Cavity A, that its End *z* is but a little Way off of the Bottom.

There are Cocks joined to every one of the Cavities; or elſe they have other Holes that are ſhut up with Screws that have Leathers on them; the chief Uſe of them is to let out the Water very clean from the Cavities, leſt they ſhould grow ruſty, when the Machine is not in Uſe.

Experiment

Experiment 5.] Pour in Water through the Tube *o p*, ſo as to fill the Cavity D; if you continue to pour in Water, it will riſe up through the Tube *s t*, and then deſcend through *q r* into the Cavity C, which is alſo filled, the Air aſcending up through *x u*, and going out through *z y*.

Turn the Machine upſide down, opening the Cocks of the Cavity C and D, the Water will deſcend into the Cavities B and A. Having again ſhut the Cocks, as alſo the Hole *y* of the Tube *z y*, ſet the Machine again the right Side upwards, and pour in Water again through the Tube *o p*, till the upper Surface of the Machine be covered with Water. Now, if the Hole *y* be opened, the Water will ſpout up to almoſt twice the Height of the Machine, and the Motion of the Water will continue, till the Cavity A be emptied of its Water. The Height of the ſpouting Water will continually diminiſh, and at laſt it will not be double the Diſtance of the Veſſels.

475 The Effect of this Machine is to be attributed to the Compreſſion of Air in the Veſſels. The Preſſure of the Atmoſphere at *o* and *y*, as alſo in the Veſſels, is equal, and theſe Preſſures deſtroy one another; therefore, they are not to be conſidered in the Examination of the Machine. When at laſt the Water is poured into the Tube *o p*, it is ſuſtained in it by the Preſſure of the Air contained in the Cavity D, and acting upon the Surface of the Water which ſtands at a ſmall Height in that Cavity; which Air, therefore, is compreſſed by the Weight of the Water, whoſe Height is *p o*. We ſpeak of the Preſſure, by which the Preſſure of the Atmoſphere is overcome. The Air in the upper Part of the Cavity B communicates with the Air above-mentioned by the Tube *st*, and is equally compreſſed, and acts with the ſame Force upon

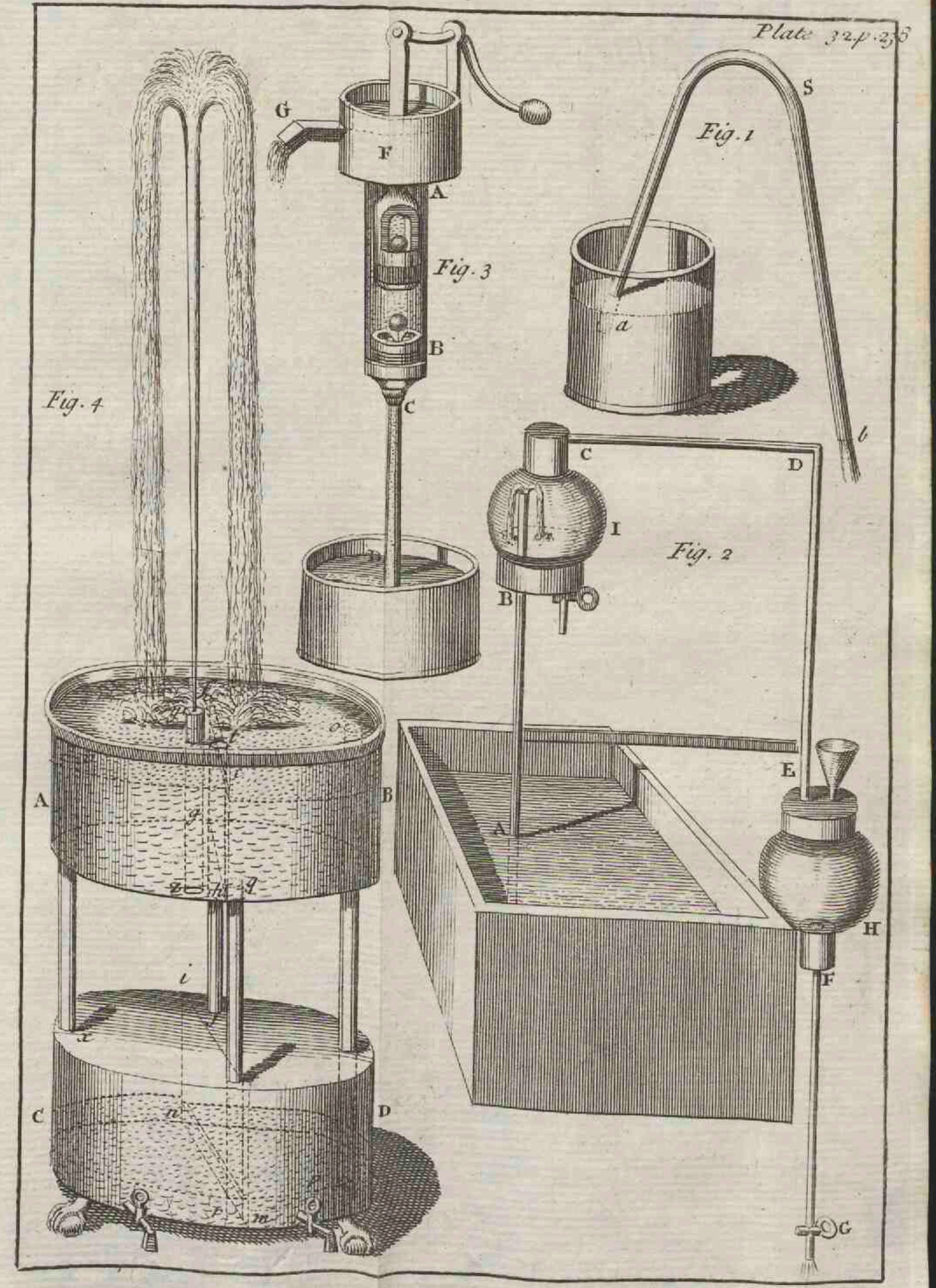
Fig. 1
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Fig. 2
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Fig. 3
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upon the Surface of the Water in that Cavity. This Preſſure is to be added to the Preſſure ariſing from the Height of the Water, in order to have the Force by which the Air is compreſſed in the Cavity C, as alſo in the upper Part of the Cavity A, by reaſon of the Communication through the Tube *x u*. The Preſſure, therefore, upon the Surface of the Water in that Cavity A, is equal to a Pillar of Water, whoſe Height is almoſt double the Weight of the whole Machine. And therefore it ſpouts as if it was preſſed by ſuch a whole Column, that is, to a Height not much wanting from the Height of that whole Column. * * 358

The Height is continually diminiſhed, for the Columns of Water, which compreſs the Air, continually become ſhorter, becauſe the Water aſcends in the Cavities C and D, and its Height is diminiſhed in the Cavity B. In the ſame Time the Cavity A is continually evacuated, and the Water aſcends through a greater Space, before it comes to *y*; therefore, it is driven to a leſs Height above *y*.

CHAP. XVII.

Of the undulatory Motion of the Air, where we ſhall treat of Sound.

I*F the Air be agitated in any Manner*, the Particles moved recede from their Place, and 476 drive the neighbouring Particles in a leſs Space; and as the Air is dilated in one Place, it is compreſſed in the Place next to it; the compreſſed Air, by the Reſtitution of the Spring, not only returns to its firſt State, but is alſo dilated by the Motion acquired by the Particles.

The Air, being firſt dilated by that Motion, is reſtored to its firſt State, and the Air is compreſſed

ſed towards other Parts. This again obtains, when the Air laſt compreſſed expands itſelf, by which a Compreſſion of Air is again produced; therefore, *from any Agitation there ariſes a Motion analogous to the Motion of a Wave on the Surface of*
*402 *Water.** The Air is compreſſed in that Manner with a Dilation following, ſo as to make what is
*703 *called a Wave of Air:* * Compreſſed Air always dilates itſelf every Way, and *the Motion of theſe*
478 *Waves is the Motion of a Sphere expanding itſelf* in the ſame Manner as the Waves move circularly
405 upon the Surface of the Water.

478 *Whilſt a Wave moves in the Air, wherever it paſſes, the Particles are removed from their Place, and return to it, running through a very ſhort Space in going and coming.*

Plate XXXIII. *Fig.* 1.] Now, to explain the Laws of this Motion, let us conceive Particles of Air to be placed at equal Diſtances, and to be in a right Line, as *a*, *b*, *c*, *d*, &c and f. Let the Wave be ſuppoſed to move along that Line. Now let us ſuppoſe it to be come forward along that Line, as far as between *b* and *p*, and that the Air is dilated between *b* and *h*, but compreſſed between *h* and *p*, as all this is repreſented in *Line* 1.

479 *The greateſt Denſity is at m, which is the Middle between h and p; and the greateſt Dilation between b and h, is in the Middle e.*

480 *Wherever the neighbouring Particles are not equally diſtant, the Motion, ariſing from Elaſticity, cauſes the leſs diſtant Particles to move towards*
*432 *thoſe that are moſt diſtant*; * and this Motion alone, abſtracting from all other Motion acquired, is to be examined.

Between b and e there is a Motion from *b* towards *e*, that is, *conſpiring with the Motion of the Waves; there is alſo ſuch a Motion between m and p.*

But

But *there is a* contrary *Motion between e and m, and it is directed from m towards e.* 482

At m and e, where the Directions of the Motions are changed, *no Action arises from the Elasticity,* because the neighbouring Parts are placed at equal Distances among themselves. 483

In the Places b, b, and p, the Difference of the neighbouring Parts is the greatest of all; and therefore, there *is the greatest Action of the Elasticity.* 484

From this it follows, that a Particle, according to its different Place in a Wave, suffers a different Action from the Elasticity by which its Motion is generated, accelerated, diminished or destroyed; therefore, the Direction of the Motion of a Particle cannot be determined from the Action of the above-mentioned Direction only, and does not always agree with that Direction, and the Motion of the single Particles is changed every Moment.

All the Particles between *b* and *p* are removed according to the Order of the Letters. The Particles between *b* and *p* continue their Motion, and the rest between *b* and *b* return towards *b*, as will be said hereafter.

These continue in the Motion by which they return, until, by the Action of the Elasticity, whose Direction is changed in the Point *e*, the Motion acquired anew be destroyed; in which Case a Particle, as *b*, returns to rest, and its first State. In the following Moment the Particle *c* comes to rest in its first State, but *p* comes forward to *q*, as in the Line 2, and successively in equal Moments, the Wave has all the Positions which are here represented in the Lines 1, 2, 3, &c. 485
and 13; and, *whilst the Wave,* from the Position in the Line 1, comes to the Position in the Line 13, *it runs thro' its whole Breadth. The Particle p,* in that Motion, *goes and returns, and* the Mo-

tion of it is made ſenſible in the Figure, and, as it plainly appears, this Particle *goes ſucceſſively through all the Situations of the Particles*; in the Waves all the Particles ſingly are agitated by the like Motion.

486 *The Motion of any Particle*, as *p*, *in its going backward and forward, is analagous to the Motion of a vibrating Pendulum, whilſt it* performs two Oſcillations, that is, *does* once *go forward and backward.* A Pendulum deſcends in its Oſcillation, and the Motion acquired conſpires with the Motion of Gravity, and is accelerated by it, until it comes down to the loweſt Part of the Arc to be deſcribed, that is, the Middle of the Way to be run through; the Pendulum goes on by the Motion acquired, which is deſtroyed by the Action of Gravity, whoſe Direction changes in this Point, whilſt the Body aſcends up the other Part of the Arc to be deſcribed: This Body returns by the ſame Laws.

The Particle *p* is moved by the Elaſticity, and this Motion is accelerated by the Action of the Elaſticity, until it comes to the Situation of the
*481 Particle *m*, the Line 1, * which Situation is ſeen in Line 4, in which the Particle *p* is, in the Middle Point of the Space, to be run through by the Motion backward and forward. By the Motion
*482 acquired, though Gravity acts againſt it, * it perſeveres in its Motion, until, by the Action of the ſaid Elaſticity, the Motion be wholly deſtroyed, which happens, when it has gone through a Space equal to that in which it was generated; then the Particle is in the Poſition which is ſeen in Line 7, which anſwers to the Situation of the Particle *h* in Line 1. Then, by the Elaſticity, the Particle returns, and is accelerated, until it has acquired the Situation of the Particle *e*, in
*482 Line 1, * as in Line 10, that is, until, as in Line

Line 4, it comes again to the Point that is in the
Middle of the Way to be run through. The Par-
ticle continues in its Return, until by the Action
of the Elaſticity, whoſe Direction is again chang-
ed *, the whole Motion is deſtroyed; and then *483
the Particle returns to its firſt Poſition, as in the
Line 13, and there, not being agitated by any
new Motion, it remains at reſt. Therefore, *the* 487
*Motion of the tremulous Body, by which the Air is
agitated, ceaſing, there are no new Waves genera-
ted*, and the Number of the Waves is the ſame as
the Number of the Agitations of that Body.

If, after two Vibrations of a Pendulum, the
Action of Gravity ſhould ceaſe, as in the Air,
after the going and returning of a Particle, the
Action of the Elaſticity on that Particle ceaſes,
the Motion of a Particle of Air would wholly
agree with the Action of a Pendulum. In the
middle Point of the Arc, which is to be run thro'
in the Oſcillation, there is no Action of Gravity,
and its Direction is changed; in the middle
Point of the Space to be gone through by the
Particle *p* in its going and coming, in which it is
in the 4th and 10th Line, the Situation of this
Particle agrees with the Situation of the Particles
m and *e* in Line 1, in which Points there is no
Action of Elaſticity, and its Direction is chang-
ged *. In a Pendulum, the more a Body oſcilla- *483
ting is diſtant from the loweſt Point or Middle of
the Arc to be deſcribed, by ſo much greater is the
Force of Gravity acting upon it; the more alſo
the Particle *p* is diſtant from the Space to be run
through, the more is the Action of the Elaſticity
upon it; and in the Lines 1, 7, and 13, the Par-
ticle is moſt diſtant from the Point above-menti-
oned, and its Situation there agrees with the
Points, *b*, *b* and *p* in the Line 1, in which the
Action of the Elaſticity is greateſt of all *. *484

According to which Law, ſince this Action of
Elaſticity increaſes with the increaſed Diſtance of
the often-mentioned middle Point, it is deter-
mined from the very Law of the Elaſticity of the
Air, whoſe Particles drive one another away with a
Force which is inverſely as the Diſtance between
*432 the Centers of the Particles *: and it is demon-
ſtrated, that the Action of Elaſticity upon ſuch
a Particle, as *p*, is increaſed or diminiſhed in Pro-
portion to the Diſtance of the middle Point of
the Space to be run through: And, therefore, alſo
in that Part there is an Analogy between the Mo-
tion of a Particle and the Motion of a Pendulum
*156 oſcilating in a Cycloid *.

If the Breadth of a Wave remaining, the Par-
ticles run out thro' a greater Space, the Compreſ-
ſion and Dilatation of the Air in the Wave will be
greater, and there will be a greater Action of Ela-
ſticity, and that greater in the ſame Ratio in which
the Space gone through in the going and coming
is increaſed: And the Motion of a Particle, as *p* in
this Caſe, differs from the Motion in the foregoing
Caſe, as the unequal Oſcillations of different Pen-
dulums differ; which, as they are performed in
*156 equal Times *, the ſame will alſo obtain here.

Therefore, a Particle, as *p*, if the Breadth of
the Wave continues, the ſame goes and comes in
the ſame Time, through whatever Space it be car-
ried out of its Place; that is, the Wave will go its
488 Breadth in the ſame Time; therefore, *all equal
Waves, whether the Air be more or leſs agitated,
are equally ſwift.*

489 Now *let* us examine unequal *Waves*; let them
be as A to B, and let the Space gone through by
the Particles in the Motion of each of them, in
going and coming, be in the ſame Ratio; in that
Caſe the Compreſſions and Dilatations in corre-
ſpondent

spondent Places will be equal; *the* Actions, therefore, from the Elasticity, don't differ in correspondent Distances from the middle Point of the Spaces to be run thro' by the Particles, in their going and coming. Therefore, those *Motions are analogous to the Motions of two Pendulums, whose Lengths are as* A *and* B, and which run thro' similar Arcs; for, in the correspondent Points of those Arcs, the Action of Gravity is the same.

In Pendulums the Action of Gravity increases as the increased Quantity of Matter; and whatever be this Quantity, the Motion is equally swift, when the Gravity is not changed: On the contrary, the Action of Elasticity is determined in the Motion of Waves, and depends upon the Distance between the Particles and the Velocity, which is generated from it; the Elasticity remaining the same, is inversly as the Quantity of Matter to be moved.* *In the Waves above-mentioned,* *65
the Quantities of Matter are as the Breadth of 490
the Waves *a* and *b*, and the Velocities generated by the Elasticity are, therefore, in correspondent Points as *b* to *a*. Therefore, these *Motions are analogous to the Motions of Pendulums* describing similar Acrs, and *moved with different Forces of Gravity, which are to one another as* B *to* A; for, in correspondent Points of similar Arcs, the Celerities arising from different Gravities are as those Gravities.

Now to compare the Motion of Waves with the Motion of Pendulums, we must consider Pendulums differing in Length, and on which different Forces of Gravity act*, and we have shewn *489
what these Causes produce singly in the Dura- 490
tion of the Vibrations.* Both these are to be *158
joined together, and the Squares of the Times 165
of the Oscillation of Pendulums, whose Motions

are analogous to the Motion of the above-men-
*480 tioned Waves, are as the Length A and B, * and
158 *190 inversely, as the Gravities B and A; * that is,
165 again directly, as A and B; the Ratio of which Ratio's is a Ratio, compounded of the Squares of the Quantities A and B. Therefore, the Times of the Oscillations are as A and B, and the Times are in the same Ratio in which the Particles of the Waves go and come; that is, the Waves run through their Breadths, which are as A to B; which Times are, therefore, as the Spaces gone thro' by the Waves, and, therefore, the Motions are equally swift. If the Space be changed thro' which the Particles go and come, the Velocity
488 of the Waves is not changed; * wherefore, the Proportion which we have put down for a Demonstration, between the Spaces gone thro' by the Particles in their going and coming, may be neglected, and the Proposition will be generally
491 true, that *Waves, whether equal or any way unequal, move with the same Velocity.*

492 This Rule will hold good, *if* the State of the Air is not changed; but *the Elasticity remaining the same, the Density of the Air often varies; and* the Elasticity *may be changed*, the Density remaining the same; lastly, both are often liable to be changed.

In the first Case, supposing both the Waves to be equal, and also the Spaces thro' which the Particles go and come, the Celerities arising from the Elasticity, which is always the same, are in-
*65 versely as the Densities; * but this Variation of
288 Celerity answers in the Motions of equal Pendu-
*492 lums, with the Variation of the Gravity *, in which Cases the Squares of the Celerities of the
*165 Vibrations are as the Gravities themselves *; therefore, *in Waves their Squares of the Celerities are inversely*

inversely as the Densities. The Supposition of the Equality of the Waves, and the Spaces gone thro' by the Particles, does not hinder this Demonstration from being universal *. *488

When the Density remains the same, but the E- 491
lasticity is changed, the Celerity arising from it va- 493
ries in the same Ratio as the Elasticity; wherefore, from the Demonstration of the foregoing Proposition in this Case, *the Squares of the Celerities of the Waves are as the Degrees of the Elasticity.*

If the Elasticity and the Density differ, the Squares 494
of the Velocities of the Waves will be in a Ratio compounded of the direct Ratio of the Elasticity *, *493
and the inverse Ratio of the Density *. *492

If the Density and the Elasticity increase or de- 495
crease in the same Ratio, the inverse Ratio of the Density will destroy the direct Ratio of the Elasticity, and *the Celerity of the Waves will not be changed.*

This last Case happens in the Compression of 496
the Air *. Therefore, *from the changed Height of* *430
the Pillar of Mercury, which is sustained in a Tube void of Air by the Pressure of the Atmosphere *, *422
which shews, that the Weight, by which the Air is compressed near the Earth, is changed, *we must* 497
not judge the Celerity of the Waves to be changed. For the same Reason, *the Waves are moved with the same Celerity in the Top of a Mountain as in a Valley,* unless there be a Change of the Elasticity itself, by reason of the Cold, which is almost always more intense on the Top of a Mountain than in a Valley; and this would occasion the Waves to move slower *. *493

It is plain also, that *the Waves move faster in* 498
Summer than in Winter *. *434

The Celerity of the Waves is compared to 499
the Celerity which a Body acquires in falling, *by determining,* from the known Height of the

Mercury, which weighs equally with the Pref-
*422 fure of the Atmofphere *, and alfo the Denfity
*442 of the Air *; *the Height of the Atmofphere, fup-pofing it every where equally denfe with the Air near the Earth; the Velocity of the Waves will be the fame as a Body could acquire in falling from half that Height.* Which Velocity, from what has been faid, may be eafily difcovered by Expe-
157 riments made upon Pendulums.
158 If the Weight, by which the Air is compreffed, be diminifh'd, the Air expands itfelf in the fame
42 Ratio; and fuppofing the Atmofphere every where of the fame Denfity, its Height does not vary, which agrees with what has been faid, that the Velocity of the Waves is the fame in different
496 Compreffions of the Atmofphere.

500 The Motion of the Air, which we confider in this Computation, arifes from Elafticity alone; and the Computation would be exact, if the Particles themfelves had not a fenfible Proportion to the Interftices between them; but if we fuppofe here that they bear a fenfible Proportion to them, the Motion of the Waves will be fwifter; for it is propagated through folid Bodies in an Inftant, which muft alfo be referred to heterogeneous Corpufcles fwimming in the Air.

501 *The Motion of Waves in the Air produces Sound*; of which, before we fpeak, we muft lay down fomething in general relating to Senfation.

502 So ftrict is the Union of the Body and the Mind, that fome Motions in the Body do, as it were, cohere with certain Ideas in the Mind, and they cannot be feparated from each other. From the Motion of the Body are new Ideas every Moment excited in the Mind, and fuch are the Ideas of all fenfible Objects; yet we find nothing common between the Motion in the Body and the Idea

Idea in the Mind. We cannot perceive what Connexion is here, nor that any Connexion is poſſible. There are an infinite Number of Things hidden from us, of which we have not ſo much as an Idea.

The undulatory Motion of the Air agitates 503
the *Tympanum*, or Drum of the Ear, by which Means a Motion is communicated to the Air contained in that Organ, which, being carried to the auditory Nerve, excites in the Mind the Idea of Sound.

The Structure of the Ear, both internal and external, is wonderful; but here we treat of the Motion of the *Air*; that it *is the Vehicle of Sound*, is proved by the following Experiment.

Experiment 1. *Plate* XXXIII. *Fig.* 2.] Take the Leaden Plate O, which has two cylindric Pillars of the ſame Metal C, C, fixed to it; join a little Bell A to the braſs Wire B D, and let it be tied with Strings to the Pillars C, C; lay the Plate O upon the braſs Plate of the Air-Pump, putting between a little Cuſhion of Cotton, or Raw-Silk; ſet a Receiver on over all this Apparatus: Cover the Receiver with a Plate that has the Collar of Leathers ſcrewed to it, through which the braſs Wire D E can ſlip up and down *; to the *440
braſs Wire you muſt faſten the Plate *e f*, ſo that, by turning the Wire round, the Bell A may be agitated. Pump out the Air from the Receiver, and ſhaking the Bell in the Manner before deſcribed, you will not hear the Sound. By turning the Wire D E, the Bell will move backward and forward ſeveral times; but we are only to obſerve that Motion in which the Plate *e f* doth not touch the Wire *b d*. Letting in the Air, the Sound will be heard as before.

From

505 From this alone, that the Air is the Vehicle of Sound, and that Sound is moved thro' it without the Air's being carried from one Place to another, it evidently follows, that in Sound there is an undulatory Motion of the Air, and that Sound arises from the tremulous Motion of Bodies. That this obtains in Cords, or Strings of musical Instruments, no Body doubts, since, by giving them a tremulous Agitation, they produce a Sound. In great Bells, and several other Bodies, this tremulous Motion is very sensible; but it will become visible by the following Experiment made upon a sounding Glass Bell.

Experiment 2.] Let the Glass Bell CC be fixed with Plaister, or Cement, to a wooden Screw, by Means of which it may be made very fast to the transverse Piece of Wood AB; this Wood must be sustained by two wooden Pillars SS, to which it is firmly join'd with Screws and Nuts. There is a Pin with a Screw upon it, that goes through one of the Pillars, just even with the Mouth of the Bell; so, by screwing it forwards or backwards, you may set it nearer to, or farther from, the Edge of the Bell. If this Distance be very small, and the Bell be struck, it will, by its tremulous Motion, strike several times against the Pin with its Edge.

506 Hence we deduce, that *a Body that is struck, continues to give a Sound some Time after the Blow*; the agitated Fibre will continue his Vibra-
*215 tion some Time, on Account of the Elasticity *; we often see, as in *Experiment* 1, that a Body gives a Sound, tho' the Air, agitated by it, has no Communication with the outward Air; whence it
507 follows, that *by the Agitation of the Air, the Fibres*

of which Bodies consist, are moved; which Motion is transferred into the external Air.

This Translation of the Sound, by the tremulous Motion of the Fibres, is very remarkable; and how the Communication of this Motion extends itself, will appear by a single Experiment.

Experiment 3. *Plate* XXXIII. *Fig.* 2.] This Experiment differs from the first only in this; that if, instead of tying the Bell to the leaden Machine C O C, it be fastened to the Ends of a brass Plate bent in the Figure of a double Gnomon, which is made fast by a Screw to the Plate of the Air-Pump, and the Air be pumped out, and the Bell shak'd in the same Manner as in the first Experiment; you will find but very little Difference between the Sound that is made, when the Air is exhausted, and when the Air is re-admitted.

The tremulous Motion of the Parts of the Bell is communicated to the brass Wire *b d*, so as to move the Strings by which the Bell is suspended, and this Motion is transferred to the bent brass Plate; the Screw, with which this Plate is joined to the brass Plate of the Air-Pump, touches the Plate, and communicates a tremulous Motion to it, by which the Air is agitated, and the Sound of the Bell is heard.

The Celerity of the Sound is the same as the Ce- 508
lerity of the Waves, which strike the Ear; and to
this must be referred what has been said of their *491
Celerity. * In respect to *Numb.* 499, it is to be 492,
observed, that the Celerity of Sound can no way 493, 494,
be determined by Calculation; for the Propor- 495,
tion between the Diameters of the Particles and 496,
in the Interstices between them, is not known, 497, 498,
neither 499,

neither how large a Space the heterogeneous Particles take up in the Air.

The Celerity of Sound may be immediately determined by an Experiment.

509 If a Flash of Fire goes off at Night with a Noise, and a Spectator stands at any known Distance from the Fire, who, with a short Pendulum, measures the Time between seeing the Light, and hearing the Sound, he will have the Celerity of the Sound; for the Motion of Light, at least, thro' the Space as such an Experiment can be made in, is instantaneous.

510 By such an Experiment made in *France*, it appear'd, that Sound run 1800 *French* Feet in a Second Minute of Time; but this Celerity is not
498 constant.

511 If at the same Time in which the Velocity of the Sound is determined by this Method, there
*492 be made the two Experiments above-named *,
499 one may, by Calculation, determine the Motion
499 of Sound by the Elasticity of the Air, and by comparing it with the Velocity immediately mention'd, you will have the Acceleration of the Sound, from the Thickness of the Particles, and
512 the heterogeneous Matter.

*491 *The Celerity of the Sound is equable*; yet in going through a greater Space, it is sometimes ac-*
*493 *celerated or retarded*, from the different Degree of Elasticity in different Places, in which there
494 are different Degrees of Heat or Cold.

513 *The Celerity of the Sound does not much differ, whether it goes with the Wind, or against the Wind.* By the Wind a certain Quantity of Air is carried from one Place to another; the Sound is accelerated as long as it moves through that Part of the Air, if the Direction of the Sound be the same with the Direction of the Wind; but as Sound moves very swift, in a very short Time it will

will run through the Air which is agitated by the Wind, and the Acceleration does not last long, which, indeed, is not very great; for the most violent Winds, which are strong enough to root up Trees, and blow down Houses, have their Celerity to the Celerity of the Sound, but about as one to 33; by the same Argument it is proved, that no sensible Retardation is occasioned by the Wind, when the Sound moves against it.

The Space which the Particles run through, as they come and go, may be increased and diminished by the Wind; *therefore, the Sound may be heard at a greater or smaller Distance, according to the Direction of the Wind.*

The Intensity of the Sound depends upon the Strokes of the Air on the auditory Nerve, and these Strokes are as the Quantities of Motion in the Air.

Whence it follows, that, *cæteris paribus, the* 515
*Intensity of the Sound is as the Space run through by the Particles in their going and coming**. *488

All Things remaining as before, if the Weight 53,
by which the Air is compressed be changed, the 63.
Celerity does not vary *, but the Density is chan- *496, *429,
ged in the same Ratio as the Weight*. 229

Therefore, *cæteris paribus, the Intensity of the* 516
Sound is as the Weight by which the Air is compressed *, that is, this Intensity increases and de- *62
creases, as the Pillar of Mercury, which is *in Æquilibrio* with the Weight of the Atmosphere.

Experiment 4. *Plate* XXXIII. *Fig.* 3.] Shake the Bell A in compressed Air * exactly in the *454.
Manner as it was shaked *in Vacuo*, in *Experim.* 1, and the Sound will be increased; which will again be diminished, if opening the Bell you let the Air return to its first State.

As

As the Intenſity of the Sound in compreſſed Air, included in a Veſſel, is greater, ſo the Fibres, of which the Glaſs V V is made, are agitated, and a greater Agitation is communicated to the external Air.

517 *If all Things remain as before, but the Elaſticity be increaſed*, the Denſity is diminiſhed in the ſame
*430 Ratio as the Elaſticity is increaſed * ; but the Celerity increaſes as the Square Root of the Ela-
*493 ſticity * ; therefore, *the Intenſity of the Sound is* directly *as the Square Root of the Elaſticity, and in-*
*64 *verſely as the Elaſticity itſelf* *; but the Ratio, compounded of theſe, is the inverſe Ratio of the
518 above-mentioned Square Root of the Elaſticity. The Intenſity of the Sound is diminiſhed; there-
519 fore, as its Velocity is increaſed, and *in Summer*, cæteris paribus, *the Intenſity of Sound is leſs than in Winter*; *yet, in Summer Bodies do more eaſily tranſmit Sound*, becauſe their Parts cohere leſs ſtrictly, as will be explained at a proper Time, and they do more eaſily acquire a tremulous Motion.

Experiment 5. *Plate* XXXIII. *Fig.* 5.] Hang up the Bell A in a Glaſs, and opening the Cock, that the Air in the Glaſs may have Communication with the external Air, let the Glaſs be ſhaked, and the Diſtance be determined when the Sound can be heard; warm the Glaſs, and repeat the Experiment, and the Sound will be heard at a greater Diſtance.

520 *The Intenſity of Sound, conſidered in general, is in a compound Ratio of the Space run thro' by the Par-*
*515 *ticles, in their going backward and forward**, *of*
*116 *the Weight compreſſing the Air* * ; *and laſtly, of the*
*517 *inverſe Ratio of the Square Root of the Elaſticity*.*

521 There is alſo a Difference in Sound, from the Number of the Vibrations of the Fibres of the Body

Body which produce the Sound, that is, of the Number of the Waves produced in a certain Time, according to the different Number of the Percussions in the Ear, the Mind receiving a different Sensation.

A Musical Tone depends upon this Number of 522 Vibrations, which is said to be the more *acute*, according as the Returns in the Air are more frequent; and more *grave*, the less the Number of the Waves is; *and the Degrees of the Sharpness of the different Sounds are to one another, as the Number of the Waves which are produced in the Air at the same Time.* 523

A Tone does not depend upon the Intensity of the 524 *Sound, and an agitated Cord gives the same Sound, whether it vibrates through a greater or a less Space.** *257

Concords arise from the Agreement between the 525 *different Motions of the Air, which affect the Auditory Nerves at the same Time.*

If two tremulous Bodies perform their Vibra- 526 tions in the same Time, there will be no Difference between their Tones and this Agreement, which is the most perfect of all, is called *Unison.*

If the Vibrations are as 1 to 2, this Consonance, 527 or Agreement, is called *Octave*, or *Diapason.*

Supposing the Vibrations as 2 to 3, that is, if 528 the second Vibration of one Body always agrees with the third of another, such a Consonance is called a *Fifth*, or *Diapente.*

Vibrations, which are as 3 to 4, give a Conso- 529 nance, which is called a *Fourth*, or *Diatessaron.*

Ditonus is, when the Returns of the Air are 530 as 4 to 5.

And *Sesquiditonus* is a Consonance, from a Con- 531 course of the fifth Vibration of one Body with the sixth of another.

A Con-

A Conſonance from the Agitation of Cords, if they be of the ſame Kind, is eaſily determined, by knowing their Dimenſions and Tenſion.

532 Cæteris paribus, *if the Lengths of two Cords are as the Number of Returns in a Conſonance, you will have the Conſonance between the Sounds*
*259 *which the Strings produce.* *

533 *The ſame* obtains, *if*, cæteris paribus, *the Dia-*
*260 *meters have the aforeſaid Proportion.* *

534 And alſo, *if*, cæteris paribus, *the Proportion of the Vibration in a Conſonance be given between the*
*258 *Square Roots of the Tenſiions.* *

535 *And generally, ſuppoſing any Cord of the ſame Kind, if the Ratio be compounded of the direct Ratio of the Lengths and of the Diameters, and the inverſe Ratio of the Square Roots of the Tenſions be the Ratio between the Numbers of Vibrations performed in the ſame Time in any Conſonance whatever, you will have that Conſonance by the*
*261 *Agitation of thoſe Cords.* *

All theſe have been experimentally tried by Muſicians; they have obſerved a very remarkable Phænomenon relating to theſe Cords, whoſe different Caſes very well deſerved to be explained.

536 *Let any Muſical Strings be ſo extended, as to perform their Vibration in equal Times; if you give Motion to the one, the other will alſo move.* Every Wave of the Air, ariſing from the tremulous Motion of the firſt String, ſtrikes the ſecond String, and gives it a little Motion; the String, from the leaſt Motion, goes backward and forward ſeveral
*257 Times *, and is moved by the Stroke of the firſt Wave, whilſt the ſecond Wave comes forward, whoſe Motion conſpires with the Motion of the
*257 String *, and accelerates it. What is ſaid of the ſecond Wave muſt alſo be referred to the other Waves that follow, and there will be an Acce-

Acceleration, 'till the Motion of both Strings be almoſt equal.

From the ſame Demonſtration it follows, that 537 *an agitated String will communicate Motion to another, which performs two or three Vibrations, whilſt the firſt performs but one.*

Now, if the agitated String performs ſeveral Vibrations, whilſt the String that is to be moved by the Air can perform but one, from the foregoing Demonſtration it would follow, that it muſt communicate a particular Motion to it. To diſcover which, it is to be obſerved, that the Duration of the Vibration and the Length of the String are reciprocal; ſo that every Thing elſe continuing as before, the determined Length can no way be ſeparated from the unchanged Duration of the Vibration. If therefore any String be ſtruck with ſeveral Strokes, by which, Motion is communicated to it, and the Strokes are more frequent than what is agreeable to the Length of the String; that Part of it, whoſe Length agrees with the Time of the communicated Vibration, will be agitated as much, and there will be, as it were, an undulatory Motion communicated to the String; the Length of the Waves in the String will depend upon the Duration of the communicated Vibration, that is, upon the Time between the Strokes.

Take two Strings, in ſuch Proportion, that one 538 *may vibrate twice whilſt the other vibrates but once*, and let the firſt String be put in Motion; the Duration of the Vibrations, which are communicated to the laſt String with the Motion of the Air, agrees with a String of half its Length *, * 259
and ſuch is the Length of the Waves in it: Therefore, *by the communicative Motion, the String is divided into two equal Parts*, and the Middle

 Point

Point is at reſt. This is confirmed by an Experiment, if you lay a Piece of Paper upon the String to which the Motion is communicated; for it will remain at reſt, if you lay it upon the Middle of the String, but any other Part of it will be affected with a tremulous Motion.

539 *If the String which is put into Motion, in order to cauſe Motion in another, performs three Vibrations, whilſt the String, to be moved, performs but one, the laſt will be divided into three Parts by the communicated Motion, and there will be two Points of Reſt*; which may be confirmed by the ſame Experiment above-mentioned. All other Caſes that have communicated Motion, which are obſerved by Muſicians, are eaſily deduced from what has been ſaid.

540 What has been ſaid of the Reflexion and In-
* 406 flexion of the Waves in Water *, may be referr'd
408 to their Reflexion in Air, the Elaſticity in this
469
410 Cauſe producing the ſame Effect as the Preſſure of the raiſed Water in that.

541 *From the Reflexion of the Sound there often ariſes a Repetition of it, which is called an Eccho.* If different Parts of the ſame Wave, expanding
* 477 itſelf into a Sphere *, ſtrike upon different Surfaces, ſo that being reflected they concur together, the Motion of the Air will be ſtronger there, and the Sound will be heard. *The ſame Sound is often repeated different times* from the different Parts of the ſame Way reflected to different Diſtances, and ſome of which alſo ſucceſſively concur at the
542 ſame Place. Such a Repetition ſometimes happens from the Reflexion being repeated.

541 *The Sound is often increaſed by Reflexion in a Tube*: The moſt perfect Figure of all that can be given to ſuch a Tube, is that of a Parabola, revolving about a Line a Quarter of an Inch diſtant from the

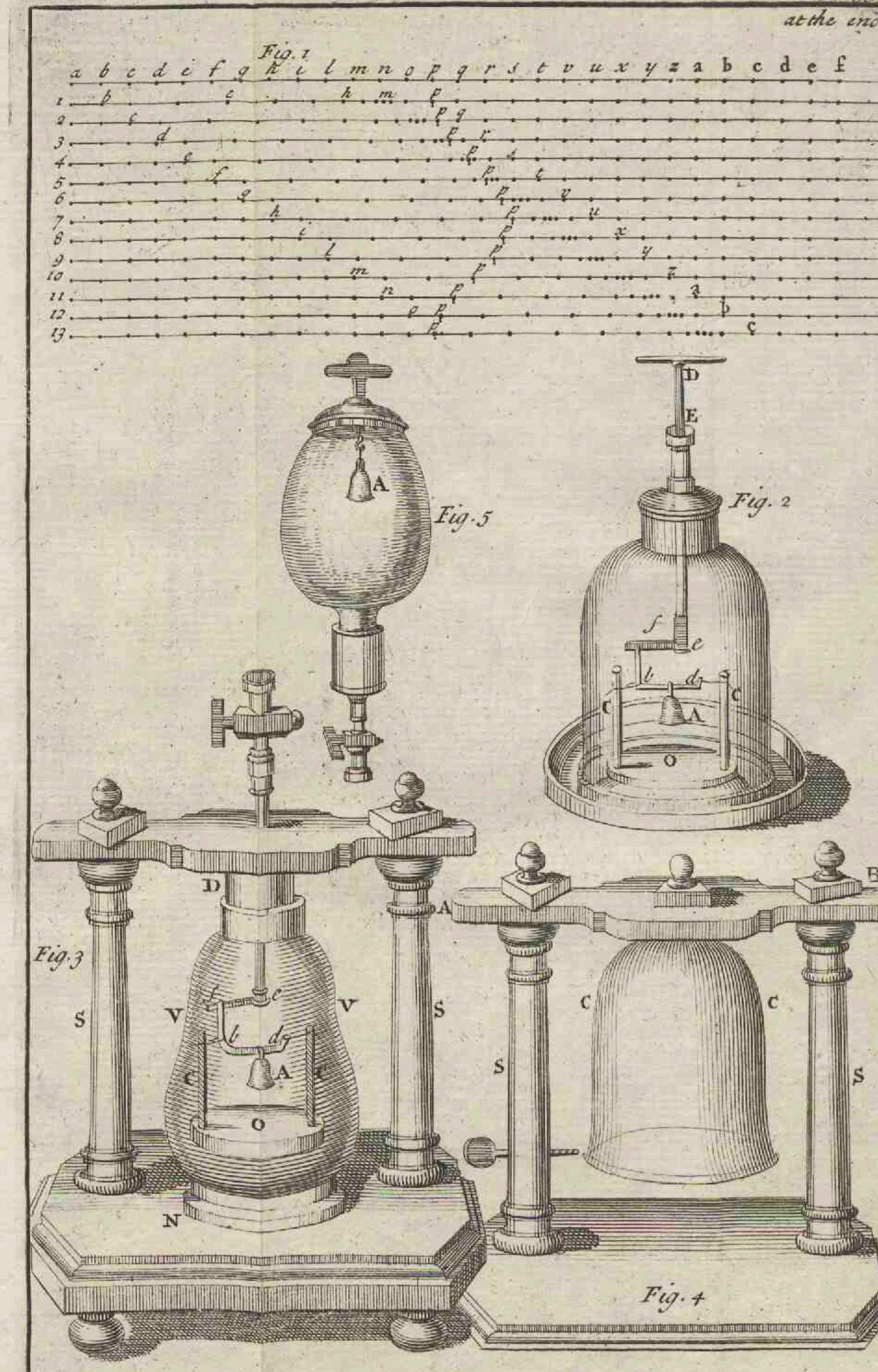
Fig. 1
a b c d e f g h i l m n o p q r s t v u x y z a b c d e f
Fig. 5
A
Fig. 2
D
E
f
e
b
d
C
A
O
Fig. 3
D
A
S
V
f
e
b
d
A
C
O
N
Fig. 4
B
C
S

the Axis. For if any one ſpeaks in ſuch a Tube, ſetting his Mouth in the Axis of the Machine, and in the Focus of the Parabola, the Waves will be ſo reflected, that every one of their Parts will acquire a Motion parallel to the Axis of the Machine, whereby the Force of the Wave, and alſo of the Sound, will be very much increaſed. There muſt be a Mouth-Piece, to fit the Lips, fixed to the End of the Tube.

FINIS.

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