The elements of natural or experimental philosophy

1886

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OF

NATURAL OR EXPERIMENTAL

PHILOSOPHY.

BY

TIBERIUS CAVALLO, F.R.S. &c.

ILLUSTRATED WITH COPPER PLATES.

IN FOUR VOLUMES.

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Printed by Luke Hansard,

FOR T. CADELL AND W. DAVIES, IN THE STRAND.

1803.

Luke Hansard, Printer,
Great Turnstile, Lincoln's-Inn Fields.

THE

PREFACE.

THE principal characteriſtic which diſtinguiſhes the human being from the reſt of the animal creation, is the inheritance of knowledge, which the individuals of each generation are able to derive from their predeceſſors.

The bee of modern times forms the cells of its hive exactly of the ſame ſhape as the bee of the remoteſt antiquity; each ſpecies of birds builds its neſt after the ſame unalterable pattern, and ſings the ſame invariable melody. The ſheep of the preſent day has no better defence againſt the wolf; nor has the fly againſt the ſpider, nor the ſmaller birds againſt the eagle, than the like animals of former times. The ſame wants, ſimilar dangers, the like defects, and unalterable cuſtoms, are the conſtant attendants of each different tribe; nor is any individual benefited by the experience or by the improvements of all its predeceſſors.

Man alone has received from his Divine Creator the ineſtimable advantage of being benefited by the knowledge of his forefathers, and of his being able to bequeath that knowledge, together with his own improvements, to his poſterity.

The accumulated experience of a long ſeries of years, accurately recorded in a vaſt many books, or traditionally imparted from one generation to the other, gradually exalts the ſtate of human beings, ſupplies their wants, increaſes their ſecurity, and promotes their happineſs. The plough, the loom, the forge, the preſs, the glaſs-houſe, and innumerable other uſeful inventions of our predeceſſors, ſucceſſively improved by conſtant uſe and experience, form the invaluable advantages of modern times; and their combined effect, actually elevates the individuals of a modern civilized nation, ſo far above the uninſtructed ſavages, as might almoſt ſeem to render them of a different ſpecies.

That experience, properly difpofed under diftinct heads, forms the various fubjects of knowledge. The arrangement, and the elucidation of each particular fubject, is called a Science. The ultimate or the practical application of it is called an Art.

Arts and fciences are too numerous and too extended, to be comprehended in their greateft extent by each fingle individual: hence is derived the divifion of labour, or the adoption of a particular branch by each fingle individual. But all thofe branches derive their origin from the fame natural powers, they are all in their principles regulated by the fame general laws of Nature, and almoft all their applications may be fubjected to calculation and demonftration. The inveftigation of their origin, and of their mutual dependence on each other, the illuftration of their principles, the methods of enlarging their limits by means of experiments and calculation, and their application to our various wants, fall under the title of NATURAL OR EXPERIMENTAL PHILOSOPHY, the ELEMENTS of which form the fubject of this Work.

In the courſe of the laſt twenty or thirty centuries, during which time (as written documents inform us) more or leſs attentive obſervations have been made on the properties of natural bodies, various theories have been formed, or different ideas have been entertained concerning the nature of thoſe bodies, or concerning the general ſubject of Natural Philoſophy; but the ſmall proportion of real facts, and the vaſtly greater proportion of vague and unwarrantable ideas which formed thoſe theories, rendered them always inſufficient, and frequently abſurd; whence confuſion of ideas, and retardation of ſcience, naturally enſued.

The nature and the fate of thoſe theories gradually cautioned the judicious part of the inquiſitive world, and ſhewed them the neceſſity of ſubſtituting experiments and ſtrict mathematical reaſoning to the ſuggeſtions of the imagination. This rational reform, or cautious mode of proceeding, ſince the 16th century, has been productive of a vaſt number of uſeful diſcoveries; and, by its having placed the progreſs of ſcience in the right channel,

channel, has enabled philoſophers to trace out the principles of ſeveral of its branches, to inveſtigate divers new ſubjects, and to open new paths to the inexhauſtible treaſures of nature.

The progreſs of experimental inveſtigation, and the mathematical mode of reaſoning, are both ſlow and laborious; but they are ſafe, and productive of true and uſeful knowledge: nor has the human being any other means of feeling his way through the dark labyrinth of Nature. It is wonderful to obſerve what manual labour, and what exalted exertions of the human mind, have been beſtowed upon the various branches of Natural Philoſophy. Thoſe profound inquiries, ſometimes fruitleſs, and at other times either directly or indirectly ſucceſsful, alternately diſplay the ſtrength and the weakneſs of the human underſtanding; but upon the whole, it muſt be acknowledged that wonderful improvements have undoubtedly been derived from thoſe extraordinary exertions; and the progreſs of ſcience within the laſt two centuries has certainly advanced with increaſing velocity.

It is not my intention to deceive the reader by aſſerting, that I have rendered all the principles of Natural Philoſophy intelligible to the meaneſt capacity; for in that caſe, I ſhould either have been obliged to omit the more abſtruſe branches of philoſophy, or the fallacy of the aſſertion would be rendered ſtaringly manifeſt in ſeveral of the following pages. Original diſcoveries of facts, or principles or laws of nature, are generally made through intricate and perplexed paths. By ſubſequent reviſion and conſideration, the ſuperfluous is removed, the defective is ſupplied, and the confuſed materials are properly arranged; whence the train of reaſoning frequently becomes ſhorter and more natural, or the nature of the ſubject is rendered more evident and more intelligible. But this ſimplification has a limit which differs in different ſubjects; nor can the comprehenſion of what depends upon a vaſt number of previous ideas, mathematically connected, be rendered attainable to ſuch perſons as are deſtitute of ſuch ideas, or whoſe mind is incapable of retaining the neceſſary chain of reaſoning.

By

By following the example of the cleareſt writers, and by conſidering each particular ſubject in different points of view, I have endeavoured to explain it with all the ſimplicity and the clearneſs which my ſlender ability could ſuggeſt. In ſeveral places I have avoided ſome abſtruſe technical formalities of order or phraſeology, and have preferred familiar expreſſions wherever it appeared practicable; but when the ſubject ſeemed leſs likely to be comprehended by the greateſt number of readers, I have always placed it in the Notes, where thoſe only who are competently qualified may read it. And here it muſt be obſerved, that, for the ſake of diſtinction, the references from the text to thoſe notes, conſiſt of the common numerical figures; whereas the references to other notes containing quotations, additional remarks, &c. conſiſt of aſteriſms, or ſuch like marks.

A few repetitions, which the reader will meet with in the courſe of the work, will, I truſt, be eaſily excuſed, conſidering that they have been thought neceſſary for promoting the elucidation of particular ſubjects. With

reſpect

respect to the termination of certain words of an entire Latin origin, it must be observed that I have indiscriminately written them either with a Latin or with an English termination, such as *radii* and *radiuses*, *media* and *meidums*, &c. for having found them used both ways by different writers, I was unwilling to adopt a decided partiality for either mode.

With respect to the disposition of the materials throughout the work, it may perhaps be necessary to mention, that my rule has been to begin with the general properties of matter, or such as constant experience shews to belong to bodies of every kind. I have afterwards proceeded to examine those which belong to a particular set of bodies, and then those of fewer or of single bodies.

The astronomical part has been naturally placed after the statement of the above-mentioned properties, since the knowledge of the appearances of the celestial bodies is not so immediately concerned with our welfare, as that of the substances which nearly ſurround us, and of which our very bodies are formed.

In the illuſtration of the various branches, a multiplicity of experiments and extended hiſtorical accounts have been carefully avoided, leſt the ſtatement of ſuperfluities ſhould have occupied the place of uſeful materials. The different ſubjects of Natural Philoſophy cannot be rendered ſufficiently intelligible without a certain extent of explanation; but at the ſame time their number would render the work too extenſive, if the limits of abſolute neceſſity were not carefully preſerved. In this, however, the Author is expoſed to a dangerous dilemma, as the ſame illuſtration which proves prolix to certain readers, is inſufficient for others. Different views of the ſame abſtruſe ſubject, though tedious to the proficient, are undoubtedly of great aſſiſtance to the novice. In this caſe the limits of ſufficiency or of inſufficiency are vague and indeterminate; and whilſt they tend to perplex the author, they afford, according to the inclination of the reader, ample ſcope for criticiſm or ſatisfaction. Natural order, ac-

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curacy of ſtatements, perſpicuity, and conciſeneſs, have been the conſtant objects of my views in the compilation of this work. I have endeavoured to ſelect from multiplicity, and to remove obſcurity. In certain places I have added new facts, in others I have ſearched for new and true explanations of natural effects. I have pointed out the defects of ſeveral particulars, and have recommended the elucidation of the ſame to the diligence of zealous ſtudents. But whether or not the performance is ſufficiently conformable to thoſe views, I humbly ſubmit it to the deciſion of the impartial and diſtinguiſhing part of my readers.

As this work is likely to fall into various hands, it may perhaps be uſeful to add a few remarks and a few directions for the uſe, not of the proficient, but of thoſe to whom the ſubject is either partially or entirely new, in order that unprofitable labour, or extravagant expectations, may in great meaſure be avoided.

Of the various readers of books in general, I ſhall briefly attempt to diſcriminate the fol-

lowing

lowing claſſes. There are ſome who imagine that the ſame velocity of reading is ſufficient for a novel, or a poetical, or an hiſtorical, or a ſcientific book; and when they find that they are not able to comprehend the latter, they conclude either that the author is obſcure, or that they themſelves have not capacity ſufficient for it. Others imagine that a ſingle careful peruſal of a ſcientific book is ſufficient to inſtruct them in a new ſubject. Laſtly, there are others who never proceed to the next page, unleſs they have thoroughly underſtood the preceding part of the work. This method, in the reading of natural philoſophy, though very proper, is by no means very pleaſing, and generally tires the ſtudent before he has read a quarter of the work.

Where a great many new ideas muſt be acquired, much attention muſt be neceſſarily beſtowed. Therefore, in the reading of novels or poetry, the only exertion of the mind which is required for a ſatisfactory peruſal, is the connection of the different parts or accounts, and a tolerable degree of attention to the beauties of the performance which

which arise from the style and the imagination of the writer; for with respect to facts and the meaning of words, they are so much like the occurrences of common life, as never to demand any exertion of the understanding. Nearly the same thing may be said with respect to the reading of history. But with scientific subjects the case is quite different; for in them the great variety of new things and new ideas, to which words of uncommon use have been appropriated, and their dependance upon each other, or upon facts of unusual occurrence, demand a continual exertion both of the memory and of the understanding; which, unless it be relieved by means of order, patience, and a competent allowance of time, will certainly prove irksome to most students.

Therefore on those accounts I beg leave to recommend the following method. Let the novice in the study of natural philosophy read this work a first time, rather slowly, but without perusing those notes which, as has been remarked above, have a numerical reference, nor caring, as he proceeds, whether

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he does or does not thoroughly underſtand or remember all the particulars. By this means he will acquire a ſuperficial idea of the ſubject, the meaning of ſeveral words will become familiar to him, and he will in all probability be delighted by the various and wonderful properties of the material world. He may then commence a ſecond peruſal, proceeding rather ſlower, endeavouring thoroughly to underſtand every part, and every figure of the plates, and examining, according to his ability, a greater or a leſſer part of the notes. During this ſecond peruſal, it would be highly uſeful to attend a courſe of experimental lectures; for by this means the various objects, machines, actions, &c. will be eaſily and permanently fixed in his mind.

During this ſecond reading he ſhould pay particular attention to the numerous technical words, the meaning of which, whenever he forgets them, may be eaſily found out by recurring to the Index at the end of the work, which has for this purpoſe been rendered

much

much more copious than is cuſtomary for books of this kind. Laſtly, the ſtudent may read over a third time, or oftener, ſuch parts only of the work as his particular inclination, or his underſtanding or his memory may render neceſſary.

T. C.

WELLS STREET,
JAN. 1ſt, 1803.

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ELEMENTS OF

NATURAL PHILOSOPHY.

CHAPTER I.

Of NATURAL PHILOSOPHY;—*its Name*;—*its Object*;—*its Axioms*;—*and the Rules of Philosophizing.*

THE word Philoſophy, though uſed by ancient authors in ſenſes ſomewhat different, does, however, in its moſt uſual acceptation, mean *the love of general knowledge.* It is divided into *moral* and *natural.* Moral philoſophy treats of the manners, the duties, and the conduct of man, conſidered as a rational and ſocial being; but the buſineſs of natural philoſophy, is to collect the hiſtory of the phenomena which take place amongſt natural things, viz. amongſt the bodies of the Univerſe; to inveſtigate their cauſes and effects; and thence to deduce ſuch natural laws, as may afterwards be applied to a variety of uſeful purpoſes*.

Natural

* The word *philoſophy* is of Greek origin. Pitagoras, a learned Greek, ſeems to have been the firſt who called himſelf philoſopher; viz. a lover of knowledge, or of wiſ-

Natural things means all bodies; and the aſſemblage or ſyſtem of them all is called the *univerſe.*

The word *phenomenon* ſignifies an appearance, or, in a more enlarged acceptation, whatever is perceived by our ſenſes*. Thus the fall of a ſtone, the evaporation of water, the ſolution of ſalt in water, a flaſh of lightning, and ſo on; are all phenomena.

As all phenomena depend on *properties* peculiar to different bodies; for it is a property of a ſtone to fall towards the earth, of the water to be evaporable, of the ſalt to be ſoluble in water, &c. therefore we ſay that the buſineſs of natural philoſophy is to examine the properties of the various bodies of the univerſe, to inveſtigate their cauſes, and thence to infer uſeful deductions.

Agreeably

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dom, from the words φίλος, a *lover* or *friend*, and σοφίας, of *knowledge* or *wiſdom.* Moral philoſophy is derived from the latin *mos*, or its plural *mores*, ſignifying manners or behaviour. It has been likewiſe called *ethics*, from the Greek ἦθος, *mos*, *manner*, *behaviour.* Natural philoſophy has alſo been called *phyſics*, *phyſiology*, and *experimental philoſophy.* The firſt of thoſe names is derived from φύσις, nature, or φυσικὴ, natural; the ſecond is derived from φύσις, *nature*, and λογος, *a diſcourſe*; the laſt denomination, which was introduced not many years ago, is obviouſly derived from the juſt method of experimental inveſtigation, which has been univerſally adopted ſince the revival of learning in Europe.

* *Phenomenon*, whoſe plural is *phenomena*, owes its origin to the Greek word φαίνω, *to appear.*

Agreeably to this, the reader will find in the courſe of this work, an account of the principal properties of natural bodies, arranged under diſtinct heads, with an explanation of their effects, and of the cauſes on which they depend, as far as has been aſcertained by means of reaſoning and experience; he will be informed of the principal hypotheſes that have been offered for the explanation of facts, whoſe cauſes have not yet been demonſtratively proved; he will find a ſtatement of the laws of nature, or of ſuch rules as have been deduced from the concurrence of ſimilar facts; and, laſtly, he will be inſtructed in the management of philoſophical inſtruments, and in the mode of performing the experiments that may be thought neceſſary either for the illuſtration of what has been already aſcertained, or for the farther inveſtigation of the properties of natural bodies.

We need not ſay much with reſpect to the end or deſign of natural philoſophy.—Its application and its uſes, or the advantages which mankind may derive therefrom, will be eaſily ſuggeſted by a very ſuperficial examination of whatever takes place about us. The properties of the air we breathe; the action and power of our limbs; the light, the ſound, and other perceptions of our ſenſes; the actions of the engines that are uſed in huſbandry, navigation, &c.; the viciſſitudes of the ſeaſons, the movements of the celeſtial bodies, and ſo forth; do all fall under the conſideration of the

the philoſopher. Our welfare, our very exiſtence, depends upon them.

A very ſlight acquaintance with the political ſtate of the world, will be ſufficient to ſhew, that the cultivation of the various branches of natural philoſophy has actually placed the Europeans and their colonies above the reſt of mankind. Their diſcoveries and improvements in aſtronomy, optics, navigation, chemiſtry, magnetiſm, mineralogy, and in the numerous arts which depend on thoſe and other branches of philoſophy, have ſupplied them with innumerable articles of uſe and luxury, have multiplied their riches, and have extended their powers to a degree even beyond the expectations of our predeceſſors.

The various properties of matter may be divided into two claſſes, viz. the *general properties*, which belong to all bodies, and the *peculiar properties*, or thoſe which belong to certain bodies only, excluſively of others.

In the firſt part of this work we ſhall examine the general properties of matter. Thoſe which belong to certain bodies only, will be treated of in the ſecond. In the third part we ſhall examine the properties of ſuch ſubſtances as may be called *hypothetical*; their exiſtence having not yet been ſatisfactorily proved. In the fourth we ſhall extend our views beyond the limits of our Earth, and ſhall examine the number, the movements, and other properties of the celeſtial bodies.

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The fifth, or last part, will contain several detached articles, such as the description of several additional experiments, machines, &c. which cannot conveniently be inserted in the preceding divisions.

The axioms of philosophy, or the axioms which have been deduced from common and constant experience, are so evident and so generally known, that it will be sufficient to mention a few of them only.

I. Nothing has no property; hence,

II. No substance, or nothing, can be produced from nothing.

III. Matter cannot be annihilated, or reduced to nothing.

Some persons may perhaps not readily admit the propriety of this axiom; seeing that a great many things appear to be utterly destroyed by the action of fire; also that water may be caused to disappear by means of evaporation, and so forth. But it must be observed, that in those cases the substances are not annihilated; but they are only dispersed, or removed from one place to another, or they are divided into particles so minute as to elude our senses. Thus when a piece of wood is placed upon the fire, the greatest part of it disappears, and a few ashes only remain, the weight and bulk of which does not amount to the hundredth part of that of the original piece of wood. Now in this case the piece of wood is divided into its component ſubſtances, which the action of the fire drives different ways: the fluid part, for inſtance, becomes ſteam, the light coaly part either adheres to the chimney or is diſperſed through the air, &c. And if, after the combuſtion, the ſcattered materials were collected together, (which may in great meaſure be done), the ſum of their weights would equal the weight of the original piece of wood.

IV. Every effect has, or is produced by, a cauſe, and is proportionate to it.

It may in general be obſerved with reſpect to thoſe axioms, that we only mean to aſſert what has been conſtantly ſhewn, and confirmed by experience, and is not contradicted either by reaſon, or by any experiment. But we do not mean to aſſert that they are as evident as the axioms of geometry; nor do we in the leaſt preſume to preſcribe limits to the agency of the Almighty Creator of every thing, whoſe power and whoſe ends are too far removed from the reach of our underſtandings.

Having ſtated the principal axioms of philoſophy, it is in the next place neceſſary to mention the rules of philoſophizing, which have been formed after mature conſideration, for the purpoſe of preventing errors as much as poſſible, and in order to lead the ſtudent of nature along the ſhorteſt and ſafeſt way, to the attainment of true and uſeful knowledge.—Thoſe rules are not more than four; viz.

I. We are to admit no more cauſes of natural things, than ſuch as are both true and ſufficient to explain the appearances.

II. Therefore to the ſame natural effects we muſt, as far as poſſible, aſſign the ſame cauſes.

III. Such qualities of bodies as are not capable of increaſe or decreaſe, and which are found to belong to all bodies within the reach of our experiments, are to be eſteemed the univerſal qualities of all bodies whatſoever.

IV. In experimental philoſophy we are to look upon propoſitions collected by general induction from phenomena, as accurately or very nearly true, notwithſtanding any contrary hypotheſes that may be imagined, till ſuch time as other phenomena occur, by which they either may be corrected, or may be ſhewn to be liable to exceptions.

With reſpect to the degree of evidence which ought to be expected in natural philoſophy, it is neceſſary to remark, that phyſical matters cannot in general be capable of ſuch abſolute certainty as the branches of mathematics.—The propoſitions of the latter ſcience are clearly deduced from a ſet of axioms ſo very ſimple and evident, as to convey perfect conviction to the mind; nor can any of them be denied without a manifeſt abſurdity. But in natural philoſophy we can only ſay, that becauſe ſome particular effects have been conſtantly produced under certain circumſtances; therefore they will moſt likely continue to be produced as long as the ſame circumſtances exiſt; and likewiſe that they do, in all probability, depend upon thoſe circumſtances. And this is what we mean by *laws of nature*; as will be more particularly defined in the next chapter.

We may, indeed, aſſume various phyſical principles, and by reaſoning upon them, we may ſtrictly demonſtrate the deduction of certain conſequences. But as the demonſtration goes no farther than to prove that ſuch conſequences muſt neceſſarily follow the principles which have been aſſumed, the conſequences themſelves can have no greater degree of certainty than the principles are poſſeſſed of; ſo that they are true, or falſe, or probable, according as the principles upon which they depend are true, or falſe, or probable. It has been found, for inſtance, that a magnet, when left at liberty, does always direct itſelf to certain parts of the world; upon which property the mariner's compaſs has been conſtructed; and it has been likewiſe obſerved, that this directive property of a natural or artificial magnet, is not obſtructed by the interpoſition or proximity of gold, or ſilver, or glaſs, or, in ſhort, of any other ſubſtance, as far as has been tried, excepting iron and ferrugineous bodies. Now aſſuming this obſervation as a principle, it naturally follows, that, iron excepted, the box of the mariner's compaſs may be made of any ſubſtance that may be moſt agreeable to the workman, or that may beſt anſwer other purpoſes. Yet it muſt be confeſſed,

confessed, that this proposition is by no means so certain as a geometrical one; and strictly speaking it may only be said to be highly probable; for though all the bodies that have been tried with this view, iron excepted, have been found not to affect the directive property of the magnet or magnetic needle; yet we are not certain that a body, or some combination of bodies, may not hereafter be discovered, which may obstruct that property.

Notwithstanding this observation, I am far from meaning to encourage scepticism; my only object being to shew that just and proper degree of conviction which ought to be annexed to physical knowledge; so that the student of this science may become neither a blind believer, nor a useless sceptic*.

Besides a strict adherence to the abovementioned rules, whoever wishes to make any proficiency in the study of nature, should make himself acquainted with the various branches of mathematics; at least with the elements of geometry, arithmetic, trigonometry, and the principal properties of the conic

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* *Scepticism* or skepticism is the doctrine of the *sceptics*, an ancient set of philosophers, whose peculiar tenet was, that all things are uncertain and incomprehensible; and that the mind is never to assent to any thing, but to remain in an absolute state of hesitation and indifference. — The word sceptic is derived from the Greek σκεπτικὸς, which signifies *considerate*, and *inquisitive*.

conic ſections; for ſince almoſt every phyſical effect depends upon motion, magnitude, and figure, it is impoſſible to calculate velocities, powers, weights, times, &c. without a competent degree of mathematical knowledge; which ſcience may in truth be called the language of nature.

CHAPTER II.

CONTAINING A GENERAL IDEA OF MATTER, AND ITS PROPERTIES.

Of the ELEMENTS; — *and the Definitions of Words that are principally uſed in Natural Philoſophy.*

THE matter or ſubſtance of the bodies which we ſee, feel, taſte, or, in ſhort, that affect our ſenſes, becomes known to us merely by its properties. We know that the ſun exiſts becauſe we ſee its luminous and circular ſhape; becauſe we feel its heat. We know that the ground exiſts becauſe we ſee it, and feel it with our limbs. We acknowledge the exiſtence of air, becauſe we feel the reſiſtance it offers to the motion of other bodies, &c. Now the ſun, the ground, the air, and all other bodies, muſt, agreeably to the firſt axiom, conſiſt of ſomething. That ſomething is called *matter*; yet we are perfectly ignorant of the intimate nature of that matter; ſince we are unable to ſay whether

whether it conſiſts of particles of any definite ſize, ſhape, and hardneſs; and whether all the bodies of the univerſe conſiſt of the ſame ſort of matter differently modified, or of different ſorts of matter; and in the latter caſe, we can form no conjecture reſpecting the poſſible number of thoſe different ſorts.

Some philoſophers have ſteadily ſupported that there is one ſort only of original matter, and that the variety of bodies ariſes from the various arrangements of that primitive matter; like paſte, bread, and biſcuits, which may be ſaid to conſiſt of the ſame matter, viz. flour.

Other philoſophers have believed that the ſorts of primitive matter, or *elements*, are two. Others, that they are three. Others again, that they are four or five, or ſix or ſeven, and ſo on. But the hiſtory of ſuppoſitions muſt not be miſtaken for the knowledge of facts*.

The truth is, that the preſent ſtate of knowledge does not furniſh us with reaſons ſufficient to determine

* Whoever wiſhes to examine the various ideas that have been entertained concerning original matter, and the number of the elements, muſt conſult the works of the following authors; but his labour will not be compenſated by any material information: Ariſtotle; Plato; Boyle, on the principles of natural bodies; Newton's Optics; Woodward's Nat. Hiſt. of the Earth, p. v.; Muſſchenbroek's Elements of Phyſ. § 61, 83, 383; Keill Introd. to Nat. Phil. Lect. viii.; Higgins on Light; Chambers's Cyclop; and Hutton's Mathem. Dict. Art. *Element.*

mine the number of the elements. Modern chemifts, indeed, find from the refult of their numerous analyfes, that all the bodies which have been fubjected to experiments, are either the fubftances that are mentioned in the following lift, or they are a combination of fome of them. Yet no great confidence fhould be placed upon their number; for befides there being great fufpicion that feveral of them are ftill refolvable into fimpler components; new fubftances are almoft daily difcovered by the prefent rapid progrefs of philofophical inveftigation; and fome of them are merely hypothetical.

The following lift contains the bare names of thofe elementary fubftances which are at prefent acknowledged by the philofophical chemifts, or fuch as chemifts have not yet been able to decompofe; but a full explanation of the fame will be found in other chapters of this work; and till then the reader is requefted not to endeavour to inveftigate the meaning of their names, or to take any farther notice of them.

Light,
Calorific, or Caloric,
The Electric Fluid,
The Magnetic Fluid,
Oxygen,
Hydrogen,
Azote,
Carbon,
Sulphur,
Phofphorus,
Radical muriatic,
Radical boracic,
Radical fluoric,
Radical fuccinic,
Radical acetic,
Radical tartaric,
Radical pyro-tartaric,
Radical oxalic,

Radical

Radical gallic,	Nickel,
Radical cytric,	Cobalt,
Radical malic,	Bifmuth,
Radical benzoic,	Antimony,
Radical pyro-lignic,	Zinc,
Radical pyro-mucic,	Iron,
Radical camphoric,	Tin,
Radical lactic,	Lead,
Radical fach-lactic,	Copper,
Radical formic,	Mercury,
Radical pruffic,	Silver,
Radical febacic,	Platina,
Radical bombic,	Gold,
Radical laccic,	Silica,
Radical fuberic,	Argill,
Radical zoonic,	Baryt,
Arfenic,	Strontian,
Molybdenite,	Lime,
Tungften,	Magnefia,
Chrome,	Jargonia,
Titanite,	Vegetable alkali,
Sylvanite,	Foffil alkali, and
Uranite,	Volatile alkali.
Manganefe,	

Though moft of the words that frequently occur in the fubject of the prefent work, are generally ufed in common language, yet the accuracy of philofophical defcriptions fuggefts the neceffity of defining their meanings with a greater degree of precifion, in order to avoid, as much as poffible, any

any ambiguity of ſenſe, or any uncertainty of expreſſion.

Beſides thoſe which are mentioned in the preſent chapter, there are ſeveral other words which deſerve likewiſe to be particularly defined; but thoſe we ſhall explain occaſionally in the courſe of the work, and when the mind of the reader may be better diſpoſed to comprehend their meanings.

Space (though it be incapable of a proper definition) may be ſaid to be that univerſal and unlimited expanſe in which all bodies are contained; and that part of ſpace, which is occupied by any particular body, is called the *place of that body.*

Space is diſtinguiſhed into *abſolute*, and *relative. Abſolute ſpace* is that which is referred to nothing, and remains always ſimilar and immoveable. *Relative ſpace* is the ſame with abſolute ſpace in magnitude and figure, but not in ſituation. Suppoſe, for example, that a ſhip ſtood perfectly immoveable in the univerſe, the ſpace which is contained within its cavity, would be called *abſolute ſpace.* But if the ſhip be in motion, then the ſame ſpace within it will be called *relative ſpace.*

Place is likewiſe diſtinguiſhed into *abſolute* and *relative*; the former being immoveable and permanent; whereas the latter refers to other bodies. Thus if a man be ſeated in a corner of a ſhip whilſt the ſhip is failing along, he is ſaid to remain in the ſame place *relatively* to the parts of the ſhip; yet he is continually changing his *abſolute place.*

Rest is the permanence of any body in the same place, and it is called *absolute rest*, or *relative rest*, as the place, which the body occupies, is either absolute or relative.

Motion, on the contrary, is a continual and successive change of place. And it is called *absolute motion* or *relative motion*, according as the change of situation is made in absolute or in relative place.

Thus, if a ship were to remain immoveable in the universe, a man sitting in a corner of it, would be said to be absolutely at rest; but if the ship be in motion whilst the man remains sitting, then this man will be said to be at rest relatively to the parts of the ship, though he is actually or absolutely in motion.—Farther, suppose that the ship were to move equably forward over a distance equal to its length, and that at the same time the man in his chair were drawn from the fore to the back part of the ship, with the same equable motion, then the man would be in motion relatively to the parts of the ship; yet he would remain in the same absolute place.

With respect to the words *matter* and *body*, we shall for the present only remark the following difference between them; viz. that the word *matter* has no relation to any determinate figure; whereas the word *body* more generally means some separate and determinate quantity of matter. Thus we say with propriety, that *the movements of the celestial bodies are difficultly determined, and the matter which forms*

forms the atmoſphere is heterogeneous; whereas it would be improper to change the places of the words *body* and *matter*, by ſaying that *the movements of the celeſtial matters are difficultly determined, and the body which forms the atmoſphere is heterogeneous.*

Time, ſtrictly ſpeaking, is incapable of definition, and the only thing we can remark with reſpect to it, is the difference between *abſolute* and *relative time*. *Abſolute time* flows equably, but does not refer to the motion of bodies. *Relative time* is that portion of abſolute time, during which a certain movement is performed, and we aſſume ſome of thoſe movements, when they are equably and ſteadily performed, as the meaſures of time. Thus that portion of abſolute time which the ſun employs in performing its apparent revolution round the earth, is called *a day*; the 24th part of that day is called *an hour*; 365 times that day is called *a year*, and ſo on.

The *properties* of a thing are thoſe *qualities* and *operations* which belong to that thing, and by which it is diſtinguiſhed from other things that do not poſſeſs the ſame properties. It is, for inſtance, a property of the ſun to be luminous, of the magnet to attract iron, &c.

The *hardneſs* of a body is that degree of reſiſtance which the body offers to any power that may be applied for the purpoſe of ſeparating its parts. Whereas *fluidity* is the want of that reſiſtance; ſo that a perfect fluid is that body whoſe parts may be ſeparated by the application of the leaſt force. It will appear from the ſequel that we are not acquainted with any perfectly hard, or any perfectly fluid, body; ſo that we can only examine the intermediate gradations, which exiſt between thoſe extremes; but thoſe gradations which are expreſſed by the words *hardneſs*, *rigidity*, *brittleneſs*, *toughneſs*, *ſoftneſs*, *clammineſs*, *fluidity*, *&c.* are incapable of preciſe definitions or limits.

Cauſe and *effect* are relative terms; the *effect* being that which is produced by the cauſe, and the *cauſe* that which produces the effect.

Cauſes as well as effects are diſtinguiſhed into *primary*, *ſecondary*, &c. or into *immediate* and *remote*. Thus when the heat of the ſun rarefies the air, that rarefaction produces wind, and that wind impels a ſhip forward. In this caſe the heat of the ſun is the caûſe of the wind; the wind is the effect of the rarefaction, and is at the ſame time the cauſe of the ſhip's motion; the motion of the ſhip is effected by the action of the wind, ſo that the wind is the immediate, and the heat of the ſun is the remote, cauſe of the ſhip's motion.

A law of nature, or *mechanical law*, is a general effect, which has been conſtantly obſerved to take place under certain determinate circumſtances*. Thus we know from conſtant and univerſal experience, that whenever a body is left to itſelf, it always

* *Mechanical* means ſomething that relates to, or is regulated by, the nature and laws of motion.

ways falls towards the centre of the earth, unlefs fome other body prevents it ; we therefore affume this obfervation as a law of nature, and exprefs it by faying, *that the various bodies of the earth tend, or gravitate, towards the centre of it.*

The exiftence or non-exiftence of a *vacuum*, meaning an extenfion entirely void of matter, has often been difputed amongft philofophers; their arguments always depending upon fome affumed hypothefis concerning the intimate nature of matter or of its ultimate particles; but as we are utterly ignorant of the nature and properties of thofe particles, their arguments cannot determine the queftion one way or the other.—The only conclufions we can make with refpect to a *vacuum*, are 1ft. that the poffibility of its exiftence can be eafily imagined; 2dly. that we are not certain whether it really exift or not; and laftly, that if it be admitted that the figure of the leaft particles of matter is unchangeable, the motions of bodies, fuch as continually take place in the univerfe, cannot be underftood without admitting the exiftence of a vacuum.

The word *infinity* has likewife been productive of numerous difputes. Many odd pofitions have been affumed for the fupport of fpecious arguments, and feveral abfurd confequences have been deduced from them. Thofe errors have principally arifen from the idea of fomething determinate, which has been annexed to the words *infinite*, or *infinity*, in-

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ſtead of ſomething indefinite or indeterminate. In conſequence of this idea, infinites have often been compared together, and one infinite has been ſaid to be the double, or treble, or the half, &c. of another infinite; whereas infinites, (in the true ſenſe of the word, which means ſomething greater or leſs than any aſſignable quantity, but not determinate) are incapable of compariſon; ſince an indeterminate quantity cannot bear any aſſignable proportion to another indeterminate quantity; and of courſe one infinite cannot be ſaid to be greater than, equal to, or leſs than, another infinite.

It has been uſually alledged, that if a line be infinitely extended one way only, and another line be infinitely extended both ways; the latter infinite line muſt be double the former infinite line, which evidently implies a limited or determinate length; namely, that the latter line has been extended on either ſide as much as the former line has been extended one way only.

Again; take the length of one inch, and ſuppoſe it to be divided into an infinite number of parts. Take alſo the length of a foot, and ſuppoſe this to be divided into an infinite number of parts. Here, they ſay, it is evident that the latter infinite is exactly equal to twelve times the former. But this, in my humble opinion, ſeems to be a miſtaken concluſion; for the expreſſions of infinity do not refer to the extenſions of one foot and one inch; but to the numbers of the parts into which thoſe extenſions have been divided; and thoſe numbers can bear no aſſignable proportion to each other; juſt becauſe they are infinite.

The fact then is, that one foot is equal to twelve times one inch; and if each of thoſe extenſions be divided into any number of parts equal to each other in length, the number of parts in the extenſion of one foot will be equal to twelve times the number of the parts that are contained in the extenſion of one inch; but this is not the meaning of dividing a foot or an inch into an infinite number of parts; therefore when the foot and the inch, are each divided into an infinite number of parts, thoſe numbers have no aſſignable proportion to each other; though the *ſum* of the former is undoubtedly equal to twelve times the ſum of the latter*.

* Numerous inſtances of an infinite number of quantities having a finite or determinate ſum, occur both in arithmetic and in geometry. In geometry it is ſhewn, that a finite line may be divided into an infinite number of parts; and it is evident that the ſum of all thoſe parts muſt be equal to the line itſelf; viz. a finite quantity. In arithmetic it is ſhewn, beſides many other inſtances, that if you take one half, and one half of that half, and one half of the laſt half, and ſo on without end, the ſum of them all is equal to one; that is $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}$, &c. $=1$.

CHAPTER III.

OF THE GENERAL, OR COMMON, PROPERTIES OF MATTER.

IT has been already remarked, that a body is diſtinguiſhed from other bodies by means of its peculiar properties. Thus we know water by its fluidity, and by its want of taſte, ſmell, and colour; gold is known by its great weight and peculiar colour; ſalt is known by its particular taſte; and ſo forth. But there are certain properties, which belong equally to water, to gold, to ſalt, and to all other bodies. Extenſion for inſtance is a property which belongs to them all; for they all are extended. So likewiſe is weight; for they all are more or leſs heavy. Such then are called *General, or Common, Properties of Matter*; and, as far as we know, they are ſix in number; viz. *extenſion, diviſibility, impenetrability, mobility, vis inertiæ, or paſſiveneſs*, and *gravitation.*

We have ſaid above, as far as we know, becauſe matter in general may poſſeſs other properties, that are not yet come to our knowledge. And the ſame obſervation may be made with reſpect to the univerſality of thoſe properties; viz. that they are ſaid to be general, becauſe no body was ever found wanting any one of them. But mankind is not acquainted with all the bodies of the univerſe, and even ſeveral of thoſe which are known to exiſt, cannot be ſubjected to experiments.

Extenſion of a body is the quantity of ſpace which a body occupies; the extremities of which, limit or circumſcribe the matter of that body. It is otherwiſe called the *magnitude*, or *ſize*, or *bulk* of a body.

A certain quantity of matter may indeed be very ſmall, or ſo fine as to penetrate the pores of moſt other bodies; but yet ſome extenſion it muſt have; and it is by the compariſon of this property that one body is ſaid to be larger than, equal to, or ſmaller than, another body. The meaſurement of a body conſiſts in the compariſon of the extenſion of that body with a certain determinate extenſion, which is aſſumed for the ſtandard, ſuch as an inch, a foot, a yard, a mile; and hence we ſay that a certain body is three feet long, another body is the hundredth part of an inch in length, and ſo on*.

A body is not only extended, but it is extended three different ways, viz. it has *length*, *breadth*, and *thickneſs*. Thus an ordinary ſheet of writing paper is about ſixteen inches long, fourteen inches broad, and

* The attempts, which have been made for the purpoſe of eſtabliſhing an invariable ſtandard of meaſure, together with the difficulties which obſtruct the perfect accompliſhment of that object; as alſo the principal meaſures which are now in uſe, will be mentioned in another part of this work.

and one hundredth part of an inch thick. Either of thoſe dimenſions might be called the length, or the breadth, or the thickneſs; but, by general cuſtom, the longeſt extenſion is called the length, the next is called the breadth, and the ſhorteſt is called the thickneſs.

The outſide of a body; the boundary, or that which lies contiguous to other bodies that are in contact with it, is called *the ſurface* of that body, and it has two dimenſions only, viz. length and breadth; but it cannot have any thickneſs, for if it had thickneſs, it would not be the outſide of the body; yet a ſurface by itſelf cannot exiſt. We indeed talk of ſurfaces independent of matter, as when we compare one ſurface with another, or deſcribe the methods of meaſuring and dividing ſurfaces. In thoſe caſes, however, the ſurfaces exiſt in our imagination only, and even then our ideas have a reference to body. In ſhort, our ſenſes cannot perceive a ſurface without the exiſtence of a body; or, more properly ſpeaking, the outſide of a body cannot exiſt without the body itſelf.

As the ſurface is the outſide or boundary of a body, ſo *a line* is the boundary of a ſurface; ſuppoſe, for inſtance, that a ſurface is divided into two parts, the common boundary of the two parts is called *a line*, which has one extenſion, viz. length only.

A *point* is the beginning, or the end, of a line, and of courſe it has no extenſion; it being defined by the mathematicians, that which has no parts nor magnitude. Thus if you divide a line into two parts, the divifion or boundary between the two parts is *a point*.

Having fhewn above that there cannot exift, or that our fenfes cannot perceive, a furface without a body, it evidently follows that neither a line nor a point can be perceived without a body. We fpeak of the line or path of a planet; we alfo fay that a ftone thrown horizontally defcribes a curve line; but in thofe cafes the meaning is, that the planet, or the ftone, has paffed through certain places; not that thofe lines do actually exift as any thing fubftantial. When we look on a fheet of paper, we fee its furface, the edge of which is a line, and the extremity of the line, or corner, of the paper, is a point. But if you remove the paper; the furface, the line, and the point, vanifh from our fight, and they can only remain in our imagination.

Divifibility of matter is the property of its being divifible into parts. Some philofophical writers have confidered it as a diftinct property of matter itfelf; but it may with more propriety be confidered as a property of extenfion; for we can eafily conceive that a given extenfion may be divided into any number of parts, let it be ever fo great; but it is by no means known whether matter is, or is not, capable of being divided *ad infinitum*, that is, without any limit.

That a certain extenfion, as an inch, or any other length, be it ever fo fmall, is capable of infinite diviſion, may be rendered evident by means of arithmetic or of geometry. We may take, for inſtance, the halves of the propoſed extenſion, then the halves of thoſe parts, then the halves of thoſe halves, and ſo on without end; for if you proceed in this manner ever ſo far, there will after all ſtill remain the halves of the laſt parts, which may be alſo divided into other halves, &c. Again, ſuppoſe the line AB in fig. 1ſt. plate 1. to be the propoſed extenſion. Through the extreme points of this line draw two indefinite lines EF, and CD, parallel to each other. In one of thoſe lines, as EF, take a point L, and from this point draw ſtraight lines to any parts of the line BD, every one of which lines will evidently cut the propoſed extenſion AB into a different point. Now as the line BD may be produced towards D without limitation, and ſtraight lines may be drawn from L to an infinite number of points in the extended line BD; therefore the extenſion AB may be divided without end, or beyond any aſſignable number of parts.

Thus far we have ſhewn that extenſion may be diviſible into an unlimited number of parts; but with reſpect to the limits of the diviſibility of matter itſelf we are perfectly in the dark. We can indeed divide certain bodies into ſurpriſingly fine and numerous particles, and the works of nature offer many fluids and ſolids of wonderful tenuity; but both our efforts, and thoſe naturally ſmall objects,

objects, advance a very short way towards infinity. Ignorant of the intimate nature of matter, we cannot assert whether it may be capable of infinite division, or whether it ultimately consists of particles of a certain size, and of perfect hardness.

I shall now add some instances of the wonderful tenuity of certain bodies, that has been produced either by art, or that has been discovered by means of microscopical observations amongst the stupendous works of nature.

The spinning of wool, silk, cotton, and such like substances, affords no bad specimens of this sort; since the thread which has been produced by this means, has often been so very fine as almost to exceed the bounds of credibility, had it not been sufficiently well authenticated. Mr. Boyle mentions, that two grains and a half of silk was spun into a thread 300 yards long.

A few years ago a lady of Lincolnshire spun a single pound of woollen-yarn into a thread 168000 yards long, which is equal to 95 English miles*. Also a single pound weight of fine cotton-yarn was lately spun, in the neighbourhood of Manchester, into a thread 134400 yards long.

The ductility of gold likewise furnishes a striking example of the great tenuity of matter amongst the productions of human ingenuity. A single grain weight of gold has been often extended into a surface

* This lady's name at that time was Miss Ives. It is now Mrs. Ayre.

a ſurface equal to 50 ſquare inches. If every ſquare inch of it be divided into ſquare particles of the hundredth part of an inch, which will be plainly viſible to the naked eye, the number of thoſe particles in one inch ſquare will be 10000; and, multiplying this number by the 50 inches, the product is 500000; that is, the grain of gold may be actually divided into at leaſt half a million of particles, each of which is perfectly apparent to the naked eye. Yet if one of thoſe particles be viewed in a good microſcope, it will appear like a large ſurface, the ten-thouſandth part of which might by this means be eaſily diſcerned.

An ingenious artiſt in London has been able to draw parallel lines upon a glaſs plate, as alſo upon ſilver, ſo near one another, that 10000 of them occupy the ſpace of one inch.—Thoſe lines can be ſeen only by the aſſiſtance of a very good microſcope.

Another workman has drawn a ſilver wire, the diameter of which does not exceed the 750th part of an inch.

But thoſe prodigies of human ingenuity will appear extremely groſs and rude, if they be compared with the immenſe ſubtility of matter which may every where be obſerved amongſt the works of nature. The animal, the vegetable, and even the mineral, kingdom, furniſh numerous examples of this ſort.

What muſt be the tenuity of the odoriferous parts of muſk, when we find that a piece of it will ſcent

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a whole room in a ſhort time, and yet it will hardly loſe any ſenſible part of its weight. But ſuppoſing it to have loſt one hundredth part of a grain weight, when this ſmall quantity is divided and diſperſed through the whole room, it muſt ſo expand itſelf as not to leave an inch ſquare of ſpace where the ſenſe of ſmell may not be affected by ſome of its particles. How ſmall muſt then be the weight and ſize of one of thoſe particles?

The human eye, unaſſiſted by glaſſes, can frequently perceive inſects ſo ſmall as to be barely diſcernible. The leaſt reflection muſt ſhew him, that the limbs, the veſſels, and other neceſſary parts of ſuch animals, muſt infinitely exceed in fineneſs every endeavour of human art. But the microſcope has diſcovered wonders, that are vaſtly ſuperior, and ſuch indeed as were utterly unknown to our forefathers, before the invention of that noble inſtrument.

Inſects have been diſcovered, ſo ſmall as not to exceed the 10000th part of an inch: ſo that 1000000000000 of them might be contained within the ſpace of one cubic inch; yet each animalcule muſt conſiſt of parts connected with each other; with veſſels, with fluids, and with organs neceſſary for its motions, for its increaſe, for its propagation, &c. How inconceivably ſmall muſt thoſe organs be? and yet they are unqueſtionably compoſed of other parts ſtill ſmaller, and ſtill farther removed from the perception of our ſenſes.

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We might eafily fill a great many pages with examples and calculations relative to this fubject; but as the pleafing narration of fuch wonderful facts is not likely to give any real information concerning the general properties of matter, which form the fubject of this part of the book, I muft refer the inquifitive reader to other works*. The confideration of this divifibility does alfo lead the mind to certain curious fpeculations. (1)

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* Boyle's book of Effluvia; Keill's Introduction to Nat. Phil.; Rohault's Phyficks; Phil. Tranf. N. 194; s'Gravefand's Phil.; Muffchenbroek's Phil. &c.

(1.) Several writers, when treating of the divifibility of matter, have mentioned two curious theorems, which I fhall fubjoin in this note, as they may be of ufe to the fpeculative philofopher. Thofe theorems are eftablifhed on the fuppofition that matter is divifible without end.

Theorem I. *A quantity of matter however fmall, and any finite fpace however large, being given; it is poffible that that matter may be diffufed through all that fpace, and fo fill it, as not to leave in it a pore, whofe diameter will exceed a given right line.*

Let the given fpace be a cube, whofe fide is AB, fig. 2d. P. 1. fo that the cube be equal to $\overline{AB,|}^3$ and let the quantity of matter be reprefented by b^3; alfo let the line D be the limit of the diameter of the pores.

The fide AB being a finite quantity, may be conceived to be divifible into parts equal to the line D. Let the number of thofe parts be reprefented by n, fo that $n\text{D}=\text{AB}$, and $n^3\,\text{D}^3=\overline{\text{AB}|}^3$. Conceive the given fpace to be divided into cubes, each of whofe fides be equal to the right line

D, and

The contemplation of thofe wonders of nature, cannot fail of impreffing on our minds a ftrong idea of humility as well as of aftonifhment.—A vaft gradation of animals perfect in their kind, but fmaller than the human being in fize and duration, defcends as far down as our eyes can poffibly difcern, even when they are affifted by the moft powerful microfcopes. This vaft gradation, inftead of exhaufting the powers of nature, fhews the probable

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D, and the number of thofe cubes will be n^3, which cubes are reprefented in the fig. by E, F, G, H. Again, let the particle b^3 be fuppofed to be divided into parts whofe number be n^3; and in each cubic fpace let there be placed one of thofe particles; by which means the matter b^3 will be diffufed through all the given fpace. Befides each particle being placed in its cell, may be formed into a concave fphere, whofe diameter may be equal to the given line D; whence it will follow, that each fphere will touch that which is next to it; and thus the quantity of matter b^2, be it ever fo fmall, will fill the given finite fpace, however large, in fuch a manner as not to leave in it a pore larger in diameter than the given line D.

Corollary. There may be a given body, whofe matter if it be reduced into a fpace abfolutely full; that fpace may be any given part of the former magnitude.

Theorem II. *There may be two bodies equal in bulk, whofe quantities of matter may be very unequal, and though they have any given ratio to each other, yet the fums of the pores or empty fpaces in thofe bodies may almoft approach the ratio of equality.*

The

bable exiſtence of animated beings vaſtly ſmaller than thoſe; nor have we the leaſt reaſon to fix a limit to the ſeries.

If we contemplate the ſtate of exiſtence of thoſe animals; of one, for inſtance, out of a large number of the ſame ſpecies, that has been born in a glaſs of dirty water; whoſe life laſts but a few hours, and whoſe ſize is leſs than the 5000th part of an inch; for ſuch animals have been actually ſeen. If we indulge our fancy by conſidering what knowledge, or what ideas, can he poſſibly entertain of man, — of the earth, — of the univerſe; we may without difficulty conclude, that, far from having any preciſe notions of our exiſtence, he may in all probability

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The demonſtration of this theorem is eaſily derived from the foregoing, for ſince the matter of a body may be conceived to be condenſable into any part of the original bulk; therefore ſuppoſing two bodies, A and B of equal bulk, to be ſuch that the matter of A be 100 times the matter of B; the matter of B may be conceived to be condenſed into one 1000000th part of its original bulk, and of courſe the matter of A will be condenſed in one hundred 1000000th parts of the ſame bulk; in which caſe the ſpaces left in the original bulk of B will be to the ſpaces left in the original bulk of A as 999999 to 999900, which numbers are nearly equal to each other.

Inſtead of the above-mentioned numbers, the proportion of the quantities of matter may be increaſed at pleaſure, and ſo may the proportion of the original bulks of the bodies to the ſpaces into which they may be conceived to be condenſable.

probability look upon the glaſs of dirty water as the boundary of the habitable world. Out of that water, tradition or his own experience, ſhews him nothing but the inevitable deſtruction of his ſpecies, and a confuſed aſſemblage of immenſe objects, whoſe nature and whoſe motions are utterly inexplicable to him. Yet he may poſſibly ſuſpect that thoſe very objects have powers infinitely ſuperior to thoſe of his own ſpecies.

Let us now follow the analogy, and let us briefly apply the ſame contemplation to ourſelves. The planets, the ſtars, the comets, and perhaps an infinity of other bodies that are far beyond the reach of our knowledge, manifeſt the exiſtence of powers infinitely above us, and perhaps even leſs comprehenſible to us than we are to the above-mentioned animalcule. Confined to the globe of this earth, which is only a ſpeck in the univerſe; and, with reſpect to us, not much better nor worſe than the glaſs of dirty water is with reſpect to thoſe inſects; how inſignificant are our powers, and how imperfect is our knowledge of nature! How little likely are we to comprehend the real order of things, and the Great Wiſdom that regulates the whole! In this ſublime inquiry the aſſiſtance of our reaſoning faculty is trifling indeed; the clew of analogy is ſhort and imperfect; and our imagination ſoon loſes itſelf in the boundleſs extent of immenſity.

Impenetrability is that property, by which a body excludes every other body from the place which itſelf

itſelf occupies. Thus one cannot drive a cubic inch of gold into a cubic inch of ſilver. You may indeed melt and incorporate the two metals into one lump; but then the lump will meaſure two cubic inches; which proves not that the gold occupies the ſame cubic inch of ſpace which is occupied by the ſilver; but that the particles of the two metals are placed contiguous to each other. Thus alſo, if a quantity of water be put into a ſtrong veſſel, for inſtance, of iron, and the veſſel be accurately ſhut up, it will not be poſſible to preſs the ſides of the veſſel towards each other; the matter which fills the cavity of it being ſufficient to reſiſt any degree of preſſure.

Though impenetrability be admitted as a general property of matter, it muſt, however, be obſerved, that in certain mixtures of two or more bodies of different natures, a loſs of bulk does actually take place; thus if a cubic inch of ſpirit of wine be mixed with a cubic inch of water, the bulk of the mixture will be ſomewhat leſs than two cubic inches; yet the weight of the mixture (provided no evaporation be allowed to take place) will be equal to the ſum of the weights of the two fluids; which indicates that one of the fluids muſt have filled up ſome of the pores or vacuities of the other fluid. It is beſides not unlikely that ſome other finer fluid may have eſcaped in the act of mixing the two bodies.

In other parts of this work we ſhall take notice

 of

of the loſs of weight and other phenomena, that take place in many caſes of mixture; but with reſpect to impenetrability itſelf, we may rather conſider it abſtractly as a property of the real quantity of matter which exiſts in bodies, independently of pores and vacuities, than as a general property, without exception, of bodies in their uſual ſtate of exiſtence.

Mobility of matter is that eſſential and general property, whereby any body is capable of being moved from one part of abſolute ſpace to another part of it. Experience conſtantly ſhews, that the force, which is required to move a body, is proportionate to its weight; therefore we conclude with ſaying, that all bodies are capable of being moved; provided an adequate force be employed to put them in motion.

It is a fact proved by conſtant and univerſal experience, that the progreſs of a body in motion is retarded preciſely in proportion to the obſtruction which the body meets with in its way. Thus if two bodies, A and B, exactly alike in ſhape, weight, and ſubſtance, be put in motion by equal impulſes, and meet with equal obſtructions; by moving, for inſtance, through the ſame medium, or by rolling over the ſame ſort of plain ſurface, thoſe two bodies will run over equal ſpaces in equal times; but if the body A meets with half the obſtruction that the body B meets with, then A will go as far again as the body B; when A meets with a quarter of the

5 obſtruction,

obſtruction, it will go four times as far as B; and in ſhort, A will percur a ſpace longer than B, by as much as its obſtruction is diminiſhed; and conſequently when the obſtruction to A's motion is entirely removed, A will go infinitely farther than B; that is, it will continue to move for ever. It therefore appears, that a body once put in motion has no power to ſtop itſelf; nor can its motion ceaſe, unleſs ſome force is exerted by ſome external power againſt it.

By the ſame ſort of reaſoning, we prove that a body at reſt has no power to put itſelf in motion, and of courſe that it will continue for ever at reſt, unleſs it be impelled by ſome external power; for ſince we find that a certain impulſe is required to move a body with a certain quickneſs, viz. ſo as to let it run over the ſpace of a mile in one minute; that with half that impulſe it will percur half a mile; with the hundredth part of the original impulſe it will percur the hundredth part of a mile; it will naturally follow, that without any impulſe at all, it will not move in the leaſt: a body therefore has no power either to put itſelf in motion if it be at reſt, or to ſtop itſelf if it be in motion: and this paſſiveneſs of matter is called the *vis inertiæ*, or *want of activity*, of bodies.

A novice in philoſophy may perhaps be induced to ſuſpect the truth or generality of this property of matter, by obſerving that a man, or other animal, can eaſily move himſelf from reſt, or ſtop his

 motion:

motion: but in this caſe it muſt be remarked, that the animal receives a general impulſe at the commencement of his life, and that all his actions, as long as he exiſts, are the conſequence of that original impulſe. I ſhall endeavour to illuſtrate this matter by an inſtance of a much leſs complicated nature.

It is very well known that a common eight-day clock, when it is once wound up, will continue to move its pendulum for a whole week, and at the end of every hour it will ſtrike a number of ſtrokes on the bell. It is evident likewiſe that thoſe motions of the pendulum, the hammer, &c. are owing to the original power or impulſe which was communicated to the machine by the perſon who wound it up; yet an ignorant man might ſay, if bodies cannot put themſelves in motion, nor can they ſtop themſelves when they are actually in motion; how does it happen that the ſtriking part of the clock puts itſelf in motion, and then ſtops itſelf at the end of every hour? The anſwer is, that the power which was communicated to the ſpring or weight of the clock, is ſo regulated by the mechaniſm, as to act by little and little, ſufficiently to keep the pendulum and the wheels in motion; and that when a particular part of one of thoſe wheels comes againſt a certain machinery, it then diſengages a portion of the other power, viz. of the ſpring or weight of the ſtriking part, which puts the hammer in action.

What

What has been ſaid of the clock will perhaps be ſufficient to remove the difficulty reſpecting the apparent ſelf-moving power of more complicated mechaniſms, ſuch as that of an animal or vegetable body. But though we are led by the analogy of much ſimpler movements, to admit the dependence of animal and vegetable motion on an original impulſe; we do not, however, preſume to explain the origin, dependence, and poſſible modifications of that impulſe; our underſtandings, and our knowledge, being as yet inſufficient to explain the nature and the laws of that original energy.

Attraction is that property whereby one body or part of matter attracts, or endeavours to get near, another body. There are ſeveral ſorts of attraction; ſuch as the magnetic attraction, which takes place between magnets and iron; the electric attraction, which is obſerved amongſt bodies in certain circumſtances, &c. Theſe attractions, however, belong to certain bodles only, and of courſe they muſt be examined in other parts of this work. But there is a ſort of attraction which belongs to bodies of every kind; it is mutual among them, and it ſeems to pervade the univerſe. It is that property whereby bodies tend, or fall, towards the centre of the earth, and it has been called *gravitation*, becauſe the quantity of that tendency in different bodies, is the meaſure of their *weight* or *gravity*.

Experience, reaſoning, and analogy, ſhew that this gravitation exiſts not only between the globe of the earth and the ſurrounding bodies, but between all parts of matter. One terreſtrial body gravitates or tends towards another terreſtrial body; the moon gravitates towards the earth; the moon, the earth, and all the planets, gravitate towards each other, and towards the ſun; and probably the ſun, with all its planetary ſyſtem, may gravitate towards ſome other object.

The motion of certain bodies which ſeem to fly away from the earth, muſt not be conſidered as an exception of this general law; for in thoſe caſes the bodies only give way to other ſurrounding bodies of a heavier nature, viz. that have a greater tendency towards the earth. Thus ſmoke, when extricated from burning bodies, goes upwards, or from the centre of the earth, becauſe the ſurrounding air, which is heavier than ſmoke, takes its place: but if the air be removed, or at leaſt it be ſo far rarefied as to become lighter than ſmoke, then the ſmoke will deſcend like a ſtone or other heavier body. Thus alſo if you drop a piece of cork into an empty veſſel, the cork will go downwards or to the bottom of the veſſel; but if afterwards you pour water into the veſſel, the cork will aſcend in order to make way for the water, which has a greater tendency towards the centre of the earth than an equal bulk of cork.

Daily and conſtant experience ſhews to every perſon, that near the ſurface of the earth, all bodies tend towards the centre of it, unleſs they are hindered by other bodies. But the reader may naturally

turally aſk, how is it known that the planets and the earth gravitate towards the ſun? The anſwer is, that from the accurate meaſurements of the motions of thoſe planets, they are found to follow the ſame laws that bodies do, which are projected in a certain manner near the ſurface of the earth, and whoſe motion is undoubtedly determined by the power of gravitation; we therefore, according to the rules of philoſophizing, attribute ſimilar cauſes to ſimilar effects, and conclude that the planets gravitate towards the ſun, in the ſame manner as ſtones, water, and other terreſtrial bodies, gravitate towards the earth.

What is the cauſe of gravitation, or how can a body act upon another body through a certain ſpace? is a queſtion which naturally preſents itſelf to the inquiſitive mind; but which we are utterly incapable to anſwer.

A variety of conjectures have been formed, and many hypothetical ſuppoſitions have been offered, for the elucidation of this queſtion; but as they are all involved in abſurdity and obſcurity, I ſhall not detain my reader with any account of them. All we can ſay is, that the effect is certain, the knowledge of its laws is highly uſeful to mankind; but its cauſe is hidden amongſt the myſteries of nature.

CHAPTER IV.

OF MOTION IN GENERAL; THE LAWS OF MOTION, AND THE PROPERTIES OF SIMPLE MOTION.

OF the general properties of matter, the firft three may be prefumed to have been fufficiently illuftrated in the preceding chapter; but the other three, viz. mobility, *vis inertiæ*, and gravitation, are the foundation of the extenfive doctrine of motion, or of mechanics; and are therefore deferving of a full and particular examination.

Almoft all the phenomena of nature are owing to motion. The appearance and difappearance of the cœleftial bodies; the increafe of animals and vegetables; the compofition and decompofition of complex fubftances, fire, &c. are all effected by motion. Therefore the laws of motion muft be looked upon as the foundation of natural philofophy; fo that without a clear comprehenfion of thofe laws, it will be impoffible to make any proficiency in the ftudy of nature.

The importance and extent of the fubject, render it neceffary to divide the materials into feveral chapters, in each of which fuch particulars will be arranged, as are more immediately connected with each other, and more conducive to concifenefs and perfpicuity.

It

It is a natural consequence of the *vis inertiæ* of matter, that whatever body is in motion, must be supposed to have been put in motion by some active force; viz. some external impulse.

This impelling force may be of two sorts. It may either communicate the impulse at first, and then cease to act, like the impulse which is given to a bullet by the discharge of a gun; or it may act irremittedly on the body in motion, like the force of gravity on a stone that is dropped from any height. For distinction sake we shall call the first simply an *impulse*, and the latter an *accelerative force.*

A body may be put in motion by one, two, or more forces at the same time, and those forces may be either all simple, or all accelerative, or some may be of one sort, and others of the other sort.

Most of the movements that commonly take place in the world, are the effect of more than one impulse; and they are never performed with perfect freedom, since they are always performed in resisting mediums. However, in order to preserve perspicuity as much as it lies in our power, we shall in the first place examine the motions arising from a simple impulse in a non-resisting medium, and shall then proceed in the examination of the more intricate causes of motion.

Three general laws of motion have been deduced from innumerable experiments and observations, by by means of the ſtricteſt philoſophical reaſoning.— They are as follows:

I. Every body will continue in its ſtate of reſt, or of moving uniformly in a ſtraight line; unleſs it be compelled to change that ſtate by forces impreſſed.

II. The change of motion is always proportional to the moving force impreſſed, and is always made according to the right line, in which that force is impreſſed.

III. Action and re-action are always equal and contrary to each other; or the actions of two bodies mutually upon each other, are always equal, and directed towards contrary parts.

The firſt of thoſe laws is evidently nothing more than the *vis inertiæ* of matter, announced in a different manner; excepting only the aſſertion of the body moving in a ſtraight, and not in a curve, line, which particular may perhaps be deſerving of ſome explanation.

The proof of this particular property has likewiſe been deduced from conſtant experience; for we find that whenever a body moves in a curve line, there always is ſome ſecondary power which forces it to deviate from the rectilinear courſe; and that deviation is exactly proportional to that ſecondary power. Thus a ſtone which is thrown horizontally would proceed horizontally in a ſtraight line, were it not drawn downwards by the

force

force of gravity; and we find by computation, that the deviation from the horizontal direction is exactly proportional to the force of gravity.

Hence the second law has been deduced, in which it is asserted that the change of motion is always proportional to the moving force impressed, and is made according to the right line in which that force is impressed; for if it were made in a crooked line, it would imply the action of a third force; and if it were not proportional to the moving force, the effect would not be adequate to the cause.

The third law may be easily illustrated by means of examples; and the least reflection on the phenomena, which commonly occur, will be sufficient to manifest the truth and universality of it.

When a man strikes one of his hands against the other, the blow is felt equally by both hands. If you strike a glass bottle with a steel hammer, the blow will be received equally by the hammer and by the glass bottle; and it is immaterial whether the hammer be moved against the bottle at rest, or the bottle be moved against the hammer at rest; yet the bottle will be broken, whereas the hammer will not, because the same blow, which is sufficient to break glass, is not sufficient to break a lump of steel.—It is for the same reason, that if a man strike his fist against another man's face, the blow, which is equally received by the fist and by the

the

the face, will produce a material hurt on the latter, but not upon the former.

If a ſtone be tied to a horſe by means of a rope, the horſe in dragging the ſtone will exert a degree of force equal to the reſiſtance of the ſtone; for the rope which is ſtretched both ways will equally pull the horſe towards the ſtone, and the ſtone towards the horſe. And, in fact, the ſtone will not follow the horſe, unleſs the power of the horſe be greater than the reſiſtance of the ſtone.

Experience likewiſe ſhews, that if a loadſtone and a piece of iron be placed on ſeparate pieces of cork, and be ſuffered to float on the ſurface of water, the attraction between them will be mutual, and they will move towards each other ſo as to meet in a place between their two original ſituations. If the loadſtone only be held faſt in its place, the iron will come all the way to meet it; and if the iron only be held faſt in its place, the magnet will advance towards the iron until it comes in contact with it.

The motion given to a boat by oars is likewiſe a convincing illuſtration of the third law; for by the action of one extremity of each oar againſt the water one way, its other end re-acts upon the boat, and impels it the contrary way.

We ſhall now examine the motion which is produced by a ſingle impulſe, which acts at firſt only,

and

and then leaves the body to proceed by itſelf, in an unreſiſting medium.

It has been already ſhewn, that in this caſe the body will continue to move uniformly; that is, it would run over equal ſpaces in equal portions of time; and ſuch would be the caſe of a bullet ſhot out of a gun, or of a ſtone thrown out by a man's hand, were they not impeded by the reſiſtance of the air, and were they not acted upon by the force of gravity. But it is now neceſſary to take notice of ſeveral particulars relative to this ſort of motion.

In the firſt place it may be aſked, how does the impelling force put the body in motion, or what does it communicate to the body? The anſwer is, that the moving force does not communicate any thing to the body; but it only moves the body through a certain ſpace in a certain time, after which the body, being left to itſelf, will continue to move at the ſame rate, viz. will continue to run over like ſpaces in the like portions of time; and that merely in conſequence of its *vis inertiæ*; of which *vis inertiæ*, however, we do not pretend to know any thing more, than that it has been found to be a general property of matter.

All the particulars which can be remarked with reſpect to the above-mentioned ſimple motion, are the relations between the time, in which a certain ſpace is deſcribed; the ſpace which is percurred in a certain time, the quantity of which ſhews the *velocity*; the quantity of matter in motion; and laftly, the *momentum*, by which word we mean the force of the body in motion, and reckon it equivalent to the impreffion that the body in motion would make on another body at reft, that fhould be prefented to it precifely in the direction of its motion.

The *momentum* has been often called *the quantity of motion*, or fimply *the motion*; but we fhall not make ufe of the laft word in this fenfe, left it fhould be miftaken for the velocity, in which fenfe it has been likewife ufed. We fhall alfo exprefs the above-mentioned four particulars by their initial letters, viz. T for the time, S for the fpace, V for the velocity, Q for the quantity of matter, and M for the momentum.

By the word *velocity* we mean nothing more than the ratio of the quantity of fpace which is run over in a certain portion of time. Thus it is faid that a body moves with the velocity of three feet per fecond; alfo that the velocity of a body A is to the velocity of another body B, as two to three; meaning that if A goes over a certain fpace, as for inftance, four miles, in a certain time, the body B will percur fix miles in the fame time; fince two is to three as four is to fix.

It is therefore evident, that in equal times the velocities are as the fpaces; but if the times be unequal, then the velocities are as the quotients of the fpaces divided by the times refpectively. Thus

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ſuppoſe that a body A paſſes over ten feet in two minutes, and another body B paſſes over eight feet in four minutes, the velocity of A will be to the velocity of B as $\frac{10}{2}$ to $\frac{8}{4}$; that is as five to two; for by dividing the ten feet by the two minutes, we find how many feet the body A runs over in *one minute*, and likewiſe by dividing the eight feet by four minutes, we find how many feet the body B runs over in the ſame time; viz. *one minute*; ſo that by the operation of dividing the ſpaces by the times reſpectively, we do nothing more than find out the ſpaces that are percurred by the two bodies in *equal times*, and then compare them together.

Before we proceed any farther, it is neceſſary to obſerve, that whenever it is ſaid that certain things are as certain other things, we only aſſert the ratio of the former to the latter; viz. that the former increaſe or decreaſe according as the latter do increaſe or decreaſe; but from ſuch aſſertions nothing real and determinate can be deduced, unleſs we have recourſe to experiments, in order to aſcertain ſome of thoſe particular things with which others are compared. Thus in the preceding paragraph, it has been aſſerted that the velocities are as the quotients of the ſpaces divided by the times; yet this aſſertion will not enable us to determine the velocity, or the ſpace run over, or the time, which is employed by a certain body in motion,

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unlefs fome of thofe particulars be previoufly known. Hence if we learn from actual experiment (viz. by meafuring the fpace with a ruler and the time by a watch), or are otherwife informed, that a body has been moving through ten feet in two feconds; then dividing the ten by two, the quotient five gives the velocity; which means that the body moves at the rate of, or percurs, five feet per fecond. If by the abovementioned proportional expreffion we wifh to find the fpace, we muft previoufly know the velocity and the time; and if we wifh to afcertain the time, we muft previoufly know the velocity and the fpace. Therefore, in general, the ufe of fuch proportional expreffions is to render certain particulars deducible, by computation, from other particulars which belong to the fame expreffion, and which have been previoufly afcertained by means of actual experiments. We fhall now proceed to explain the other particulars which relate to the above-mentioned fimple or equable motion.

The *fpace* is as the velocity multiplied by the time; (that is, S is as V T) for if a body move with the velocity of three feet per minute, it is evident that it muft pafs over twice three, or fix, feet, in two minutes; three times three, or nine, feet, in three minutes; four times three, or twelve feet, in four minutes; and, in fhort, the fpace is as the product of the velocity, or rate of going, multiplied by the time.

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The *time* is as the ſpace divided by the velocity; (viz. T is as $\frac{S}{V}$) for if a body, for inſtance, runs over 12 feet when its velocity is three feet per minute, it is evident that in order to find the number of minutes, that the body has employed in paſſing over 12 feet of ſpace, we muſt ſay, by the common rule of three, if the body paſſes over three feet in one minute, how many minutes will it employ in paſſing over 12 feet; which proportion is ſtated thus; 3 : 1 : : 12 :, and as the ſecond term is unity, we need only divide the 12 by 3; (viz. the ſpace by the velocity) and the quotient 4 is the time ſought.

The *momentum*, and the *quantity of matter*, are the two laſt particulars which remain to be examined with reſpect to this ſort of motion. It has already been mentioned, that the momentum is the force of the body in motion, and is equivalent to the impreſſion it would make on another body that ſhould be placed at reſt directly before it.

According to the fourth axiom, every effect muſt be produced by an adequate cauſe; therefore if a body be cauſed to move with a certain velocity by means of a certain impulſe, the double of that impulſe will be required to make it move with the double of that velocity; three times that impulſe to let it move with three times the original velocity; and, in ſhort, the moving force or impulſe muſt be proportionate to the velocity. And for the ſame reaſon, the reſiſtance, which muſt be oppoſed to the ſaid body in order to ſtop it, muſt likewiſe be proportionate to the velocity of the body.

Now let two diſtinct bodies, A and B, move with equal velocities; but let the quantity of matter in B be the double of the quantity of matter in A; and it is evident that the momentum of B muſt be double the momentum of A; for if we imagine B to be divided into two equal parts, each of thoſe parts muſt have a momentum equal to the momentum of A; (A being equal to the half of B) and of courſe both halves together muſt have a momentum double of the momentum of A.

If the body B be ſuppoſed to move as faſt again as A, or with the double of its former velocity, it follows, from what has been mentioned above, that its momentum muſt be double of its former momentum; but before its momentum was double the momentum of A, therefore now its momentum muſt be quadruple the momentum of A; that is, it muſt be multiplied by two on account of its double quantity of matter, and again by two on account of its double velocity; which is as much as to ſay that the momentum is as the product of the quantity of matter multiplied by the

the velocity; (viz. M is as Q V.)—Or we may consider it as a definition, and say that by the *momentum* we mean the product of the quantity of matter by the velocity *.

If the quantity of matter in B, instead of being double, be supposed to be treble, or quadruple, or the half, or other multiple, of the quantity of matter in A; the same mode of reasoning will shew that its momentum must be treble, or quadruple, or the half, or any other multiple respectively of the momentum of A, when the velocities of A and B are equal; but that those momentums must be multiplied by the velocities when the velocities of the bodies A and B are unequal; which proves that the proposition is universally true.

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* The measure of the momentums of bodies, under the title of *vis matrix*, or *vis viva*, when moving with different velocities, produced some years ago a long and loud dispute amongst the learned in Europe. The intricacy of the arguments would render a statement of the question too long for this work, and it would besides be attended with little or no profit to the beginner; I shall therefore refer such of my readers as are desirous of being informed relatively to this question, to two excellent tracts; the first of which is entitled *An Essay on Quantity*, by the Reverend Mr. Reid, in the 45th vol. of the Phil. Trans. the second is *An Inquiry into the Measure of the Force of Bodies in Motion*, by Dr. Irwin, Phil. Trans. for 1745.

Laſtly, the quantity of matter is as the momentum divided by the velocity; (viz. Q is as $\frac{M}{V}$); for let V in the preceding proportional expreſſion (M as V Q) be repreſented by the number 2; then that proportional expreſſion will become M as 2 Q; meaning that the momentum is as twice the quantity of matter; but if the momentum is as twice the quantity of matter, therefore, taking the halves of thoſe quantities, (for the halves, or the quarters, or any other like parts, or multiples of two quantities, have the ſame proportion to each other as the quantities themſelves. Euclid. Elem. B. v. prop. 15.) half the momentum will be as the quantity of matter, which is expreſſed thus; Q is as $\frac{M}{2}$. Again, if the velocity be repreſented by any other number, as by 12, the proportion M as V Q, will become M as 12 Q, and, taking the 12th part of thoſe two quantities, we ſay, that ſince the momentum is as 12 times the quantity of matter, therefore the 12th part of the momemtum is as the quantity of matter, which is expreſſed thus; Q as $\frac{M}{12}$; but the velocity is repreſented by the number 12 in the laſt ſuppoſition; by the number 2 in the preceding ſuppoſition, and may be repreſented by any other number; therefore, univerſally, the quantity of matter is as the quotient of the momentum divided by the velocity.

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I ſhall now collect all the propoſitions, or laws, which belong to ſimple motion, under one point of view, and, for the ſake of perſpicuity, I ſhall expreſs them both in the conciſe way, by uſing the initial letters, and in words.

V is as $\frac{S}{T}$; S is as V T; T is as $\frac{S}{V}$; M is as V Q; and Q is as $\frac{M}{V}$.

The ſame expreſſed in words.—In *ſimple motion, viz. when a body is put in motion by a ſingle impulſe, which acts at firſt, and then leaves the body to proceed by itſelf in a non-reſiſting medium; or when ſeveral bodies are thus ſeparately put in motion; the velocities are as the ſpaces divided by the times; the ſpaces are as the velocities multiplied by the times; the times are as the ſpaces divided by the velocities; the momentums are as the velocities multiplied by the quantities of matter; and, laſtly, the quantities of matter are as the momentums divided by the velocities.*

Thus, conſidering the importance of the ſubject, I have endeavoured to demonſtrate the particulars relative to ſimple motion, in as familiar a manner, and as little encumbered with mathematical expreſſions, as the ſubject ſeemed to admit, purpoſely to adapt them to the capacity of beginners. And I muſt earneſtly entreat the reader to make himſelf maſter of the contents of this chapter before he proceeds to the next.

CHAPTER V.

OF THE MOTION ARISING FROM CENTRIPETAL, AND CENTRIFUGAL, FORCES; AND OF THE CENTRE OF GRAVITY.

A *Centripetal force* is that power which compels bodies to move, or to tend towards a point, which is called the *centre of attraction*. *A centrifugal force*, on the contrary, is that power which compels bodies to recede from a point, which is called the *centre of repulsion*. *Gravitation*, or that power, by which bodies are forced to fall towards the centre of the earth, is a centripetal force, and will serve us as an example for the illustration of the general theory.

But though bodies direct their course towards the centre of the earth, yet the attractive power must not be considered as a peculiar property of that centre, or of any particular body near it. Attraction is a property which belongs to matter in general, and is proportionate to the quantity of it. The parts of the earth mutually gravitate towards, or attract, each other;—a stone attracts another stone, or any other body; the earth attracts a stone, as well as the latter attracts the former, and all bodies, in short, mutually attract each other; nor are we acquainted with any particle of matter which may be be ſaid to be deſtitute of attraction towards the whole aſſemblage of terreſtrial bodies. That, *cæteris paribus*, the attractive force is proportionate to the quantity of matter, may be eaſily proved; for let A, B, and C be three bodies equal in every reſpect; and if A attract C with a certain force, (for inſtance, a force equal to one ounce) it is evident that B, its equal, muſt likewiſe attract C with the force of one ounce; and, of courſe, A and B together, or a body equal to thoſe two, muſt attract C with the force of two ounces. Again, if we take ten equal bodies, it is evident, that two of them will attract another diſtinct body with twice the force of one of them only, as alſo that four, or five, or ſix of thoſe equal bodies will attract the other body with four, or five, or ſix times reſpectively the force of one of them only, and ſo forth; which evidently ſhews the generality of the propoſition.

It is in conſequence of this truth, that when a body A prevents another body B from falling towards the centre of the earth, the former is preſſed by the latter, and that preſſure is proportionate to the quantity of matter in B. Now, that preſſure is called the *weight* of the body B, and the quantity of it is expreſſed by comparing it with a certain arbitrary ſtandard weight, which may be called an ounce, a pound, a grain, &c. So that when a certain body A is ſaid to weigh three pounds, whilſt another body B weighs one pound,

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the meaning is, that the quantity of matter in A, and of courſe its attraction towards the earth, is treble the quantity of matter in B, or the attraction of B towards the earth.

Since attraction is a general property of matter, it may be aſked, why do we not perceive any attraction between the bodies which uſually ſurround us, as for inſtance between two flints, or two pieces of lead? The anſwer is, that the attractive force of matter in general is too ſmall to become perceptible, excepting when the bodies, or one of them, is very large, as is the caſe between the earth and a flint, or other body; for if you ſuppoſe that a flint ſtone A be equal to the 1000000000000000th part of the whole earth, and likewiſe ſuppoſe that another body B is attracted by the earth with a force equivalent to one pound; then it follows that the body B muſt be attracted by the flint ſtone A with a force equivalent to the 1000000000000000th part of a pound; which is too ſmall to produce any ſenſible effect. Yet, notwithſtanding this, the accuracy and improvements of the preſent age, have found means of rendering the attraction between bodies of no great ſize, ſufficiently ſenſible; but the account of ſuch experiments will be found in another part of this work.

Conſidering that the attraction is mutual between bodies, as between a ſtone and the earth, it may be aſked, why does not the earth move towards the ſtone at the ſame time that the ſtone moves towards the earth? The anſwer is, that the earth, agreeably to the theory, muſt actually move towards the ſtone, but its motion is too ſmall to be perceived by our ſenſes; for if we ſuppoſe that the earth is at leaſt 1000000000000000000000000 times larger than the ſtone, the attraction of the earth for the ſtone, muſt be to the attraction of the latter for the former, as that immenſe number is to unity. Now ſince the effects are always proportionate to their cauſes, it follows, that if in a certain time the ſtone moves through 1000 feet in its deſcent towards the earth, the earth muſt in the ſame time move towards the ſtone through $\frac{1000}{1000000000000000000000000}$ parts of a foot; or (which is the ſame thing) through the 1000000000000000000000th part of a foot; a quantity vaſtly too ſmall for our perception.

Were the two bodies not ſo diſproportionate, they would both be ſeen to move towards each other. Thus if two equal bodies, as A and B fig. 3. Plate I. be placed at a certain diſtance of each other, and be then left at liberty, viz. free from any obſtruction, they will move towards each other, and will meet at a point C midway between their original ſituations. But if the bodies be unequal; for inſtance A in fig. 4. Plate I. be three times as big as B, then they will meet at a point C, which is as much nearer the original ſituation of A, than that of B, as the body A is bigger

bigger than the body B; viz. AC will be equal to one third part of BC; for since the quantity of matter in A is equal to three times the quantity of matter in B, the attraction of the former must be three times as great as the attraction of the latter, consequently the space run over by the body B must be three times as great as the space run over by the body A, in the same time.

It is evident that the like reasoning may be applied to bodies that bear any proportion to each other; hence we conclude that *the distances of the original situations of the bodies from the point C, where if left at liberty they will meet in consequence of their mutual attraction, are inversely as their quantities of matter*; viz. as the quantity of matter in A, is to the quantity of matter in B, so is the distance BC, to the distance AC.

The point C is called *the centre of gravity* of those two bodies; being in fact the point, or centre, towards which they gravitate, and where they will actually meet, if not disturbed by any external force or impediment.

What has been observed with respect to the two bodies, may be easily applied to the mutual attraction of three, or four, or, in short, of any number of bodies; there being always a centre of gravity which is common to them all. Such also is the case with a single body; viz. there is a point in any single body, which is its centre of gravity, towards which, if the body were divided into dif-

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ferent parts, thoſe parts would gravitate. The nature and properties of the centre of gravity will be farther noticed in the next chapter.

Since the attractive power is proportionate to the quantity of matter, it follows, *that all ſorts of bodies, however different they may be in their weights, if they begin to move towards the earth from the ſame height, at the ſame time; they muſt be equally accelerated; that is, they muſt all deſcend through the like ſpace in the ſame portion of time*; for though a body A be twice as heavy as another body B, if you imagine that the former is divided into two equal parts, each of thoſe parts muſt be equal to B, and of courſe it muſt move through an equal ſpace, as B, in the ſame time. Now it is evident, that when the two parts of A are joined together, the effect muſt be the ſame. The like reaſoning may be extended to bodies, whoſe quantities of matter bear any other proportion to each other. Hence all ſorts of bodies, when left at liberty, would fall from the ſame height to the ground preciſely in the ſame time, were they not unequally reſiſted by the air through which they move. I ſay unequally reſiſted, becauſe that reſiſtance is in proportion not to the quantity of matter, but to the ſurface, when the quantities of matter are equal. This may be ſatisfactorily proved by a variety of experiments. Take, for inſtance, a ſmall quantity of cotton, ſpread it as much as you can, then let it fall from your hand to the ground, and you will find that the

the cotton will employ three, four, or more, ſeconds of time in that deſcent. But if you take up that cotton and compreſs it into a very ſmall compaſs, you will find that on repeating the experiment, the ſame quantity of cotton will deſcend to the ground in leſs than a ſecond. Thus alſo if you drop from the ſame height at the ſame time a guinea and a common gold leaf, the guinea will come to the ground incomparably quicker than the gold leaf. But if you compreſs the leaf ſo as to form it into a ſmall lump, and repeat the experiment with this lump and the guinea, they will be found to touch the ground nearly at the ſame moment *.

The converſe of the laſt propoſition is likewiſe evident; namely, *that if bodies, in falling from the ſame height towards the centre of the earth, deſcribe equal ſpaces in the ſame portion of time, the attraction*

* This propoſition is confirmed in a manner leſs eaſy indeed, but more evident and concluſive, by means of a tall glaſs receiver, having a mechaniſm at its upper end, from which a guinea and a feather, or other light body, may be dropped at the ſame time. When this glaſs receiver is ſet ſtraight up, and is exhauſted of air, in the manner which will be deſcribed hereafter, the above-mentioned guinea and feather, will, on being diſengaged, arrive at the bottom of the receiver at the ſame moment preciſely. But if the receiver be not well exhauſted of air, then the feather will arrive at the bottom later than the guinea; and much more ſo when the receiver is quite full of air.

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must be proportionate to their quantities of matter; otherwise the spaces, &c. would not be equal.

Hitherto we have taken notice of the properties which naturally arise from the attraction being proportionate to the quantity of matter. It is now necessary to examine the actual motion of bodies which move towards a centre of attraction.

The great difference between the simple impulse, mentioned in the preceding chapter, and a centripetal, or centrifugal, force, is that the former produces equable motion; that is, such as compels bodies to describe equal spaces in equal portions of time; whilst the latter produces unequable motion; viz. it compels bodies to describe unequal spaces in equal portions of time.

This inequality arises from the continual action of the latter power; for a centripetal, or centrifugal, force, does not act at first only; but it does continually act upon, and impel, the bodies in motion; that is, the centripetal, towards the centre of attraction, and the centrifugal, from the centre of repulsion.

The attraction of the earth, or gravitating power, has been found, from a variety of facts, which will be mentioned hereafter, to decrease in proportion as the squares of the distances from the centre of the earth, increase; or, in other words, the force of gravity at different heights is inversely as the squares of the distances from the centre of the earth. At a height, for instance, as far from the

the ſurface of the earth as the ſurface is from the centre, the force of gravity is a quarter of what it is at the ſurface; for the diſtances being as one to two, their ſquares are one and four; therefore, as one is to four, ſo is the force of gravity at the above-mentioned height, to the force of gravity at the ſurface.

This diminution of intenſity in the proportion of the ſquares of the diſtances from the centre of emanation, ſeems to take place not only with the force of gravity, but likewiſe with all ſorts of emanations from a centre, ſuch as light, ſound, &c. as far however as we are able to judge from the preſent ſtate of knowledge; for with the decreaſe either of ſound or of light, this law has not been aſcertained to any great degree of accuracy.

But, independently of actual experiments, it may be ſtrictly demonſtrated, that *emanations, which proceed in ſtraight lines from a centre, and do not meet with any obſtruction, muſt decreaſe in intenſity inverſely as the ſquares of the diſtances from the centre.* (1)

Bodies

(1) Let A, fig. 5. Plate I. be the centre of emanation (for inſtance the flame of a candle.) Let OPB*v* be a ſquare hole, and drawing ſtraight lines from A to the corners of this ſquare, produce them indefinitely towards I, H, E, *r*.

In the firſt place it is evident that the light which paſſes through the ſquare hole OPB*v*, will fill all the ſpace between

Bodies that are left to fall from any height, will move faſter and faſter the nearer they come to the ſurface of the earth; for if the force of gravity acted upon a body only at the commencement of

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tween the four ſtraight lines AH, AI, A*r*, and AE. Secondly, it is alſo evident that if a plane ſurface be placed at E, parallel to the ſquare OPB*v*, all that part of it which lies between the aforeſaid ſtraight lines; viz. IHE*r*, will be illuminated by the light which paſſes through OPB*v*; but as the plane IHE*r* is larger than OPB*v*, the light upon it cannot be ſo denſe as at OPB*v*; and for the ſame reaſon, if a plane be ſituated at D, parallel to OPB*v*, the light upon it will be leſs denſe than at OPB*v*, but more denſe than at IHE*r*, &c. Thirdly, it is alſo evident that the planes IHE*r*, KGD*s*, LFC*x*, are ſquare figures, ſince the hole OPB*v* has been ſuppoſed to be a ſquare. Therefore, the only thing which remains to be proved, is, that if the diſtance AC be equal to twice the diſtance AB, the area of the ſquare LFC*x* is four times as large as the area OPB*v*; that if AD be equal to three times AB, the area KGD*s* is nine times as large as OPB*v*; or, in ſhort, that the areas OB, LC, KD, &c. are as the ſquares of the diſtances from A, which is eaſily done; for ABP, ACF, being equiangular triangles (Eucl. p. 29. B. I.) we have (Eucl. p. 4. B. VI.) AB : AC : : PB : FC; but PB and FC are the homologous ſides of the ſimilar plane figures OPB*v*, LFC*x*; and (Eucl. p. 20. B. VI.) thoſe figures are as the ſquares, or in the duplicate proportion, of their homologous ſides; therefore OPB*v*: LFC*x* : : $\overline{PB}|^2 : \overline{FC}|^2 :: \overline{AB}|^2 : \overline{AC}|^2$. And the like reaſoning may be applied to the other ſquares KGD*s*, &c.

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its descent, the body would, (according to the laws of simple motion, Chap. IV.) continue to describe equal spaces in equal portions of time. But the very next moment the force of gravity impels the body again, in consequence of which the body's velocity must be doubled; since the second impulse is equal to the first, and the first remains unaltered. For the same reason on the third moment the body's velocity will be trebled, and so on. Or, speaking more properly, the velocity will increase as the time increases, viz. *the velocity will be as the time*; the meaning of which is, that the velocity at the end of two seconds is to the velocity at the end of three seconds, as two to three; or the velocity at the end of one minute is to the velocity at the end of one hour, as one is to sixty, &c. *.

The spaces described by such descending bodies cannot be proportionate simply to the times of descent; for that would be the case if the velocity remained unaltered; but, the velocity increasing

* The velocities are as the times when the gravitating power remains unaltered, or with the same gravitating power; but if two distinct gravitating powers be compared together, then the velocities will be as the products of the times multiplied by the gravitating forces respectively; it being evident that a double force will produce a double effect, a treble force will produce a treble effect, &c. Hence when the times are equal, or in the same time, the velocities are as the gravitating, or the impelling, forces.

continually,

continually, it is evident *that the ſpaces muſt be as the times multiplied by the velocities*; for a double velocity will force the body to move through a double ſpace in an equal portion of time, and through a quadruple ſpace in twice that time; alſo a quadruple velocity will force the body to move through a quadruple ſpace in an equal portion of time, and through eight times that ſpace in twice that time; and ſo on in any proportion. But it has been ſhewn above that the velocities are as the times; therefore to ſay that the ſpaces are as the times multiplied by the velocities, is the ſame thing as to ſay that the ſpaces are as the times multiplied by the times, or as the ſquares of the times; and for the ſame reaſon it is the ſame thing as to ſay that the ſpaces are as the velocities multiplied by the velocities, or as the ſquares of the velocities *.

This property of deſcending bodies, (viz. that they run through ſpaces which are as the ſquares of the times) has been uſually demonſtrated in a different way by the philoſophical writers. Their demonſtration may, perhaps, appear more ſatisfactory than that of the preceding paragraphs to ſome of my readers; I ſhall therefore ſubjoin it, eſpecially as it proves at the ſame time another law relative to the velocity of deſcending bodies.

* Therefore in equal times the ſpaces are as the impelling, or gravitating, forces. See the laſt note.

Let AB, fig. 6. Plate I. represent the time, during which a body is descending, and let BC represent the velocity acquired at the end of that time. Complete the triangle ABC, and the parallelogram ABCD. Also suppose the time to be divided into innumerable particles, *ei*, *im*, *mp*, *po*, &c. and draw *ef*, *ik*, *mn*, &c. all parallel to the base BC. Then, since the velocity of the descending body has been gradually increasing from the commencement of the motion, and BC represents the ultimate velocity; therefore the parallel lines *ef*, *ik*, *mn*, &c. will represent the velocities at the ends of the respective times A*e*, A*i*, A*m*, &c. Moreover, since the velocity during an indefinitely small particle of time, may be considered as uniform; therefore the right line *ef* will be as the velocity of the body in the indefinitely small particle of time *ei*; *ik* will be as the velocity in the particle of time *im*, and so forth. Now the space passed over in any time with any velocity is as the velocity multiplied by the time; viz. as the rectangle under that time and velocity; hence the space passed over in the time *ei* with the velocity *ef*, will be as the rectangle *if*; the space passed over in the time *im* with the velocity *ik*, will be as the rectangle *mk*; the space passed over in the time *mp* with the velocity *mn*, will be as the rectangle *pn*, and so on. Therefore the space passed over in the sum of all those times, will be as the sum of all those rectangles. But since the particles of time

time are infinitely ſmall, the ſum of all the rectangles will be equal to the triangle ABC. Now ſince the ſpace paſſed over by a moving body in the time AB with a uniform velocity BC, is as the rectangle ABCD, (viz. as the time multiplied by the velocity) and this rectangle is equal to twice the triangle ABC (Eucl. p. 31. B. I.) therefore the ſpace paſſed over in a given time by a body falling from reſt, is equal to half the ſpace paſſed over in the ſame time with an uniform velocity, equal to that which is acquired by the deſcending body at the end of its fall.

Since the ſpace run over by a falling body in the time repreſented by AB, fig. 7. Plate I. with the velocity BC is as the triangular ABC, and the ſpace run over in any other time AD, and velocity DE, is repreſented by the triangle ADE; thoſe ſpaces muſt be as the ſquares of the times AB, AD; for the ſimilar triangles ABC, and ADE, are as the ſquares of their homologous ſides, viz. ABC is to ADE as the ſquare of AB is to the ſquare of AD, (Eucl. p. 29. B. VI.)

In fig. the 8th. Plate I. the ſpaces, which are deſcribed by deſcending bodies in ſucceſſive equal portions of time, are repreſented, for the purpoſe of impreſſing with greater efficacy on the mind of the reader, the principal law of gravitation. The line AB repreſents the path of a body, which is let fall from A, and deſcends towards the ground at B. The diviſions on the line AB denote the places of

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the body at the end of one ſecond, two ſeconds, &c. which equal portions of time are marked on the left hand ſide; whilſt the numbers on the right expreſs the feet percurred, or real diſtances from A to the firſt diviſion, from A to the ſecond diviſion, and ſo on. It appears, therefore, that in one ſecond the body has deſcended through 16,087 feet, that in two ſeconds it has deſcended through four times 16,087, or 64,348 feet, &c.

It may alſo be obſerved, that the ſpaces run through during each ſingle ſecond, are as the odd numbers 1, 3, 5, 7, &c.; that is, if the ſpace percurred in the firſt *ſecond* be called one, the ſpace percurred during the ſecond *ſecond* only will be three times as great, the ſpace percurred in the third *ſecond* will be five times as great, and ſo on. In fact, if we ſubtract 16,087 from 64,348, the remainder, 48,261, is equal to three times 16,087; if we ſubtract 64,348 from 144,783, the remainder, 80,435, is equal to five times 16,087, &c.

It has been ſhewn above that the force of gravity at equal diſtances from the centre of the earth is proportionate to the quantity of matter; but it muſt be obſerved, that when the diſtances are unequal, then the gravitating forces, or weights, of bodies, are as the quotients of the quantities of matter divided by the ſquares of the diſtances reſpectively, or, which is the ſame thing, the weights of bodies are ſaid to be as the reſpective quantities of matter directly, and the ſquares of the reſpective diſtances inverſely;

inverſely; ſince the gravitating force has been ſhewn to decreaſe inverſely as the ſquares of the diſtances from the centre of attraction. Thus if a body A, which is five times as big as another body B, is ſituated at the diſtance of 4000 miles from the centre of the earth, whilſt B is ſituated at 6000 miles diſtance, then the weight of A will be to the weight of B as the quotient of five divided by the ſquare of 4000, is to the quotient of one divided by the ſquare of 6000; viz. as $\frac{5}{16000000}$ is to $\frac{1}{36000000}$ *.

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* Suppoſe it be required to find how much a leaden ball, which on the ſurface of the earth weighs twenty pounds, will weigh at the top of a mountain which is three miles high.

The ſemidiameter of the earth is known to be about 3985 miles, to which we add the height of the mountain, viz. three miles, and we have the two diſtances; that is from the centre of the earth to the ſurface, 3985 miles, and from that centre to the top of the mountain 3988 miles. The ſquares of thoſe numbers are 15880225 and 15904144. Then ſay as 15904144 is to 15880225, ſo is twenty pounds to a fourth proportional, which by the common rule of three (viz. by multiplying 15880225 by 20, and dividing the product by 15904144) will be found to be 19,969; or 19 pounds and $15\frac{1}{2}$ ounces, which is the weight of the leaden ball at the top of the mountain, viz. nearly half an ounce leſs than on the ſurface of the earth.

It muſt not, however, be imagined that the leaden ball, which is balanced by a counterpoiſe of twenty pounds in

 a pair

In the preceding explanations and examples, the ſpaces and velocities of deſcending bodies have been calculated on the ſuppoſition that the force acts uniformly; viz. that during the deſcent of the body from A, fig. 8. Plate I. towards the ground, the attraction of the earth does not increaſe; which ſuppoſition, ſtrictly ſpeaking, is not true; for it has already been ſhewn, that the force of gravity decreaſes inverſely as the ſquares of the diſtances from the centre of the earth; ſo that the nearer the body comes to the ground, the ſtronger its gravitation will be. However, in ſhort diſtances from the ſurface of the earth, that increaſe of gravity is ſo very trifling, that for common purpoſes it may be ſafely neglected. But as the ſame theory is applicable to all ſorts of gravitating powers, and as very great diſtances may ſometimes enter the calculation, it will be proper to ſubjoin the method of calculating the velocities which are acquired by bodies deſcending towards a centre

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a pair of ſcales on the ſurface of the earth, will appear lighter at the top of the mountain; for this will not be the caſe, becauſe the counterpoiſe itſelf will loſe an equal portion of its weight by being ſituated on the top of the mountain; and of courſe the equilibrium of the ſcales will not be diſturbed. But if the leaden ball in queſtion be weighed in one of thoſe weighing inſtruments which are made with a ſpiral ſteel ſpring, then indeed the decreaſe of its weight at the top of the mountain will be clearly perceived, provided the weighing inſtrument be ſufficiently accurate.

centre of attraction, when the increase of the attractive power is taken into the account (2).

It is evident that, since the continual action of the force of gravity accelerates the motion of a descending body, it must continually retard the motion

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(2) This problem is taken from Dr. Saunderson's Method of Fluxions.

PROBLEM.

Let S. fig. 9. Plate I. be the centre of the earth, B any point in its surface, and let the force of gravity in all places be reciprocally as the squares of their distances from the centre of the earth: *it is required to determine the velocity of a heavy body at the surface of the earth, which it acquires in falling from any given altitude AB.*

Let x be any indeterminate distance from the centre of the earth, and let v be the velocity of the falling body at that distance. Let $\frac{1}{\overline{SB}|^2}$ represent the force of gravity at B, and consequently $\frac{1}{xx}$ its force at the distance x. First then it is plain, that after the falling body is arrived at the distance x, and then descends further through any infinitely small space as $\dot{x}$, the time of that infinitely small descent will be as $\frac{\dot{x}}{v}$; that is, it will be as the space directly, and as the velocity inversely; and the infinitely small acquisition made by the velocity in that descent will be as the time $\frac{\dot{x}}{v}$ and the accelerating force $\frac{1}{xx}$ jointly; that is, $\dot{v}$ will be as $\frac{\dot{x}}{vxx}$; therefore $v\dot{v}$ will be as $\frac{\dot{x}}{xx}$; therefore the fluents of these fluxions, which are generated in equal times, motion of an aſcending body. A ball, for inſtance, which is projected upwards, will be gradually retarded by the gravitating force, which acts in a contrary direction.

The foregoing explanations relatively to the laws of gravity, or of a centripetal force, may be eaſily applied to the explanation of the properties of a repulſive, or centrifugal, force; for in fact the ſame reaſoning

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times, will be proportionable; that is, $\frac{1}{2}vv$ will be as $\frac{1}{SB}-\frac{1}{SA}$, or as $\frac{SA-SB}{SB\times SA}$, or as $\frac{AB}{SB\times SA}$: therefore, ſince the quantities 2 and SB are conſtant, vv will be as $\frac{AB}{AS}$.

This being diſcovered, let DB be the height from which a body will fall to the ſurface of the earth in one *ſecond* of time; and ſince during ſo ſmall a deſcent, the force of gravity may be looked upon as uniform, it is evident that a body falling from D to B will acquire a velocity which will carry it uniformly through the ſpace 2BD in a ſecond of time. Let 2BD repreſent this velocity; then muſt every other velocity be repreſented by the ſpace through which it will carry a body in a ſecond of time. Now to find the velocity acquired in falling from A to B, I ſay as $\frac{DB}{DS}$ is to $\frac{AB}{AS}$, ſo is $4DB^2$ (the ſquare of the velocity acquired in falling from D to B) to $4DB\times DS\times\frac{AB}{AS}$, (the ſquare of the velocity acquired in falling from A to B) $=4mm\times\frac{AB}{AS}$, ſuppoſing m to be a mean proportional between DB and DS. Therefore a body falling from A to B acquires a velocity reaſoning will do for the one as for the other, changing only the word *attraction* for *repulſion*, and the word *acceleration* for *retardation*. Thus the velocity of a body which is receding from a centre of repulſion, is retarded in proportion as the time increaſes; or the velocities are ſaid to be inverſely as the times. Alſo the ſpaces decreaſe, or are, inverſely, as the ſquares of the times.

If a body be thrown perpendicularly upwards, that is, in a direction from the centre of the earth, with the velocity which it acquired by falling in a given time, it will arrive at the ſame height from

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velocity that would carry it through the ſpace $2\,m \times \frac{\overline{AB}|^{\frac{1}{2}}}{\overline{AS}|^{\frac{1}{2}}}$ in a ſecond of time.

Scholium. DB and DS having been aſcertained by means of experiments, $2\,m$ is thereby found to be about ſeven Engliſh miles.

Corollary 1. If AB be infinite, the quantity $\frac{AB}{AS}$ becomes equal to one, hence it goes out of the queſtion; and therefore a body falling from an infinite height, will acquire at the ſurface of the earth but a finite velocity; viz. ſuch a velocity as will carry it uniformly through ſeven miles in one ſecond of time.

Coroll. 2. Therefore, *e converſo*, if a body be thrown upwards with ſuch a velocity, it will never return, but it will aſcend for ever.

Coroll. 3. After the ſame manner we may determine the velocity of a falling body, whatever be the law of gravity.

which

which it fell, in the same time; and will lose all its momentum. And when bodies are thrown perpendicularly upwards, the heights of their ascents are as the squares of their velocities, or as the squares of the times of their ascending.

N. B. Throughout this chapter no notice has been taken of the resistance, which the air offers to the motion of bodies.

CHAPTER VI.

THE METHOD OF ASCERTAINING THE SITUATION OF THE CENTRE OF GRAVITY, AND AN ENUMERATION OF ITS PRINCIPAL PROPERTIES.

THE definition and the nature of the centre of gravity having been shewn in the preceding pages, we shall in the present chapter shew the method of finding its situation in a system of bodies, as well as in a single body or figure; after which we shall state its various properties, the knowledge of which is of the utmost importance in the study of natural philosophy, and especially in mechanics.

When the common centre of gravity of two bodies is to be determined, their quantities of matter and distance from each other being known;

draw a ſtraight line from the centre of gravity of one of the bodies, as A, fig. 10. Plate I. to the centre of gravity of the other, B. (which centres we ſhall, for the preſent, ſuppoſe to be known; for it will preſently be ſhewn how to find the centre of gravity of a ſingle body.) Then divide this line in E, ſo that its parts BE, AE may be to each other in the proportion of A to B, and E is the centre ſought. For example, let A weigh 3 pounds, B 2 pounds, and let the diſtance AB be 20 feet. Say as 3 is to 2, ſo is BE to AE; then by compoſition (Eucl. p. 18. B. V.) ſay as 3 *plus* 2, viz. 5, is to 2 ſo is BE *plus* AE, viz. 20 feet, to a fourth proportional, which, by the common rule of three, is found to be 8, and is equal to AE; ſo that the centre of gravity E is 8 feet diſtant from A.

When the centre of gravity of three bodies, as A, B, and D, fig. 11. Plate I. is to be determined, their quantities of matter and diſtances being known; you muſt in the firſt place find the centre of gravity E between any two of thoſe bodies, as of A and B, after the manner mentioned above. Then imagine that the two bodies A and B are both collected in the point E, and laſtly find the centre of gravity between E and D, which will be the common centre of gravity of the three bodies; viz. draw the ſtraight line DE, then as the ſum of the matter in A and B, is to the matter in D, ſo is DC to CE; and, by compoſition, ſay as the ſum of the matter in A, B and D, is to D, ſo is DE to CE;

which

which gives C, the common centre of gravity of the three bodies.

In the same manner the centre of gravity of four, or more, bodies may be determined; viz. by conceiving the matter of three of those bodies to be collected in the common centre of gravity of those three bodies, and then finding the common centre of gravity of the last mentioned centre and the fourth body, &c.

The centre of gravity of a single body may be easily determined by the following general method, viz. by supposing the body to be divided into two or more parts, and then finding the common centre of gravity of those parts, which will be the centre of gravity of the body itself. But in certain regular figures, such as a circular surface, a sphere, a cube, &c. it is evident that the centre of the figure must coincide with the centre of gravity; for if in the circle, for instance, fig. 12. Plate I. you divide the area into two equal parts ADB and AEB, it is evident that the common centre of gravity of those two equal parts must be somewhere in the line AB; and if, by cutting the circle in any other direction ED, you divide the area into two other equal parts EAD, and DBE, it is evident that the common centre of gravity of those two parts must be somewhere in the diameter ED; therefore the centre of gravity of the circle must be in the intersection of the two diameters AB, ED; viz. at C, which is the centre of the circle.

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In a right lined plane triangle, as ABD, fig. 13. Plate I. the centre of gravity may be eafily found by dividing any two of its fides, as AD and BD, each into two equal parts at F and E, and by drawing ftraight lines from thofe points of divifion to the oppofite angles; the interfection C of thofe two lines being the centre of gravity of the triangle; for fince the line AE divides the triangle into two equal parts, (Eucl. p. 1. B. VI.) the common centre of gravity of thofe two parts muft be fomewhere in the line AE, and for the fame reafon the common centre of gravity of the two parts ABF, and FBD muft be fomewhere in the line BF; therefore the centre of gravity of the whole triangle muft be at C, the interfection of the two lines AE, BF.

If the figure be terminated by more than three ftraight lines, as ABCDE, fig. 14. Plate I. its centre of gravity may be found by dividing it into any convenient number of triangles, as ABC, BCF, DFE, then by finding the centre of gravity of each triangle, and laftly by finding the common centre of gravity of all the triangles, which is to be done in the fame manner as the centre of gravity between three or more bodies was determined above. In a fimilar manner the centre of gravity of irregular folids may frequently be found.

There are however feveral figures in which the centre of gravity cannot be eafily found by the above defcribed methods, at leaft not with great accuracy. But a more general, and accurate method

method of finding the centre of gravity, is derived from the doctrine of fluxions, which will be found explained in the note (1).

The

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(1) Imagine that at D, E, F, G, H, fig. 15. of Plate I. there are so many weights affixed to the inflexible line AB; and let C be the centre of gravity of the said line and weights together; so that when the point C rests upon a fulcrum, neither end will preponderate, and of course the whole loaded line will remain perfectly balanced.

It has been shewn in the preceding pages that the momentum or force of any weight, as H, (viz. the body suspended at H) to raise the opposite end A of the line, or lever, is expressed by the product of its quantity of matter multiplied by its distance from the fulcrum, or centre of gravity C; viz. by $H \times HC$ (for by the letters D, E, F, &c. we express the weights of the respective bodies). Therefore, since the line with all the weights is perfectly balanced, it follows that the sum of the momenta of all the weights which lie on one side of C, must be equal to the sum of all the momenta, which lie on the other side of C; viz. $H \times HC + G \times GC + F \times FC = D \times DC + E \times EC$; that is $H \times \overline{BC - BH} + G \times \overline{BC - BG} + F \times \overline{BC - BF} = E \times \overline{EB - BC} + D \times \overline{DB - BC}$; or $H \times BC - H \times BH + G \times BC - G \times BG + F \times BC - F \times BF = E \times EB - E \times BC + D \times DB - D \times BC$. Then by transposition we have $H \times BC + G \times BC + F \times BC + E \times BC + D \times BC = H \times BH + G \times BG + F \times BF + E \times EB + D \times DB$; which equation by division becomes $BC = \frac{H \times BH + G \times BG + F \times BF + E \times EB + D \times DB}{H + G + F + E + D}$ which shews that the distance of the centre of gravity, C, from the

The principal properties of the centre of gravity are as follows:

I. *If two bodies be connected together by means of an*

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the extremity B, is equal to the quotient of the ſum of the products of all the weights multiplied each by its diſtance from B, divided by the ſum of all the weights.

Hence we derive the following general rule for finding the centre of gravity in a ſyſtem of bodies; viz. *Aſſume a point at one extremity of the ſyſtem; multiply the weight of each body by its diſtance from that point; divide the ſum of the products by the ſum of the weights, and the quotient will expreſs the diſtance of the centre of gravity from the aſſumed point.*

Take notice that if the above-mentioned point be aſſumed not at the extremity of the ſyſtem, but any where between the bodies, as between D and H, fig. 15. plate I; then the products by their reſpective diſtances, of the bodies on one ſide of the aſſumed point, muſt be conſidered as negative, whilſt the other are conſidered as poſitive: and they muſt be all added together agreeably to the common algebraical rule for adding poſitive and negative quantities together. The reſult then, according as it turns out poſitive or negative, will ſhew the diſtance of the centre of gravity from the aſſumed point, either on the poſitive or on the negative ſide of that point.

The rule may be applied to the caſes of ſingle bodies, as for inſtance, for finding the centre of gravity of a triangle, of a cone, of a ſphere, &c. by imagining the ſaid figute to be reſolved into an infinite number of parts; for by the method of fluxions, the ſum of the momenta, as alſo of the weights of all thoſe parts, may be eaſily aſcertained.

Thus

an inflexible line or rod, as A and B, fig. 22. Plate I. and the line or rod be supported by a prop, or (as it is commonly

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Thus let the figure be a plane, as A D C, fig. 16. Plate I. whose axis is A B, and whose parts are supposed to be endued with gravity. Imagine this figure to be resolved into an infinite number of weights F G, fg, &c. all perpendicular, or all alike inclined, to the axis A B, and let x represent the distance A E of the little weight F G from A. Then the breadth of one of those weights is denoted by $\dot{x}$ (the fluxion of the axis A E); therefore one of those infinitely small weights is expressed by $FG \times \dot{x}$; the fluent of which, when x becomes equal to the whole axis A B, is the sum of all the weights. Farther, if one of those weights be multiplied by its distance from A, the product, $FG \times \dot{x} \times x$, will express its momentum; and the fluent of this expression, when x becomes equal to the whole axis A B, is the sum of all the momenta. Therefore, agreeably to the general rule, if the fluent of $FG \times x\dot{x}$ be divided by the fluent of $FG \times \dot{x}$, the quotient will express the distance of the centre of gravity from A on the axis A B.

If the figure be a solid, imagine it to be divided into an infinite number of sections, or small weights, all perpendicular, or all alike inclined, to the axis; put the expression which denotes one of those sections, instead of F G in the above fluxional expressions; then proceed as above directed. Or, which is the same thing, call one of those sections S; then divide the fluent of $Sx\dot{x}$, by the fluent of $S\dot{x}$, and the quotient will express the distance of the centre of gravity.

The only practical difficulty consists in finding the value of F G; or of the above mentioned section S in a solid figure, which commonly called) *a fulcrum, placed under the centre of gravity C; the bodies will remain motionless.*

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which value must, by means of the equation of the figure, be expressed in terms wherein x is the only variable letter. But the following examples will point out the mode of proceeding.

Example I. *To find the centre of gravity of a straight line, or very slender cylinder, AB, fig. 17. plate I. whose parts may be supposed to be endued with gravity.*

Let its length AB be called e, and supposing the line to be divided into an infinite number of little parts or weights. One of those parts is denoted simply by $\dot{x}$; (for the breadth FG is nothing) and the fluent of $\dot{x}$ is x. Also the momentum of one of those particles is expressed by $x\dot{x}$, whose fluent is $\frac{1}{2}x^2$. Therefore, dividing the latter fluent by the former, we have $\frac{\frac{1}{2}x^2}{x}$, which when $x=e$, becomes $\frac{\frac{1}{2}e^2}{e}$ or $\frac{e}{2}$, and shews that the centre of gravity is at C in the middle of the line; viz. its distance from either extremity is equal to half the length of the line.

Example II. *To find the centre of gravity in a triangle ABC, fig. 18. Pl. I. where the axis AD=a, base BC=b; EF is parallel to the base, and =y; and let AO be called x.*

From the similarity of the triangles we have AD : BC :: AO : EF; viz. $a:b::x:\frac{bx}{a}=y$. Therefore the infinitely small weight EF is denoted by $y\dot{x}$, or by $\frac{bx\dot{x}}{a}$, whose fluent is $\frac{bx^2}{2a}$; which (when x is equal the whole axis AD) becomes $\frac{ba^2}{2a}$ or $\frac{ba}{2}$. Also the momentum of the little weight $\frac{bx\dot{x}}{a}$

In this situation no one of the bodies will have more tendency towards the earth than the other; for

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is denoted by $\frac{bx^2\dot{x}}{a}$; the fluent of which is $\frac{bx^3}{3a}$, or (when x is equal AD) $\frac{ba^2}{3}$. Then $\frac{ba^2}{3}$ divided by $\frac{ba}{2}$, quotes $\frac{2a}{3}$, which is the distance of the centre of gravity from the vertex A; viz. $\frac{2}{3}$ of the axis AD.

It needs hardly be mentioned that the centre of gravity must necessarily be in the axis.

Example III. *To find the centre of gravity in a parabolic figure ABC, fig.* 19. *Plate I.*

Put the axis AD $= a$; abscis AF $= x$; and ordinate EF $= y$. By conics we know that the square of the ordinate EF is equal to the product of the parameter multiplied by the abscis, viz. $y^2 = px$, hence $y = p^{\frac{1}{2}}x^{\frac{1}{2}}$, and EG $= 2y = 2p^{\frac{1}{2}}x^{\frac{1}{2}}$; which being multiplied by $\dot{x}$, viz. $2p^{\frac{1}{2}}x^{\frac{1}{2}}\dot{x}$, represents one of the infinitely small weights, into which the parabola is supposed to be resolved; and its fluent $\frac{2p^{\frac{1}{2}}x^{\frac{3}{2}}}{\frac{3}{2}}$, or $\frac{4}{3}p^{\frac{1}{2}}x^{\frac{3}{2}}$ represents the sum of all the weights. Farther $2p^{\frac{1}{2}}x^{\frac{1}{2}}x\dot{x}$; or $2p^{\frac{1}{2}}x^{\frac{3}{2}}\dot{x}$ is the momentum of the little weight; whose fluent, which represents the sum of all the momenta, is $\frac{2p^{\frac{1}{2}}x^{\frac{5}{2}}}{\frac{5}{2}}$, or $\frac{4}{5}p^{\frac{1}{2}}x^{\frac{5}{2}}$. Then dividing the latter fluent by the former, we have $\frac{4}{5}p^{\frac{1}{2}}x^{\frac{5}{2}} \div \frac{4}{3}p^{\frac{1}{2}}x^{\frac{3}{2}} = \frac{3}{5}x$, or (when x is equal to the whole axis AD) $= \frac{3}{5}a$; so that the centre of gravity is distant from the vertex A, $\frac{3}{5}$ of the whole axis AD.

Example

for ſince C is their common centre of gravity, the diſtances A C and CB, are inverſely as the weights of

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Example IV. *To find the centre of gravity of a right cone ABC, fig. 20. Plate I.*

Put the axis or altitude AD$=a$, diameter of the baſe BC$=b$; abſciſs AO$=x$ and ordinate EO$=y$.

Then from the ſimilarity of the triangles AOE, ADB, we have AD : DB : : AO : EO; viz. $a:\frac{b}{2}::x:y=\frac{bx}{2a}$, and, putting c for the circumference of a circle whoſe diameter is unity, the circumference of a circle whoſe diameter is EF, or $2y$, will be $2cy$; whoſe area is $2cy\times\frac{1}{2}y$; viz. cy^2, or (by ſubſtituting $\frac{bx}{2a}$ for its equal y) $\frac{cb^2x^2}{4a^2}$. Therefore $\frac{cb^2x^2}{4a^2}\dot{x}$ repreſents one of the infinitely ſmall weights into which the cone is ſuppoſed to be reſolved, and its fluent, $\frac{cb^2x^3}{12a^2}$, is the ſum of all thoſe weights. Alſo $\frac{cb^2x^2}{4a^2}x\dot{x}$ is the momentum of the little weight, and its fluent, $\frac{cb^2x^4}{16a^2}$, is the ſum of all the momenta. Then dividing the latter fluent by the former we have $\frac{cb^2x^4}{16a^2}\div\frac{cb^2x^3}{12a^2}=\frac{3}{4}x$; or (when x becomes equal to the whole axis AD) $\frac{3}{4}a$; which ſhews that the diſtance of the centre of gravity from A on the axis AD is equal to $\frac{3}{4}$ of the whole axis AD.

Example V. *To find the centre of gravity of an hemiſphere ABO, fig. 21. Plate I.*

Put the axis or radius AD$=a$; DP$=x$; and MP, which is parallel to the baſe, $=y$. Then PMD being a right angled triangle, we have $\overline{MP}|^2=\overline{MD}|^2-\overline{DP}|^2$; viz.

 $yy=$

of thofe bodies; viz. AC is to CB as the body B is to the body A. Now fhould the rod be moved from its fituation AB into the fituation FE, the body B would defcribe the arch BE, and the body A would defcribe the arch AF, which arches reprefent the velocities of thofe bodies; for they are the fpaces through which they move in

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$yy = a^2 - x^2$. And, putting c for the circumference of a circle whofe diameter is unity, the circumference of a circle whofe diameter is ME, or $2y$, will be $2cy$, and its area will be $2cy \times \frac{1}{2}y$; viz. cy^2, or (by fubftituting for yy its value as found above, viz. $a^2 - x^2$) $ca^2 - cx^2$; and this is a fection of the hemifphere parallel to the bafe. Then $\overline{ca^2 - cx^2} \times \dot{x}$ is one of the infinitely fmall weights into which the hemifphere is fuppofed to be divided; and its fluent $ca^2x - \frac{cx^3}{3}$ is the fum of all thofe weights. Alfo $\overline{ca^2 - cx^2} \times x\dot{x}$ is the fluxion of the momentum of the fmall weight; the fluent of which, viz. $\frac{ca^2x^2}{2} - \frac{cx^4}{4}$, is the fum of all the momenta. And, when x is equal to the whole axis AD, thofe two fluents become $\left(ca^3 - \frac{ca^3}{3}\right.$, or $\left.\frac{3ca^3 - ca^3}{3}, \text{ or}\right) \frac{2ca^3}{3}$; and $\left(\frac{ca^4}{2} - \frac{ca^4}{4}\right.$; or $\left.\frac{2ca^4 - ca^4}{4}, \text{ or}\right) \frac{ca^4}{4}$. Then, dividing the latter fluent by the former, we have $\frac{ca^4}{4} \div \frac{2ca^3}{3} = \frac{3ca^4}{8ca^3} = \frac{3}{8}a$; fo that the centre of gravity is diftant from the point D, $\frac{3}{8}$ of the axis, or of the radius, AD.

the

the ſame time. But it has been demonſtrated by the geometricians, that thoſe arches bear the ſame proportion to each other as the radii or diſtances CB, CA; viz. BE is to AF as BC is to AC; and it has been ſhewn above, that BC is to AC as the body A is to the body B; therefore it follows, that the arch BE is to the arch AF as the body A is to the body B. But of four proportional quantities, the product of the extremes is equal to the product of the means; therefore the product of the body B, multiplied by the arch BE (which conſtitutes the momentum of B) is equal to the product of the body A multiplied by the arch AF (which conſtitutes the momentum of A); ſo that their momentums being equal, thoſe bodies will balance each other, and of courſe they will remain at reſt. It is evident that the ſame reaſoning is applicable to the common centre of gravity of any number of bodies, as alſo to the centre of gravity of a ſingle body; viz. that if a ſyſtem of bodies, that are connected together, or a ſingle body, be placed with the centre of gravity on a fulcrum, that ſyſtem, or ſingle body, will remain perfectly balanced thereon, and as ſteady as if it were placed upon a flat horizontal ſurface.

II. *The ſtate, whether of reſt or motion, of the common centre of gravity of two bodies, will not be altered by the mutual action of thoſe bodies upon each other.*

For, in the firſt place, ſuppoſe that the centre of

 gravity

gravity is at reſt; and that the two bodies, in conſequence of their mutual attraction, approach each other in a certain time; it follows from the foregoing theory, that the ſpaces through which they move muſt be inverſely as their weights, and of courſe, the remaining ſpaces between their laſt ſituations and the centre of gravity, will remain in the ſame proportion to each other as the original diſtances; therefore the centre of gravity will not be moved from its original place.—An example will render this explanation more evident.

Let the body A, fig. 23, Plate I. weigh 2 pounds, and B, 6 pounds. The diſtances of thoſe bodies from the centre of gravity C, are inverſely as the weights of thoſe bodies; viz. BC, is 10 feet, and AC, 30 feet; (that is, as one to three) becauſe the weights are as three to one. Now ſuppoſe that in conſequence of their mutual attraction, thoſe bodies begin to move towards each other; and if in one minute B comes to the place D, having paſſed over the diſtance BD, equal to one foot; the body A muſt in the ſame time have paſſed over 3 feet, and muſt have arrived at E; then taking away BD from BC; viz. one foot from 10 feet, there remains DC, equal to 9 feet; alſo taking away AE from AC; viz. 3 feet from 30, there remains CE, equal to 27 feet. Now thoſe two remaining diſtances; viz. 9, and 27, are the one to the other as one to three; therefore, &c.

In the ſecond place, if the two bodies, together with

with their common centre of gravity, be in motion, the peculiar motion, which arises from their mutual attraction, cannot alter the course of their common centre of gravity; because, as has been said above, they must move towards the centre C in the inverse ratio of their weights.

III. The same reasoning may be evidently applied to the case of three, or four, or, in short, of any number of bodies; as also to the case of repulsion; changing, in the latter case only, the name of *centre of attraction*, for *centre of repulsion*; and reckoning the motion of the bodies not towards, but from that centre. Hence it may in general be concluded, that *the state of the common centre of gravity of different bodies will not be altered by the mutual action of those bodies upon each other.*

IV. *If any two bodies be carried towards the same parts with equal or unequal velocities, the sum of the momentums of both the bodies will be equal to the momentum that would arise if both the bodies moved with the velocity of their common centre of gravity.*

When the velocities of the bodies A and B are equal, the proposition is self-evident; for then their common centre of gravity must move with the like velocity. But whether the velocities be equal or unequal, the proposition may be easily demonstrated (2).

V. *If*

(2) Let A and B, fig. 1. Plate II. be two bodies moving towards D. Let C be their common centre of

gravity,

V. *If two bodies be carried towards contrary parts, the difference of their momentums towards those contrary parts*

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gravity, and whilst A moves as far as *a*, let B move as far as *b*, and C as far as *c*. Since the spaces A *a*, B *b*, and C *c*, express the respective velocities of the two bodies, and of their centre of gravity, all we have to prove is, that the sum of the products of A multiplied by A *a*, and of B multiplied by B *b*, is equal to the product of A plus B, multiplied by C *c*; viz. that $A \times Aa + B \times Bb = \overline{A+B} \times Cc$.

Since C is the centre of gravity, A is to B as B C is to A C, and as *b c* is to *a c*. Then alternately $BC : bc :: AC : ac$, and conversely $BC : BC - bc :: AC : AC - ac$. But $BC - bc$ is equal to $Cc - Bb$; and $AC - ac$ is equal to $Aa - Cc$; therefore $BC : AC :: Cc - Bb : Aa - Cc$. But it has been shewn above, that $A : B :: BC : AC$; therefore $A : B :: Cc - Bb : Aa - Cc$. Now, since of four proportional quantities, the product of the extremes is equal to the product of the means, we have $A \times Aa - A \times Cc = B \times Cc - B \times Bb$, and by transposition $A \times Aa + B \times Bb = \overline{A + B} \times Cc$.

When the bodies do not move in the same straight line, the demonstration is the same; excepting only that in this case the velocity is to be reckoned not upon the path which is actually described by the bodies, but upon the path of their common centre of gravity. Suppose, for instance, that the bodies A and B, fig. 2. Plate II. move towards D, whilst their common centre of gravity C moves in the line C D; also that in a certain time those bodies have moved as far as the places *a* and *b* respectively, at the same time that their common centre of gravity has moved from C to *c*.

From

parts (which is equivalent to the ſum of their momentums towards the ſame part) will be equal to the momentum that would ariſe if both the bodies were carried towards the ſame part with the velocity of their common centre of gravity (3).

VI. What

From A, B, *a*, and *b*, drop BH, AF, *ag* and *b*K, all perpendicular to HD, the direction of the centre of gravity. Then F*g* will repreſent the velocity of A, and HK, the velocity of B; for thoſe are the real advances the bodies have made towards D. Now from the property of the centre of gravity we have A : B : : BC : AC : : (ſince the right angled triangles ACF, BCH, are equiangular, and conſequently ſimilar, by Eucl. p. 15, B. I. and p. 4. B. VI.) HC : CF : : K*c* : *cg*. Then the demonſtration proceeds in the ſame manner as for the preceding caſe.

(3) Suppoſe that the body A, fig. 3. Plate II. moves from A to *a*, at the ſame time that the body B moves in a contrary direction from B to *b*, whilſt their common centre of gravity moves from C to *c*. Then their reſpective velocities are repreſented by A*a*, B*b*, and C*c*. Now, in order to demonſtrate the propoſition, we muſt prove that A multiplied by A*a*, minus B, multiplied by B*b*, is equal to A multiplied by C*c*, plus B multiplied by C*c*; or, $A \times Aa - B \times Bb = \overline{A + B} \times Cc$.

From the nature of the centre of gravity, we have A : B : : BC : AC : : *bc* : *ac*; hence alternately BC : *bc* : : AC : *ac*, and converſely BC : BC — *bc* : : AC : AC — *ac*. But it has been ſhewn that A : B : : BC : AC; therefore by ſubſtitution and alternation, we have A : B : : BC — *bc* : AC — *ac*.

Farther,

VI. What has been ſaid in the preceding paragraphs with reſpect to the centre of gravity of two bodies, may be applied to the centre of gravity of three, or four, or, in ſhort, of any number of bodies; for it follows from the preceding propoſitions, that two or more of the bodies may be conceived to be concentrated into their common centre of gravity; hence the caſe may always be reduced to that of two bodies only.

If it be aſked why, in the computation of the centre of gravity, we took no notice of the decreaſe of the attractive force according to the ſquares of the diſtances; the anſwer is, that in that caſe the diſtance being one and the ſame; viz. (the diſtance of the body A from the body B, Fig. 22. Plate I. is the ſame as the diſtance of B from A,) the computation is not altered by it*.

Farther, $BC - bc$ is equal to $Bb + Cc$, and $AC - ac$ is equal to $Aa - Cc$; therefore $A : B :: Bb + Cc : Aa - Cc$; of which four proportional quantities the product of the extremes muſt be equal to the product of the means; viz. $A \times Aa - A \times Cc = B \times Bb + B \times Cc$; and, by tranſpoſition, $A \times Aa - B \times Bb = \overline{A+B} \times Cc$.

* For inſtance, if we ſay that the attraction of A towards B is as the weight divided by the ſquare of the diſtance; viz. $\frac{A}{\overline{AB}|^2}$, and that the attraction of B towards A is as the weight of B divided by the ſquare of the diſtance; viz $\frac{B}{\overline{AB}|^2}$; thoſe two fractions have the ſame denomina-

tor,

CHAPTER VII.

THE THEORY OF PERCUTIENT BODIES, THAT MOVE WITH EQUABLE MOTION.

THOUGH the present part of this work treats expresly of such properties as belong to all bodies, without noticing the particular qualities which distinguish one body from another; yet in this chapter it will be necessary to take notice of one peculiarity only; namely, of the different effects which, in the collision of bodies, are produced by their being *elastic* or *non-elastic*; the meaning of which words will be explained in the following definitions.

1. A body *perfectly hard* is that whose figure is not in the least altered by the stroke, or collision, of another body.

2. A body *perfectly soft* is that whose figure is altered by the least impression, and which is destitute of the power of recovering its original figure.

3. An *elastic body* is that which yields to the impression of another body, but afterwards recovers its figure. And,

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tor, $\overline{AB}|^2$; consequently, to say that $AC : BC :: \frac{B}{\overline{AB}|^2} : \frac{A}{\overline{AB}|^2}$ is the same thing as to say, that $AC : BC :: B : A$.

4. It is called *perfectly elastic* when it recovers its original figure entirely, and with the same force with which it lost it; otherwise it is called *imperfectly elastic*.

5. One body is said to strike *directly* on another body, when the right line, in which it moves, passes through the centre of gravity of the other body, and is perpendicular to the surface of that other body.

Though there are innumerable gradations from a body perfectly hard, to one perfectly soft; or between the latter and a body perfectly elastic; yet we cannot say with certainty that a body perfectly possessed of any of the above mentioned qualities does actually exist. It is however certain that our endeavours have not been able to deprive certain bodies of the least degree of their elasticity, by mechanical means.

The object of the theory of percutient bodies is to determine the momentums, the velocities, and the directions of bodies after their meeting; which we shall lay down, and explain, in the following propositions. But it must be observed, that throughout this chapter we only speak of bodies which move with equable motion, that is, of such as describe equal spaces in equal portions of time; and we do likewise suppose that the bodies move in a non-resisting medium, and that they are not influenced by any other action, excepting the single impulse, which puts them in motion: for though ſuch ſimple and regular movements never take place in nature; yet when their theory is once eſtabliſhed, the complicated caſes, wherein the reſiſtances of mediums and other interfering cauſes, are comprehended, may be more commodiouſly examined; and proper allowances may be made agreeably to the nature of thoſe cauſes.

I. *If bodies moving in the ſame ſtraight line, ſtrike againſt each other, the ſtate of their common centre of gravity will not thereby be altered; viz. it will either remain at reſt, or it will continue to move in the ſame ſtraight line, exactly as it did before the meeting of the bodies.*

This propoſition is ſo evidently deduced from the properties of the centre of gravity, as mentioned at N° II and III. in the preceding chapter, that nothing more needs be ſaid about it in this place.

II. *Let there be two non-elaſtic bodies; and if one of them move in a ſtraight line, whilſt the other is at reſt in that line, or is moving in the ſame direction, but at a ſlower rate, or is moving in the contrary direction; viz. towards the body firſt mentioned; then thoſe bodies muſt neceſſarily meet or ſtrike directly againſt each other, and after the ſtroke they will either remain at reſt, or they will move on together, conjointly with their common centre of gravity.—Their momentum after the ſtroke will be equal to the ſum of their momentums before the ſtroke, if they both moved in the ſame direction, but it will be equal to the difference of their momentums if they*

moved

moved in contrary directions.—Their velocity after the ſtroke will be equal to the quotient that ariſes from dividing the ſum of their momentums, if they both moved the ſame way, or the difference of their momentums, if they moved in contrary directions, by the ſum of their quantities of matter.

That in any of the above mentioned caſes the two bodies muſt meet, and ſtrike againſt each other, is ſo very evident as not to require any farther illuſtration.

That after the ſtroke thoſe two bodies muſt either remain at reſt, or they muſt move together, conjointly with their common centre of gravity, is likewiſe evident; for as the bodies are not elaſtic, there exiſts no power that can occaſion their ſeparation.

With reſpect to the momentum, it may be obſerved, that when the two bodies meet, whatever portion of momentum is loſt by one of them muſt be acquired by the other; ſince, according to the third law of motion, action and re-action are always equal and contrary to each other; therefore, if before the ſtroke the bodies moved the ſame way, their joint momentum after the ſtroke will be equal to the ſum of their momentums before the ſtroke. If one of the bodies was at reſt, then, as its momentum is equal to nothing, the joint momentum will be equal to the momentum of the other body before the meeting. If the bodies moved towards each other, then their momentum after the meeting will be equal to the difference of their former momentums; and if in this caſe their momentums are equal, then their difference vaniſhes; hence the bodies will remain motionleſs after their meeting.

The laſt part of the propoſition is likewiſe evident; ſince it has already been ſhewn, that in equable motion, the velocity is equal to the quotient of the momentum divided by the quantity of matter.

When the weights and velocities of the two bodies before their meeting are known, their velocity after the meeting may be determined by the following general method.

Let A and B, in fig. 4, 5, 6, and 7, of Plate II. which repreſent the above mentioned caſes, be the two bodies; let C be their common centre of gravity, and D the place of their meeting. Make DE equal to DC; ſo that the point D may be between C and E; then DE will repreſent the velocity of the two bodies after their meeting; for, ſince the bodies after the concurſe move together conjointly with their common centre of gravity; and ſince it has been proved in the preceding propoſition, and at N° II. of the preceding chapter, that the ſtate of the common centre of gravity of the two bodies is not altered by their mutual action upon each other; therefore the velocity of their common centre of gravity after their meeting, muſt be equal to its velocity before the meeting; viz. DE muſt be equal to CD, and is the ſame as the velocity

velocity of the two bodies after their meeting, because then they move together with their common centre of gravity.

Of the above mentioned figures, it may be easily perceived, that the 4th shews when both the bodies move the same way; the 5th represents the case in which B is at rest before the stroke, and of course the two points B and D coincide; the 6th shews when the two bodies move towards each other; and the 7th shews when the two bodies move towards each other with equal momentums, in which case, after their meeting, they will remain at rest. The respective velocities of those two bodies are represented in all the four figures, by A D and B D; for they run over those distances in the same time; and A B is the difference of those velocities. Also their respective momentums are represented by the product of the weight of A multiplied by A D, and the product of the weight of B multiplied by B D. The momentum of both the bodies together after their meeting, is represented by the product to their joint weight multiplied by D E (1).

Since

(1) The following is an example of the numerical computation of the first case, fig. 4, which will be sufficient to indicate the manner of calculating the other cases.

Let A weigh 10 pounds, and move at the rate of 4 feet per minute.

Let

Since when one of the bodies is at reſt, the velocity after the meeting is equal to the quotient of the velocity of the moving body, divided by the ſum of the quantities of matter of both the bodies; it follows that the larger the body at reſt is, the ſmaller will the velocity be after the meeting. For

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Let B weigh 6 pounds, and move at the rate of 2 feet per minute.

And let the diſtance A B be 32 feet.

The centre of gravity is found by ſaying 16 : 32 : : 10 : $BC = \frac{32 \times 10}{16} = 20$ feet; hence $AC = 12$ feet. (See page 75.)

Put $BD = x$, and AD will be equal to $32 + x$. Then the time employed by A in moving from A to D, is equal to the quotient of the ſpace $32 + x$, divided by its velocity; viz. it is $\frac{32 + x}{4}$. And the time employed by B in moving from B to D, is equal to the quotient of the ſpace x, divided by the velocity of B; viz. it is $\frac{x}{2}$. But ſince the bodies meet at D, thoſe times muſt be equal; that is, $\frac{32 + x}{4} = \frac{x}{2}$; hence $64 + 2x = 4x$; and $x = 32 = BD$.

Then $DE = DB + BC = 32 + 20 = 52$ feet; that is, after the meeting, the two bodies will move from D to E; (viz. over 52 feet) in as much time as each of them employed in going to D; that is, 16 minutes. Therefore, to find how many feet per minute the bodies will run over after the meeting, divide 52 by 16, and the quotient $3\frac{1}{4}$ ſhews that they will move at the rate of $3\frac{1}{4}$ feet per minute.

 inſtance,

inſtance, if the moving body A weigh one pound, and move at the rate of one foot per minute, whilſt the body B at reſt weigh one pound alſo, the velocity after the concurſe will be half a foot per minute; half a foot being the quotient of one foot, divided by the ſum of their quantities of matter; viz. 2 pounds. If *cæteris paribus* B weigh 10 pounds, then the velocity after the concurſe will be the 11th part of a foot per minute. If B weigh 100000 pounds, then the velocity after the concurſe will be the 100001th part of a foot per minute; and in ſhort, when B is infinitely bigger than A, the velocity after the concurſe will be infinitely ſmall, which is the ſame thing as to ſay, that in that caſe, after the ſtroke, the bodies will remain at reſt. And ſuch is the caſe when a non-elaſtic body ſtrikes againſt an immoveable obſtacle.

III. *If a body in motion ſtrikes directly againſt another body, the magnitude of the ſtroke is proportional to the momentum loſt, at the concurſe, by the more powerful body.*

According to the third law of motion, action and re-action are equal and contrary to each other; therefore whatever momentum is loſt by one of the bodies, is acquired by the other. Or the magnitude of this acquired momentum (which is the effect of the ſtroke) is as the momentum loſt by the more powerful body; it being by the quantity of of the effect that we meaſure the quantity of the action.

IV. *When a given body ſtrikes directly againſt another given body, if the latter be at reſt, the quantity of the ſtroke is proportional to the velocity of the former body.—If the ſecond body be moving in the ſame direction with the firſt, but at a ſlower rate, the magnitude of the ſtroke will be the ſame as if the ſecond body ſtood ſtill, and the firſt impinged upon it with a velocity equal to the difference of their velocities.—And laſtly, if the bodies move directly towards each other, the magnitude of the ſtroke is the ſame as if one of the bodies ſtood at reſt, and the other ſtruck it with the ſum of their velocities.*

The momentum of a given body is proportionate to its velocity; for with a double velocity the momentum is double, with a treble velocity the momentum is treble, and ſo on; therefore, as long as the body remains the ſame, the magnitude of the ſtroke, being proportional to the momentum, muſt likewiſe be proportional to the velocity. And when one of the bodies is at reſt, the magnitude of the ſtroke is evidently proportional to the velocity of the moving body.

V. *It follows from the foregoing theory, that the mutual actions of bodies, which are incloſed in a certain ſpace, are exactly the ſame, whether that ſpace be at reſt or move on uniformly and directly.*

For if the motion of the ſpace adds to the velocity of thoſe bodies within it, which move the ſame way, it takes away an equal portion of velocity from thofe bodies within it which move the contrary way; thus, all the motions of the bodies in a fhip are performed in the fame manner, and the fame effects are produced on each other, whether the fhip be at reft or move uniformly forwards.

The attentive reader muft have perceived, that in the explanation of the preceding cafes the dif-tances have been reckoned from the centres of the bodies; whereas it is evident that the thickneffes of the bodies muft be deducted from thofe dif-tances; for the bodies do not ftrike againft each other with their centres, but with their furfaces. It muft be obferved, however, that when the dif-tances are very great, and the bodies proportionate-ly very fmall, it is immaterial whether the diftances be reckoned from the centres or from the furfaces of the bodies. But if great accuracy be wanted, the thicknefs of the bodies may be eafily allowed for in the computation.

A fimilar obfervation may be made with refpect to the fhapes of the bodies; viz. that they have been reprefented as being globular, for the pur-pofe of rendering the explanation fhort and per-fpicuous.

Having hitherto treated of unelaftic bodies, that is, of fuch bodies as are either perfectly hard or perfectly foft, it is now neceffary to ftate the rules of congrefs which take place with elaftic bodies.

At

At the commencement of the preſent chapter, elaſticity has been ſaid to be that property by which bodies yield or ſuffer their figure to be altered by the preſſure of other bodies; but which, on the removal of the preſſure, recover their original figure of their own accord.

This recovery of the figure is performed with greater or leſs quickneſs, and with more or leſs exactneſs, in different bodies; which differences conſtitute the various degrees of elaſticity. And thoſe bodies, which recover their figure completely, and as quickly as they loſt it, are ſaid to be *perfectly elaſtic*.

Though we are acquainted with the effects and the laws of elaſticity, in a manner ſufficient to render that property ſubſervient to our purpoſes, yet the cauſe of that property is by no means underſtood; nor has any hypotheſis been offered in explanation of it, which may be ſaid to be ſufficiently plauſible.

Let a ſtring, A B, fig. 8. Plate II. be ſtretched between, and be firmly faſtened on, two immoveable ſupports at A and B; and if, by applying a finger at C, this ſtring be pulled towards D, the ſtring will be found to reſiſt that effort with a force greater and greater, the farther it is pulled from its original ſtraight ſituation. When diſengaged from the finger, the ſtring will not only return to its original ſtraight ſituation, but it will go beyond it; viz. towards E, and nearly as far from the ſtraight ſituation A C B, as A D B is from it, after which it will again bend itſelf towards D, and ſo on, vibrating backwards and forwards, but deviating continually leſs and leſs from the ſtraight ſituation, until at laſt it remains at reſt in its original ſituation A C B.

Now when the ſtring is firſt diſengaged from the finger, its elaſtic force draws the part D towards C; but this force decreaſes in proportion as the part D comes nearer to C, and when the part D is arrived at C, that is, when the ſtring is in its ſtraight ſituation, the above-mentioned force is infinitely little or is equal to nothing; but by that time the part D, having been impelled by a continual though decreaſing force, will have acquired a momentum which carries it towards E; viz. beyond the ſtraight ſituation; but as ſoon as the ſtring goes beyond C, the elaſtic force begins to act again in a direction contrary to that of the momentum. This action becomes ſtronger and ſtronger the farther the middle part of the ſtring goes from the ſtraight direction, and of courſe it gradually diminiſhes the above-mentioned momentum, until at laſt the momentum being entirely ſpent, the ſtring begins again to move towards C, in virtue of the elaſtic force, and ſo on.

The extent of the vibrations becomes continually ſhorter and ſhorter, on account of the reſiſtance of the air, and of the want of perfect pliability, or of perfect elaſticity, in the parts of the ſtring; which

which two caufes continually tend to diminifh the momentum, &c.

If the ftring be moved out of its ftraight direction, not by the application of a finger, but by a body E, fig. 9. Plate II. falling upon it at C from the height H, it will be readily underftood that the momentum of the body will eafily impel the ftring towards D; but the re-action of the ftring on the body will gradually diminifh the momentum of the latter, and the farther the ftring is carried from the ftraight fituation, the ftronger will its re-action be, until at laft the body, having loft all its momentum, will be carried back again towards C by the elaftic force of the ftring; and in its way back, the conftant though decreafing action of this elafticity from D to C, will give it a momentum which will carry it up towards H; and the body would afcend precifely up to H, were it not for the above-mentioned caufes of obftruction; viz. the refiftance of the air, &c.

It is almoft fuperfluous to obferve that the greater the height is from which the body E falls upon the ftring, the farther will the ftring be removed from its ftraight fituation, and of courfe the ftronger will its re-action be.

This explanation of the elafticity of the ftring may be applied to all forts of elaftic bodies; for the furface of every one of them will be bent more or lefs by any ftroke or preffure, and will afterwards recover its original form by re-acting the contrary way with a force, which, in perfectly elaftic bodies, is equal to the preffure or ftroke received.

If inftead of the ftring, whofe ends are immoveable, &c. we imagine that two equal bodies like A and B, fig. 10. Plate II. being impelled by equal and contrary forces, directly ftrike againft each other at C, it is evident that the contiguous furfaces of both will be bent inwardly, and that the elaftic force of A will drive the body B back from C towards B, (as the ftring did in the preceding cafe) at the fame time that the elaftic force of the body B will impel A back with equal force from C towards A, fo that in this cafe the bodies after the ftroke will recede from each other; whereas had they been non-elaftic, they would have remained ftationary at the place of their congrefs C.

We muft now determine the effects produced after the congrefs of bodies that are *perfectly* elaftic; from which the laws of congrefs amongft thofe that are *imperfectly* elaftic may afterwards be eafily deduced.

VI. *When two bodies that are perfectly elaftic ftrike directly on each other, their relative velocity* (by which is meant the excefs whereby the velocity of the fwifter body exceeds that of the flower) *will be the fame before and after the ftroke; viz. they will recede from each other with the fame velocity with which they approached before the ftroke.*

It has been fhewn above, that when two given bodies ftrike on one another, the magnitude of the

ftroke

ſtroke is proportional to their reſpective velocities. By the definition, a perfectly elaſtic body has been ſaid to be that which recovers its figure with the ſame force with which it loſt it. Therefore in perfectly elaſtic bodies the reſtoring force is equal to the compreſſing force: ſo that if the momentums of the bodies produced a certain compreſſion, the elaſtic force muſt re-act on the bodies with the like force; hence the bodies will be forced to recede from each other with the ſame velocity wherewith they approached each other.

It is a natural conſequence of this demonſtration, *that in equal times taken before and after the ſtroke, the diſtances of the bodies from one another will be equal, and therefore in equal times their diſtances from their common centre of gravity will likewiſe be equal.*

Thus much being premiſed, the laws of congreſs in bodies that are perfectly elaſtic may be eaſily determined. But in order to comprehend the ſolution of the following caſes, the reader ſhould recollect and keep in view the following particulars, which have all been ſufficiently proved in the preceding pages; viz. 1ſt, that the diſtances of two bodies from their common centre of gravity are inverſely as their weights: 2dly, that the ſtate of reſt or of uniform motion of the centre of gravity of bodies is not altered by the mutual action of thoſe bodies on each other: 3dly, that in bodies that are perfectly elaſtic, the reſtoring is equal to the compreſſing preffing force: 4thly and laftly, that the diftances of the bodies from each other, and from their common centre of gravity, are equal in equal times taken before and after the ftroke.

Now let A and B, fig. 10. Plate II. be two equal bodies perfectly elaftic; C is their common centre of gravity, which, fince the bodies are equal, ftands midway between them. Let both the bodies be impelled with equal force, directly towards each other, in confequence of which they will move in a certain time (for inftance, a minute) from their refpective places to C, where they meet. After the impulfe their elafticity will impel each body back towards its original place, fo that at the end of one minute from the time of their meeting they will be found precifely where they were a minute before their meeting, viz. at A and B.

Let A and B, fig. 11. Plate II. be two equal and perfectly elaftic bodies, as in the laft cafe, and let one of them, for inftance B, be at reft, whilft the other body A moves towards it, fo as to reach it in one minute's time. Here AB reprefents the velocity of it, and CB reprefents the velocity of the centre of gravity C; for in the fame time that A comes from the place A to the place B, C muft come from C to B, therefore at the end of one minute after the ftroke the centre of gravity muft be at F, viz. as far from B as its original fituation C was from the place B a minute before the ftroke; but a minute after the ftroke the body A muft muſt be as far from the centre of gravity, viz. from F, as it ſtood a minute before the ſtroke; therefore it muſt ſtand at B. Now as the ſame reaſoning may be applied to any other equal times taken before and after the ſtroke, ſuch as half a minute before and half a minute after, &c. therefore in this caſe the body A after the ſtroke will remain ſtationary at B, and the body B will move on with the velocity that the body A had before the ſtroke.

Having thus explained two of the ſimpleſt caſes of congreſs in a ſeparate manner for the ſake of perſpicuity, I ſhall now give one general rule for the ſolution of all the other caſes, which are delineated in Plate II. from fig. the 12th to fig. the 20th incluſively, in which figures A and B repreſent the two bodies; C is their common centre of gravity; D the place at which they meet. A D expreſſes the velocity of A; B D the velocity of B; and C D that of the centre of gravity.—Then the rule for determining the velocities after the ſtroke is as follows:

Take a point E in the line A B, produced if neceſſary, ſo that the diſtance C E be equal to C D; then after the ſtroke the right line E A will expreſs the velocity of the body A from E towards A, and the right line E B will expreſs the velocity of B from E towards B.

In any one of thoſe caſes the centre of gravity C muſt move from C to D, and after the ſtroke, from D forwards to a diſtance D F equal to D C in

a portion

a portion of time equal to that in which A and B employ for coming from the places A and B to D.

Make F*a* equal to C A; and ſince in equal times, taken before and after the ſtroke, the diſtances of the bodies from the common centre of gravity are equal, therefore when the centre of gravity is at F the body A will be at *a*; ſo that after the ſtroke it will move from D towards *a*, and D*a*, which it has paſſed over in that time, will repreſent its velocity. But becauſe C E is equal CD, or to F D, and C A, is equal to F*a*, the difference of the right lines C E, C A will be equal to the difference of the right lines FD, F*a*; viz. EA will be equal to D*a*. But D*a* repreſents the velocity of the body A after the impulſe, therefore its velocity will alſo be repreſented by EA. And ſince the relative velocity of the bodies before and after the ſtroke is the ſame, and E A repreſents the velocity of the moving body A; therefore the velocity of the body B, moving from E towards B, is of courſe repreſented by the right line E B.

For the better illuſtration of this theory, I ſhall briefly mention the meaning of the figures which exhibit the various caſes of the congreſs of bodies that are perfectly elaſtic, to every one of which the preceding explanation is equally applicable.

Fig. 12. is the caſe when B is larger than A, (which is indicated by C, the ſituation of the centre of gravity) B is at reſt, and A ſtrikes againſt it. In this caſe, after the ſtroke, both the bodies will recede

cede from the point D, with the velocities EA and E B.

Fig. 13. ſhews when A is larger and runs againſt B at reſt; in which caſe, after the ſtroke, both bodies will move towards F.

Fig. 14. is the caſe when A is larger than B, both bodies are in motion the ſame way, and meet at D, &c.

Fig. 15. the ſame as the preceding, excepting that A is leſs than B.

In fig. 16. A and B meet at D, where A remains at reſt.

In fig. 17. after the ſtroke the equal bodies A and B recede with interchanged velocities.

In fig. 18. the bodies are proportional to their velocities, in which caſe the points C, F, D, and E, coincide.

In fig. 19. A remains ſtationary at the place of congreſs D.

In fig. 20. though the bodies A and B meet at D between the places A and B, yet after the ſtroke both bodies will move towards F. (2)

After

(2) The method of making the numerical computation of thoſe caſes will be ſhewn by the following example, which is adapted to the caſe repreſented in fig. 14, to which the reader is requeſted to direct his eye; though for want of room the parts of that figure be not drawn in the due proportion.

Let

After the explanation of the preceding cafes, the method of determining the velocities after the ftroke, when the bodies are not perfectly elaftic, may be eafily underftood.

Thus

Let A and B be two perfectly elaftic bodies. A weighs 2 pounds, and moves at the rate of 8 feet per fecond. B weighs one pound, and moves the fame way at the rate of 5 feet per fecond; and let the diftance A B be 12 feet.

1. To find the centre of gravity C, we have A + B : B :: AB : CA, viz. 3 : 1 :: 12 : 4; fo that AC = 4, and CB = AB — AC = 8.

2. To find the diftance BD, put BD = x; and fince the diftances AD and BD are run over in the fame time, the former at the rate of 8, and the latter at the rate of 5 feet, per fecond; therefore we have $\frac{x}{5} = \frac{x+12}{8}$; hence $8x = 5x + 60$; and $3x = 60$; or $x = 20$ = BD.

3. If the diftance BD, viz. 20, be divided by the velocity of B, viz. 5, the quotient 4 is the number of feconds, during which the bodies moved from their refpective places A and B, to the place of their congrefs D.

4. EC = CD = CB + BD = 8 + 20 = 28; and EA = EC — AC = 28 — 4 = 24; which being divided by 4 (the number of feconds found above) gives 6 for the velocity of A after the ftroke, in the direction from E towards A.

Alfo EB = EC + CB = 28 + 8 = 36; which, being divided by 4 (the number of feconds, &c.) gives 9 for the velocity of B after the ftroke in the direction from E towards B. So that after the ftroke, the bodies A and B will both continue to move the fame way, but the former at

Thus let A and B, fig. 21 and 22, Plate II. be two bodies imperfectly elaſtic; C their common centre of gravity; D the place of their meeting. Divide AC in *a*, ſo that AC may be to *a*C, as the force compreſſing the body A is to the force whereby it reſtores itſelf. Alſo divide BC in *b*; ſo that BC may be to *b*C as the force compreſſing the body B is to the force whereby it reſtores itſelf. Take CE equal to CD; and laſtly the right line E*a* will expreſs the velocity of A after the ſtroke, in the direction from E towards *a*, and the right line E*b* will expreſs the velocity of B after the ſtroke in the direction from E towards D.

There being perhaps no body in nature which may be ſaid to be perfectly elaſtic, the rules given for determining the velocities of bodies that are perfectly elaſtic, cannot be verified experimentally; but the deviation of the experimental reſult from the rules is proportionate to what the bodies want

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at the rate of 6, and the latter at the rate of 9 feet per ſecond.

In the like manner may the other caſes be calculated.—I have given this method of adapting the calculation to the figures, or of expreſſing the parts of the diagrams, by means of numbers, in preference to the complex rules which have been given for this purpoſe by certain learned writers, becauſe the latter are ſeldom remembered, and are difficultly applied to the ſolution of the various caſes of impact amongſt bodies that are poſſeſſed of perfect elaſticity.

of

of perfect elasticity; and this deviation is taken into the account after the manner mentioned above.

The precise degree of elasticity of which any particular body is possessed, must be ascertained by means of actual experiments on the body itself, which experiments differ according to the various nature of the bodies.

When more than two bodies move in the same straight line, the determination of the velocity of each body, after the various impacts with each other, cannot be comprehended under any general rules, the variety of cases being too great, and sometimes very intricate; yet when any particular case presents itself, the preceding rules will be found sufficient for the solution of it, viz. for ascertaining the velocities, &c. But in the solution of such cases, the operator must take care to calculate, in the first place, the velocities of those two bodies which appear from the circumstances of the case to meet first; then to substitute those new velocities which are the real velocities of those two bodies after their meeting, and with them to calculate (according as any one of those bodies is concerned with the second stroke) the velocities after the second congress, and so forth.

Most of the foregoing cases, both of perfectly and of imperfectly elastic bodies, might be expressed in the form of *canons*, (that is, of particular rules) and by stating bodies of various weights, and moving with various velocities, the numbers

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of thoſe canons might be increaſed without end. The following three paragraphs contain three ſuch canons by way of examples, which the reader will eaſily perceive to be nothing more than particular caſes expreſſed in words.

When two bodies, perfectly elaſtic and equal, are carried towards the ſame part, after the meeting (if their velocities be ſuch as to admit of their ſtriking againſt each other) they will continue to move in the ſame direction but with interchanged velocities. But if they be carried towards contrary parts, then after the meeting, they will go back with interchanged velocities.

If any number of equal and perfectly elaſtic bodies lie at reſt, contiguous to each other in the ſame ſtraight line, and another body equal to one of them ſtrike the firſt of them in the ſame ſtraight line with any velocity; then after the ſtroke the ſtriking body and all the reſt will remain at reſt, and the laſt body only will move on with the velocity of the ſtriking body.—In this caſe the bodies act as if they were ſeparate; viz. when A, fig. 23, Plate II. ſtrikes directly againſt B, after the ſtroke A will remain at reſt, and B would move on with the velocity that A had if the other equal body C ſtood not contiguous; but as C is contiguous to it, B communicates its velocity to C, and remains itſelf at reſt; and in the like manner C communicates the ſame velocity to D, and D to E; which laſt body E will in conſequence of it be forced to move towards F with the velocity that A had at firſt.

If every thing remain, as in the preceding case, excepting that now two bodies A *and* B, *fig.* 24, *Plate* II. *contiguous to each other, be impelled towards* C, *then after the stroke,* A, B *and* C *will remain at rest, and* D *and* E *will move off towards* F *with the velocity that* A *and* B *had at first.*

CHAPTER VIII.

OF COMPOUND MOTION; OF THE COMPOSITION AND RESOLUTION OF FORCES; AND OF OBLIQUE IMPULSES.

BY *compound motion* is understood that movement of bodies which arises from more than one impulse; for in such cases the velocity and the direction of the body put in motion, arise from the concurrence of all the impulses, and participate of them all, under certain determinate laws, which will be specified in the following propositions.

I. *When a body is impelled at the same time by two forces in different directions, the body will move not in any one of them, but in a direction between those two.*

Thus if a body A, fig. 25, Plate II. be impelled by two forces, viz, one which by itself would drive it in the direction AB, and another, which by itself would drive it in the direction AD; then the body A being

A being impelled by both thoſe forces or impulſes, at the ſame time, will move in a direction A C, between A D and A B; for ſince, according to the ſecond law of motion, the change of motion is always made in the direction of the right line in which that force is impreſſed, therefore the motion of the body along the line A D is altered from the direction A D, to another direction towards A B, by the other impulſe, which acts in the direction A B. And for the ſame reaſon, the motion of the body along the line A B is changed for another direction towards A D by the impulſe which acts in that direction. Therefore the motion ariſing from thoſe two impulſes muſt have a direction between A D and A B.—But it will be ſhewn in the following propoſition, how much this new direction will deviate from A D, and from A B.

II. *When a body is impelled at the ſame time by two forces in different directions, if two lines be drawn from the place in which the body receives the double impulſe, in the directions of thoſe impulſes; and the lengths of thoſe lines be made proportionate to the impelling forces; alſo through the end of each of thoſe lines a line be drawn parallel to the other, a parallelogram will thereby be formed; and if a diagonal line be drawn from the place where the body receives the double impulſe to the oppoſite corner of the parallelogram, the length and ſituation of that diagonal will repreſent the velocity and the direction of the body's motion, ariſing from the double impulſe.*

 Thus,

Thus, ſuppoſe that the body A, fig. 26, Plate II. be impelled in the direction A D, by a force which would enable it to move at the rate of 4 feet per ſecond; alſo that at the ſame time the ſame body be impelled by another force in the direction A B, which would enable it to move at the rate of 3 feet per ſecond. Make A D equal to four, and A B equal to three (for inſtance, inches; or you may uſe any other dimenſion to repreſent feet). Through D draw D C parallel to A B; and through B draw B C parallel to A D; by which means the parallelogram A B D C will be formed. Laſtly, draw the diagonal A C, and A C is the direction in which the body which is impelled by the above-mentioned two impulſes, will move. Alſo the length A C will expreſs the velocity of the body; ſo that if AC be found, either by calculation or by meaſuring the diagram, to be 5 inches long (1); we conclude that

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(1) The length and direction of A C; viz. the angles it makes with AD and AB, may be eaſily found by trigonometry; it being the ſolution of a plane triangle, in which two ſides, and the angle between thoſe two ſides, are known.

The direction of the impulſes being given, the angle DAB is alſo known; for it is the angle which the directions of the two impulſes make with each other. The angle ADC is likewiſe known, becauſe it is the complement of the angle DAB to two right angles. The lines AD and DC (=AB) are to each other in the proportion that the body will move at the rate of 5 feet per ſecond, ſince in the dimenſions of A B and A D, inches were employed for repreſenting feet.

That the body thus impelled by the two forces muſt move along the line A C, is eaſily deduced from the ſecond law of motion; for ſince the change of motion is proportionate to the moving force impreſſed; if from any point *c* in the diagonal A C you draw two right lines, viz. *dc* parallel to A B, and *bc* parallel A D, thoſe two lines will repreſent the deviations of the body's motion from the directions A D and AB; ſince by law the 2d, the change of motion is made in the direction of the moving force impreſſed. And thoſe two lines are proportional to the impelling forces,

portion of the two impulſes, and may be repreſented by any dimenſions, as inches, feet, &c. Therefore in the triangle ADC the ſides AD, DC, and the included angle D, are known. Hence by trigonometry we have $AD + DC : AD - DC :: \text{tangent } \frac{DAC + DCA}{2} :: \text{tangent } \frac{DCA - DAC}{2}$; whence we obtain half the ſum of the angles at the baſe, viz. of the angles D C A and D A C. Then half the ſum, plus half the difference, is equal to the greater of thoſe angles; viz. DCA; and half the ſum, minus half the difference, is equal to the other angle DAC, which gives the direction of AC: thus all the angles will be known. Laſtly, ſay, as the ſine of the angle DCA is to the ſine of the angle ADC, ſo is AD to a fourth proportional, which is equal to AC. Hence both the direction and the length of AC will be known.

or to the lines A B and A D, which represent those forces; viz. *dc* is to *bc*, as AB is to AD; because (by Eucl. p. 24. B. vi.) the parallelogram A*dcb* is similar to the parallelogram ABDC.

If it be said that the body thus impelled will at any time be found at any other place *o* out of the diagonal A C, draw *om* parallel to A D, and *od* parallel to AB; then *om* and *od*, which represent the deviations, &c. ought to be proportional to the forces which occasion those deviations, viz. *o m* ought to bear the same proportion to *o d* as AD does to AB. But this is not the case, because the parallelogram A *d m o* is not similar to the parallelogram ADBC. Therefore the body, &c. must move along the diagonal AC, and in no other direction.

III. *When a body is impelled at the same time by three forces in three different directions, the velocity and the direction of the body's motion, which arises therefrom, must be determined by first ascertaining the course which would be produced by any two of those forces, according to the preceding proposition; and then by finding the course last found, and the third force, which will be the course sought.*

Thus if a body A, fig. 27, Plate II. be impelled by three forces; viz. with a force which by itself would enable it to move in the direction A B at the rate of four feet per minute; by a second force, which by itself would enable it to move in the direction AC at the rate of three feet per minute; and lastly, by a third force, which by itself would enable it to move at the rate of five feet per minute

nute in the direction A D. Make the lengths of the right lines proportionate to the forces, viz. A B four, A C three, and A D five, inches, or feet, &c. long. Then imagine as if the body were impelled by the firſt and ſecond forces only, and, by the preceding propoſition, find the compound motion ariſing therefrom, viz. through B draw B E parallel to A C, and draw C E through C parallel to A B; and the diagonal A E will repreſent the direction and the velocity of the motion reſulting from thoſe two forces. Then after the ſame manner find the compound motion reſulting from the force repreſented by A E, and the third force repreſented by A D; viz. by drawing through E and D the lines E F and D F, reſpectively parallel to A D and to A E, as alſo the diagonal A F; and this diagonal A F will repreſent the courſe of the body, viz, the velocity and direction of its motion, ariſing from the above-mentioned three impulſes.

The demonſtration of this propoſition is ſo evident a conſequence of the preceding propoſition, that it will be needleſs to detain the reader with a repetition of almoſt entirely the ſame words.

It appears likewiſe, that the like reaſoning may be extended to the caſe of four or five, or in ſhort, of any number of impulſes.

Notwithſtanding the apparent multiplicity and intricacy of ſuch caſes, an obvious remark will furniſh a general rule, by means of which the place of the body at any time may be eaſily determined in all cases; viz. whether the impulses be single, or accelerative like the force of gravity, or whether some of them be of the former, and others of the latter sort.

The observation which furnishes the rule is, that at the end of a certain time the body which is impelled by two forces, will be found precisely at the place where it would be found if the two forces acted one after the other; the time however must not be doubled. For instance, in the case of fig. 26, Plate II. the body A is impelled by two forces, viz. one in the direction AD, which alone would drive it as far as D in one second, and another force in the direction AB, which alone would drive it to B in one second. Now if you imagine that those two forces be applied one after the other, viz. that when the body is at D, the other force impels it in the direction DC parallel to AB, and as far from D as B is from A; then C is the place where the body will be driven in one second by the compound action of both the forces.

This observation is evidently applicable to the case of four or more impulses; and hence we derive the following general rule for finding the place or situation of a body after a certain time, when the body is impelled by any given number of given forces.

Rule. *Imagine as if the body were impelled by the given forces, not at once, but successively one after the other, in directions parallel to their original directions, and*

and each in an equal portion of time; and the last situation is the place where the body will be driven in the like portions of time, by the joint action of all the forces at the same time.

Thus in the case of fig. 27, Plate II. the first force by itself would impel the body to B in one minute; the second force would by itself impel it from B to E in one minute (B E being equal and parallel to A C); and the third force alone would impel it from E to F in one minute (E F being parallel and equal to A D); therefore the joint action of all the three forces will drive the body from A to F in one minute.

If, instead of one minute, any other portion of time be made use of, the figures arising therefrom will always be similar, so that whether the figure be larger or smaller the point F will always be in the right line A F; which likewise shews that when a body is impelled by single impulses (viz. such as produce equable motion) let their number be what it may, the course of the body between its original place A and the place F, where it will be found at the end of any time, is always rectilinear; hence the right line A F represents, as we have already observed, the direction and velocity of the body's motion.

Sometimes the directions and the strength of the impulses are so circumstanced as to produce no motion on the body; in which case the forces are said to be balanced in opposite directions; and to those

cases

cafes the above-mentioned rule is equally applicable. Thus if a body A, fig. 28, Plate II. be impelled by a force which in one minute's time would drive it in the direction A B as far as B, and likewife by another force equal to the former, which by itfelf would drive it in the direction A C oppofite to the direction A B, to a place C, as far from A as B is from A; it is evident that thofe two equal, but oppofite, impulfes, acting at the fame time, will not produce the leaft motion in the body, for they deftroy each other.

Likewife if a body be impelled by three powers in three different directions, and the compound courfe which would be produced by two of thofe forces be equal and oppofite to the third force, the body will not be moved by thofe three forces.— Thus if the body A, fig. 29, Plate II. be impelled in the direction A B by a force which in one minute would enable it to go as far as B; alfo by a fecond force, which in one minute would enable it to go in the direction A C as far as C; and laftly, by a third force, which in one minute would enable it to move in the direction AE as far as E. Find the courfe of the body which would arife from the joint action of the two forces AB and AC, viz. by drawing B D parallel to A C, and C D parallel to A B, and joining A D. Now if this diagonal A D happens to be equal to AE and oppofite to it, that is, in the fame right line, then the body A will not be moved by the joint action of thofe three forces; for the

the two forces A B and A C are equivalent to the force reprefented by A D; but this force A D is equal and oppofite to the force A E; therefore the fum of the two forces A B and A C, is likewife equal and oppofite to A E; hence the body will not be moved from its original place A.

Since A B D C is a parallelogram, the line B D is equal and parallel to A C, as alfo the line D C to A B; and, in cafe of an equilibrium or balance of the three forces, A D has been fhewn to be equal to, and in the fame right line with, A E, which is the fame thing as to fay that A D is parallel to A E. Therefore we eftablifh the following propofition, which is of great ufe in mechanics.

IV. *If a body be impelled by three powers, or, (which is the fame thing) it be drawn by three powers, in three different directions, and thofe powers balance each other fo as to leave the body at reft; then thofe powers muft have the fame proportion to each other as have the right lines* (A B, B D, *and* A D) *drawn parallel to their directions, and terminated by their mutual concourfe. And* vice verfa, *if the lines drawn parallel to the directions of the three forces, and terminated by their mutual concourfe, bear to each other the fame proportion that the forces bear to each other, then the body will remain at reft* (2).

It

(2) By trigonometry, the fides of plane triangles are as the fines of their oppofite angles. Therefore in the triangle

It will be hardly neceſſary to obſerve, that a balance of forces may take place amongſt any number of ſuch forces; ſo that a body may remain perfectly at reſt, though acted upon by four or five, or any other number of forces. In this caſe the forces are ſometimes called *preſſures*; for in fact they only preſs upon the body without being able to move it out of its place.

V. *As the joint action of ſeveral impulſes compel a body to perform a certain courſe, ſo whenever we obſerve any particular courſe of a body, we may imagine that courſe to be produced by the joint action of two or more impulſes that are equivalent to that ſingle impulſe.*

Thus finding that a body A, fig. 26, Plate II. has moved from A to C, we may imagine either that the body has been impelled by a ſingle force in the direction of A C, and proportionate to the length of A C, or that it has been impelled by two forces at once, viz. by one in the direction of A D,

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angle A B D, fig. 29, Plate II. A B is to B D as the ſine of the angle A D B, or D A C, is to the ſine of the angle D A B; hence any two powers will be to each other reciprocally as the ſines of the angles, which the lines repreſenting their directions, make with the direction of the third power. Farther, A D is to A B as the ſine of the angle A B D is to the ſine of B D A, or D A C; and in like manner the power acting according to A E is to the power acting according to A C, as the ſine of the angle A C D is to the ſine of the angle A D C, or B A D.

and

and proportionate to the length of AD; and by another force in the direction of AB or DC, and proportionate to AB or DC. Therefore, *if two ſides of any triangle* (as AD and DC) *repreſent both the quantities and the directions of two forces acting from a given point, then the third ſide* (as AC) *of the triangle will repreſent both the quantity and the direction of a third force, which, acting from the ſame point, will be equivalent to the other two, and* vice verſa.

Thus alſo in fig. 27, Plate II. finding that the body A has moved along the line AF from A to F in a certain time; we may imagine, 1ſt, that the body has been impelled by a ſingle force in the direction and quantity repreſented by AF; or 2dly, that it has been impelled by two forces, viz. the one repreſented by AD, and the other repreſented by AE; or thirdly, that it has been impelled by three forces, viz. thoſe repreſented by AD, AB, and AC; or laſtly, that it has been impelled by any other number of forces in any directions; provided all thoſe forces be equivalent to the ſingle force which is repreſented by AF.

This ſuppoſition of a body having been impelled by two or more forces to perform a certain courſe; or, on the contrary, the ſuppoſition that a body has been impelled by a ſingle force, when the body is actually known to have been impelled by ſeveral forces, which are, however, equivalent to that ſingle force; has been called *the compoſition, and reſolution*

of

of forces; and is of great uſe in mechanics, as will be ſhewn in the ſequel.

In the preceding pages we have laid down the laws relative to the congreſs, or impact, of bodies, when the bodies ſtrike in a direction perpendicular to each other. It is now neceſſary to examine thoſe caſes in which the bodies ſtrike in an oblique direction, the effects of which could not have been conveniently explained previouſly to the doctrine of the compoſition and reſolution of forces, ſince it depends principally on that doctrine.

It has been ſhewn, that if a body A, fig. 26, Plate II. be ſtruck by two powers at the ſame time, viz. by one in the direction from A towards B, and by the other in the direction from A towards D; the body will thereby be forced to deſcribe the diagonal A C. Now let this motion be reverſed, viz. imagine that the two powers or bodies K and L, are at reſt, and that the body A, advancing from C, along the line C A, ſtrikes againſt thoſe two bodies at the ſame time; the conſequence will be, that both the bodies will be moved from their places, ſince they are both ſtruck; that the impulſe will be divided amongſt them in the proportion of the line A B, which is perpendicular to the body K; to the line A D, which is perpendicular to the body L; and laſtly, that the body L will be impelled towards Q, whilſt the body K is impelled towards P.

It

It is evident that the force of the impulſe muſt be divided amongſt the two bodies; ſo that the greater is the quantity of it which is communicated to the one, the ſmaller muſt be the quantity of it which is communicated to the other; alſo each of thoſe quantities muſt be leſs than the whole original force of the body A; otherwiſe there would be an accumulation of force without any adequate cauſe, which is not poſſible.

The force is not only divided amongſt the two bodies K and L, but it is divided in the proportion of the lines BA to DA, which is eaſily proved thus: Since any force may be reſolved into two or more forces, therefore if we divide the force repreſented by the line A C into two forces, ſuch as that one of them cannot poſſibly act upon the body K, whilſt the other acts directly againſt it, we ſhall by that means determine the queſtion. Draw A D parallel to the ſurface of the body K at the point of congreſs; from C drop C D perpendicular to A D, and through the point of congreſs draw A B parallel to D C, which A B being perpendicular to A D, muſt likewiſe be perpendicular to the body K at the point of congreſs. Thus the force A C is reſolved into the two forces A D and D C, or A B; the former of which cannot have any action upon the body K, whilſt the latter acts entirely upon it. For inſtance, imagine that inſtead of a body moving from C towards A, two bodies move towards that point, viz. one in the direction D A and the other in the direction B A, and it is evident

that

that of thoſe two powers, the one in the direction B A will act entirely and directly upon the body K, whilſt the other in the direction D A paſſes by it, and of courſe cannot affect it.

By the like reaſoning it is proved, that when the force repreſented by C A is reſolved into two other forces, viz. A B, which is parallel to the ſurface of the body L at the point of congreſs, and B C, or its equal A D, which is perpendicular to it, the latter only will act upon the body L; therefore the force which acts upon the body K is repreſented by A B, or its equal D C; and the force which acts upon the body L is repreſented by the line A D, or its equal B C; and thoſe two forces are equivalent to the force A C.

The inclination of the direction of the ſtroke to the body K, or to the line A D, which is parallel to the ſurface of it at the point of congreſs, is repreſented by the angle D A C; and the inclination of the ſtroke to the body L, or to the line A B, is repreſented by the angle C A B. Now (by trigonometry) when A C is made radius, D C, or its equal A B, becomes the ſine of the angle of inclination D A C; and B C, or its equal A D, becomes the ſine of the angle of inclination B A C; therefore *the effect of the oblique force* C A, *is to the effect that would be produced by the ſame force coming in a perpendicular direction, as the ſine of the angle of inclination is to radius*; which is a general and uſeful law in the computation of oblique impulſes.

In

In the present instance the proportion of the oblique force C A to move the body K, is to that of the same force coming in a perpendicular direction, as the sine D C is to radius A C; and for the body L it is as the sine B C to radius A C.

If in the above-mentioned case we imagine that one of the bodies be removed, whilst the other is fixed, we shall then form the case represented by fig. 1. Plate III. in which the body A, moving in the direction A C, strikes obliquely at C on the firm obstacle B F; where it is plain that the magnitude of the oblique stroke is to the magnitude of the same stroke if it had come in a direction perpendicular to the obstacle, as the sine of inclination, or of *incidence*, (viz. as the perpendicular A B) is to the radius A C.

If a body perfectly elastic as A, *fig.* 1. *Plate* III. *strike obliquely at* C *on the firm obstacle* B F, *then after the stroke this body will be reflected from that obstacle in the direction* C E, *in such a manner as to form the angle of reflection* E C F, *equal to the angle of incidence* A C B.

The oblique force A C being resolved into two forces, viz. D C perpendicular to the obstacle and A D parallel to it; the effect on the plain is the same as if the body had advanced towards it directly from D, and according to the laws of congress between perfectly elastic bodies, (chap. vii.) the body A after the stroke would be sent back in the direction C D. But of the two forces into which the original force of the body A was resolved, this body retains the one represented by A D, since this force was not concerned in striking the obstacle; therefore after the stroke the body A is actuated by two forces, viz. one represented by C D, equal to A B, equal to E F; and the other represented by C F, equal to D A, equal to D E, hence it must move in the diagonal C E; and since the lines C F, F E, are respectively equal to the lines CB, B A, and the angles at B and F are equal, because they are right-angles; therefore (Eucl. p. iv. B. I.) the triangle EFC is in every respect equal to the triangle ACB; consequently the angle of reflection ECF is equal to the angle of incidence ACB.

Some writers call the angle ACD the angle of incidence, and the angle DCE the angle of reflection; viz. the angles made by the body with the perpendicular DC; this however does not alter the proposition, for the angle ACD is likewise equal to the angle DCE; those angles being the complements of the equal angles ACB, ECF, to two right angles.

In any case, whenever two bodies strike obliquely against each other, whether one or both be in motion, their directions, velocities, and momentums after the stroke may be easily determined from what has been explained in the last paragraphs, together with what has been delivered concerning the direct impact of elastic and non-elastic bodies in Chap. VII.

VII. And the following example will ſhew the application.

Imagine that two non-elaſtic bodies, A and B fig. 2. Plate III. moving, the former in the direction AC, the latter in the direction BD, do meet at CD. Let the line MG be drawn through their centres and the point of contact. From the original ſituations of thoſe bodies, viz. from A and B, drop AM and BN perpendicular on MG. Then the force of each body may be reſolved into two forces, viz. that of A into AM, and MC; and that of B into BN and ND.

Of its two forces, A retains the force AM, whilſt the force MC is exerted againſt the other body. Of the two forces belonging to the body B, the force BN is retained by it, whilſt the force ND is exerted againſt the other body. Therefore the action of thoſe bodies upon each other is exactly the ſame as if they moved directly one from M and the other from N; hence whether they would, after the ſtroke, proceed both the ſame way, or different ways, and at what rate, muſt be determined by the rules of direct impact (chap vii.) But when their velocities have been thus determined; for inſtance, it be found that if the bodies had moved directly from M and N, after the ſtroke the body A would have moved as far as O, whilſt the body B would have moved as far as G. Then it muſt be recollected, that, in the preſent caſe of oblique colliſion, the body A has retained the force

 AM;

AM; therefore after the ſtroke the body A is actuated by two forces, viz. one equal and parallel to AM, and another force, which is equal and parallel to CO; in conſequence of which this body muſt run a compound courſe, which is found thus: Through the centre C draw CI equal and parallel to AM; through I draw IE equal and parallel to CO; then the diagonal CE exhibits the velocity and the direction of the body A after the oblique concurſe.

With reſpect to the body B, it has been ſaid that at the concurſe this body retains the force BN, and that, if the bodies had moved directly towards each other, B would, after the ſtroke, have moved from D to G. Therefore through D draw DH equal and parallel to BN, and through H draw HF equal and parallel to DG; and laſtly, the diagonal DF will repreſent the velocity and the direction of the body B after the oblique concurſe.

This is the caſe when the bodies are perfectly hard or non-elaſtic. But if they be perfectly elaſtic, then ſuppoſe it be found by the rules for elaſtic bodies that, after the ſuppoſed direct concurſe, the body A would have been ſent back to Q in the ſame time that B would have been ſent back to R. Then after the oblique ſtroke the body A will be actuated by two forces, viz. one equal and parallel to AM, and the other equal and parallel to CQ: And the body B will likewiſe be actuated by two forces, viz. one equal and parallel to BN, and the other equal and parallel to DR. Therefore in fig. 3, through Q draw QZ equal and parallel to AM, alſo through Z draw IZ equal and parallel to CQ; then the diagonal CZ repreſents the direction and velocity of the elaſtic body A after the oblique concurſe.

Again through R draw the line RX equal and parallel to BN, and through X draw the line XY equal and parallel to DR, then the diagonal DX will repreſent the velocity and direction of the elaſtic body B after the oblique concurſe.

In ſhort, the caſes repreſented in fig. 2, and fig, 3, differ only in this, namely, that the bodies are ſuppoſed to be perfectly hard in the former, but perfectly elaſtic in the latter.

We might now proceed to examine the particulars relative to the congreſs of three or more bodies, as alſo of bodies of different ſhapes, for hitherto we have ſuppoſed the bodies to be quite ſpherical, &c. but this we ſhall omit, firſt, becauſe the reader may, by a little exertion of his ingenuity, eaſily derive it from what has been already explained; and ſecondly, becauſe the particular examination of all the branches of compound motion would ſwell the ſize of the work far beyond the limits of an elementary book *.

* For further information relative to this ſubject, the reader may conſult the 2d book of s'Graveſande's Mat. Elem. of Nat. Phil. edited by Deſaguliers.

CHAPTER IX.

OF CURVILINEAR MOTIONS.

HITHERTO we have confidered the compound motion which arifes from fimple impulfes, or fuch as produce equable motion. It will now be neceffary to apply the above-mentioned rules to the cafes of that fort of compound motion, which arifes from the joint action of a fimple and of an accelerative or continuate force; in which cafe it will be found, that the body will defcribe not a ftraight courfe, as when it is impelled by fimple impulfes, but it will defcribe curve lines, which differ according as the proportion of the forces differs; excepting however when the forces act in the fame direction, or directly oppofite to each other, in which two cafes, the motion of the body will always be rectilinear.

Imagine that the body A, fig. 4. Plate III. is impelled from A towards H, with fuch a force as by itfelf would enable it to run over the equal fpaces AB, BF, FG, &c. in equal portions of time; for inftance, each of thofe diftances in one minute. Imagine likewife that an attractive (confequently, an accelerative) force, continually draws the fame body A towards the centre C, in fuch a manner as as by itſelf would enable it to run over the unequal ſpaces AI, IK, KL, LM, in equal portions of time, viz. a minute each.

Now, the joint action of both thoſe forces, muſt compel the body A to run the compound and curvilinear courſe A N O P, &c.—Through B draw the line B C, that is, in the direction of the centre of attraction;—through I draw I N parallel to A B; and it is evident, from what has been ſaid above, that at the end of the firſt minute the body will be found at N. Now if at this period the attractive force ceaſed to act, the body would run on in the direction N R, by the firſt law of motion. But ſince the attractive force continues to act, the body at the end of the ſecond minute will be found at O; for the like reaſon, at the end of the third minute it will be found at P, and ſo on. The courſe then ANOP is not ſtraight; but it conſiſts of the lines AN, N O, O P, &c. forming certain angles with each other.

If inſtead of finding the place of the body at the end of every minute, we had determined its place at the end of every half minute; then each of thoſe lines AN, NO, &c. would have been reſolved into two lines containing an angle. And in the ſame manner, if we had determined the ſituation of the body at the end of every thouſandth part of a minute, each of the lines AN, NO, &c. would have been reſolved into a thouſand lines inclined to each other; but ſince the attractive force acts not at intervals, but conſtantly and unremittedly; therefore, the real path of the body is a polygonal courſe, conſiſting of an infinite number of ſides; or more juſtly ſpeaking, it is a continuate curve line, which paſſes through the points A, N, O, P, &c. as is ſhewn by the dotted line.

The curvature of the path ANOP of a body, which is acted upon at the ſame time by an equable, and by an accelerative force, varies according to the proportion of the two forces. Thus if the equable impulſe be increaſed or diminiſhed, in ſuch a manner as by itſelf would enable the body to paſs over ſpaces longer or ſhorter than AB, BF, FG, &c. in the like equal portions of time, as were ſuppoſed above, the attractive force remaining the ſame; then the curvature of the path will be increaſed or diminiſhed accordingly, as is ſhewn in fig. 5 and 6. P. III.

When the two forces are in a certain ratio to each other, then the courſe or path of the body is a circle; in other proportions within a certain limit, the path becomes elliptical, or an oval more or leſs extended; and in other proportions beyond that limit the path becomes an open curve, or ſuch as never returns to itſelf. Such curves are called *parabolas* or *hyperbolas*, and their properties, as well as thoſe of the *ellipſis*, are deſcribed by all the writers on conic ſections.

In

In fig. 4, the centre of attraction C, has been placed not very far from the direction AH of the equable force. But when this centre is very far from it, the right lines CB, CF, CG, &c. will become nearly parallel, and in many cases, they may without error, be considered as being actually parallel.

A case of this sort is represented in fig. 7. Plate III. where the centre of attraction is so remote from AG, that the right lines BC, FC, GC, &c. which proceed from it, are not to be distinguished from parallel lines. In this case, if the spaces AI, AK, AL, &c. be as the squares of the times, viz. as the squares of one minute, of two minutes, of three minutes, &c.; whilst the spaces AB, AF, AG, &c. or their equals IN, KO, LP, &c. be simply as the times, then the curve or path of the body, ANOP, is a sort of curve called a *parabola*, which is more or less open according as the projectile or equable force is more or less powerful. And such is the path which is described by all bodies that are projected obliquely near the surface of the earth, viz. cannon balls, stones thrown by the hand or other engine, and in short by all sorts of projectiles; excepting however that deviation from the parabolic curvature, which is occasioned by the resistance of the air; and which in certain cases is very considerable. For near the surface of the earth, the spaces described by descending bodies, are as the squares

of

of the times, (according to what has been ſhewn in Chap. V.) and the centre of attraction is about 4000 miles below the ſurface. (1.)

The

(1.) The admirable doctrine of curvilinear motion deſerves the greateſt attention of the philoſopher, ſince it unfolds the grandeſt phenomena of nature. It comprehends almoſt all the movements which take place in the world. It meaſures and aſcertains every particular relative to the motions of cœleſtial bodies.—It leads the human mind, through ſafe paths, to the inveſtigation and knowledge of the moſt complicated appearances, and the moſt abſtruſe ſubjects. I ſhall, therefore, in this place endeavour to explain this doctrine in as conciſe and comprehenſive a manner, as the nature of the ſubject, and the limits of the work, may ſeem to allow.

Of Equable Motion in Circular Orbits.

A *centripetal force*, in its full meaning, is that whereby a body in motion is continually drawn from its rectilinear courſe, towards ſome centre. This force may likewiſe be the action of a ſtring holding the body; or it may be its coherence with another revolving body, or it may be the gravitating power, &c.

A *centrifugal force* is the re-action or reſiſtance, which a moving body exerts to prevent its being turned out of its way, and whereby it endeavours to continue its motion in the ſame direction; and as re-action is always equal and contrary to action, ſo is the centrifugal to the centripetal force. The centrifugal force ariſes from the inertia of matter; for the body that moves round a centre, would fly off in the direction of the laſt moment, or laſt particle

of

The paths of bodies that move round a centre of attraction, are poſſeſſed of ſeveral remarkable properties;

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of its curvilinear courſe, viz. in a tangent to the curve, were the action of the centripetal force to be ſuſpended. The equality of the two forces, viz. of the centripetal in oppoſition to the centrifugal force, may be more eaſily conceived, by imagining that a revolving body is detained within its circular orbit by a ſtring; for this ſtring muſt equally endeavour to draw the body towards the centre of attraction, and the centre of attraction towards the body.

Since the centripetal force is proportionate to the ſpace which the body deſcribes in a given time by the action of that force, it is evident that the centripetal as well as the centrifugal force, may be repreſented by the naſcent lines BC, *bc*, fig. 8. Plate III. for whilſt the body deſcribes the infinitely ſmall tangent AB, the ſpace which the centripetal force compels it to paſs through in the ſame time, is equal to BC.

N.B. The lines BC, *b c*, as well as AB, A *b*, are drawn ſo large, merely for the ſake of illuſtration; whereas by naſcent or evanaſcent lines, we mean lines of the ſame nature, but indefinitely ſmall, and near the point A.—The ſame thing muſt be underſtood of other lines or quantities, which are naſcent or evaneſcent in the following propoſitions.

Propoſition I. *In a very ſmall arch, the ſine, the tangent, the chord, and the arch itſelf, are to each other nearly in the ratio of equality.*

The right-angled triangles ABE, and ACD, fig. 9, Plate III. are ſimilar; therefore, AE : AD : : BE : CD, when the arch BFD, or angle BAD becomes very ſmall, or perties; that is their periods, their velocities, their diſtances from the centre of attraction, &c. follow certain invariable laws, the knowledge of which is exceedingly uſeful in the inveſtigation of natural phenomena.

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or ſmaller than any given quantity, the point E will approach the point D indefinitely near; ſo that the difference between AE and AD will almoſt vaniſh, and of courſe the difference between BE (the ſine) and CD (the tangent) will likewiſe nearly vaniſh; viz. the ſine and the tangent will become nearly equal. And ſince the chord BD, and the arch BFD, are each of them longer than the ſine, and ſhorter than the tangent; therefore in very ſmall arches, the ſine, the tangent, the chord, and the arch itſelf, are nearly equal.

Prop. II. *In a circle the evaneſcent, or infinitely ſmall ſubtenſes of the angle of contact, are as the ſquares of the conterminal arches.*

In fig. 8. Plate III. BC, and bc drawn perpendicular to the tangent Ab, are the ſubtenſes of the angle of contact bAc, made by the tangent Ab, and the circumference ACD; which ſubtenſes muſt be imagined to be very near the point A; in which caſe we ſhall prove them to be to each other as the ſquares of the conterminal arches AC, Ac.

In conſequence of the paralleliſm of the lines AD, BC, bc, and of Ab, mC, nc; the line BC is equal to Am, and bc is equal to An. (By Eucl. p. 8. B. VI.) AD : AC :: AC : Am; and AD : Ac :: Ac : An; therefore AD $\times$ Am = $\overline{AC}|^2$; and AD $\times$ An = $\overline{Ac}|^2$. Hence we have $\overline{AC}|^2 : \overline{Ac}|^2 :: Am \times AD : An \times AD :: Am : An ::$ BC : bc.

Here

phenomena. Those laws will be found mathematically deduced from a few well established principles,

Here AC, A *c*, may be taken for the arches as well as for the chords which subtend those arches; since, by the preceding proposition, they are nearly equal.

Corollory. Since AD : AC : : AC : A *m* (= B C); we have $BC = \frac{\overline{AC}|^2}{AD}$.

Prop. III. *In the first or nascent state of circular motion, the projectile force infinitely exceeds the centripetal force.*

In fig. 10. Plate III. the circle ACD represents the orbit of the body A, moving equably along the said circumference; viz. the body A is impelled by a projectile force, in the direction AH perpendicular to AN, and is at the same time constantly acted upon by an attractive force in the direction towards the centre N; those two forces being so adjusted, or being in such proportion to each other, as to keep the body in the circular orbit ACDA.

In the very small arch AC, the line AB is to the line A *m* (= BC) as the force of projection is to the attractive, or centripetal, force, at the distance A N; for whilst A B represents the equable movement which arises from the projectile force in a certain time, BC represents the deviation from that course, or the force whereby the body is drawn towards the centre of attraction in the same time.

Now, by the preceding proposition BC (=A*m*) : AC : : AC : AD; and when the arch AC becomes extremely small, or is in its nascent state, then the diameter A D becomes infinitely greater than A C; and of course A C, or A B (which by p. 1. is nearly equal to it) becomes infinitely greater ples, in the note immediately under this paragraph, But the principal of them will be proved experimentally in the sequel. For the present, the reader

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greater than BC, or Am; viz. the projectile force infinitely greater than the central force.

In order to compare, and to demonstrate with more expedition, the proportions relative to the velocities, the forces, &c. of bodies revolving equably in different circular orbits, as ACD. and ILO, fig. 10, it will be useful to substitute letters instead of those particulars, and whilst the capital letters are applied to the body A moving in the circular orbit ACD, the small letters of the same name will denote the same things with respect to the body I moving in the circular orbit I L O. Therefore let

F, f, stand for the central *forces*.

V, v, for the circular *velocities*, or for any arches A C, I L; since in equable motions the spaces passed over in a given time are as the velocities.

T, t, stand for the periodical *times*.

D, d, for the diameters, and

P, p, for the *peripheries* or circular orbits.

The meanings of those letters will be easily remembered, since they are the initials of what they are meant to represent.

Prop. IV. *The central forces are as the squares of the velocities directly, and as the diameters inversely.*

By the substitution of the above-mentioned letters, the equation of the corollary to prop. 2d. becomes $F = \frac{V^2}{D}$, or $f = \frac{v^2}{d}$; hence $F : f :: \frac{V^2}{D} : \frac{v^2}{d}$.

Prop.

reader would do well if he fixed in his mind two of thoſe laws, which are as follows, and whoſe uſe is very extenſive.

1ſt.

Prop. V. *In different circular orbits the central forces are as the diameters directly, and as the ſquares of the periodical times inverſely.*

In equable motions the velocity is expreſſed by the quotient of the ſpace divided by the time; and in circular motion the periphery is the ſpace; hence $V = \frac{P}{T}$. Since, by p. iv. $F = \frac{v^2}{D}$; we have $FD = V^2$, and $\overline{FD}|^{\frac{1}{2}} = V = \frac{P}{T}$; therefore $\overline{FD}|^{\frac{1}{2}} \times T = P$; and $FDT^2 = P^2$. But the periphery of any circle is equal to 3,1416 multiplied by the diameter; therefore $P^2 = \overline{3{,}1416}|^2 \times D^2 = FDT^2$; and $\overline{3{,}1416}|^2 \times D = FT^2$; hence we have the force $F = \frac{\overline{3{,}1416}|^2 \times D}{T^2}$ for the circular orbit ACD fig. 10, and $f = \frac{\overline{3{,}1416}|^2 \times d}{t^2}$ for any other circular orbit as I L O; therefore $F : f :: \frac{\overline{3{,}1416}|^2 D}{T^2} : \frac{\overline{3{,}1416}|^2 d}{t^2} :: \frac{D}{T^2} : \frac{d}{t^2}$.

Prop. VI. *When the revolving bodies deſcribe equal areas in equal times, then the central forces are as the cubes of the diameters.*

In this caſe the area is repreſented by VD, which being equal to the other area vd, we have $V : v :: d : D$, and $V^2 : v^2 :: d^2 : D^2$. But (by prop. IV.) $F : f :: \frac{V^2}{D} : \frac{v^2}{d}$; hence $\frac{V^2}{D} : \frac{v^2}{d} :: V^2 d : v^2 D$; therefore $F : f :: d^2 d : D^2 D :: d^3 : D^3$.

Prop.

1ſt. *When bodies revolve in equal circles, the central forces are as the ſquares of the velocities; or a double projectile force balances a duadruple force of centripetal*

Prop. VII. *When the periodical times are equal, the central forces are as the radii, viz. as the diſtances from the centre of attraction, and vice verſa.*

When $T = t$, the analogy of Prop. V. (viz. $F : f :: \frac{D}{T^2} : \frac{d}{t^2}$) becomes $F : f :: D : d :: \frac{D}{2} : \frac{d}{2}$ (viz. as the radii); the converſe of this propoſition is alſo evident, viz. that when $F : f :: D : d$, the periodical times muſt be equal.

Prop. VIII. *When the diameters are equal, the central forces are as the ſquares of the velocities.*

For (by Prop. IV.) $F : f :: \frac{V^2}{D} : \frac{v^2}{d}$; therefore when $D = d$, this analogy becomes $F : f :: V^2 : v^2$; viz. when the circles are equal, or in the ſame circle, the forces are as the ſquares of the velocities.

Prop. IX. *When the diameters, or diſtances, and of courſe the circles, are equal, the central forces are inverſely as the ſquares of the periodical times.*

Since in that caſe the analogy of Prop. V. (viz. $F : f :: \frac{D}{T^2} : \frac{d}{t^2}$) becomes $F : f :: \frac{1}{T^2} : \frac{1}{t^2} :: t^2 : T^2$.

Prop. X. *When the diameters are equal, the periodical times are inverſely as the velocities.*

It appears from Prop. V. that $V : v :: \frac{P}{T} : \frac{p}{t} :: \frac{D}{T} : \frac{d}{t}$ (ſince the diameters of circles bear to each other the ſame *centripetal attraction.* For instance, if a body, which is impelled with a certain velocity, and is attracted with

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same proportion as their peripheries). Now when $D = d$, then the preceding analogy becomes $V : v :: \frac{1}{T} : \frac{1}{t}$; or $T : t :: v : V$.

Prop. XI. *When the velocities are equal, the forces are inversely, as the diameters.*

For in that case, the analogy of p. IV. (viz. $F : f :: \frac{V^2}{D} : \frac{v^2}{d}$) becomes $F : f :: \frac{1}{D} : \frac{1}{d}$; or $F : f :: d : D$.

Prop. XII. *When the velocities are equal, the periodical times are as the diameters; or as the peripheries, which is the same thing.*

By prop. V. $V : v :: \frac{P}{T} : \frac{p}{t}$; and when $V = v$, then $\frac{P}{T} = \frac{p}{t}$, or $P\,t = p\,T$; whence we have $T : t :: P : p :: D : d$.

Prop. XIII. *When the central forces are equal, the periodical times are as the square roots of the distances, or of the diameters.*

In prop. V. it has been shewn that $F : f :: \frac{D}{T^2} : \frac{d}{t^2}$. Now when $F = f$; then $\frac{D}{T^2} = \frac{d}{t^2}$; or $D\,t^2 = d\,T^2$ which gives the analogy $T^2 : t^2 :: D : d$; and of course $T : t :: D^{\frac{1}{2}} : d^{\frac{1}{2}}$.

Prop. XIV. *When the central forces are equal, the squares of the velocities are as the distances; and the periodical times are as the velocities.*

By prop. IV. $F : f :: \frac{V^2}{D} : \frac{v^2}{d}$; and when $F = f$; then with a certain central force, describe a circle round the centre of attraction; then if the velocity be doubled, or tripled, the attractive force must be four

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then $\frac{V^2}{D} = \frac{v^2}{d}$ and $V^2\ d = v^2\ D$; which gives the analogy $V^2 : v^2 :: D : d :: \frac{D}{2} : \frac{d}{2}$ (the distances being the halves of the diameters). Also by p. XIII. $T : t :: D^{\frac{1}{2}} : d^{\frac{1}{2}}$, and by the last analogy, $V : v :: D^{\frac{1}{2}} : d^{\frac{1}{2}}$; therefore $T : t :: V : v$.

Prop. XV. *When the central forces are inversely as the squares of the diameters, or of the distances; then the squares of the periodical times are as the cubes of the distances.*

Imagine the central forces to be as some power, m, of the distances; viz. $F : f :: D^m : d^m$. Now by prop. V. $F : f :: \frac{D}{T^2} : \frac{d}{t^2}$; therefore $D^m : d^m :: \frac{D}{T^2} : \frac{d}{t^2} :: D\, t^2 : d\, T^2$; and $\frac{D^m}{D} : \frac{d^m}{d} :: t^2 : T^2$; or $D^{m-1} : d^{m-1} :: t^2 : T^2$; and $D^{\frac{m-1}{2}} : d^{\frac{m-1}{2}} :: t : T$; hence $D^{\frac{1-m}{2}} : d^{\frac{1-m}{2}} :: T : t$.

Now when the forces are equal then the power, m, vanishes, or $m = 0$, and then the last analogy becomes $D^{\frac{1}{2}} : d^{\frac{1}{2}} :: T : t$, which is the same thing as was shewn in prop. XIII. When the forces are as the distances, then m is the first power, or $m = 1$, and in that case the above-mentioned analogy becomes $D^0 : d^0 :: 1 : 1 :: T : t$, and of course $T = t$, which is the case of prop. VII.—Lastly, when the forces are inversely as the squares of the distances, then $m = -2$ and the above-mentioned analogy, becomes $D^{\frac{3}{2}} : d^{\frac{3}{2}} :: T : t$; or $D^3 : d^3 :: T^2 : t^2$; or $\frac{D^3}{2} : \frac{d^3}{2} :: T^2 : t^2$.

The

four or nine times as ſtrong as it was before, in order to let the body move in the ſame circle.

2d. *When bodies move in unequal circular orbits, ſo*

The planets of our ſolar ſyſtem follow this grand law of nature. The ſquares of their periodical times are as the cubes of their diſtances from the common centre of attraction, which is very near the centre of the ſun, as will be ſhewn hereafter; and thus Newton's hypotheſis of mutual and univerſal attraction amongſt the bodies of the univerſe is ſhewn to be ſo conſonant with the ſtricteſt mathematical reaſoning, and with all the appearances, that none but the ignorant can refuſe their aſſent to it.

This doctrine of circular movements, which I have exhibited in 15 propoſitions, might have been condenſed into a narrower compaſs, had not my principal object been to render the comprehenſion of it eaſy to the reader; I have been taught by experience, that in many inſtances it is far more laborious to deduce every particular caſe from one comprehenſive propoſition, than to read a particular propoſition for every ſingle caſe.

Having, in the preceding propoſitions, ſtated the proportions between the forces, the velocities, and the periodical times, of bodies that revolve in circular orbits; it is now neceſſary to render thoſe propoſitions practically uſeful, by ſhewing in what manner they may be employed for the determination of any particular caſe; ſince it has already been remarked, that the knowledge of the proportion which certain things bear to each other, will not enable us to determine any abſolute quantity, unleſs ſome of the particulars be previouſly determined by means of actual experiments.

ſo that the ſquares of the times of their revolutions are as the cubes of their diſtances from the centres of thoſe circles, then the central forces are inverſely as the ſquares of the diſtances ; and vice verſa.

The

Therefore in order to render the expreſſions of the meaſures of the above-mentioned forces, velocities, &c. more eaſy and conciſe, we ſhall endeavour to involve in them only one unknown quantity, viz. a certain power of the radius or diſtance of the revolving body from the centre of attraction ; for when this quantity becomes known either from experiments or by deduction from other known quantities, we may thereby eaſily determine all the other particulars.

I. The attractive force is meaſured by the velocity which may be uniformly generated in a given time, which time we ſhall call one, (meaning one ſecond, or one minute, or, in ſhort, the unity of any other diviſion of time that may be uſed in the computation,) and ſhall expreſs this force or velocity by r^n, (viz. an indeterminate power n of the radius r of the orbit).

II. It has been ſhewn in page 67, that a body, which begins to move from reſt and proceeds towards a centre of attraction, will at the end of any given time acquire ſuch velocity as would enable it to move equably through twice that ſpace in an equal portion of time, if the action of the attractive force were ſuſpended. Therefore the diſtance through which a body will deſcend towards the centre of attraction in the above-mentioned time one, is $\frac{1}{2} r^n$.

III. Hence if A C fig. 10, Plate III. be an arch deſcribed in a certain time T, the diſtance A m, which the body would

The foregoing theory of curvilinear motion is very extenſive, ſince it applies to a great variety of terreſtrial as well as celeſtial phenomena. But in the

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would deſcend through towards the centre of attraction N in the ſame time, will be $= \frac{\frac{1}{2} r^n T^2}{1^2}$; for ſince the ſpaces which are thus deſcended, as the ſquares of the times, we have $1^2 : T^2 :: \frac{1}{2} r^n : \frac{\frac{1}{2} r^n T^2}{1^2}$.

IV. The part AC of the circumference, which is deſcribed by the body in the time T, is $= \frac{T r^{\frac{n+1}{2}}}{1}$; ſince in the circle ACD, we have $\overline{AC}|^2 = AD \times Am$ (Eucl. p. 8. B. VI.) $= 2AN \times Am = AN \times 2Am = \frac{T^2 r r^n}{1^2} = \frac{T^2 r^{n+1}}{1^2}$; therefore $AC = \frac{\overline{T^2 r^{n+1}}|^{\frac{1}{2}}}{1} = \frac{T r^{\frac{n+1}{2}}}{1}$.

V. The velocity with which the body moves in the circle is $= r^{\frac{n+1}{2}}$. By prop. IV. the ſquare of the velocity is as the product of the diameter, or of the radius, multiplied by the force; and according to the above-mentioned notation, (§. I.) the force is expreſſed by r^n; therefore the ſquare of the velocity is $= r r^n = r^{n+1}$, and of courſe the velocity itſelf is expreſſed by $r^{\frac{n+1}{2}}$.

VI. The periodical time, or time of a whole revolution, is $= 2 c r^{\frac{1-n}{2}}$ (c being $= 3{,}1416$, &c. that is the circumference of a circle whoſe diameter is one.) For ſince the calculation of the particulars which relate to those phenomena, certain circumstances generally interfere,

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(by §. IV.) $\frac{T\, r^{\frac{r+1}{2}}}{1}$ is the part of the orbit which is described in the time T, the part which is described in the time 1, must evidently be $r^{\frac{n+1}{2}}$. Then, the spaces described with a uniform motion being as the times, it will be $r^{\frac{n+1}{2}} : 2rc$ (= the whole circumference) :: 1 : $2cr^{\frac{1-n}{2}}$.

VII. The space through which the body must descend towards the centre of attraction, in order to acquire a velocity equal to that with which it revolves, is equal to half the radius, viz. $\frac{1}{2} r$. For in Chap. V. it has been shewn, that the spaces described by descending bodies are, as the squares of the times, or of the velocities. It has also been shewn (§. II.) that the velocity r^n is acquired by a descent through $\frac{1}{2} r^n$. At present we wish to know how low a body must descend, to acquire a velocity equal to $r^{\frac{n+1}{2}}$ (§. V.) hence we say, as the square of r^n is to the square of $r^{\frac{n+1}{2}}$; so is $\frac{1}{2} r^n$ to a fourth proportional; viz. $r^{2n} : r^{n+1} :: \frac{1}{2} r^n : \frac{\frac{1}{2} r^{2n+1}}{r^{2n}} = \frac{\frac{1}{2} r r^{2n}}{r^{2n}} = \frac{1}{2} r$.

Thus we have expressed the measures of the velocities, periodical times, &c. in a general yet simple manner. They may be applied to any attractive power, and to any periodical revolution; the only quantity which needs be known, interfere, which render the reſult of the calculations ſomewhat different from the obſervations; that is, of the experiments. In terreſtrial affairs, the

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known, being the value of r''. But for the ſake of illuſtration, we ſhall now apply it to the force of terreſtrial gravity; in which caſe it is known, that a body near the ſurface of the earth, will deſcend from reſt 16,087 feet in the firſt ſecond of time, (which is the time 1); therefore, $\frac{1}{2} r'' = 16{,}087$ feet, and $r'' = 32{,}174$. (§. II.) Hence by ſubſtituting thoſe values for $\frac{1}{2} r^n$ and r^n reſpectively, the above-mentioned meaſures will be expreſſed in known terms.

Example 1. The velocity of a body that revolves round the earth but near the ſurface of it, is (by §. V.) $r^{\frac{n+1}{2}}$; which, by ſubſtituting, 32,174 for r'', becomes $\overline{32{,}174r}|^{\frac{1}{2}}$; and this becomes (ſince the ſemi-diameter or radius r of the earth is known to be nearly 21000000 feet) $\overline{32{,}174 \times 21000000}|^{\frac{1}{2}}$ $= 25993{,}3$ feet per ſecond; ſo that a body moving with that velocity, would revolve continually round the earth; that velocity being juſt ſufficient to balance the force of gravity; but this velocity is about 30 times as great as the initial velocity of a cannon ball.

N. B. No notice of the reſiſtance of the air has been taken in this example, or will be taken in the following examples of this note.

The periodical time of the ſame body under the ſame circumſtances, is (by §.VI.) $2\,c\,r^{\frac{1-n}{2}} = 2c\,\frac{\overline{r}|^{\frac{1}{2}}}{\overline{r''}|^{\frac{1}{2}}} = 2 \times$ $3{,}1416 \times \frac{4582{,}5}{5{,}67} = 5087''{,}5$; or 1 hour, $24'$, $47''{,}5$.

Example

the reſiſtance of the air is one of the principal obtruders. The movements of the cœleſtial bodies are

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Example 2. By prop. IX. when the diſtances are equal or in the ſame circle, the central forces are inverſely as the ſquares of the periodical times; and, by the preceding example, the velocity which near the ſurface of the earth is equivalent to gravity, is = 25993,3 feet per ſecond. Therefore, we ſay as the ſquare of the earth's diurnal rotation round its axis, is to the ſquare of the periodical time of the body mentioned in the preceding example, (viz. of 1^h $24'47''$, 5, or nearly $85'$); ſo is the force of gravity (which we ſhall call 1) to the centrifugal force of bodies near the equator of the earth; viz. $2073600'$ (= the ſquare of 24 hours) : $7225'$: : 1 : 0,003485 = the centrifugal force near the equator; viz. the force by which bodies that are near the equator, are attracted towards the centre, is to the force with which they endeavour to fly off, in conſequence of the earth's diurnal rotation round its axis, as 1 is to 0,003485; or as 1000000 to 3485; viz. the former is almoſt 300 times more powerful than the latter.

By this means we may determine the centrifugal force of bodies in different latitudes; for as the earth turns round its axis, it is evident that thoſe bodies on the ſurface of it, which lie nearer to the axis, or, which is the ſame thing, are nearer to the poles, perform circles ſmaller than thoſe which lie nearer to the equator; though they are all performed in the ſame time, viz. 24 hours. Hence (by prop. VII.) the periodical times being equal, or the ſame, the central forces are as the radii of the circles, and as in different latitudes the radii are equal to the coſines of the latitudes, therefore, as the radius is to the coſine of a given latitude, are generally influenced by more than one centre of attraction. Thus the moon is attracted by the earth and likewise by the sun. The planets are attracted

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latitude, so is the centrifugal force of bodies situated at the equator, to the centrifugal force of bodies at that given latitude. Now as the cosines grow shorter and shorter, the nearer they come to the poles, so the tendency of bodies to fly off from the surface of the earth is greatest at the equator, but it diminishes as you approach the poles; and hence we see why the earth has been found by means of undoubted measurements and other observations, to be an oblate spheroid, whose polar diameter is the shortest. And this furnishes a strong evidence of the earth's daily rotation about its axis.

Example 3. The mean distance of the moon from the centre of the earth is, 1267200000 feet, or about 60 semidiameters of the earth. Also the force of gravity at different distances, is inversely as the squares of the distances, and the radius of the earth is 21000000 feet; therefore, as the square of 1267200000, is to the square of 21000000, so is the force of gravity at the surface of the earth, to the force of gravity at the distance of the moon, viz. 160579 5840000000000 : 441000000000000 : : 1 : 0,000274; so that the force of gravity at the surface of the earth, is to the force of gravity at the moon as 1 is to 0,000274; or as 1000000 to 274. And since near the earth falling bodies pass over 16,087 feet in the first second of time; therefore, we say, 1000000 : 274 : : 16,087 : 0,0044 of a foot; which shews that the moon, should its velocity cease at once, would fall towards the earth, and in the first second of time would descend through not more than $\frac{44}{10000}$ ths of a foot.

Farther.

attracted by the ſun, and likewiſe by each other, &c.

On this account we might now extend our examination to the caſes in which two or three, or more

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Farther. By prop. XV. when the central forces are inverſely as the ſquares of the diameters, then the cubes of the diſtances are as the ſquares of the periodical times. Therefore the diſtance of the body, which circulates near the ſurface of the earth (Example 1.) being one ſemidiameter of the earth, and the diſtance of the moon being 60 ſemidiameters; alſo the period of the former being 84′,8 we may find the period of the latter by ſaying $1^3 : \overline{60}|^3 :: \overline{84',8}|^2 :$ to the ſquare of the moon's period; viz. 1 : 216000 :: 7191 : 1553256000; the ſquare root of which, viz. 39411″,3 or 27 days 8 hours 51′,3, is the period of the moon's revolution round her orbit, which is nearly equal to what the aſtronomers reckon it, viz. $27^d\ 7^h\ 34'$; and it would have come out exactly like it, had the diſtances been ſtated with exactneſs; and had we likewiſe taken into the account certain circumſtances, which interfere with that period, which however we have purpoſely avoided in this example, for the ſake of brevity.

Similar calculations may be inſtituted with reſpect to all the planets of our ſolar ſyſtem, and the reſult of the calculations will be found to coincide wonderfully well with the appearances; which, as we have already remarked, is a ſtrong confirmation of the Newtonian theory of univerſal gravitation.

Example 4. Let a ball of one pound weight be faſtened to a ſtring 2 feet long, and be whirled about a centre ſo as to deſcribe each revolution in half a ſecond. In this caſe the more centres of attraction act upon the ſame body; but this inveſtigation we ſhall omit on two accounts, viz. firſt, becauſe the ſubject is too intricate and

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the orbit or circumference of the circle is $4 \times 3,1416 = 12,5664$. The velocity of the ball is 25,1328 feet per ſecond. In order to determine the centrifugal force of the ball thus revolving, viz. the force with which the ſtring is ſtretched by it, compared with the force of gravity (which is $= 1$), we make uſe of the analogy of prop. V. (viz. $\frac{D}{T^2} : \frac{d}{t^2} :: F : f$) which, by ſubſtituting $2r$ for D; $2cr^{\frac{1-n}{2}}\overline{}|^2$ for T^2; 4 for d; 0″,25 for t^2; and 1 for F; becomes $\frac{r^n}{2c^2}\left(= 2r \div 2cr^{\frac{1-n}{2}}\overline{}|^2\right) : \frac{d}{t^2} :: 1 : f = \frac{2dc^2}{r^n t^2} = \frac{dc^2}{\frac{1}{2}r^n t^2}$, equal to $\frac{4 \times 9,86}{16,087 \times 0,25} = \frac{39,44}{4,02175} = 9,8$ which is the meaſure of the central force of the body in queſtion; this force therefore is to to the force of gravity as 9,8 to 1; ſo that ſince the body weighs one pound when quieſcent, viz. it ſtretches the ſtring with the weight of one pound; therefore when revolving according to the ſuppoſition, it will ſtretch the ſtring with the force of 9,8 pounds.

Now this central force may be called centripetal or centrifugal, according as it is applied to the tenacity of the parts of the ſtring, or to the force of the body; ſo that the body is ſaid to be retained by a centripetal force 9,8 times as great as the force of terreſtrial gravity; or it may be ſaid that the centrifugal force of the revolving body ſtretches the ſtring as much as if a weight of 9,8 pounds were ſimply ſuſpended to it.

Of

and extenſive; and ſecondly, becauſe in moſt natural phenomena, the diſturbing cauſe which ariſes from the action of a ſecond or a third, or in general

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Of the motion of bodies about a centre of attraction, but in curves differing from circles.

It has been ſufficiently ſhewn that a certain determinate velocity is required to confine the movement of a body in a circular orbit round a centre of attraction; whence it follows, that with a greater or a leſſer velocity bodies will move in curve lines different from circles. Thoſe curves appear to be the conic ſections; and ſince, ſtrictly ſpeaking, the circle is likewiſe a conic ſection, therefore it may be concluded, that in general the movements of bodies round any centre of attraction are performed in curves of the conic kind, provided the bodies do not meet with any obſtructing medium, or other attraction, in their way; for under ſuch circumſtances, their paths may degenerate into ſpirals, or other curves of a more intricate nature.

The movements of the cœleſtial bodies are not ſtrictly circular, though they do not deviate much from that figure; excepting however the comets which move either in very eccentric elipſes, or elſe in parabolas or hyperbolas; and therefore in the laſt two caſes they can never return to the ſame parts of the heavens; but they muſt continually recede from the common centre of attraction, which, in our ſolar ſyſtem, is not far from the centre of the ſun.

With reſpect to the theory of circular movements, I have endeavoured to demonſtrate the principles, and to illuſtrate the practical operations in a manner ſufficiently extenſive; being perſuaded that if that branch of compound motion be well underſtood, the reader (provided he is acquainted with the neral of more than one centre of attraction, is not very conſiderable; yet in the courſe of this work, the method of taking the above-mentioned circumſtances

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the principal properties of the conic ſections) will eaſily comprehend what follows; I ſhall therefore endeavour to explain the nature of the movements in curves of the conic kind, in a manner more comprehenſive and conciſe.

In Fig. 11. Plate III. ACD repreſents a circular orbit, A*z* S repreſents an elliptical orbit, A *r* E a parabolic, and AKF an hyperbolic orbit, of bodies moving with certain velocities under the influence of the centre of attraction N, which is the centre of the circle, and the focus of the conic ſections.

Let AB, perpendicular to AD, repreſent the velocity which is neceſſary to retain the body in the circular orbit, and let this velocity be called 1; for we ſhall compare the other degrees of velocity with this unity. Alſo let a body be projected from A in the direction AI with any other degree of velocity n. It is now neceſſary to determine the nature of the curve which will be deſcribed with this other velocity n, or rather it is required to aſcertain what the value of n muſt be in order to produce each particular conic ſection.

Draw mK parallel to AI, interſecting the circle as well as the other curves. Let AN be denoted by d; the ſemi-tranſverſe axis of any of the conic ſections, by a; the ſemi-conjugate, by b; and Am ($=$BC$=$G$z$$=Hr$$=$IK) by x. Then the ordinate mC in the circle will be $=\overline{2dx-xx}|^{\frac{1}{2}}$, but the ordinate mz of the ellipſis, and mK of the hyperbola may be both repreſented by $\frac{b}{a}\ \overline{\times 2ax \mp xx}|^{\frac{1}{2}}$.

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cumſtances into the account, will in many caſes be ſufficiently pointed out.

It is however proper to obſerve, that the various circumſtances which obſtruct or influence the movements

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The fluxions of thoſe ordinates are $\frac{d\dot{x} - x\dot{x}}{\overline{2dx - xx}|^{\frac{1}{2}}}$ and $\frac{b}{a} \times \frac{a\dot{x} \mp x\dot{x}}{\overline{2ax \mp xx}|^{\frac{1}{2}}}$ which fluxions are to each other as the velocities in every point of their reſpective curves in the direction AI; and in the like proportion are the quantities $\frac{d - x}{\overline{2d - x}|^{\frac{1}{2}}}$ and $\frac{b}{a} \times \frac{a \mp x}{\overline{2a \mp x}|^{\frac{1}{2}}}$ thoſe quantities being the above mentioned fluxions divided by the ſame quantity, $\frac{\dot{x}}{\overline{x}|^{\frac{1}{2}}}$.

Now when the point in the curve approaches the point A ſo near as to coincide with it, then A*m* vaniſhes, or $x = 0$; and the above expreſſions become $\frac{d}{\overline{2d}|^{\frac{1}{2}}}$ and $\frac{b}{a} \times \frac{a}{\overline{2a}|^{\frac{1}{2}}}$; ſo that at the point A the velocity which retains the body in the circular orbit, is to the velocity which retains the body in the ellipſis or the hyperbola, as $\frac{d}{\overline{2a}|^{\frac{1}{2}}} : \frac{b}{a} \times \frac{a}{\overline{2a}|^{\frac{1}{2}}}$ $:: d^{\frac{1}{2}} : \frac{b}{a^{\frac{1}{2}}} :: 1 : n$; therefore $n d^{\frac{1}{2}} = \frac{b}{a^{\frac{1}{2}}}$; and $nnd = \frac{bb}{a}$, or $annd = bb$. When $x = d =$ AN, then $2y$ is the parameter, and (ſince the parameter is a third proportional to the tranſverſe and conjugate diameters) $2a : 2b :: 2b :$ $2y$, movements of bodies, are far from being all known, or fully underſtood. Beſides, even thoſe that are known,

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$2y$, or $a : b :: b : y \pm \frac{bb}{a} = \frac{b}{a} \times \overline{2ax \mp xx}|^{\frac{1}{2}} = \frac{b}{a} \times \overline{2ad \mp dd}|^{\frac{1}{2}} = \overline{\frac{2ab^2d \mp b^2d^2}{aa}}|^{\frac{1}{2}}$; which equation being ſquared, becomes $\frac{2ab^2d - b^2d^2}{aa} = \frac{b^4}{a^2}$ for the ellipſis, and $\frac{2ab^2d + b^2d^2}{a^2} = \frac{b^4}{a^2}$ for the hyperbola. And being divided by $\frac{b^2}{a^2}$, thoſe expreſſions become $2ad - d^2 = b^2 = annd$ for the ellipſis, and $2ad + d^2 = b^2 = annd$, for the hyperbola. Therefore

In the ellipſis: the ſemi tranſverſe axis is $a = \frac{d}{2 - n^2}$; the ſemiconjugate axis is $b = \frac{nd}{\overline{2 - nn}|^{\frac{1}{2}}}$

In the hyperbola: the ſemitranſverſe is $a = \frac{d}{n^2 - 2}$; the ſemiconjugate is $b = \frac{nd}{\overline{nn - 2}|^{\frac{1}{2}}}$

Having determined thoſe values of the tranverſe and conjugate diameters, wherein n is the only indeterminate value, we may, by making certain ſubſtitutions inſtead of n, aſcertain what the value of n muſt be in order to produce one curve or another.

Thus by making $n = 1$, each of the above values becomes equal to d; therefore the two diameters become equal to each other, the curve is of courſe a circle. And in known, are moſtly fluctuating in the intenſity of their actions. Much light has undoubtedly been thrown

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in fact the velocity which retains the revolving body in a circular orbit, has been called 1, or unity.

If we make $n = \overline{2}|^{\frac{1}{2}}$, then $a = \frac{d}{2-nn} = \frac{d}{2-2} = \frac{d}{0}$, which is an algebraical expreſſion of infinity. And all the other expreſſions will likewiſe become infinite; hence, the tranſverſe and conjugate diameters in that caſe becoming infinite, the curve is the parabola.

If we make n equal to a quantity leſs than the ſquare root of 2 (viz. leſs than the ſquare root of twice that velocity which is required to retain the body in a circular orbit;) then the values $\frac{d}{2-n^2}$ and $\frac{nd}{\overline{2-nn}|^{\frac{1}{2}}}$ viz. of a and b, will be poſitive; whereas, by the ſame ſubſtitution, the value $\frac{nd}{\overline{nn-2}|^{\frac{1}{2}}}$ becomes impoſſible; which ſhews, that when n is leſs than the ſquare root of 2, the curve can only be the ellipſis.

Laſtly, if we make n equal to any thing greater than the ſquare root of 2; then the values of a and b for the hyperbola become poſitive; whereas thoſe for the ellipſis become impoſſible; hence in this caſe the curve muſt be the hyperbola.

We ſhall conclude this ſubject with the following general propoſition, which, together with its corollaries, is applicable to a variety of natural phenomena.

In all determinate orbits, deſcribed by bodies revolving with certain velocities in non reſiſting mediums, about a centre of attraction, the areas, which are deſcribed by a ſtraight line connecting thrown on this ſubject by the ingenuity of ſcientific perſons during the two laſt centuries, yet a great deal ſtill remains to be done, and a vaſt field of ſpeculation offers itſelf to the induſtry of future philoſophers.

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connecting the centre of attraction and the revolving body, lie in one invariable plane, and are always proportional to the times in which they are deſcribed.

Imagine the time to be divided into equal particles, and that a body moving round the centre of attraction N, fig. 12, Plate III. runs over the ſpace A B in the firſt particle of time. It is evident that, were the body left to itſelf, it would proceed ſtraight to H, deſcribing BH, equal to A B, in the ſecond particle of time; but at B imagine that the body receives a ſingle inſtantaneous impulſe from the centre of attraction N in the direction BN, ſufficient to change its direction from BH to BC. Through H draw CH parallel to BN, which will meet BC in C; and, agreeably to the laws of compound motion, at the end of the ſecond particle of time, the body will be found at C in the ſame plane with the triangle ANB. Draw the lines NC, NH, and the triangle NBH will be equal to the triangle NBC, ſince they ſtand on the ſame baſe NB, and between the ſame parallels NB, CH (Eucl. p. 37, B. I.) It will likewiſe be equal to the triangle ABN, ſince they have equal baſes and the ſame altitude (Eucl. p. 1. B. VI.) By the very ſame mode of reaſoning it may be proved, that, if the centripetal force act upon the body at the end of each ſucceſſive particle of time, ſo as to let the body deſcribe the ſpaces CD, DE, EF, &c. thoſe ſpaces will all lie in the ſame plane; the triangles ANB, BNC, CND, DNE, &c.

 will

In the present ſtate of the world the improvements of ſcience ſeldom die with individuals. The accumulation of knowledge by leading the underſtanding, and by furniſhing tools to the ſenſes, promotes the diſcovery of farther truths, and the inexhauſtible

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will be all equal, and will be deſcribed in equal times. Conſequently two or three, or any number of them, will be deſcribed in two or three, or the like number of particles of time, viz. they are as the times.

Now imagine that thoſe triangles are infinitely increaſed in number, and diminiſhed in ſize; then the polygonal path ABCDEF, will become a continuate curve; for the conſtant action of the centre of attraction will be continually drawing the body away from the direction of the tangent at every point of the curve. And it is evident that the ſectoral areas of the ſaid curve, or number of infinitely ſmall triangles, muſt be proportional to the times in which they are deſcribed, and that the curve muſt lie in one immoveable plain.

Corollary 1. *The velocities in different parts of the orbit are inverſely as the perpendiculars dropped from the centre of attraction on the tangents to the orbit at thoſe parts or points.* For ſince the velocities are as the baſes AB, BC, CD, &c. of equal triangles, they muſt be inverſely as the heights of thoſe triangles, (Eucl. p. 15, B. VI. and p. 38, B. I.) which are the ſame as the perpendiculars dropped from the centre N, on the tangents to the orbit at thoſe points.

Corollary 2. *The times in which equal parts, or arches of the orbit are deſcribed, are directly as thoſe perpendiculars to the tangents.* For when the arches, or baſes of the triangles, are equal, the triangles are as their altitudes; that is, exhaustible fund of nature offers on all sides innumerable objects of investigation to the inquisitive mind.

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is, as the above-mentioned perpendiculars. But they are likewise as the times; therefore, &c.

Corollary 3. If, by drawing lines parallel to the chords AB, BC, of any two contiguous and evanescent arches described in equal times, the parallelogram be completed, the diagonal BG, when produced, will pass through the centre of attraction N, which proves the converse of the proposition; viz. that *when the areas, which are described by a straight line, connecting a moving body and a certain point, are proportional to the times in which they are described, then the body is under the influence of a centripetal force tending to that point.*

Corollary 4. *In every point of the orbit the centripetal force is as the sagitta, or versed sine, of the indefinitely small arch at that point.*—The centripetal force at B is as BG, because BG is equal to CH, and CH is the deviation from the straight direction AH, which has been occasioned by the centripetal force. And the half of BG, viz. B O, is the sagitta, or versed sine, of the indefinitely small arch ABC.

CHAPTER X.

OF THE DESCENT OF BODIES UPON INCLINED PLANES; AND THE DOCTRINE OF PENDULUMS.

Prop. I. WHEN *a body is placed upon an inclined plane, the force of gravity which urges that body downwards, acts with a power so much less, than if the body descended freely and perpendicularly downwards, as the elevation of the plane is less than its length.*

If BD, fig. 1, Plate IV. be an horizontal plane, and a body A be laid upon it, this body will remain motionless; for though the power of gravity, or (which is the same thing) its own weight, draws it towards the centre of the earth, yet the plane DB supports it exactly in that direction; hence no motion can arise.

But if the plane be inclined a little to the horizon, as in fig. 2, Plate IV. then the body will descend gently towards the lower end D. And if the inclination of the plane be increased, as in fig. 3, Plate IV. the body will run down towards D with greater quickness.

In the two last cases; or, in general, whenever the plane is inclined to the horizon, the action of gravity is not entirely but partially counteracted by the plane. For if, from the centre A of the body, in the figures 2 and 3, you draw two lines, viz. AG per-

perpendicular to the horizon, and AF perpendicular to the plane; the whole force of gravity, which is represented by the line AE, is resolved into two forces; viz. AF and EF, whereof AF being perpendicular to the plane, is that part of the gravitating power which is counteracted by the inclined plane; or that part of the weight of the body which is supported by the plane BD; and EF represents the other part of the gravitating power, which urges the body downwards along the surface of the plane. Therefore the force of gravity which moves the body, is diminished in the proportion of AE to EF. But the triangles AFE, EDG, and BDC, are equiangular, and of course similar (because the angles at F, C, and G are right, and the angle AEF is equal to the angle DEG, by Eucl. p. 15. B. I.; as also equal to the angle DBC, by Eucl. p. 29. B. I.) Hence we have AE to EF, as DB to BC; viz. as the length of the plane is to its elevation, or as the whole force of gravity is to that part of it which urges the body down along the inclined plane*.

Prop. II. *The space which is described by a body descending freely from rest towards the earth, is to the space which it will describe upon the surface of an in-*

* If (by trigonometry) DB be made radius, BC becomes the sine of the angle of inclination BDC; *therefore the whole force of gravity is said to be to that part of it which urges a body down an inclined plane, as radius is to the sine of the plane's inclination to the horizon.*

clined plane in the ſame time as the length of the plane is to its elevation, or as radius is to the ſine of the plane's inclination to the horizon.

The force of gravity, which urges a body down along the ſurface of an inclined plane, is diminiſhed by the partial counteraction of the inclined plane; but its nature is not otherwiſe changed; viz. it acts conſtantly and unremittedly. Hence the velocity of the body is continually accelerated, and the ſpaces it runs over are alſo proportional to the ſquares of the times; though thoſe ſpaces will not be ſo long as if the body deſcended freely and perpendicularly towards the ground.

Now in order to aſcertain how much the ſpace, which is deſcribed by a body running down an inclined plane in a certain time, is ſhorter than the ſpace through which it would deſcend freely and perpendicularly in the ſame time, we muſt recollect what has been proved in page 64, relatively to the ſpaces, which are deſcribed in the ſame time, by bodies that are acted upon by different central forces; namely, that in equal times, the ſpaces are as the forces; then, ſince the whole force of gravity is to that force which draws a body down the inclined plane, as radius is to the ſine of the plane's inclination. Therefore the ſpace deſcribed by a body which deſcends freely, is to the ſpace which a body will deſcribe on an inclined plane, in the ſame time as radius is to the ſine of the plane's inclination, or as the length of the plane is to its altitude.

Example.

Example. Let the length BD of the inclined plane, fig. 2, Plate IV. be 10 feet, and its elevation BC, 4 feet. It is known from experiment, that in the first second of time, a body will descend freely from rest through 16,087 feet. Therefore, by the rule of three, we say, as 10 feet are to 4 feet, so are 16,087 feet to a fourth proportional, viz. 10 : 4 : : 16,087 : $\left(\frac{4 \times 16,087}{10} = \right)$ 6,435 feet, which shews that a body running down the inclined plane BD, would pass over little less than six feet and a half, or 6,435 feet, in the first second of time.

Prop. III. *If upon the elevation* BC, *fig.* 4, *Plate* IV. *of the plane* BD, *as a diameter, the semicircle* BEGC *be described, the part* BE *of the inclined plane, which is cut off by the semicircle, is that part of the plane over which a body will descend, in the same time that another body will descend freely and perpendicularly along the diameter of the circle, viz. from* B *to* C, *which is the altitude of the plane, or sine of its inclination to the horizon.*

The triangle BEC is equiangular, and of course similar, to the triangle BDC (for the angle at B is common to both, and the angle BEC is by Eucl. p. 31. B. III. a right angle, and therefore equal to the right angle BCD) hence we have BD to BC as BC is to BE. But, by the preceding proposition, the space descended freely and perpendicularly, is to the space run over an inclined plane in the same time, as the length of the

the plane is to its elevation, viz. as BD is to BC; therefore, the ſpace run freely and perpendicularly, is to the ſpace run over the inclined plane, likewiſe as BC is to BE. And ſince BC is the ſpace freely deſcended by a body in a certain time, BE muſt be the ſpace which is run down by a body on the inclined plane in the ſame time.

Cor. A very uſeful and remarkable conſequence is derived from this propoſition, namely, *that a body will deſcend from* B *over any chord whatſoever as* BE, *or* BF, *or* BG, *of the ſemicircle* BEFC, *exactly in the ſame time, viz. in the ſame time that it would deſcend freely from* B *to* C. For if you imagine the inclined plane to be BH inſtead of BD; then by this propoſition, the body will deſcend either from B to F, or from B to C in the ſame time; and again, if you imagine the inclined plane to be BI, then by this propoſition, the body will deſcend either from B to G, or from B to C, in the ſame time. And, in ſhort, the ſame thing may be proved of any other chord of the ſemicircle.

Prop. IV. *The time of a body's deſcending along the whole length of an inclined plane, is to the time of its deſcending freely and perpendicularly along the altitude of the plane, as the length of the plane is to its altitude; or as the whole force of gravity is to that part of it which acts upon the plane.*

The ſpaces run over the plane being as the ſquares of the times, we have the ſquare of the time of paſſing over BD, fig. 4, Plate IV. to the ſquare of the

the time of paſſing over BE, as BD is to BE. But BD is to BC as BC is BE, viz. BD, BC, and BE are three lines in continuate geometrical proportion; therefore (Eucl. p. 20, B. VI.) BD is to BE, as the ſquare of BD is to the ſquare of BC. It has been ſhewn above, that the ſquare of the time of paſſing over BD, is to the ſquare of the time of paſſing over BE, as BD to BE; therefore thoſe ſquares of the times are to each other as the ſquare of BD to the ſquare of BC; and of courſe the ſquare roots of theſe four proportional quantities are likewiſe proportional (Eucl. p. 22, B. VI.) viz. the time of a body's deſcending from B to D is to the time of its deſcending freely and perpendicularly from B to E, or from B to C, as BD is to BC, or as the length of the plane is to its altitude; or (by the 1ſt propoſition of this chapter) as the whole force of gravity is to that part of it which acts upon the plane.

Prop. V. *A body by deſcending from a certain height to the ſame horizontal line, will acquire the ſame velocity whether the deſcent be made perpendicularly, or obliquely, over an inclined plane, or over many ſucceſſive inclined planes, or laſtly over a curve ſurface.*

1ſt. In page 64, it has been ſhewn, that the velocity of a body deſcending freely towards a centre of attraction, is as the product of the attractive force multiplied by the time; and by the preceding propoſition it has been proved, that on an inclined plane the force of gravity is diminiſhed in proportion as the time of the body's running down the

the whole length of the plane, is increaſed, viz. when the force of gravity is half as ſtrong as it would be in free ſpace, the time is doubled; and when the force is one-third as ſtrong, the time is trebled, &c. therefore the product of the time by the force is always the ſame; for $\frac{1}{2}$ multiplied by 2 is equal to $\frac{1}{3}$ multiplied by 3, is equal to $\frac{1}{4}$ multiplied by 4, &c. hence the velocity being as that product, muſt, of courſe, be always the ſame, or a conſtant quantity. For example, ſuppoſe, that when the body deſcends perpendicularly down from B to C, fig. 4, Plate IV. the whole force of gravity acts upon it. Let us call that whole force 1, and let the time employed by the body in coming down from B to C be one minute, then the velocity acquired by that deſcent is repreſented by the product of the time by the force, viz. 1 by 1, which makes one. Now when the body deſcends from the ſame altitude B, to the ſame horizontal line DC, over the inclined plane BD, the force of gravity which draws it downwards is diminiſhed; for inſtance, ſuppoſe it to act with a quarter of its original power, then the time of the body's deſcending from B to D will be four minutes, and the velocity acquired by that deſcent, being as the product of the force by the time, is as the product of $\frac{1}{4}$ by 4, which is one, or the ſame as when the body deſcends perpendicularly down from B to C.

2dly. Suppoſe that the body deſcends from the ſame altitude E to the ſame horizontal line DC, fig.

fig. 5, Plate IV. along the contiguous inclined planes EF, FG, GD; by the time it arrives at D it will have acquired the ſame velocity as if it had deſcended perpendicularly from B to C, or from E perpendicularly down to the horizontal line DC; for, by the firſt part of this propoſition, it will acquire the ſame velocity whether it deſcends from E to F or from K to F, and by adding to both the plane FG, it follows that the body will acquire the ſame velocity whether it deſcends along the ſingle plane KG, or along the two contiguous planes EF, FG. And by the like reaſoning it will be proved, that the body will acquire the ſame velocity whether it deſcends along the ſingle plane BD, or along the contiguous two planes KG, GD, or along the contiguous three planes EF, FG, GD, &c.

3dly. If the number of contiguous planes be ſuppoſed infinite, and their lengths infinitely ſmall, they will conſtitute a curve line, like BH; whence it follows, that a body by its deſcent along the curve line BH, or any other curve, will acquire the ſame velocity as if it deſcended perpendicularly from B to C.

Prop. VI. *Let a circle be perpendicular to the horizon, and if two chords be drawn from any two points in the circumference, to the point in which the circle touches the horizon; the velocities which are acquired by the deſcents of two bodies along thoſe chords, will be as the lengths of the chords reſpectively.*

It

It has been ſhewn by the preceding propoſition, that a body will acquire the ſame velocity whether it deſcends from B to D. fig. 6, Plate IV. or from E to D; D being the point of contact with the horizontal plane GI; and likewiſe the ſame velocity will be acquired by deſcending from C to D, or from F to D; ſo that the velocities, which are acquired by deſcending along thoſe chords, are reſpectively the ſame as the velocities acquired by deſcending perpendicularly from E and F to D. And (from what has been ſhewn in p. 65) thoſe velocities are as the ſquare roots of ED and FD. Now (Eucl. p. 8. B. VI.) AD is to DB as DB is to ED; therefore (Eucl. p. 20. B. IV.) AD is to ED, as the ſquare of AD is to the ſquare of DB, and, for the ſame reaſons, AD is to FD, as the ſquare of AD is to the ſquare of CD. Hence, alternately, AD is to the ſquare of AD, as ED is to the ſquare of BD; and AD is to the ſquare of AD, as FD is to the ſquare of CD; therefore ED is to the ſquare of BD, as FD is to the ſquare of CD; that is, alternately, ED is to FD as the ſquare of BD is to the ſquare of CD; and of courſe the ſquare root of ED is to the ſquare root of FD as BD is to CD, and as the velocity acquired by deſcending along BD is to the velocity acquired by deſcending along CD.

Prop. VII. *If there be two planes of unequal lengths, but equally inclined to the horizon, the times of deſcent along*

along the whole lengths of thoſe planes will be as the ſquare roots of their lengths reſpectively.

Let BD and EF, fig. 7, Plate IV. be two planes of unequal lengths, but equally inclined to the horizon; and it follows from prop. IV. of this chapter, that the time of deſcent along the plane BD is to the time of the perpendicular deſcent along BC, as BD is to BC; alſo that the time of deſcent along E F is to the time of deſcent along E C, as E F is to E C. The times of the perpendicular deſcents along B C and E C are as the reſpective ſquare roots of B C and EC (ſee page 65.) Now the triangles BDC and EFC being equiangular, and therefore ſimilar (Eucl. p. 4. B. VI.) we have BC to EC as BD to EF, and of courſe the ſquare root of BC is the ſquare root of EC, as the ſquare root of BD is to the ſquare root of EF; viz. as the time of deſcent along BD is to the time of deſcent along EF.

Cor. The ſame thing muſt be underſtood (as it may eaſily be derived from the above propoſition) of two or more contiguous planes ſimilarly ſituated, as B I D, E H F; and likewiſe of two curve ſurfaces that are ſimilar and ſimilarly ſituated; ſince thoſe curves may be conceived to conſiſt of an infinite number of planes ſimilarly ſituated.

Thus much will ſuffice for the preſent with reſpect to the properties of inclined planes, in which we we have ſuppoſed the bodies to be ſpherical, and the planes as well as the bodies to be perfectly ſmooth and not obſtructed, either by friction or by the reſiſtance of the air. We ſhall now explain the properties of pendulums or pendulous bodies; a pendulum being a body hanging at the end of a ſtring, like A, fig. 8, Plate IV. and moveable about a fixed *point of ſuſpenſion* C. A pendulum however may conſiſt of a ſingle body ſuſpended without any ſtring, ſuch as a rod of wood or other matter ſuſpended by one end, &c. but in the following propoſitions a pendulum muſt be underſtood to be according to the former definition, viz. a body ſuſpended at the end of a ſtring, &c. and the ſtring muſt be ſuppoſed to be void of weight, as alſo to move with perfect freedom about the point of ſuſpenſion, unleſs the contrary be mentioned.

Prop. VIII. *If a pendulum be moved to any diſtance from its natural and perpendicular direction, and there be let go, it will deſcend towards the perpendicular, then it will aſcend on the oppoſite ſide nearly as far from the perpendicular, as the place whence it began to deſcend; after which it will again deſcend towards the perpendicular, and thus it will keep moving backwards and forwards for a conſiderable time; and it would continue to move in that manner for ever, were it not for the reſiſtance of the air, and the friction at the point of ſuſpenſion, which always prevent its aſcending to the ſame height as that from which it laſtly began to deſcend.*

Thus

Thus the pendulum, fig. 9, Plate IV. being moved from the perpendicular direction CB to the ſituation AC, and there left to itſelf, will deſcend along the arch AB with an accelerated motion, in the ſame manner as if it deſcended over a curve ſurface AB; for it is evidently the ſame thing whether a body deſcends along ſuch a ſurface, or is confined by the ſtring CB, ſo as to deſcribe the ſame curve AB. By the time the body arrives at the loweſt point B, it will acquire the ſame velocity as if it had deſcended perpendicularly from E to B, (by Prop. V.) This velocity (if the retardation ariſing from the reſiſtance of the air and the friction be removed) will carry it beyond the point B with a retarded motion in an equal portion of time, as far as D (ſee page 71) viz. as far from B as A is from B. It will then deſcend again with an accelerated motion towards B, and ſo on. For ſince the velocity of the pendulum in its aſcent is retarded by the ſame uniformly acting power, which accelerates it in its deſcent, namely, by the force of gravity, there muſt be the ſame time employed in deſtroying as in generating any momentum.

It likewiſe follows from this conſideration, that the weight of the pendulum cannot alter its time of deſcent or aſcent; for it has been ſhewn above, that bodies of different weights will move through equal ſpaces in equal times, towards a centre of attraction, provided the attractive force be the ſame. And the motion of a pendulum is evidently owing to the gravitating power.

The whole motion of the pendulum one way is called a *vibration* or *oſcillation*. Thus the motion of the pendulum from A to D is one vibration; from D to A is another vibration, and ſo on. The body which hangs by the ſtring, is commonly called the *bob* of the pendulum.

This property of the pendulum is fully confirmed by a variety of experiments. A pendulum, if once moved out of its perpendicular ſituation, and then left to itſelf, will move forwards and backwards for a conſiderable time (in ſome caſes, for many hours); but every vibration will be a little ſhorter than the preceding, until at laſt the pendulum will entirely ceaſe to move. That this gradual retardation is entirely owing to the reſiſtance of the air, and to the friction at the point of ſuſpenſion, is proved by obſerving that the ſame pendulum has been found to continue its vibrations longer and longer, in proportion as thoſe cauſes of obſtruction have been diminiſhed; hence we conclude, that if thoſe cauſes could be entirely removed, the pendulum would continue to vibrate for ever.

Prop. IX. *The velocity of a pendulum in its loweſt point is as the chord of the arch which it has deſcribed in its deſcent.*

Thus if there be two pendulums of equal lengths, as CF and CA, fig. 10, Plate IV. and the former of them deſcends from F, whilſt the latter deſcends from

rom A; then at the loweſt point B the velocity of the former will be to the velocity of the latter, as the chord or ſtraight line FB is to the chord or ſtraight line AB, or as the velocities acquired by the perpendicular deſcents GB EB; which is an evident application of the propoſitions V. and VI. of this Chapter.

Prop. X. *The very ſmall vibrations of the ſame pendulum are performed in times nearly equal; but the vibrations through longer and unequal arches are performed in times ſenſibly different.*

It is evident (from cor. to prop. III.) that if the pendulous body, inſtead of vibrating along circular arches, could move along the chords of thoſe arches, the ſemi-vibrations, whether long or ſhort, would be all performed in equal times; viz. each in the time that a body would employ in deſcending perpendicularly along the diameter of the circle, or twice the length of the pendulum. For inſtance, in fig. 10, Plate IV. the pendulous body would deſcend from F to B or from A to B along the chords or ſtraight lines FB or AB, exactly in the ſame time, viz. the time it would employ in the perpendicular deſcent from H to B; and ſince the deſcent from A to B, or from F to B, is half a vibration, therefore each whole vibration would be performed in twice that time.

But ſince the body vibrates not along the chords but along the arches, therefore the unequal vibrations cannot be performed in equal times (ſee prop. IV.);

IV.); yet in very ſmall arches the chords are nearly equal to the arches that are ſubtended by them, (ſee prop. I. of the note in p. 139.) therefore the vibrations along very ſmall arches, though of unequal lengths, are performed in times nearly equal.

Prop. XI. *As the diameter of a circle is to its circumference, ſo is the time of a heavy body's deſcent from reſt through half the length of a pendulum to the time of one of the ſmalleſt vibrations of that pendulum.*

The demonſtration of this propoſition depends upon certain difficult mathematical principles; we ſhall therefore ſubjoin it by way of a note, for the information of thoſe who are qualified to read it; and ſhall now proceed to ſhew the uſe of this curious propoſition by means of examples (1).

This

(1) The analogy which is announced in the above propoſition, is deduced from the properties of a curve called the *cycloid.* It is therefore neceſſary, in the firſt place, to ſhew the nature and principal properties of that curve, from which the above mentioned analogy may afterwards be derived.

If a circle, as AB, fig. 11, Plate IV. reſting on a right line AL, touches it in a point A; and if this circle be rolled along the ſaid line, until the ſame point A in the circumference, which firſt touched the line AL, comes again in contact with it in another point L; or till the circle AB, by rolling along the line AL, has performed a whole revolution; then the point A will, by its two-fold motion, deſcribe the curve ACDIL, which is called a *cycloid.*

The circle ABC is called the *generating circle*, AL is the *baſe*, and DF, erected perpendicularly in the middle of the

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This propoſition ſhews the proportionality of four quantities; viz. the diameter of a circle, its circumference, the time which a heavy body employs in falling from reſt through a certain ſpace, and

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the baſe, and extended from the baſe to the curve, is the *axis* of the cycloid.—ABC, DGF, HIK, repreſent the generating circle in different ſituations.

From this generation of the cycloid, the following particles are obviouſly derived.

I. *The baſe AL is equal to the circumference of the generating circle.*

II. *The axis DF is equal to the diameter of the generating circle.*

III. *The part KL of the baſe, viz. the part between one extremity of it and the place which touches the generating circle in any ſituation of it, is equal to the correſponding arch IK, or GF, of the generating circle; the ordinate IE being parallel to the baſe.*

IV. *FK, or its equal ME, is equal to the remaining arch IH, or GD.*

V. *The chord IK is perpendicular to the curve at I.*

VI. *The chord IH, being perpendicular to IK, (for the angle HIK in the ſemicircle is a right angle) is a tangent to the curve at the point I.*

VII. *The tangent IH of the curve at I, or chord of the circular arch HI, is equal and parallel to the chord DG. Alſo IK is equal and parallel to FG.*

VIII. *The length of the ſemicycloid DIL is equal to twice the diameter DF of the generating circle; and any cycloidal arch ID, cut off by a line IE parallel to the baſe, is equal to* and the time of a ſmall oſcillation of a pendulum, whoſe length is equal to twice that ſpace.

It is very well known that the diameter of a circle is to its circumference, as one is to 3,1415 nearly;

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twice the chord DG of the correſponding circular arch DG, which is cut off by the ſame line IE.

Draw PT indefinitely near and parallel to IE, which will cut the circle DGF in Q. Join DQ produce DG to meet TP in S; from Q draw QO perpendicular to DS, and draw GR, a tangent to the circle at G, and RD a tangent at D. Then, ſince PT is indefinitely near to EI; GS is equal to the increment IT of the curve, whilſt GO is the increment of the chord DG; for DQ being nearly equal to DO, muſt exceed DG by the increment, or additional part GO. And this increment or addition to the chord has been made at the ſame time that the curve DI has been increaſed of the part IT, equal to GS.

Now the triangles DRG, GQS, being ſimilar (ſince DR is parallel to QS, and the angles at the vertex G are equal), and DR being equal to RG, QS muſt be equal to QG; hence GO is likewiſe equal to OS, and of courſe GS is equal to twice GO; but GS is equal to the increment of the curve, and GO is equal to the contemporaneous increment of the chord DG; therefore the increment of the curve is equal to twice the increment of the chord. And as this reaſoning is applicable to any point of the curve from D to L, therefore we conclude, that ſince from the upper point D to the loweſt L, the curve increaſes twice as faſt as the correſponding chord of the circle DGF, therefore any arch DI of the curve is equal to twice the correſponding chord DG; and at L where the correſponding chord nearly; therefore if one of the other particulars be known, we may find out the fourth by means of the common rule of three.

Example 1st. The time in which a body will descend from rest through 16,087 feet, viz. (one second)

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chord is DF, the curve or semicycloid DIL is equal to twice DF, viz. twice the diameter of the generating circle.

IX. *CA, CB, fig. 12, Plate IV. represent two equal semicycloidal cheeks set contiguous to each other with their bases CE, CK, in the same direction. BDA is an inverted cycloid equal to the cycloid of which CA or CB is the half, and its base reaches from the vertex B of one semicycloid to the vertex A of the other. At C suspend a pendulum CLI, whose length is equal to one of the semicycloids. As this pendulum vibrates in the plane of the cycloids, its string will apply itself first to one and then to the other of those cheeks, by which means the end I of the pendulum will move precisely in the curve BDA; viz. in a cycloid.*

It is evident from the construction, that BA is the base of the cycloid BDA; that BF = FA = CE = CK, and that CD = CLI = CLB = BID = twice the diameter of the generating circle FGD, or EHB.

In any situation of the pendulum, as CLI draw LH through the point where the contact between the string of the pendulum and the cycloidal cheek terminates, and draw IY through the end of the pendulum, both parallel to the base BA; and join the points B, H, G, F, with the lines BH, and GF.

Since CLB is equal to CLI the disengaged part LI of the string must be equal to LB, and of course equal to twice ſecond) being given, to find the time in which a pendulum of twice that length (viz. of 32,174 feet) will perform one of its leaſt vibrations.

Here we have $1 : 3{,}1415 :: 1'' :$ to a fourth proportional, viz. to $3'',1415$, which is the time in which

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twice the chord HB (by § VIII). But BH is equal and parallel to the tangent LM (by §. VII); therefore HB is equal to ML, and conſequently LM is equal to MI; hence the parallels HL, IG are equidiſtant from the baſe BA, and cut off equal arches HZB, FSG, from the generating ſemicircles; therefore the chord FG is equal and parallel to the chord HB, and to MI. Alſo MF is equal to IG, thoſe lines being the oppoſite ſides of a parallelogram. Now as BM is equal to HL, and (by § IV.) equal to the arch HZB, or to the arch FSG; the remainder MF, equal to IG, will be equal to the remaining part GD of the ſemicircle; which proves that the extremity I of the pendulum is always in the cycloidal curve ADB.

For the ſake of brevity we ſhall call the pendulum which vibrates in a cycloid, *a cycloidal pendulum.*

X. *The velocity of a cycloidal pendulum in its loweſt point is proportional to the ſpace paſſed through; viz. to the arch of the cycloid which the pendulum has deſcribed in its deſcent.*

Thus in fig. 12, Plate IV. If the pendulum begin to deſcend from I; at D, its velocity will be as the arch ID; and if it begin to deſcend from B, then when it arrives at the loweſt point D, its velocity will be as the arch BID; which we are now going to prove.

It has been ſhewn in chap. X. prop V. that a body will acquire the ſame velocity whether it deſcends obliquely from which the pendulum of 32,174 feet will perform each of its very ſmall vibrations; viz. little more than three ſeconds.

Example 2. The time in which a body will deſcend from reſt through 16,087 feet (viz. one ſecond) being given, to find the time in which a pendulum of four feet will perform one of its leaſt vibrations.

Here

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from I to D, or perpendicularly from Y to D. Alſo the ſquare of the velocity of a falling body is as the ſpace paſſed through, or the velocity is as the ſquare root of the ſpace; therefore the velocity acquired by the pendulum I, in its deſcent from I to D, is as the ſquare root of YD, viz. as $\sqrt{YD}$. But (Eucl. p. 8. B. VI.) DY : DG : : DG : FD; therefore $DY = \frac{DG \times DG}{FD}$. Now FD being an invariable quantity, DY muſt increaſe or decreaſe according as the ſquare of DG increaſes or decreaſes; or the ſquare root of DY (viz. the velocity in queſtion) is as DG, which is equal to half the cycloidal arch DI; hence the velocity is as the cycloidal arch.

XI. *All the vibrations of a cycloidal pendulum, whether long or ſhort, are performed in equal times.*

In all ſorts of motion, as we have abundantly ſhewn, the ſpace is as the product of the time multiplied by the velocity; viz. S is as TV, which gives the following analogy, S : V : : T : 1. But it has been juſt ſhewn, that in the caſe of a cycloidal vibration, the ſpace is as the velocity; therefore the time muſt be as unity, or always the ſame.

 XII. *If*

Here half the length of the pendulum is 2 feet; therefore in the firſt place we muſt find out what time a body will employ in deſcending from reſt through 2 feet; and ſince the ſpaces paſſed over by deſcending bodies, are as the ſquares of the times, (ſee page 65) we ſay as 16,087 feet are to two feet, ſo is the ſquare of one ſecond to the ſquare of the time ſought; viz. 16,087 : 2 : : 1″ : $\left(\frac{1 \times 2}{16,087} = \right)$ 0,1243, the ſquare root of which, viz. 0″,352, is the time of a body's deſcent through 2 feet.

XII. *If a cycloidal pendulum begin to deſcend from any point L, fig. 13, Plate IV. towards the vertex V; its velocity at any point M (viz. the velocity acquired by deſcending from L to M) will be as the ſquare root of the difference of the ſquares of the two arches VL, and VM* $\left(\text{viz. as } \overline{\overline{VL}|^2 - \overline{VM}|^2}\right|^{\frac{1}{2}}\Big)$; *or it will be as the ſine of a circular arch whoſe radius is equal to VL, and whoſe coſine is equal to VM.*

Through the points L and M draw LR, MS, parallel to the baſe AB, which lines will cut the generating circle in O and Q. And draw the chords VO, VQ.

The velocity of the pendulum at the point M, after a deſcent from L, is equal to the velocity that would be acquired by a body deſcending perpendicularly from R to S (by prop. V. of this chap.); and this velocity is as the ſquare root of the ſpace RS; or as $\overline{RV - SV}|^{\frac{1}{2}}$; or as $\overline{\overline{VO}|^2 - \overline{VQ}|^2}|^{\frac{1}{2}}$; or, laſtly, as $\overline{\overline{VL}|^2 - \overline{VM}|^2}|^{\frac{1}{2}}$. (See the demonſtration of the propoſition laſt but one.)

Produce

2 feet. This time being found, we then ſay, after the manner of the preceding example; 1 : 3,1415 : : 0″,352 : $\left(\frac{3{,}1415 \times 0'',352}{1} =\right)$ 1″,1 nearly the time in which a pendulum of 4 feet performs its leaſt vibrations.

Example 3. The time in which a pendulum of 39,1196 inches performs each of its ſmall vibrations (viz. one ſecond) being given, to find the ſpace through which a body will deſcend from reſt in the ſame time.

Firſt

Produce the axis DV towards Z; at V erect VL perpendicular to DZ, and equal to the length of the cycloidal arch VML. Let the lengths VM, VL, in the ſtraight line VL, be made reſpectively equal to the lengths VM, VL, of the cycloidal arch. With the centre V and radius VL, draw the ſemicircle LZP. At M on the radius erect MX perpendicular to it, which will meet the circumference at X, and laſtly join VX.

Then MX is the ſine of a circular arch, whoſe radius is VX or VL, which is equal to the cycloidal arch VL, and whoſe coſine is VM, which is equal to the cycloidal arch VM. (By Eucl. p. 47. B. I.) MX is equal to $\overline{\overline{VX}^2 - \overline{VM}^2}|^{\frac{1}{2}}$, or to $\overline{\overline{VL}^2 - \overline{VM}^2}|$.

XIII. *If when the pendulum begins to deſcend from L along the cycloid, another body be ſuppoſed to move in the ſemicircle LZP from L towards Z with a uniform velocity, equal to the pendulum's greateſt velocity; (viz. that which the pendulum acquires by deſcending from L to the vertex V;)*

then

First we say 3,1415 : 1 : : 1″ : 0″,3183, the time in which a body will descend through a space equal to half the length of the pendulum, viz. through 19,5598 inches.

Then,

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then any circular arch XY will be described by the above-mentioned body with that uniform velocity, in the same time that the cycloidal arch which is intercepted between the two corresponding points M and N, is run over by the pendulum with its usual accelerated velocity.

Draw the line *mx* parallel, and indefinitely near, to the sine MX. Through X draw X*r* parallel to the radius VL; and in the cycloidal arch take M*m* equal to M*m* in the radius.

The arch X*x*, being indefinitely small, may be considered as a right line. Then the right angled triangles V M X, X *x r*, being similar, (because the angles *r* X *x* and M X V are equal, for each of them is the complement of V X *r* to a right angle), we have M X : VX (or VZ, or VL) : : X *r* (or M *m*) : X *x*.

Now the velocity of the pendulum at M (by § XII. of this note) is as MX; therefore the extremely small space M *m* in the arch, may without error be supposed to be described with that velocity. Also (by § X. of this note) the greatest velocity acquired by the pendulum in its descent from L to V, is as the arch LV, or as its equal, the radius LV, and is the same velocity with which the circular arch is equably described; therefore, the analogy of the preceding paragraph is, by substitution, converted into the following: viz. as the velocity with which the circular arch X *x* is described, is to the velocity with which the small cycloidal arch M *m* is described by the pendulum, as X *x* is to

Then, ſince the ſpaces deſcribed by deſcending bodies, are as the ſquares of the times, we ſay, as the ſquare of 0″,3183 is to the ſquare of one ſecond, ſo are 19,5598 inches to the ſpace through which a body will deſcend in one ſecond; viz. 0,10131489 : 1 :: 19,5598 : 193,06 inches, or 16,083 feet; the ſpace through which a body will deſcend from reſt in one ſecond.

In

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to M*m*; ſo that thoſe ſmall lines are as the velocities with which they are deſcribed. But when the ſpaces are as the velocities, the times muſt be equal; therefore, the circular arch X*x* is deſcribed in the ſame time that the correſponding cycloidal arch M*m* is deſcribed by the pendulum. Now as the ſame thing may be ſaid of all other correſponding parts between X and Y, and M and N; therefore the whole circular arch XY is deſcribed in the ſame time in which the correſponding cycloidal arch MN is deſcribed. Hence the whole cycloidal arch LV, and quadrant LZ, are deſcribed in the ſame time.

XIV. *The time of a complete oſcillation of a cycloidal pendulum, is to the time in which a body would deſcend perpendicularly along the axis of the ſame cycloid, as the circumference of a circle is to its diameter.*

In the firſt place, it is evident that the time in which the ſemicircle LZP is deſcribed in the manner mentioned above, is to the time in which the radius LV could be deſcribed with the ſame equable velocity, as the circumference of a circle is to its diameter. But the time in which the ſemicircle LZP is deſcribed, is equal to the time in which the pendulum will make a complete cycloidal oſcillation from L

to

In the preceding examples the calculations have not been carried on to a great number of decimals, purpofely to avoid prolixity; the object being only to fhew the method of performing the calculations; but in many cafes it will be neceffary to extend the operation to a greater degree of accuracy. It is likewife neceffary that the reader be informed of the real length of the pendulum, which vibrates feconds,

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to P. And the time in which LV (or its equal twice OV) could be defcribed with that fame velocity with which the circle is defcribed, is equal to the time of defcent along the chord OV, or along the axis DV (fee chap. V. and prop. VI. of this chap.); therefore, the above-mentioned analogy is, by fubftitution, converted into the following. The time of a complete cycloidal ofcillation, is to the time in which a body would defcend perpendicularly along the axis of the fame cycloid, as the circumference of a circle is to its diameter.

This propofition evidently confirms prop. the 11th of this note; for fince every cycloidal vibration is in the fame ratio to the time of defcent through the axis, as the invariable ratio of the circumference of a circle to its diameter, they muft be all performed in the fame time.

The very fmall vibrations of a common circular pendulum, may without any fenfible error be fuppofed to follow the fame laws as thofe of the cycloidal pendulum; for near the vertex V, that is, when the arch of vibration does not exceed two or three degrees, the curvature of the cycloid coincides with the curvature of a circle whofe radius is equal to CV. viz. the length of the pendulum. This is evidently ſeconds, or of the real ſpace which is paſſed over by deſcending bodies in a given time; (ſince the one may be eaſily deduced from the other) in order that he may ground his calculations on as accurate a foundation, as the preſent ſtate of knowledge can admit of.

In different parts of the world, the pendulum which vibrates ſeconds, is not of the ſame length. It

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evidently ſhewn by the figure itſelf; for when the pendulum vibrates not far from the perpendicular CV, its ſtring does hardly touch the cycloidal cheeks CA, CB; and of courſe its extremity V. muſt deſcribe a circular arch very nearly.

XV. *The times of ſimilar oſcillations of different pendulums, when the force of gravity is ſuppoſed to vary, are as the ſquare roots of the lengths of the reſpective pendulums directly, and as the ſquare roots of the reſpective gravitating forces inverſely.*

By the preceding propoſition the time of a cycloidal vibration is to the time of perpendicular deſcent along the axis, in an invariable ratio; that is, the former is as the latter. Now the time of that perpendicular deſcent is directly as the ſquare root of the axis (or of its double, viz. the length of the pendulum), and inverſely as the ſquare root of the force of gravity; for when the gravitating force is invariable, the time of perpendicular deſcent has been ſhewn to be as the ſquare root of the ſpace; and when the time is invariable, (viz. in the ſame time) the ſquare root of the ſpace has been ſhewn to be as the ſquare root of the velocity, or of the gravitating force; therefore when they are both variable, the ſquare root of the ſpace or length is as the time multiplied

It is a little longer on places that are ſituated nearer to the poles, and ſhorter in ſituations that are nearer to the equator, (the reaſon of which will be ſhewn hereafter). This difference, however, is known with ſufficient accuracy. But moſt philoſophical writers differ with reſpect to the length of the pendulum which vibrates ſeconds in the ſame latitude; and of courſe with reſpect to the real

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multiplied by the ſquare root of the gravitating force; or the time is as the ſquare root of the ſpace divided by the ſquare root of the gravitating force; that is, as the ſquare root of the length of the pendulum directly, and the ſquare root of the force of gravity inverſely.

Independently of the cycloid, the time of any circular oſcillation may be found out by means of the following problem, which is given by Profeſſor Saunderſon in his Method of Fluxions.

XVI. *To find the exact time of one of the leaſt oſcillations of a given pendulum ſwinging in an arch of a circle; and to find, without any ſenſible error, alſo the time of any other oſcillation.*

Let a pendulum ND, fig. 14, Plate IV. vibrate in the arch ADC of a circle whoſe diameter is ID; and ſuppoſe it to be at the point E in its aſcent from D to C. Let $\sqrt{BF}$ expreſs the velocity acquired by a heavy body in falling from B to F, (AC,EE, being the parallel chords of the arches ADC, EDE, which interſect the diameter in B and F), and conſequently the velocity of the pendulum at the point E. Now $\frac{1}{2}\sqrt{ID}$ expreſſes the velocity acquired by deſcending through $\frac{1}{4}$ ID, and ſince a body with that velocity would real ſpace which is paſſed over by deſcending bodies in a given time.—In the works of the moſt eminent philoſophers of this country, I find the length of the pendulum, which vibrates ſeconds in or near London, ſtated differently as follows: inches 39,2; 39,14; 39,128; 39,125; 39,1196, &c.

The

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would deſcribe uniformly a ſpace equal to $\frac{1}{2}$ ID in the ſame time in which it would fall through $\frac{1}{4}$ ID. Divide the ſpace $\frac{1}{2}$ ID by the velocity $\frac{1}{2}\sqrt{ID}$, and the quotient $\sqrt{ID}$, expreſſes the time wherein a heavy body would fall through a $\frac{1}{4}$ ID; viz. through half the length of the pendulum.

Draw *ee* indefinitely near to EE; then E*e* may be conſidered as the fluxion of the arch DE; and $\frac{Ee}{\sqrt{BF}}$ will expreſs the time wherein the ſmall arch E*e* is deſcribed by the pendulum, or the fluxion of the time of a vibration. But $Ee = \frac{\frac{1}{2}ID \times Ff}{\sqrt{IF \times FD}}$ (for, calling the radius ND, r; FE, y; and FD, x; we ſhall have $E e = \overline{\dot{x}^2+\dot{y}^2}|^{\frac{1}{2}}$. But $y^2 = 2rx - xx$, whoſe fluxion is $2y\dot{y} = 2r\dot{x} - 2x\dot{x}$; hence $\dot{y} = \frac{r\dot{x} - x\dot{x}}{y}$; or $\dot{y}^2 = \frac{r^2\dot{x}^2 - 2rx\dot{x}^2 + x^2\dot{x}^2}{y^2} = \frac{r^2\dot{x}^2 - y^2\dot{x}^2}{y^2} = \frac{r^2\dot{x}^2}{y^2} - \dot{x}^2$. Therefore $\dot{x}^2 + \dot{y}^2 = \frac{r^2\dot{x}^2}{y^2}$, and $\overline{\dot{x}^2+\dot{y}^2}|^{\frac{1}{2}} = \frac{r\dot{x}}{y} = \frac{r\dot{x}}{\sqrt{2rx - xx}} = \frac{\frac{1}{2}ID \times Ff}{\sqrt{IF \times FD}} = Ee$) $= \frac{\sqrt{ID}}{\sqrt{IF}} \times \sqrt{ID} \times \frac{\frac{1}{2}Ff}{\sqrt{FD}}$; therefore, $\frac{Ee}{\sqrt{BF}} = \frac{\sqrt{ID}}{\sqrt{IF}} \times \sqrt{ID} \times \frac{\frac{1}{2}Ff}{\sqrt{BF \times FD}}$. Biſect DB

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The late Mr. John Whitehurst, an ingenious member of the Royal Society, seems to have contrived

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in K, and KD in L; and when the arch ADC is small, the quantity IF cannot differ sensibly from IK, nor $\frac{\sqrt{ID}}{\sqrt{IF}}$ from $\frac{IL}{IK}$. Therefore $\frac{Ee}{\sqrt{BF}}$ is very nearly equal to $\frac{IL}{IK}$ $\times \sqrt{ID} \times \frac{\frac{1}{2} Ff}{\sqrt{BF \times FD}}$.

Upon the diameter BD describe the circle BGDG, cutting the chords EE, *ee* in G and *g*; then will the fluxion of the arch DG be $Gg = \frac{\frac{1}{2} BD \times Ff}{\sqrt{FB \times FD}}$; consequently $\frac{Gg}{BD}$ $= \frac{\frac{1}{2} Ff}{\sqrt{FB \times FD}}$; and therefore the fluxion of the time of vibration through DE will be $\frac{Ee}{\sqrt{BF}} = \frac{IL}{IK} \times \sqrt{ID} \times$ $\frac{Gg}{BD}$; which in fact is the time of the pendulum's moving from E to *e*.

But the fluent of this last fluxion is $\frac{IL}{IK} \times \sqrt{ID} \times$ $\frac{DGB}{BD}$; this, therefore, is the time of half a vibration or motion of the pendulum from D to C. And the time of a whole vibration through the arch ADC is $\frac{IL}{IK} \times \sqrt{ID} \times$ $\frac{BGDGB}{BD}$.

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trived and performed the leaſt exceptionable experiments relatively to this ſubject. The reſult of his

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When the arch of vibration is indefinitely ſmall, the quantity $\frac{IL}{IK}$ becomes $= 1$; and the time (T) of one of the leaſt vibrations, will become $T = \sqrt{ID} \times \frac{BGDGB}{BD}$; and therefore $BD : BGDGB :: \sqrt{ID} : T$; that is, as the diameter of a circle is to its circumference, ſo is the time ($\sqrt{ID}$) of the deſcent through half the length of the pendulum, to the time of one of the leaſt oſcillations of the pendulum; which is the ſame analogy as was derived from the properties of the cycloid. Wherefore the time of oſcillation in a cycloid, and in an indefinitely ſmall arch of a circle, is the ſame, viz. $T = 1$ ſecond, when the length of the pendulum is 39,1196 inches; as has been proved experimentally.

Therefore, the time of an oſcillation in a circular arch in general, is $T \times \frac{IL}{IK}$, or (ſince $IL = IK + KL$) the general expreſſion of the time of vibration through any arch ADC of a circle will be $T + T \times \frac{KL}{IK}$. And $T \times \frac{KL}{IK}$ is the exceſs of the time of vibration in a circular arch, above the time of vibration in the arch of a cycloid, or above the time of the leaſt circular oſcillation; the lengths of the pendulums being equal.

In order to adapt the preceding expreſſions to the practical calculation, it is neceſſary to obſerve that BD is the verſed ſine of CD, viz. of half the arch of vibration. DK is the half of that verſed ſine; KL is a quarter of it; and ID is his experiments ſhews, that the length of the pendulum which vibrates ſeconds in London, at 113 feet

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twice the length of the pendulum; hence if we call the verſed ſine of half the arch of vibration a, and call the length of the pendulum b; then the above ſtated expreſſion $T + T \times \frac{KL}{IK}$ will become $T + T \times \frac{\frac{1}{4}a}{2b - \frac{1}{2}a}$, or $T + T \times \frac{a}{8b - 2a}$.

Example 1. Suppoſe it be required to find the time in which a pendulum, that performs each of its ſmalleſt vibrations in one ſecond, will perform its vibrations in an arch of 120°.

In this caſe the length of the pendulum is $b = 39{,}1196$; the ſemiarch of vibration is 60°; and its verſed ſine (which is taken from the trigonometrical tables, and is reduced in the proportion of the tabular radius to the length of the pendulum; by ſaying, as the tabular radius is to the length of the pendulum, ſo is the tabular verſed ſine to the verſed ſine in queſtion) is $19{,}5598 = a$; therefore the time ſought is $T + T \times \frac{a}{8b - 2a} = 1'' + 1'' \times \frac{19{,}5598}{273{,}8372} = 1''{,}0714$; then if the number of ſeconds in 24 hours, viz. 86400″ be divided by the time of one vibration laſt found; viz. by 1″,0714, the quotient 80735 is the number of vibrations which the pendulum will perform in 24 hours, when it vibrates along the arch of 120°; whereas when the ſame pendulum performs very ſmall vibrations, it will vibrate exactly ſeconds, viz. it will perform 86400 vibrations in 24 hours.

Example 2. Suppoſe it be required to find the time of one vibration, when the above-mentioned pendulum vibrates through feet above the level of the sea, in the temperature of 60° of Fahrenheit's thermometer, and when the barometer

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through a semicircle. In this case the versed sine is equal to the radius, or to the length of the pendulum, viz. $a = b$; consequently the expression $T + T \times \frac{a}{8b - 2a}$ becomes $1'' + 1'' \times \frac{a}{8b - 2b} = 1'' + \frac{1''}{6} = 1'',166666$; so that the time of a small vibration of the pendulum, whose length is 39,1196 inches, is to the time of one of its vibrations along a semicircle as 1 is to 1,16666, which is nearly in the proportion of 6 to 7.

We shall conclude this long note with the demonstration of another curious property of the cycloid.

XVII. *If two points be given in a vertical plane, but not both in the same line perpendicular to the horizon, a body will descend from the upper point to the lower in the shortest time possible, if it be caused to move along the arch of a cycloid, which passes through those points, and whose base is an horizontal line that passes through the upper point.*

Thus if the two points be A and B, fig. 15, Plate IV. and it be required that a body should descend from A to B in the shortest time possible; this object will be obtained by causing the body to descend not along the straight line AB, as it might at first sight be imagined, nor along an arch of a circle, or other curve; but along the cycloid ADB, which passes through the given points A and B, and whose base is the horizontal line AO.—On account of this remarkable property, the cycloid is called *the line of swiftest descent.*—We shall divide the demonstration of this property into three parts.

barometer is at 30 inches, is 39,1196 inches; whence it follows that the ſpace which is paſſed over by bodies

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1. If a certain line, as ACDB, be the line of ſwifteſt deſcent between two points A and B; it follows that a body, after its deſcent from A as far as C, will continue to deſcend quicker along the ſame line from C to D, than along any other line, as CED; for if this be denied, then it muſt be admitted that the body will deſcend faſter along the line ACEDB, than along the line ACFDB; conſequently the line ACFDB is not the line of ſwifteſt deſcent, which is contrary to the hypotheſis.

2. Let ADGB, fig. 16, Plate IV. be a curve between the two given points A and B; let DE, EG, be two indefinitely ſmall and contiguous portions of it. Through the points D, E, and G, draw DL, EO, GP, perpendicular to the baſe AC; and through D draw DH parallel to the baſe. Now if this curve be ſuch that the velocity with which the indefinitely ſmall portion D E is paſſed over by a body after its deſcent from A to D, be always proportional to $\frac{DH \times a}{DE}$ (*a* being a certain invariable line or quantity); then the body after its deſcent from A to D, will deſcend along the curve from D to G in leſs time than along any other way DFG; and of courſe this curve will be the line of ſwifteſt deſcent.

Through F draw FQ parallel to EG, and let FQ be ſuppoſed to be paſſed over with the ſame velocity as EG; draw FN perpendicular to DE, as alſo ME and GQ perpendicular to FQ, then the triangle FNE being ſimilar to DEH, as alſo FME ſimilar to GEI, we have DE : DH : : FE : $NE = \frac{DH \times FE}{DE}$; and GE : EI : : FE : $FM = \frac{EI \times FE}{GE}$.

Hence

bodies defcending perpendicularly, in the firft fecond of time, is 16,087 feet.—This length of a *fecond*

Hence $NE : FM :: \frac{DH \times FE}{DE} : \frac{EI \times FE}{GE} :: \frac{DH}{DE} : \frac{EI}{GE} :: \frac{DH \times a}{DE} : \frac{EI \times a}{GE}$; viz. NE is to FM as the velocity with which NE is paffed over to the velocity with which FM is paffed over: whence NE, FM, are paffed over in equal times. And fince MQ is equal to EG, the time of defcent through MQ will be equal to the time of defcent through EG; fo that the time of defcent through FQ will be equal to the time of defcent through NEG. But fince the angle MQG is a right one, FG is greater than FQ; fo that the time through FG will be greater than the time through FQ, or through NEG; and fince DF is greater than DN, the time through DF will be greater than the time through DN. Whence the time of defcent along DF, FG, will be greater than the time of defcent along DN, NG. A heavy body therefore, after its fall from A to D, will defcend from D to G along the curve DEG, in lefs time than along any other line; confequently the curve ADEGB is the line of fwifteft defcent between the points A and B.

3. Let ADEM, fig. 17, Plate IV. be a cycloid whofe bafe is the horizontal line AG. Through any point D in it draw DQ parallel to the bafe AG, and cutting the generating circle at N and the axis at Q. Draw the chords GN, NM; through D draw DL perpendicular to the bafe; and draw OE indefinitely near and parallel to LD. Now the indefinitely fmall part DE of the curve may be confidered as a right line coinciding with the tangent at D, and

second pendulum is certainly not mathematically exact, yet it may be considered as such for all common purposes; for it is not likely to differ from the truth by more than $\frac{1}{1000}$th part of an inch.*

XII. *The*

it may likewise be supposed to be described by a body descending from A, with the same velocity which the body has acquired by its descent from A to D; for the acceleration of velocity through that indefinitely small space may be considered as next to nothing. Now we shall prove that this cycloid has the property of the above-mentioned curve, viz. that the velocity with which the small portion DE is described by a body falling from A, is always proportional to $\frac{DH \times a}{DE}$; (a denoting the axis GM of the cycloid).

From the above-mentioned properties of the cycloid, the small line DE, coinciding with the tangent at D, is parallel to the chord NM. Whence the triangles DHE, NQM, and GMN, are equiangular and of course similar; therefore $DE : DH :: GM\ (=a) : GN = \frac{DH \times a}{DE}$. But GN is as the volocity which is acquired by the heavy body in its descent from G to Q, or from L to D; viz. as the velocity with which the indefinitely small line DE is passed over; therefore the cycloid, having the property of the above-mentioned curve, is the line of swiftest descent, &c.

* See Mr. Whitehurst's attempt towards obtaining invariable measures of length, capacity, and weight. Also Sir George Shuckburg Evelyn's excellent paper on the standard of weight and measure, in the Philosophical Transactions for the year 1798.

XII. *The times in which ſimilar vibrations (viz. vibrations through arches of the ſame number of degrees) of different pendulums are performed, are as the ſquare roots of the lengths of the pendulums.*

Thus if the pendulum AB, fig. 18, Plate IV. be four times as long as the pendulum CD, then the time of a vibration of the former will be double the time of a ſimilar vibration of the latter. For (by cor. to prop. VII. of this chap.) the vibrations, and of courſe the ſemivibrations, being ſimilar and ſimilarly ſituated, the time of the pendulum's deſcent along the arch GB is to the time of the other pendulum's deſcent along the arch HD, as the ſquare root of GB is to the ſquare root of HD. But the circumferences of circles, or ſimilar portions of the circumferences, are as their radii; therefore the ſquare roots of ſimilar portions of the circumferences are as the ſquare roots of the radii; conſequently the times of ſimilar vibrations are as the ſquare roots of the radii, or of the lengths of the pendulums.

Throughout the preſent chapter the force of gravity has been ſuppoſed invariable; but when that is not the caſe, as for inſtance, when a pendulum, which vibrates near the ſurface of the earth, is compared with a pendulum on the top of a very high mountain, or with a pendulum which vibrates on an inclined plane; in which caſes the action of the gravitating force on the pendulum's is not the ſame, then *the time of vibration is as the quotient of the* *square root of the length of the pendulum divided by the square root of the gravitating force.*

This propofition will be found demonftrated in the note.

CHAPTER XI.

OF THE CENTRE OF OSCILLATION, AND CENTRE OF PERCUSSION.

THE attentive reader muft undoubtedly have remarked, that though in the preceding chapter much has been faid with refpect to the length of the pendulum, yet no mention has been made of the point from which that length, or diftance from the point of fufpenfion, fhould be meafured. The reafon of this omiffion is, that the determination of that point, which is called *the centre of ofcillation*, requires a very particular confideration; fuch indeed as could not without obfcurity be introduced in the preceding chapter. We fhall now endeavour to elucidate the nature of that point, and to lay down the methods of determining its fituation or diftance from the point of fufpenfion in pendulums of different lengths and fhapes.

When

When the pendulum consists of a spherical body fastened to a string, a person unacquainted with the subject might at first sight imagine that the length of the pendulum must be estimated from the point of suspension to the centre of the ball. But this is not the case; for in fact the real length of the pendulum is greater than that distance, the reason of which is, that the spherical body does not move in a straight line, but it moves in a circular arch; in consequence of which, that half of it which is farthest from the point of suspension, runs through a longer space than the other half which is nearer to the point of suspension; hence the two halves of the ball, though containing equal quantities of matter, do actually move with different velocities, therefore their momentums are not equal; and it is in consequence of this inequality that the centre of oscillation does not lie between the two hemispheres; that is, in the centre of the ball; but it lies within the lower hemisphere, viz. that which has the greater momentum. Now from this it naturally follows, that if the ball of the pendulum could be concentrated in one point, that point would be the centre of oscillation; so that the centre of oscillation is that point wherein all the matter (and of course the forces of all the particles) of the body or bodies that may be joined together to form a pendulum, may be conceived to be condensed.

The *centre of percussion* is that part or point of a pendulous body, which will make the greatest impression preffion on an obftacle that may be oppofed to it whilft vibrating; for if the obftacle be oppofed to it at different diftances from the point of fufpenfion, the ftroke, or percuffion, will not be equally powerful; and it will foon appear that this centre of percuffion does not coincide with the centre of gravity.

Let the body AB, fig. 1, Plate V. N. I, confifting of two equal balls faftened to a ftiff rod, move in a direction parallel to itfelf, and it is evident that the two balls muft have equal momentums, fince their quantities of matter are equal, and they move with equal velocities. Now if in its way, as at N. II, an obftacle C be oppofed exactly againft its middle E, the body will thereby be effectually ftopped, nor can either end of it move forwards, for they exactly balance each other, the middle of this body being its centre of gravity. Now fhould an obftacle be oppofed to this body, not againft its middle, but nearer to one end, as at N. III, then the ftroke being not in the direction of the centre of gravity, is in fact an oblique ftroke, in which cafe, agreeably to the laws of congrefs which have been delivered in chap. VIII. a part only of the momentum will be fpent upon the obftacle, and the body advancing the end A, which is fartheft from the obftacle, as fhewn by the dotted reprefentation, will proceed with that part of the momentum which has not been fpent upon the obftacle; confequently in this cafe the percuffion is not fo powerful as in the foregoing. Therefore there is a certain point in a moving

moving body which makes a ſtronger impreſſion on an obſtacle than any other part of it.—In the preſent caſe, indeed, this point coincides with the centre of gravity; becauſe the two ends of the body before the ſtroke moved with equal velocities. But in a pendulum the caſe is different; for let the ſame body of fig. 1, Plate V. be ſuſpended by the addition of a line AS, fig. 2, Plate V. which line we ſhall ſuppoſe to be void of weight and flexibility, and let it vibrate round the point of ſuſpenſion S. It is evident that now the two balls will not move with equal velocities; for the ball B, by deſcribing a longer arch than the ball A in the ſame time, will have a greater momentum; and of courſe the point where the forces of the two balls balance each other, which is the centre of percuſſion, lies nearer to the lower ball B; conſequently this point does not coincide with the centre of gravity of the body AB; but it is that point wherein the forces of all the parts of the body may be conceived to be concentrated. Hence the centre of oſcillation and the centre of percuſſion coincide; or rather they are exactly the ſame point, whoſe two names only allude, the former to the time of vibration, and the latter to its ſtriking force.

If in fig. 1, Plate V. the balls A and B be not equal, their common centre of gravity will not be in the middle at E, but it will lie nearer to the heavier body, as at D, ſuppoſing B to be the heavier body; ſo that the diſtances BD, AD, may be inverſely

versely as the weights of those bodies. Now when the above-mentioned body is formed into a pendulum, as in fig. 2, though the weights A and B be equal, yet by their moving in different arches, viz. with different velocities, their forces or momentums become actually unequal; therefore in order to find the point where the forces balance each other, so that when an obstacle is opposed to that point, the moving pendulum may be effectually stopped, and no part of it may preponderate, in which case the obstacle will receive the greatest impression; we must find first the momentums of the two bodies A and B, then the distances of those bodies from the centre of percussion, or of equal forces, must be inversely as those momentums. Thus the velocities of A and B are represented by the similar arches which they describe, and those arches are as the radii SA, SB. Therefore the momentum of A is the product of its quantity of matter multiplied by SA, and the momentum of B is the product of its quantity of matter multiplied by SB; consequently AD must be to BD, as the weight of B multiplied by SB is to the weight of A multiplied by AS. Then D is the centre of percussion. And since, when four quantities are proportional, the product of the two extremes is equal to the product of the two means; therefore if the weight of A multiplied by AS, be again multiplied by AD, the product must be equal to the product of the weight of B multiplied by BS, and again multiplied

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by BD; that is, *the product of the body on one side of the centre of oscillation multiplied by both its distance from the point of suspension and its distance from the centre of oscillation, is equal to the product of the body on the other side of the centre of oscillation, multiplied both by its distance from the point of suspension, and its distance from the centre of oscillation.*

The same reasoning may evidently be applied to a pendulum consisting of more than two bodies connected together, or to the different parts of the same pendulous body; hence we form the following general law.

If the weight of each part of a simple or compound pendulum be multiplied both by its distance from the centre of suspension, and its distance from the centre of oscillation or percussion, the sums of the products, on each side of the centre of oscillation, will be equal to each other.

From this law the rule for determining the distance of the centre of oscillation from the point of suspension is easily deduced; but the application of it is attended with considerable difficulty, on which account we shall subjoin it in the note (1), and shall now proceed to shew an experimental or mechanical

(1) Let a pendulum consist of any number of parts or small bodies A, B, C, D, E, joined together; let a, b, c, d, e, stand for their respective distances from the point of suspension; and x for the distance of the centre of oscillation from the point of suspension.

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chanical method of finding the centre of oscillation, which method is general and easy, at the same time that it admits of sufficient accuracy.

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The distances of those parts, or bodies, from the centre of oscillation will be $x-a$, $x-b$, $x-c$, $d-x$, $e-x$; D and E being supposed to be farther from the point of suspension, than the centre of oscillation is. By multiplying every one of those bodies, both by its distance from the centre of suspension and its distance from the centre of oscillation, we have, agreeably to the above-mentioned law, the equation $Aax - Aaa + Bbx - Bbb + Ccx - Ccc = Ddd - Ddx + Eee - Eex$; which, by transposition and division, is resolved into the following; viz.

$$x = \frac{Aaa + Bbb + Ccc + Ddd + Eee}{Aa + Bb + Cc + Dd + Ee}.$$

Should any of the bodies, as for instance A and B, in the preceding instance, be situated above the centre of suspension, then their distances will be negative, viz. $-a$, $-b$, though their squares aa, bb, are always positive. In this case the value of x is $= \frac{Aaa + Bbb + Ccc + Ddd + Eee}{-Aa - Bb + Cc + Dd + Ee}$.

Since the centre of gravity of a body or system of bodies, is that point wherein all their matter may be conceived to be condensed, therefore the product of all the matter or sum of the different weights A, B, C, D, E, multiplied by the distance of the common centre of gravity from the point of suspension, is equal to the sum of the products of each body multiplied by its distance from the point of suspension. Hence the above stated value of x becomes $\overline{Aaa + Bbb + Ccc + Ddd + Eee}$ divided by the product of the whole body or sum of the weights, multiplied by the distance of the

The body whose centre of oscillation, or (which is the same) of percussion, is to be ascertained, must be suspended to a pin or other support, but as freely

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the centre of gravity from the point of suspension. And being expressed entirely in words, it forms the following general

Rule 1. *If all the bodies or parts of a body, that forms a pendulum, be multiplied each by the square of its distance from the point or axis of suspension, and the sum of the products be divided by the product of the whole weight of the pendulum, multiplied by the distance of the centre of gravity from the point of suspension; the quotient will be the distance of the centre of oscillation or percussion from the point of suspension.*

The situation of the centre of oscillation may also be found by means of another rule, which we shall likewise lay down, and shall demonstrate; since in some cases this rule will be found preferable to the first.

Rule 2. *If the sum of the products of all the parts or weights, multiplied each by the square of its distance from the centre of gravity, or from a line passing through the centre of gravity parallel to the axis of vibration, be divided by the product of the whole mass or body, multiplied by the distance of the centre of gravity from the point of suspension, the quotient will be the distance of the centre of oscillation from the centre of gravity; which being added to the distance of the centre of gravity from the point of suspension, will be the distance of the centre of oscillation from the point of suspension.*

Let C A B fig. 4, Plate V. represent any sort of body regular or irregular, suspended at C; O its centre of oscillation; G its centre of gravity; C O B its axis or right line passing through the point of suspension, and centres of gravity freely as may be practicable; and being once moved out of the perpendicular situation, must be suffered to perform very short vibrations; viz. so small as to be just discernable. Then by keeping an eye on

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vity and oscillation. This body may be conceived to consist of an indefinite number of extremely small parts or weights. Let W be one of those small weights; join WC and WG, and from W drop WF perpendicular to CO. Then the product of W, by the square of its distance from C, is $W \times \overline{CW}^2$. But (Eucl. p. 47. B. I.) $\overline{CW}^2 = \overline{WF}^2 + \overline{CF}^2$; and $\overline{GW}^2 = \overline{GF}^2 + \overline{WF}^2$. (Eucl. p. 7. B. II.) $\overline{CG}^2 + \overline{GF}^2 = \overline{2CG \times GF} + \overline{CF}^2$; and by transposition $\overline{CF}^2 = \overline{GF}^2 + \overline{CG}^2 - \overline{2CG \times GF}$. Then by substitution (viz. by putting instead of $\overline{CF}^2$, its equal $\overline{GF}^2 + \overline{CG}^2 - \overline{2CG \times GF}$) the above stated equation becomes $\overline{CW}^2 = \overline{WF}^2 + \overline{GF}^2 + \overline{CG}^2 - \overline{2CG \times GF} =$ (putting $\overline{GW}^2$ for its equal $\overline{GF}^2 + \overline{WF}^2$) $\overline{GW}^2 + \overline{CG}^2 - \overline{2CG \times GF}$. And multiplying both sides by W, we have the sum of all the products $W \times \overline{CW}^2 =$ the sum of all the $W \times \overline{GW}^2 +$ all the $W \times \overline{CG}^2 -$ the sum of all the $W \times \overline{2CG \times GF}$.

But by the nature of the centre of gravity the sum of all the $W \times GF$ is $= 0$; for those which are one side of the axis must balance those which are on the other side; and of course all the $W \times 2\,\overline{CG \times GF}$ also become $= 0$. Therefore there remains the sum of all the $W \times \overline{CW}^2 =$ sum of all the $W \times \overline{GW}^2 +$ sum of all the $W \times \overline{CG}^2 =$ sum of all the $W \times \overline{GW}^2 +$ the whole body $\times \overline{CG}^2$. Or (taking away the sum of all the $W \times \overline{GW}^2$ from both sides on a clock or watch with a seconds hand, the observer must count the vibrations, and, if possible, even the part of a vibration, that are performed by that

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sides of the equation) the sum of all the $W \times \overline{CG}|^2 =$ the whole body $\times \overline{CG}|^2$.

Then $CO = \frac{\text{sum of all the } W \times \overline{CW}|^2}{\text{whole body} \times CG}$ (by rule the 1st) $= CG + \frac{\text{sum of all the } W \times \overline{GW}|^2}{\text{the whole body} \times CG}$. And lastly, $GO = CO - CG = \frac{\text{sum of all the } W \times \overline{GW}|^2}{\text{the whole body} \times CG}$; which is rule the 2d.

In the application of the above-mentioned rules, it is frequently very difficult to find the sum of the products of all the weights multiplied by the squares of their respective distances. The method of fluxions is undoubtedly the most extensive, as it may be applied to all such figures or bodies as have some regularity of shape, or such as may be expressed by an algebraical equation. But in some cases the irregularity of form is so very great, that the centre of oscillation can only be found out by means of the above-described mechanical method.

In order to find the sum of the weights, &c. you must consider an indefinitely small part, or increment, or fluxion, of the figure, as being a small weight, and multiply it by the square of its distance from the centre of suspension or axis of vibration, according to rule the 1st, or else multiply it by the square of its distance from the centre of gravity, or from a line passing through the centre of gravity, and parallel to the axis of vibration, according to rule the 2d.; then the that pendulum in one minute, and note the number. N. B. Should the pendulum appear likely to ſtop before the expiration of the minute, a gentle and

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the fluent of that expreſſion will be the ſum of the products of all the weights, multiplied by the ſquares of their reſpective diſtances, either from the axis of vibration, or from the centre of gravity, &c. Laſtly, this fluent muſt be divided by the product of the whole body (to be had by common menſuration) multiplied by the diſtance of the centre of gravity, from the point of ſuſpenſion; and the quotient will be the diſtance of the centre of oſcillation either from the point of ſuſpenſion, or from the centre of gravity, according as the operation was performed either by rule the firſt, or rule the ſecond.

A few examples will render the application of this method more intelligible.

Example 1. Let CB, fig. 5, Plate V. be a right line, or very ſlender cylinder ſuſpended at C; and call it a, (meaning either its length or weight, for the one is proportionate to the other) G is its centre of gravity. Now if you call any part of this line x, reckoning from C, then the increment or fluxion of x is $\dot{x}$, which $\dot{x}$ may be conſidered as one of the vaſt many weights which form the whole line or ſlender cylinder. The product of this weight by the ſquare of its diſtance from C is $x^2\dot{x}$, and the fluent of this expreſſion is $\frac{x^3}{3}$, which, when x repreſents the whole extent CB, becomes $\frac{a^3}{3}$, and is the ſum of the products of all the weights multiplied by the ſquares of their reſpective diſtances from C. We and dexterous application of a finger once or twice, will increaſe a little its vibrations, and prolong its action without altering the time of vibration.— For

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We muſt now find the product of the whole line multiplied by the diſtance of the centre of gravity G from C. But CG $= \frac{a}{2}$, therefore the product in queſtion is $a \times \frac{a}{2}$, or $\frac{a^2}{2}$. Laſtly, divide the above fluent $\frac{a^3}{3}$ by the laſt product, viz. $\frac{a^3}{3} \div \frac{a^2}{2}$, and the quotient $\frac{2a}{3}$ is the diſtance of the centre of oſcillation O from the point of ſuſpenſion C; ſo that CO is equal to $\frac{2}{3}$ of CB.

Example 2. Let AB, fig. 6, Plate V. be a right line or very ſlender cylinder faſtened to a line GO void of weight, and ſuſpended at O. The ends A and B are equidiſtant from O, and the axis of vibration is perpendicular to the plane which paſſes through ABOG; ſo that every part of the given line from A to G, or from B to G, is at a different diſtance from the axis of ſuſpenſion. Put OG $= a$, and GP $= x$, whoſe fluxion is $\dot{x}$, and is a particle or ſmall weight of the given line, which multiplied by the ſquare of OP, which is (Eucl. p. 47. B. I.) $= \overline{a^2 + x^2}$, becomes $a^2\dot{x} + x^2\dot{x}$. The fluent of this expreſſion is $a^2x + \frac{x^3}{3}$.

The product of the body GP by the diſtance of the centre of gravity G from O is (GP×OG) ax. Therefore the diſtance of the centre of oſcillation is $\left(\frac{a^2x + \frac{x^3}{3}}{ax} =\right) a + \frac{x^2}{3a}$

For the ſake of greater accuracy, the attentive obſerver may count the number of vibrations for a longer time, as, for inſtance, during two, or three, or

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$\frac{x^2}{3a}$; which when GP = GB, becomes $OG + \frac{\overline{BG}|^2}{3OG}$; ſo that the centre of oſcillation is at C, viz. lower than G by the quantity $\frac{\overline{BG}|^2}{3OG}$.

Example 3. Let the poſition be exactly as in the preceding example, excepting only that the axis of ſuſpenſion or of vibration, which was then perpendicular, be now parallel, to the line AB, as in fig. 7, Plate V. and in this caſe, the centre of oſcillation will coincide with the centre of gravity G; for here, all the parts of the given line, as A, G, P, B, &c. are equidiſtant from the axis of ſuſpenſion; ſo that the weight $\dot{x}$ multiplied by the ſquare of its diſtance from the axis of vibration DOC, becomes $a^2\dot{x}$; the fluent of which is a^2x, and this fluent divided by ax, quotes a; that is OG for the diſtance of the centre of oſcillation.

Example 4. Let the pendulum conſiſt of an iſoſceles triangle ABC, fig. 8, Plate V. ſuſpended at A, and the axis of vibration parallel to BC. Put the altitude AD = a; baſe BC = b; and AF = x. Through F draw GH parallel to the baſe. Then $a : b :: x : \frac{bx}{a}$ = GH; and $\frac{bx\dot{x}}{a}$ is its fluxion, which multiplied by the ſquare of AF, viz. by x^2, becomes $\frac{bx^3\dot{x}}{a}$. The fluent of this expreſſion is $\frac{bx^4}{4a}$.

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or four minutes, and then taking the half, or third part, or fourth part of the number; for that part will

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The triangle ABC $= \frac{ab}{2}$: and the diſtance of its centre of gravity from A is $= \frac{2a}{3}$; hence the product of the triangle by this diſtance is $\left(\frac{ab}{2} \times \frac{2a}{3}\right) = \frac{a^2b}{3}$. Therefore divide the above fluent, $\frac{bx^4}{4a}$, by $\frac{a^2b}{3}$, and the quotient is $\frac{3x^4}{4a^3}$; which when x is equal to the altitude AD $= a$, becomes $\frac{3a}{4}$; ſo that the diſtance of the centre of oſcillation from A is equal to $\frac{3}{4}$ of the altitude of the triangle.

Example 5. Let the pendulum conſiſt of a ſpherical body ſuſpended at O, fig. 9, Plate V. by means of a line OD, which line weighs ſo little with reſpect to the body, that its weight may be conſidered as $= 0$. Imagine DERD to be a ſection of the ſphere through its axis, and perpendicular to the axis of oſcillation KL. GE the radius perpendicular to DR. G the centre of gravity, and V the centre of oſcillation.

Let SFPS be any concentric circle; and put the ordinate PM$=y$; GP$=x$; $c=$ the circumference of a circle whoſe radius is one, and draw NM parallel to GR. Suppoſe a cylindric ſurface to ſtand on the circumference SFPS, and to be terminated by the ſurface of the ſphere; then the circumference SFPS$=cx$, and the juſt mentioned cylindric ſurface will be $= 2cyx$.

The diſtance of the particles in each ſection of this cylindric ſurface, from the centre of gravity of the ſection,

 or

will be the number of vibrations anſwering to one minute.

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or of a line paſſing through G parallel to KL, is $GP = x$; therefore the fluxion of the weight or ſurface is $2cyx\dot{x}$, which multiplied by the ſquare of the diſtance GP, viz. by x^2, gives the fluxion $2cyx^3\dot{x}$; whoſe fluent is $\frac{cyx^4}{2}$.

In order to expunge from this expreſſion one of the variable letters, it muſt be conſidered that in a circular arch the ſine is to the coſine as the fluxion of the latter is to the fluxion of the former; for in fig. 10, Plate V. where $BE = y$ is the ſine; $AE = x$ is the coſine; if you draw CF indefinitely near and parallel to BE, and BD parallel to AG, BD becomes $\dot{x}$, or the fluxion of the coſine, and CD becomes $\dot{y}$, or the fluxion of the ſine; and ſince the right-angled triangles ABE, BCD, are equiangular (the angles CBD, EBA, being equal, becauſe each of them is the complement of ABD to a right angle) and ſimilar, we have AE to EB, as CD to DB, viz. $x : y :: \dot{y} : \dot{x} = \frac{y\dot{y}}{x}$. Alſo if the radius be called a; ſince the ſquare of AB, or a^2, is equal to $x^2 + y^2$; $x = \sqrt{a^2 - y^2}$. Now by ſubſtitution the fluent $\frac{cyx^4}{2}$ becomes $\frac{2}{3}ca^2y^3 - \frac{1}{5}cy^5$ (for the fluxion of the former; viz. $2cyx^3\dot{x} = 2cyx^3 \times \frac{y\dot{y}}{x} = 2cyx^3 \times \frac{y\dot{y}}{\sqrt{a^2 - y^2}} = 2cy \times \overline{a^2 - y^2}|^{\frac{3}{2}} \times \frac{y\dot{y}}{\sqrt{a^2 - y^2}} = 2cy^2\dot{y} \times \overline{a^2 - y^2} = 2ca^2y^2\dot{y} - 2cy^4\dot{y}$; and the fluent of this laſt expreſſion is $\frac{2}{3}ca^2y^3 - \frac{2}{5}cy^5$). And when $y = a =$ radius, this fluent becomes $\frac{2}{3}ca^5 - \frac{2}{5}ca^5 = \frac{4}{15}ca^5$.

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With that number of vibrations, performed in one minute, the diſtance of the centre of oſcillation from

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The ſolidity of a ſphere, whoſe radius is a, is expreſſed by $\frac{2}{3}ca^3$, which multiplied by the diſtance $GO=d$, in fig. 9. becomes $\frac{2}{3}ca^3d$.

Laſtly, divide the above fluent by the laſt product; viz. divide $\frac{4}{15}ca^5$ by $\frac{2}{3}ca^3d$, and the quotient $\frac{2a^2}{5d}$ is the diſtance of the centre of oſcillation V from the centre of gravity G, and of courſe $OV = d + \frac{2a^2}{5d}$.

Should the point of ſuſpenſion be ſituated cloſe to the ſurface, as at D; then the diſtance between the centres of ſuſpenſion and of gravity would become equal to radius, viz. $d=a$; and in that caſe the diſtance between the centres of oſcillation and of gravity will be $\frac{2a}{5}$; and the diſtance between the centres of oſcillation and ſuſpenſion will be $\frac{7a}{5}$; that is $\frac{7}{10}$ of the diameter of the ſphere.

It plainly appears from the foregoing explanations and examples, that when the ſtring of a pendulum is ſhortened, every thing elſe remaining unaltered, the centre of oſcillation changes its place; unleſs indeed the weight of the pendulum or bob be ſuppoſed to be condenſed in one point, which caſe can have place only in the imagination.

Conſequently what has been demonſtrated reſpecting the cycloidal pendulum muſt be conſidered as a matter merely of uſeful ſpeculation, ſince from it we derive the time in which a circular pendulum performs its vibrations. But in practice from the point of suspension is determined by means of the following easy calculation.

Divide sixty seconds by the number of vibrations, which the pendulum in question has performed in one minute, and the quotient is the time of one vibration. Square this time, (viz. multiply it by itself) and multiply its square by the length of the pendulum that vibrates seconds, viz. by 39,1196 inches, and the last product shews the distance in inches of the centre of oscillation or percussion from the point of suspension in the pendulum in question.

Example 1. Let a cylinderical stick AB, fig. 3, Plate V. of about a yard in length, be suspended at A, and be caused to vibrate. Having observed that it performs 76 vibrations in a minute, it is required thereby to find the distance of its centre of oscillation from the point of suspension A.

Divide 60 seconds by 76 vibrations, and the quotient, 0″,79 nearly (viz. 79 hundreths of a second) is the time in which the pendulum in question performs one vibration. Then since the lengths of pendulums are as the squares of the times of vibration; therefore say as the square of one second,

practice a cycloidal pendulum would not perform all its vibrations in equal times; because by the application of the string to the cycloidal cheeks, the free part of the string would be shortened, and the centre of oscillation would change its place continually.

ſecond, which is one, is to the ſquare of 0,79 hundredth parts of a ſecond, viz. 0,6241; ſo is the length of the pendulum which vibrates ſeconds, viz. 39,1196 to the length ſought; that is, 1 : 0,6241 :: 39,1196; where ſince the firſt number is unity, you need, according to the preceding rule, only multiply 39,1196 by 0,6241; and the product 24,4 is the diſtance ſought; ſo that the centre of oſcillation C in the ſtick AB is 24 inches and 4 tenths diſtant from its extremity A; viz. about two thirds of its length.

Example 2. An irregular body ſuſpended by one end has been found to perform 20 vibrations in a minute. Required the diſtance of its centre of oſcillation from the point of ſuſpenſion?

Here the time of one vibration is $(\frac{60}{20})$ 3 ſeconds; the ſquare of which is 9; and 39,1196, multiplied by 9, gives 352,0764 inches, or nearly 29 feet, for the diſtance ſought.

CHAPTER XII.

OF THE MECHANICAL POWERS.

THE preceding chapters contain the doctrine of motion in a manner rather extenſive for an elementary work. The abſtract mode in which this ſubject has been delivered, may poſſibly have deterred the novice from the ſtudy of natural philoſophy. Perhaps he expected that after every theoretical chapter his attention ſhould be relieved by ſome experimental application of the doctrine. But if ſuch had been the plan, either the work would have been protracted to an immoderate length, or many uſeful branches of the theory would have been ſuppreſſed.

The importance of the doctrine of motion, and its being the foundation of almoſt all the phenomena of nature, were the motives which placed it before every other branch of natural philoſophy, and the reader may perhaps be pleaſed to hear, that whoever underſtands the leading principles of the foregoing theory, will meet with very little difficulty in the peruſal of the following parts of philoſophy.

losophy. He will alſo find that the doctrine of motion, which he may formerly have looked upon as a difficult and almoſt a uſeleſs ſubject of ſpeculation, is of general and extenſive application. Every tool, every engine of art, every œconomical machine, all the inſtruments of huſbandry, and of navigation, the celeſtial bodies, &c. are conſtructed, and act conformably to the laws of motion.

The knowledge of this doctrine anſwers two extenſive objects. It ſerves to explain natural appearances, and it furniſhes the human being with uſeful machines, which enable him to accompliſh ſuch effects, as without that aſſiſtance would be utterly out of his power.—The application to natural phenomena will be inſtanced in almoſt every chapter of this work.—The ſecond object will be conſidered immediately.

Mechanics, in its full and extenſive meaning, is the ſcience which treats of *quantity*, of *extenſion*, and of *motion*. Therefore it conſiders the ſtate of bodies either at reſt or in motion. That branch of it which conſiders the ſtate of bodies at reſt, as their equilibrium when connected with one another, their preſſure, weight, &c. is called *Statics*. That which treats of motion, is called *Dynamics*. Both thoſe expreſſions are, however, uſed in treating of ſolid bodies; for the mechanics of fluids has two denominations analogous to the above. It is called *Hydroſtatics*,

Hydroſtatics, when it treats of the equilibrium or quieſcent ſtate, and *Hydrodynamics* or *Hydraulics*, when it treats of the motion, of fluids.

What belongs excluſively to fluids will be noticed in the ſecond part of theſe elements. The equilibrium of ſolids has been ſufficiently examined in the preceding pages, and will be taken farther notice of in the following; ſince in treating of motion, of actions, of forces, &c. it will naturally appear that when thoſe forces are equal and oppoſite to each other, an equilibrium takes place.

The active application of the doctrine of motion conſiſts in the conſtruction of machines for the purpoſes of overcoming reſiſtances, or of moving bodies. Thus if a man wiſh to remove a ſtone of a ton weight from a certain place, for which purpoſe he finds his ſtrength inadequate, he makes uſe of a long pole, which being applied in a certain manner, actually enables him to move the ſtone. Thus alſo another perſon may wiſh to convey ſome heavy article to the top of his houſe, he makes uſe of a ſet of pullies with a rope, &c. and by that means eaſily accompliſhes his object.

Infinite is the number, and the variety of machines; but they all conſiſt of certain parts or ſimple mechaniſms, variouſly combined and connected with each other. Of thoſe ſimple machines we can reckon no more than ſix or at moſt ſeven; viz. the *Lever*, the *Wheel* and *Axle*, the moveable *Pul-*

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ley or *Syſtem of Pulleys*, the *Inclined Plane*, the *Wedge*, and the *Screw* *.

The action or the effect of every one of thoſe mechanical powers, depends upon one and the ſame principle; which has been fully explained in chapter IV, V, and VI; but we ſhall for the ſake of perſpicuity briefly repeat it in the following three or four paragraphs, wherein the attentive reader will find the principles or analyſis of all ſorts of machines.

The force or momentum of a body in motion, is to be derived not merely from its quantity of matter, or only from its velocity, but from both conjointly; for the heavier any body is, the greater power is required to ſtop it or to move it; and on the other hand the ſwifter it moves, the greater is its force, or the ſtronger oppoſition muſt be made to ſtop it. Therefore, the force or momentum, is the product of the weight or quantity of matter by the velocity. Thus if a body weighing 10 pounds move

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* The writers on mechanics do not agree with reſpect to the number of the mechanical powers. Some exclude the inclined plane from the number; whilſt others reckon it one of the principal, and conſider the wedge and the ſcrew as only ſpecies of it. The balance has been likewiſe reckoned a peculiar mechanical power. But it has been rejected by others, either on account of its being nothing more than a lever, or becauſe by the uſe of a balance no additional power is obtained, which advantage ought in truth to be the characteriſtic property of a mechanical power.

move at the rate of 12 feet per ſecond, and another body weighing 5 pounds move at the rate of 24 feet per ſecond, their momentums will be equal; that is, they will ſtrike an obſtacle with equal force, or an equal power muſt be exerted to ſtop them; for the product of 10 by 12, viz. 120, is equal to the product of 5 by 24.

The forces of bodies acting on each other by the interpoſition of machines is derived from the ſame principle. Thus the two bodies A and B, fig. 11, Plate V. are connected with each other by the interpoſition of an inflexible rod AB (the ſimpleſt of all machines) which reſts upon the prop or fixed point F. If the rod move out of its horizontal ſituation into the oblique poſition CFE, the body A will be forced to deſcribe the arch AE, whilſt the body B deſcribes the arch BC; and thoſe arches, being deſcribed in the ſame time, will repreſent the velocities of thoſe bodies reſpectively; therefore, the momentum of A is to the momentum of B, as the weight of A multiplied by the arch AE, is to the weight of B multiplied by the arch BC.

The velocities of A and B are likewiſe repreſented by their diſtances from F; for the arches AE, BC, are as their radii FA, FB. Thoſe velocities are alſo repreſented by the perpendiculars EG, CD; for ſince the triangles EFG, CDF, are equiangular and ſimilar, (the angles at G and D being right, and thoſe at F being equal) we have EF to FC, as

GE

GE to CD. Therefore the reſpective momentums of A and B may be repreſented either by $A \times AE$, and $B \times BC$; or by $A \times AF$, and $B \times BF$; or laſtly by $A \times EG$, and $B \times CD$.

Note. The laſt expreſſion is uſed when the motion of bodies that are ſo circumſtanced, reſults from the action of gravity; viz. when one body recedes from, whilſt the other approaches, the centre of the earth; becauſe gravity acts in that direction.

This is the principle of all ſorts of mechaniſms; ſo that in every machine the following particulars muſt be indiſpenſably found. 1ſt. One or more bodies muſt be moved one way, whilſt one or more bodies move the contrary way. One of thoſe bodies or ſets of bodies is called the *weight*, and the other is called the *power*, or they may be called *oppoſite powers*. 2dly. If the product of the weight of one of thoſe powers, multiplied by the ſpace it moves through in a certain time, be equal to the product of the weight of the oppoſite power multiplied by the ſpace it moves through in the ſame time; then the oppoſite momentums being equal, the machine will remain motionleſs. But if one of thoſe products or momentums exceeds the other, then the former is ſaid to preponderate, and the machine will move in the direction of the preponderating power; whilſt the oppoſite power will be forced to move the contrary way. And the preponderance is repreſented by the exceſs of one momentum over the

the other; for inſtance, if one of the above-mentioned products or momentums be 24, and the other 12, then the former is ſaid to be double the latter; or that the former is to the latter as two to one.

By a ſtrict adherence to thoſe particulars, the attentive reader will be enabled to eſtimate the power and effect of every machine, excepting, however, the obſtruction which ariſes from the imperfection of materials and of workmanſhip; as will fully appear from the following paragraphs.

In the explanation of the properties of the mechanical powers, we ſuppoſe the rods, poles, planes, ropes, &c. to be deſtitute of weight, roughneſs, adheſive property, and any imperfection; for when the properties of thoſe powers have been eſtabliſhed, we ſhall then point out the allowances proper to be made on the ſcore of friction, irregularity of figure, &c.

THE LEVER.

A lever is a bar of wood, or metal, or other ſolid ſubſtance, one part of which is ſupported by or reſts againſt a ſteady prop, called the *fulcrum*, about which, as the centre of motion, the lever is moveable.

The uſe of this machine is to overcome a given obſtacle, by means of a given power.—Thus if the ſtone A, fig. 12, Plate V. weighing 1000 pounds, be required to be lifted up (ſo as to paſs a rope under

der it, or for ſome other purpoſe), by means of the ordinary ſtrength of a man, which may be reckoned equal to 100 pounds weight; a pole or lever CE is placed with one end under the ſtone at E; it is reſted upon a ſtone or other ſteady body at B, and the man preſſes the lever down at C. In this caſe the man's ſtrength is equal to the tenth part of the ſtone's weight, therefore its velocity muſt be ten times greater than that of the ſtone; that is, the part BC of the lever muſt be ten times as long as the part BE, in order that the power and the weight may balance each other; and if CB is a little longer than ten times BE, then the ſtone will be raiſed. Indeed in this caſe the part CB needs not be ſo long; for as the ſtone is not to be entirely lifted from the ground, a leſſer momentum is required on the part of the power at C.

In general, to find the proper length of the lever, we need only multiply the weight by that part of the lever which is between it and the fulcrum; and divide the product by the power; for the quotient will be the length BC, which is neceſſary to form an equilibrium, and of courſe a little more than that length will be ſufficient to overcome the obſtacle.

If when the length of the lever is given, you wiſh to find what power will be neceſſary to overcome a known obſtacle or weight; multiply the weight by that part of the lever which is between it and the fulcrum, then divide the product by the other part of the lever, and the quotient is the anſwer.

The poſſible different ſituations of the weight, the fulcrum, and the power, are not more than three; hence ariſe three kinds of levers; to all of which, however, the preceding calculations are equally applicable. Thoſe ſpecies are, 1. when the fulcrum is placed between the weight and the power, as the one already deſcribed. 2. When the fulcrum is at one end, the power at the other end, and the weight between them, as in fig. 13, Plate V. And 3. When the fulcrum is at one end, the weight at the other end, and the power between them, as in fig. 14, Plate V.

Some writers add a fourth ſpecies, viz. the bent lever; but as this differs only in ſhape from the others, it does not conſtitute a proper difference of kind.

Hitherto we have ſuppoſed that the weight and the power act in directions perpendicular to the arms of the lever; but when this is not the caſe, the diſtances of the power and of the weight from the centre of motion muſt not be reckoned by the diſtances of the points of ſuſpenſion from that centre, but by the lengths of the perpendiculars let fall from the centre of motion on the lines of the direction of the forces. For inſtance, in fig. 15, Plate V. the power at P, acts by means of the ſtring PB, on the end B of the lever, in a direction BP, PB, oblique to the lever; and in eſtimating the momentum of the power, you muſt multiply the force or power applied to the ſtring, not by the length CB, but by the length CD, of the perpendicular, let fall from the centre of motion C, on the line BP, which is the *line of direction* of the power.

Thus alſo in the bent lever ABC, fig. 16, Plate V. whoſe centre of motion is at B; the momentums of D and E are the weight of D multiplied by BG, and the weight of E multiplied by BF. The reaſon of the laſt remark is eaſily derived from the compoſition and reſolution of forces (ſee chap. VIII.) Therefore we may in general ſay, that *in any ſort of lever, and in whatever directions the power and the weight act on it, if their quantities be inverſely as the perpendiculars let fall from the centre of motion on their reſpective directions, they will be in equilibrio; that is, balance each other.*

It will be hardly neceſſary to remark, that when the lever is loaded with ſeveral weights at different diſtances from the centre of motion, the momentum on each ſide of the centre of motion is equal to the ſum of the products of all the weights on that ſide multiplied each by its diſtance from the centre of motion. Thus in fig. 17, Plate V. the momentum of the ſide AD is equal to the ſum of the products of E multiplied by DA, F multiplied by GA, and H multiplied by OA; and the momentum of the ſide AB is equal to the ſum of C multiplied by BA, and K multiplied by LA.

 The

The uſe of the lever is ſo general and ſo extenſive, that levers of all ſorts and varieties are to be found in almoſt every mechaniſm;—in the works of nature as well as thoſe of human ingenuity.

The bones of a human arm, AC, fig. 18, Plate V. and indeed the greateſt number of the moveable bones of animals, are levers of the third kind. In fig. 18, D is the centre of motion; the power (viz. the inſertion of the muſcle BC, the contraction of which moves the arm) is at C, and the effect is produced, or the weight is lifted, at A.

In this natural lever the power is not advantageouſly ſituated; for as it lies very near the centre of motion, it muſt be much greater than the weight which is to be lifted at A. But the loſs of power is abundantly compenſated by other advantages, the principal of which is the compactneſs of the limb.

The *iron crow*, fig. 19, Plate V. which is commonly uſed by carpenters, blackſmiths, ſtone-maſons, &c. is a bent lever, flattened at A. It is bent a little in order that the weight may be leſs apt to ſlip off; and it is flattened for the purpoſe of its being more eaſily admitted into narrow crevices.

The *common balance*, fig. 20, Plate V. or pair of ſcales, is a lever, whoſe fulcrum or centre of motion is in the middle, and the weights are ſuſpended at the two extremities; but as thoſe extremities are equidiſtant from the fulcrum, the velocities of the weights are equal; and of courſe when neither end of the beam preponderates, the oppoſite weights muſt

muſt be equal; but when one of the weights exceeds the other, then that arm to which the former is ſuſpended, will deſcend, &c. And this is all the uſe that can be made of the balance, viz. to find when two weights are equal or unequal*.

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* Common balances are ſubject to many imperfections, the principal of which are as follows:

1ſt. A balance is frequently in equilibrio, when the oppoſite weights in its ſcales are not equal. This ariſes from the points of ſuſpenſion being not equidiſtant from the centre of motion; in which caſe the empty ſcales may be made to balance each other; yet when equal weights are put in them, thoſe weights will not balance each other; for as they are ſuſpended at unequal diſtances from the centre of motion, their momentums are actually unequal.

2dly. The beam is frequently made too ſlight; in which caſe it is apt to be bent more or leſs by the weights that are put into the ſcales; and of courſe the apparent equilibrium cannot be depended upon.

3dly. Balances ſeldom are ſufficiently ſenſible. This defect ariſes from various cauſes, as from the great weight of the beam, from roughneſs and friction at the point of ſuſpenſion, from the centre of gravity of the beam being conſiderably below the centre of motion, &c.

Balances have been made in this country and elſewhere, of a wonderful degree of ſenſibility; viz. capable of having their equilibrium diſturbed by ſo ſmall a quantity as $\frac{1}{1600000}$ part of the weight in each ſcale. See the *Journal de Phiſique*, vol. 33d. and the Phil. Tranſ. for the year 1798, p. 148. And I have heard of ſcales even of a greater degree of ſenſibility.

 The

The *ſteelyard*, fig. 21, Plate V. (which many writers called by the latin name *ſtatera romana*) is a lever of the firſt kind, whoſe fulcrum or centre of motion is at A; the weight B is ſuſpended always at the ſame diſtance CA from the centre of motion; but the power or counterpoiſe E may be ſhifted from one point to another all along the arm AD; by which means a great variety of weights may be balanced by the ſame counterpoiſe E, whoſe momentum increaſes in the proportion of its diſtance from A. The whole length of the arm AD is marked with numbered diviſions, each of which indicates the weight of B, which is balanced by the counterpoiſe E, when E is placed at that particular diviſion. Thoſe diviſions are aſcertained by trial; for the two arms of the ſteelyard being unequal in weight, their momentums, when loaden with the weights B and E, cannot be eſtimated merely by the products of thoſe weights multiplied each by its diſtance from A.

The ſteelyard was rendered more perfect by Mr. B. Martin, a philoſophical inſtrument maker of very diſtinguiſhed ability, who fixed a weight C to the ſhort end of the beam (as is ſhewn in fig 22, Plate V.) capable of juſt balancing the oppoſite arm AD; in which caſe the momentums of E and B are equal to their weights multiplied by their reſpective diſtances from A; conſequently the diviſions on the arm AD may be eaſily determined by meaſurement.

When

When a lever is ſupported at its two extremities A and C, fig. 23, Plate V. and the weight W is ſuſpended at a point B between A and C, thoſe two points A and C may be alternately conſidered one as the power and the other as the fulcrum; from which conſideration it appears that the proportion of the weight which is ſupported by one of thoſe props, is to the other in the inverſe proportion of the diſtances AB, BC; hence when a weight is carried by means of a pole between two men, in the manner commonly practiſed by draymen when they carry a caſk of beer, the weight may be made to bear harder upon one of the men than upon the other; by placing it nearer to the one than to the other.

THE WHEEL AND AXLE.

The wheel and axle (by ſome called *axis in peritrochio*) conſiſts of a cylinder, AB, fig. 1, Plate VI. and a wheel DF faſtened to the cylinder, and all moveable round the common axis, which is ſupported at its two ends B and G.

In this mechanical power, the weight C is raiſed by a rope which coils about the axle, and the power E is applied to the circumference of the wheel. Here it is plain that the velocity of the weight is to the velocity of the power, as the circumference of the axis is to the circumference of the wheel, or (becauſe circles are as their diameters) as the diameter of the axle is to the diameter of the wheel;

 hence

hence the equilibrium in this mechaniſm takes place when the weight of E is to the weight of C, as the diameter of the axle is to the diameter of the wheel.

Inſtead of the power E, the wheel may be furniſhed with little handles or ſpokes, as repreſented in the figure, which may be moved by hand. Or long ſpokes may be fixed through the axis, and the hands of one or more men may be applied to the ends of thoſe ſpokes, as in fig. 2, for the effect will be the ſame as if there were a wheel; which is ſo evident as not to need any farther illuſtration.

Cranes for raiſing great weights, capſtans, and windlaſs, ſuch as are uſed on board of ſhips, are engines of this ſort.

Fig. 3, Plate VI. repreſents a very powerful engine, nearly of this ſort. ABKI, and CIDH, are two cylinders of unequal diameters, (but the difference of thoſe diameters muſt not be very great) firmly connected together and moveable by means of the handle F round the common axis EG, whoſe extremities reſt upon two ſupports. The ſame rope is faſtened with one end at D, and is wound round the ſmall cylinder CD; then it deſcends and paſſes round the pulley Z, to the frame of which the weight W is ſuſpended; and laſtly, the other end of the rope is faſtened at A to the larger cylinder. Now by moving the cylinders round, the rope will unwind itſelf from the ſmall cylinder, and will coil itſelf round the large cylinder, as is clearly ſhewn

ſhewn by fig. 4, which repreſents the cylinders as ſeen by an eye placed in the direction of the axis. If the cylinders were of equal diameters, the lower part Z of the rope, or the weight W which is ſuſpended to it, would not be moved; for in that caſe, as much of the rope as is diſengaged from one cylinder at each revolution, would be coiled round the other cylinder; but the cylinders being of unequal diameters, it is evident that at each revolution of the handle F, more of the rope will be coiled round the cylinder ABIK than will be diſengaged from the cylinder CDIH; and of courſe the weight W will be raiſed.

THE MOVEABLE PULLEY, OR SYSTEM OF PULLEYS.

The pulley is a thick circular piece of wood, or metal, or other ſolid matter, moveable round a centre pin or axis, which is fixed in a block or frame, in the manner repreſented by A. fig. 5, Plate VI. In this fig. the frame is faſtened to a ſteady beam; a rope is paſſed over the pulley, to one end of which the weight W is ſuſpended, and the power P is applied to the other end of the rope. In this caſe it is evident, that in order to raiſe the weight, the power P muſt move downwards as much as the weight W moves upwards; or in other words, that their velocities are equal; hence no advantage is gained by this mechaniſm, excepting the conveniency of changing the direction of the motion; ſo ſo that the action of this pulley is exactly analagous to that of the balance. Therefore the third mechanical power is not ſaid to be the pulley in general, but it is ſaid to conſiſt of a *moveable pulley*, or *moveable pulleys*, as ſhewn in the figures 6, 7, and 8, Plate VI. for in thoſe caſes, power is evidently gained.

In fig. 6, the rope is faſtened to the hook at F; it paſſes round the pulley BD, to the block of which the weight W is ſuſpended, and is then held by the power at E. When the power pulls the rope, the block, with the weight, are raiſed, and the rope is ſhortened on both ſides; for inſtance, when the pulley, block, &c. are at the dotted ſituation *m*, the rope has been ſhortened of the lengths AB, CD; viz. double the height *m*D; and that quantity of rope has been drawn by the power; therefore in order to pull the weight up from the ſituation W to that of the dotted repreſentation, the power muſt have moved through twice that ſpace; that is, with double the velocity of the weight; hence the equilibrium in this caſe takes place when the power is to the weight as one is to two.

Fig. 7, repreſents the ſame caſe, excepting only that in this the direction of the power E is changed by the interpoſition of a fixed pulley F; ſo that if W weigh two pounds, the power, or oppoſite weight E, muſt weigh one pound to balance it; and then, if a little more weight be added to E, the weight W will be raiſed.

In

In fig. 8, there is a block or frame containing three pulleys, and having the weight W faſtened to its hook; there is alſo another block faſtened to a ſteady beam, and containing three other pulleys. The ſame rope paſſes through them all, and is faſtened with one end to the upper block, whilſt the power E is applied to its other end. Here it is evident, that in raiſing the weight, the rope muſt be ſhortened at *a*, *b*, *c*, *d*, *e*, and *f*; viz. ſix times as much as the weight is raiſed; and of courſe the power E muſt move with ſix times the velocity of the weight; therefore the equilibrium takes place when E is the ſixth part of W; viz. if W weighs ſix pounds, E needs not weigh more than one pound, in order to balance the weight W; but if E weigh a little more than one pound, then the weight W will be raiſed*. The like reaſoning may be extended to any other number of pulleys.

Fig.

* The circumferences of pulleys are generally grooved, or hollowed, in order to receive and retain the rope. The axis, or centre pin, is ſometimes fixed to the block, and the pulley moves round it; and at other times the axis is fixed in the pulley, and its two ends move in two holes made in the block.

A great degree of friction is the principal defect to which this mechanical power is liable, and which ariſes from three cauſes; viz. from the diameter of the axis bearing a conſiderable proportion to that of the pulley, from the pulley's rubbing againſt the ſides of the block, and from the ropes not being ſufficiently pliable.

The

Fig. 9, of Plate VI. reprefents another variety of this mechanical power. It confifts of one fixt, and one moveable pulley; but in fact each of thofe pulleys performs the office of three pulleys, for it confifts of three grooves of unequal diameters, as is fhewn by the lateral reprefentation of one of them at R. The fame rope which is faftened with one extremity to one of the blocks, paffes fucceffively over the fix grooves, and the power is applied at its other end E.

In order to underftand the action of this conftruction, it muft be confidered, that in the combination of fig. 8, where the pulleys are all of the fame diameter, each pulley muft move fafter than the preceding pulley, becaufe a greater length of rope muft pafs over each pulley than over the preceding pulley, as may be eafily comprehended by infpecting

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The principal contrivances, which have been made for the purpofe of diminifhing thofe caufes of obftruction, will be mentioned in the next chapter.

In the defcription of this mechanical power we have confidered the ropes as acting always perpendicular to the horizon; but when that is not the cafe, as for inftance, it would be, if in fig. 7, Plate VI. the hook S and the pulley F were placed at a greater diftance from each other; then the velocity of the weight is to be eftimated not by the length of the rope which is drawn, but by the perpendicular height to which the weight is raifed. And the fame thing muft be underftood with refpect to the direction of the power.

inſpecting fig. the 7th, where it is evident, that if the weight be raiſed one foot, the ropes muſt be ſhortened of a foot each, viz. a foot from B to A, and another foot from D to C; hence whilſt the length A B paſſes over the pulley B D, twice that length muſt paſs over the pulley F; ſo that the pulley F, if equal in diameter to BD, muſt make two revolutions, whilſt the pulley BD makes one revolution. It is alſo evident, that if the pulley F were of double the circumference, or, which is the ſame thing, of double the diameter of BD, then each of the pulleys would make one revolution in the ſame time. Now returning to the conſtruction of fig. 9, it will be eaſily comprehended, that as the three grooves of the upper pulley, as alſo the three of the lower pulley, belong to one ſolid body, they muſt revolve in the ſame time; therefore, their diameters, or their circumferences, muſt be made in the proportion of the quantity of rope, which muſt paſs over them in the ſame time. Thus whilſt one foot length of the rope paſſes over the firſt groove *a*, two feet of rope muſt paſs over the ſecond groove *b*, three feet of rope muſt paſs over the third groove *c*, and ſo forth. Therefore, the diameter of the ſecond groove *b*, muſt be twice the diameter of the firſt groove *a*; the diameter of the third groove *c*, muſt be three times that of the firſt *a*; the diameter of the fourth groove *d*, muſt be four times that of *a*; &c. or, in other words, the

the diameters of the grooves *a*, *b*, *c*, *d*, *e*, *f*, muſt be in arithmetic progreſſion; the difference of the terms being equal to the diameter of the firſt or ſmalleſt groove.

It is evident, that in this conſtruction, in order to raiſe the weight, ſix ropes muſt be ſhortened, and of courſe the power muſt move through ſix times the ſpace that the weight moves through, conſequently the equilibrium takes place when the power is equal to the ſixth part of the weight W.

The principal advantage which is attributed to this conſtruction, is the reduction of friction; for in this, there are only two axes and four ſurfaces which rub againſt the blocks; whereas in the conſtruction of fig. 8, where the pulleys are all ſeparate, there are ſix axes and 12 ſurfaces which rub againſt the blocks. But, in my opinion, this advantage is more than compenſated by the imperfections which are peculiar to this conſtruction; for, in the firſt place, if the grooves are not made exactly in arithmetic progreſſion, or if they become otherwiſe by the accumulation of dirt, &c. then the rope muſt partly ſlide over them, which will occaſion a conſiderable degree of friction; and ſecondly, even when the grooves are of the proper dimenſions, if the rope happens to ſtretch more in one place than in another, which is generally the caſe, then the above-mentioned ſliding and friction will alſo take place.

THE

THE INCLINED PLANE.

A plane ſuperficies inclined to the horizon, is another mechanical power; its uſe being to raiſe weights from one level to another, by the application of much leſs force than would be neceſſary to raiſe them perpendicularly. Thus in fig. 10, Plate VI. AB repreſents a plane inclined to the horizontal plane AC; where if the weight D be rolled upwards from A to B, the force neceſſary for the purpoſe will be found to be much leſs than that which would be required to raiſe it directly and perpendicularly from C to B.

In this caſe the effect which is produced, conſiſts in the raiſing of the weight from the level of AC to the level of B; but to effect this, the power muſt have moved from A to B; (for the power acts in that direction, whilſt the weight or gravity of the body acts in the direction of the perpendicular CB;) therefore the velocity of the weight in this engine, being to the velocity of the power, as the perpendicular height BC of the plane is to its length AB, the equilibrium takes place when the weight is to the power, as the length of the plane is to its perpendicular height.

This property may be clearly ſhewn by the following experiment:—Let AB, fig. 11, Plate VI. be a plane moveable upon the horizontal plane AC; viz. ſo as to admit of its being placed at any required

quired angle of inclination, which is eafily accomplifhed by means of a hinge at A, and a prop between the two planes. The upper part of the plane muft be furnifhed with a pulley B, over which a ftring may eafily run. Let the cylindrical weight D be made to turn upon flender pins in the frame F, in which the hook *e* is faftened with a ftring *e*BH, which paffing over the pulley B, holds the weight E fufpended at its other extremity.— The pulley fhould be fituated fo that the rope *e*B, may be parallel to the plane.

This plane may be fixed at any angle of inclination, and it will always be found, that if the weight of the body E be to the weight of the body D, together with that of its frame F, as the perpendicular height CB of the plane is to its length AB, the power E will juft fupport the cylinder D, with its frame F upon the plane, and the leaft touch of a finger will caufe the cylinder D to afcend or defcend; the counterpoife or power E moving at the fame time the contrary way.

It is evident, that the fmaller the angle of inclination is, the lefs force is required to draw up the weight D; and of courfe when the angle of inclination vanifhes or becomes nothing, the leaft force will be fufficient to move the body; that is, when the plane AB becomes parallel to the horizon, or upon an horizontal plane, the heavieft body might be moved with the leaft power, were it not for the

friction

friction, which is occasioned by the irregularity of the contiguous surfaces, &c. (1.)

THE

(1.) The above-mentioned explanation of the property of the inclined plane, applies only to one direction of the power; namely when the power acts in a direction parallel to the plane; but the general theory will be found in the following proposition:

When a body or weight W is sustained upon a plane, which is inclined to the horizon; viz. when the power P is just sufficient to balance the weight upon that plane; then the power is to the weight, as the sine of the plane's inclination is to the sine which the direction of the power makes with a line perpendicular to the plane.

Let AB fig. 12, Plate VI. be the plane inclined to the horizon AC, and let a weight at O be supported partly by the plane, and partly by a power which acts in the direction OV. Through O draw EOC perpendicular to AB, and at C, where EC meets the horizontal plane, erect CV perpendicular to the horizon, to meet the direction of the power as at V.

Now the body W, situated at O, is balanced, or kept at rest, by three powers, which (see prop. IV. chap. VIII.) have the same proportion to each other as have the right lines parallel to their respective directions, and terminated by their mutual concurse; namely, by the power which is as OV; by the gravitating power, which is as VC; and by the reaction of the plane, which is as OC; hence the power is to the weight, viz. P : W :: OV : VC; or (since the sides of plane triangles are as the sines of their opposite angles) P : W :: sin. OCV, or BAC: (for those angles are equal since the right-angled triangles BOC, and BAC

 have

THE WEDGE.

The wedge has been juſtly conſidered as a ſpecies of inclined plane; for it conſiſts of two inclined

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have a common angle at B) : ſin. VOE, or VOC; for thoſe angles being the complement of each other to two right angles, have the ſame ſine.

From this propoſition the following corollaries are evidently deduced:

1. Since P : W : : ſin. B A C : ſin. V O C; it will be $P : \frac{1}{\text{ſin. VOC}} :: W : \frac{1}{\text{ſin. BAC}}$; therefore if the weight W, and the inclination of the plane, or ſin. BAC remain the ſame, the power muſt increaſe or decreaſe inverſely as the ſine of VOC; hence when the direction of the power is perpendicular to EC, or parallel to the plane AB, then the ſine of VOC, being the ſine of a right angle, is the greateſt ſine poſſible, and, of courſe in that caſe the power P, which is required to ſuſtain the weight W, is the leaſt poſſible; or, which amounts to the ſame thing, then the greateſt weight may be ſuſtained by a given power. Alſo when the direction of the power coincides with OC, namely when the power acts in a direction perpendicular to the plane, then the angle VOC vaniſhes, and the power muſt be infinitely great.

2. If the direction of the power be parallel to, or coincide with the plane, then the equilibrium takes place when the power is to the weight : : OB : BC : : (Eucl. p. 8. B. VI.) BC : BA; viz. as the elevation of the plane is to its length; or as the ſine of its inclination is to radius.

3. If the direction of the power be OR; that is, parallel to the horizon, then the equilibrium takes place when the power

inclined planes joined baſe to baſe, as ſhewn in fig. 1, Plate VII. where AB or GC is the thickneſs of the wedge at its back, upon which the force or power is applied (be it the ſtroke of a mallet, or any other preſſure); the middle line FD is the axis or height of the wedge; DG and DC are the lengths of its ſlant ſides; and OD is its edge, which is to be forced into the wood or other ſolid; ſince the uſe of this inſtrument is for cleaving of wood, ſtone, and other ſolid ſubſtances; or, in general, for ſeparating any two contiguous ſurfaces.

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power is to the weight :: OR : CR :: (ſince the triangles ORC and BAC are ſimilar) BC : CA; viz. as the elevation of the plane is to its baſe.

4. The power muſt ſuſtain the whole weight, when its direction is perpendicular to the horizon.

5. The power is to the preſſure on the plane :: OV : OC :: ſin. OCV : ſin. OVC :: ſin. BAC : ſin. OVC.

6. The preceding analogy, by alternation, becomes P : ſin. BAC :: preſſ. : ſin. OVC, from which it appears that when the power and the inclination of the plane, or angle BAC, remain invariable, the preſſure on the plane muſt increaſe or decreaſe according as the ſine of OVC increaſes or decreaſes; therefore when the direction of the power is parallel to the baſe, and OVC becomes a right angle, whoſe ſine is the greateſt, then the preſſure on the plane will likewiſe be greateſt.

7. When the direction of the power is parallel to the plane, P : preſſure :: OB : OC :: BC : AC.

8. When the direction of the power is parallel to the baſe AC, then P : preſſure :: OR : OC :: BC : BA.

 Hence,

Hence its application is very extenſive, and in fact, ſciſſars, knives, nails, chiſels, hatchets, &c. are nothing but wedges under different ſhapes.

Strictly ſpeaking, in the geometrical language, a wedge may be called a *triangular priſm*; for it may be conceived to be generated by the motion of a plane triangle in a direction parallel to itſelf, as that of the triangle GCD, from GCD to ABO. And it is called an *iſoſceles* or *ſcalene* wedge, according as the generating triangle, or face, GCD, is *iſoſceles* or *ſcalene.*

The action of the wedge, is evidently derived from that of the inclined plane; yet a variety of circumſtances has rendered the inveſtigation of the power of the wedge more perplexing than that of any other mechanical power *.

The moſt rational theory ſhews, 1. *That when the preſſures on the ſides of the iſoſceles wedge are equal and act in directions perpendicular to thoſe ſides, the equilibrium takes place, when the force on the back of the wedge is to the ſum of the preſſures on the ſides, as* GF, *viz. half the thickneſs of the back, is to either of its ſlant ſides,* GD, *or* CD. 2. *That when the preſ-ſures*

* The proportion between the power, which is applied to the back of the wedge, and the effect which is produced on the ſides, has been ſtated differently by different authors. Thoſe who wiſh to examine the reaſons of thoſe different opinions, may conſult *Rowning's Comp. Syſt. of Phil.* P. I. chap. 10; and *Ludlam's Mathem. Eſſays.*

ſures are equal, but act in directions equally inclined to the ſides of the iſoſceles wedge, the equilibrium takes place when the force on the back is to the ſum of the reſiſtances upon the ſides, as the product of the ſine of half the vertical angle GDC *of the wedge, multiplied by the ſine of the angle which the directions of the reſiſtances make with the ſides, to the ſquare of radius. And* 3. *that when in a ſcalene wedge three forces acting perpendicularly upon its three ſides, keep each other in equilibrio, thoſe three forces are reſpectively proportional to the ſides.*

The three parts of this propoſition will be found demonſtrated in the note (2).

From

(2) In order to demonſtrate the firſt part of the abovementioned propoſition, let AKD, fig. 2, Plate VII. repreſent the face of an iſoſceles wedge. B and E are two obſtacles, which preſs upon its two ſides in directions perpendicular to thoſe ſides. Suppoſe the wedge to be impelled downwards as far as the dotted repreſentation GLF, in conſequence of which the obſtacles B and E muſt be driven to the places O and M. Through O and M draw OI and MQ parallel to the middle line or axis CD of the wedge; which lines will meet thoſe ſides in two points I, Q. Join I, Q, as alſo O, M, with the lines IQ, OM. Then it is evident from the paralleliſm of the lines, that OM is equal to IQ; hence the part IQ of the wedge muſt have advanced as far as OM; therefore YN, or IO, or QM, repreſents the velocity of the wedge (that is of the power); whilſt BO and EM repreſent the velocities of the obſtacles.

 Now

From this it follows that by the addition of a little more force on the back of the wedge, than that which is ſufficient to form the equilibrium, the reſiſtances will be overcome, &c.

It

Now the triangles IOB, and ACD are equiangular; (the angles at C and B being right, and the angle BIO equal to CDA; Eucl. p. 29. and 32. B. I.) and of courſe ſimilar (Eucl. p. 4. B. VI.) therefore conſidering half the wedge and one obſtacle, OB : OI : : AC : AD; that is, the velocity of the obſtacle B is to the velocity of the power, as half the thickneſs of the wedge is to its ſlant ſide. Likewiſe for the ſame reaſons we ſay that the velocity of the preſſing obſtacle E is to that of the power, as half the thickneſs of the wedge is to its ſlant ſide. Therefore, by adding thoſe proportional quantities, we ſay that the velocity of the obſtacle B plus the velocity of the other obſtacle E, is to the velocity of twice the half wedge, (viz. of the whole wedge) as the whole length AK of the back, is to the ſum of the ſides AD, DK; or as half the length of the back is to one ſide.

But when oppoſite powers, which act upon each other, are inverſely as their velocities, they form an equilibrium; therefore when the power on the back of the wedge is to the ſum of the reſiſtances on the ſides, as half the length of the back is to one ſlant ſide, the wedge remains motionleſs, which is the firſt part of the propoſition.

In order to prove the ſecond part of the propoſition; let ABC, fig. 3, Plate VII. be the face of an iſoſceles wedge, HC its height or middle line, E and *e* two obſtacles which preſs upon, or are to be removed, in the directions EF, *ef* equally inclined to the ſides of the wedge. Let the force, repreſented

It alſo appears that the ſmaller the angle GDC is, the leſs force will be required to drive the wedge into any ſolid ſubſtance.

We

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repreſented by the line EF, be reſolved into two other forces; viz. FD parallel, and DE perpendicular, to AC; then the former of thoſe forces, being parallel to the ſide of the wedge, cannot have any power upon it; therefore the original force EF will have juſt the ſame effect upon the wedge as the leſſer perpendicular force DE; the former being to the latter as radius to the ſine of the inclination of the force EF to the ſide AC. But, by the firſt part of this propoſition, this perpendicular force DE is to the power on the back of the wedge which balances it, as AC is to AH, or as radius to the ſine of the angle ACH, (viz. half the angle at the vertex of the wedge) therefore, by compounding thoſe ratios, EF × ED : power on the back × ED : : force EF : power on the back : : ſquare of radius : ſine of half the vertical angle × ſine of the inclination of the reſiſtance.

The oblique force *ef* on the other ſide of the wedge, being equal to EF, will require another power equal to the former on the back of the wedge, to balance it; therefore the ſum of the reſiſtances on the ſides of the wedge is to the whole power on the back, as the ſquare of radius is to the product of the ſine of half the vertical angle multiplied by the ſine of the inclination of the reſiſtances to the ſides of the wedge.

In order to prove the laſt part of the propoſition, let GD, GF, and GE, fig. 4, Plate VII. repreſent the directions of the three forces perpendicular to the ſides of the ſcalene wedge A B C. Produce E G ſtraight towards O, and through F draw FO parallel to DG. Then ſince thoſe three forces balance each other, they muſt be (by prop. IV.

 of

We ſhall laſtly obſerve, that when the wood ſplits below the edge of the wedge, as is ſhewn by fig. 5, Plate VII. which is generally the caſe; then the ſide of the wedge muſt be conſidered as equal to either ſide of the cleft; for in fact if we ſuppoſe that the wedge is lengthened downwards to the very apex of the cleft; the effect will be the ſame.

THE SCREW.

The ſcrew is the laſt mechanical power that remains to be deſcribed. This is likewiſe conſidered as a ſpecies of inclined plane; it being in fact nothing

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of chap. VIII.) reſpectively proportional to the three ſides of the triangle GOF; but this triangle GOF is equiangular, and therefore (Eucl. p. 4. B VI.) ſimilar, to the triangle ABC; therefore the three forces are likewiſe reſpectively proportional to the three ſides of the triangle or wedge ABC.

The triangles GOF and ABC are equiangular; for the four angles of the quatrilateral figure AEGF are equal to four right angles (Eucl. p. 32. B. I.) and ſince the angles at E and F are right, the two angles FAE and FGE muſt be equal to two right angles; that is, equal to FGE plus FGO. Therefore taking away the common angle FGE, there remains the angle FAE equal to FGO. Alſo in the like manner is proved that the angle DBE is equal to the angle OGD, and likewiſe equal (Eucl. p. 29. B. I.) to FOG. And ſince the two angles at O and G of the triangle FGO are reſpectively equal to the two angles at A and B of the triangle ABC; the third angle of the former muſt be equal to the third angle of the latter.

thing more than an inclined plane coiled round a cylinder; and the nut or perforated body which moves up or down a ſcrew, moves up or down an inclined plane in a circular, inſtead of a rectilinear, direction.

Either the ſcrew A may be moved forwards and backwards in a fixed nut as in fig. 6, Plate VII. or the ſcrew A remains fixt, and the nut BC, or perforated piece, is made to move upon the ſcrew as in fig. 7.—By way of diſtinction A is called the *male ſcrew*, and the nut B with its perforation ſhaped like an hollow ſcrew, is called the *female ſcrew.*—The ſpiral projections *e*, *f*, *g*, &c. are called the *threads* of the ſcrew.

The power which moves this moſt uſeful, and moſt powerful engine, is applied either to one end of the ſcrew, which is generally furniſhed with a ſort of head or projection, or to the end of a lever which is fixed either in the head of the ſcrew as in fig. 6. or in the nut BC, as in fig. 7. And then indeed it may with more propriety be called an engine compounded of a ſcrew and a lever.

In all caſes the equilibrium takes place between the effect which is produced at the end of the ſcrew or at the nut, and the power, when the former is to the latter as the circumference deſcribed by the power in one revolution, is to the diſtance between two contiguous threads of the ſcrew. Thus ſuppoſing that the diſtance between the threads be half an inch and the length of the lever CD be 12 inches;

inches; the circle deſcribed by the end D of the lever will be about 76 inches, or 152 times the diſtance between two contiguous threads; therefore if the power at the end D of the lever be equivalent to one pound, it will balance a preſſure of 152 pounds acting againſt the end of the ſcrew in fig. 6; or it will ſupport a weight of 152 pounds on the board B, fig. 7, &c.

The reaſon of this is ſo evidently dependent on the properties of the inclined plane, that nothing more needs here be ſaid about it.

The leaſt reflection on the preceding explanation of the nature and properties of the mechanical powers will ſufficiently prove that, ſtrictly ſpeaking, the real and original mechanical powers are not more than two in number; namely, the lever and the inclined plane; ſo that all the others are only ſpecies of thoſe two; the balance, the wheel and axle, and the pulley, being ſpecies of lever; and the wedge with the ſcrew being ſpecies of inclined plane. It is however immaterial whether thoſe powers be reckoned all primitive and diſtinct from each other, or not; for the theory remains always true and the ſame. The only advantage which might be derived from the idea of the original mechanical powers being only two, is that their properties might, in that caſe, be explained in a much more conciſe manner; yet it is to be obſerved that, after a certain limit, theories became obſcure and perplexing, in in proportion as they are rendered more concise and comprehensive.

Before we quit the present chapter it will be proper to make the following remark, the object of which is to prevent the establishment of wrong notions in the mind of the reader, with respect to the powers of the above-mentioned engines.

Beginners in this branch of natural philosophy frequently imagine that by means of the mechanical powers, a real increase of power is obtained; whereas this is not true. For instance, if a man be just able to convey 100 weight from the bottom to the top of his house in one minute's time, no mechanical engine will enable him to convey 300 weight to the same height in the same time; but the engine will enable him to convey the 300 weight in three minutes; which amounts to the same thing as to say that the man could, without the engine, carry the 300 weight by going three times to the top of the house, and carrying 100 weight at a time, provided the load admitted of its being so divided. Therefore the engine increases the effect of a given force by lengthening the time of the operation; or (since uniform velocity is proportional to the time) by increasing the velocity of that force or power.

Thus again, if any active force is able to raise a weight of 10 pounds with a given velocity, it will be found impossible, by the use of any instrument,

to

to make the ſame force raiſe a weight of 20 pounds, or in general a weight more than 10 pounds, with the ſame velocity; but it may, by the aid of the inſtruments, be made to raiſe the weight of 20 pounds with half that velocity; or, which is the ſame thing, it may be made to raiſe it to half the height in the ſame time; for it is not the power or force, but the momentum, (viz. the product of the force by the velocity) that may be increaſed or diminiſhed by the uſe of thoſe engines.

The power, or acting force, is ſo far from being increaſed by any machine, that a certain part of it is always loſt in overcoming the reſiſtance of mediums, the friction, or other unavoidable imperfections of machines. And this loſs in ſome compound engines is very conſiderable.

CHAPTER XIII.

OF COMPOUND ENGINES; OF THE MOVING POWERS; AND OF FRICTION.

ALL the inſtruments or machines which connect an active force with a certain effect, however complicated they may be, will, upon examination, be found to conſiſt of the already deſcribed mechanical powers. Thoſe component ſimple mechaniſms are frequently varied in ſhape; their connection is infinitely diverſified; but their nature and their properties remain invariably the ſame.

Various are the powers which have been employed as firſt movers of machines; but the principal of them are, 1. The natural ſtrength of a man or number of men. 2. The ſtrength of other animals, and principally of horſes. 3. The force of running water. 4. The force of the wind. 5. The elaſtic force of the ſteam of boiling water. 6. The elaſtic force of ſprings. 7. The ſimple weight of heavy bodies.

A great part of moſt machines relates to the power itſelf; viz. it conſiſts of contrivances neceſſary for the generation, application, preſervation, and renovation, of the active power or force. The effect

effect which is to be produced by that active force, is derived from the proper application of the above-mentioned simple mechanical powers; which divide, or concentrate, or regulate, the original force. As for the effects which are produced by machines, it is impossible to ascertain their number, or even to arrange them under general and comprehensive titles.

The best machine for the production of any particular effect, is that, which (all circumstances of situation, materials, &c. being considered) will produce that effect in the simplest, steadiest, safest, and cheapest manner possible.

It is not my intention to describe the principal machines, that are now in use amongst the enlightened nations of the world; that being incompatible with the nature and the limits of the present work. Those persons who may be desirous of examining the peculiar constructions of the various engines of arts, manufactures, navigation, aeconomy, &c. will find a great variety of books written expressly on the subject, in almost every language of Europe; but in none more so than in the French.

In the present chapter, the methods of computing the powers and the effects of machines in general, will be briefly stated; and the description of a few mechanisms will be inserted merely for the purpose of exemplifying the application of the methods; whence the reader may be enabled to judge

judge of the power and effect of any other machine that may fall under his examination.

A compound engine either consists of one simple mechanical power repeated two or more times; or, it consists of several simple mechanical powers variously combined, and connected with each other. In any case, the power and the effect must be estimated from the result of the effects of all the component simple mechanisms separately considered, which is done in the following manner:

Find what proportion the power bears to the effect in each simple mechanism; put all those ratios one under the other, and find their sum, which sum will express the proportion between the power and the effect of the whole compound engine*. Thus suppose that a machine is compounded of three simple mechanical powers, viz. a lever, an inclined plane, and a moveable pulley; and suppose that a power applied to one end of the lever will produce a double effect at the other end; for instance, one pound will balance two pounds; then the proportion of the power to the effect, is as one to two. Suppose also that in the inclined plane, the power is

* The sum of two or more ratios is obtained by multiplying the antecedents together and the consequents together; and the two products will form a ratio, which is called the sum of the given ratios. Thus the ratio of 2 to 3, plus the ratio of 2 to 5; plus the ratio of 4 to 7, is equal to the ratio of 16 to 105; viz. of $2 \times 2 \times 4$ to $3 \times 5 \times 7$.

is to the effect as three to ſeven ; and laſtly, that in the pulley, the power is to the effect as one to two. Now thoſe three ratios being written one under the other thus and multiplied. viz. all the antecedents together, and all the conſequents

1	:	2
3	:	7
1	:	2
3	:	28

together, the two products thence ariſing will exhibit the proportion which the power bears to the effect in the whole compound engine ; viz. that a power of 3 pounds will balance a weight of 28 pounds.

Otherwiſe the effect of a compound engine may be computed by conſidering the velocities of the power and of the effect; for they are to each other inverſely as their velocities, viz. the power is to the effect as the velocity of the latter is to that of the former. Thus in a certain compound engine I find that the power muſt move through 500 feet, whilſt the weight moves through 3 feet; I therefore conclude that a power of 3 pounds will balance a weight of 500 pounds in that machine, and of courſe a little more than 3 pounds will raiſe the 500 pounds weight.

Fig. 8, Plate VII. repreſents an engine conſiſting of three levers CD, DG, GH, each of which moves round a pin or axis fixed to a ſteady poſt, and are diſpoſed ſo as to act upon each other. Let CA, DE, GK be each one foot long, and AD, EG, KH be each three feet long, then the weight I of one pound will produce a preſſure of three pounds at G on the end of the next lever, and this force will produce an effect equal to 3 times 3, or 9 pounds, on the end D. Laſtly, the end D of the third lever CD being preſſed downwards with a force of 9 pounds, will balance or keep ſuſpended at the oppoſite end C the weight W, of 3 times 9, viz. of 27 pounds. And a little addition of power to the end H will enable the engine to lift up the weight W*.

By increaſing either the number of levers, or the difference of length between the two parts of each lever, the effect at the end C may be increaſed to any degree. But the great defect of this engine is, that the end C with the weight W can be raiſed a very ſhort way.

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* The effect of this compound engine may be calculated according to the preceding rule, by ſetting down the proportion between the power and the effect in each of the three levers; then multiplying the antecedents together and the conſequents together, their products will give the anſwer. Thus ſince the two parts of each lever are as one to three, therefore we have - - 1 : 3

1 : 3

1 : 3

1 : 27

which ſhews, that in order to form the equilibrium, the power I muſt be to the weight W as 1 to 27, the ſame as above.—The ſame thing may be done with other engines.

 Fig.

Fig. 9, Plate VII. represents a combination of four pulleys, three of which are moveable and one is fixt. But this combination must not be reckoned a single mechanical power, because the same rope does not run over all the pulleys. It is, therefore, a repetition of one and the same mechanical power.

Three ropes are fastened to the beam EF, at G, H, and K. The first rope goes round the pulley A, to the block of which the weight W is fastened; and is then tied to the hook of the block of B. The second rope goes round the pulley B, and is fastened to the hook of the block of C. The third rope goes round the pulley C, as also round the fixt pulley D; and holds the counterpoise or power I.

The pulley D being fixt to the beam, does nothing more than change the direction of the motion; therefore if the power I weigh one pound, it will balance a weight of two pounds affixed to the block of the pulley C. Then the pulley C acting with a power of two pounds, will balance a weight of twice two, or of 4 pounds affixed to the block of the pulley B, and this will balance a weight W of twice 4, or 8 pounds, affixed to the block of the pulley A; so that in order to pull up the weight W of 8 pounds, the power I needs be very little heavier than one pound.

This engine is subject to the same inconvenience as the preceding; viz. the weight W can be raised but a very little way.

Fig.

Fig. 10, Plate VII. reprefents a combination of five wheels A, B, C, D, E; each of which turns round a centre-pin, which is fuppofed to be fixed to a fteady frame. Thofe wheels are connected with each other in the following manner: The wheel A has a fmall wheel or *pinion* o faftened to, and concentric with, itfelf. This pinion is furnifhed with teeth, which move between the teeth on the circumference of the next wheel, which is likewife furnifhed with a pinion which acts in a fimilar manner on the next wheel, and fo on, excepting the laft, which has an axle inftead of a pinion, and round this axle a rope is applied, to which the weight W is fufpended.—The power I is applied to the circumference of the firft wheel.

This engine confifts of a repetition of the wheel and axle; for the pinion of each wheel is in fact its axle, excepting that inftead of acting immediately upon the weight by means of a rope, here it exerts its force againft the next wheel by means of its teeth.

Let the circumference of each wheel be equal to five times the circumference of its pinion. Then if a weight I of one pound be fufpended to the circumference of the wheel A, the pinion o will act on the circumference of the fecond wheel with a force equal to five times the power I, viz. equal to 5 pounds, and this force of 5 pounds on the circumference of the fecond wheel will enable its pinion to act on the circumference of the third wheel with a force equal to 5 times 5, viz. 25 pounds. After the ſame manner the force of 25 pounds on the circumference of the third wheel will enable its pinion to act on the circumference of the fourth wheel with a force equal to 5 times 25; viz. 125 pounds, in conſequence of which force applied to the circumference of the fourth wheel, the pinion of that wheel will act on the circumference of the laſt wheel with a force equal to 5 times 125; viz. 625 pounds, which force will balance a weight W of 5 times 625, viz. of 3125 pounds. Therefore the power I of one pound will balance the weight W of 3125 pounds.

Fig. 11, Plate VII. repreſents an engine compounded of a lever, a ſcrew, and a wheel and axle. The lever AB is moved by the application of a hand to the handle A. As the lever AB turns the axis with the ſcrew D, which is all fixed together, the ſcrew D, working into the teeth of the wheel C, will move this round its axis E, in conſequence of which the weight W will be drawn up or let down according as the lever AB is turned one way or the other. Let the power which is communicated by the hand be equivalent to one pound; then if the circumference which is percurred by the handle A, be equal to 100 times the diſtance between two contiguous threads of the ſcrew D, this ſcrew will act on the circumference of the wheel C with a force equal to 100 pounds; and if the diameter of the wheel C be to the diameter of

the

the axle E as 8 to one, then the power of 100 pounds on the circumference of the wheel C will act with a force equal to 8 times 100; viz. of 800 pounds on the circumference of the axle E, about which the rope of the weight W is wound. Therefore it appears that with this engine a weight or power of one pound will balance a weight W of 800 pounds.

A ſcrew, like D, ſituated ſo as only to turn round an axis, but without moving backwards and forwards, and always working on the circumference of a wheel, as C, is uſually called *an endleſs ſcrew.*

Fig. 12, Plate VII. repreſents an improved crane for raiſing of goods or heavy weights. This deſcription has been taken from the appendix to Mr. Ferguſon's Lectures, which I have preferred to other deſcriptions of ſimilar engines; firſt, on account of the improvements it contains, which will naturally ſhew that a variety of collateral objects muſt be kept in view by the contrivers of ſuch machines; and ſecondly, for the purpoſe of making the reader acquainted with the meaning of the principal terms that are uſed in mechanics.

A is the great wheel of this engine, and B its axle, on which the rope C winds. This rope goes over a pulley D in the end of the arm of the gib * E, and draws up the weight F, as the

* *Gib*, a projecting tranſverſe beam.

winch* G is turned round. H is the largeſt trundle†, I the next, and K is the axis of the ſmalleſt trundle, which is ſuppoſed to be hid from view by the upright ſupporter L. A trundle M is turned by the great wheel, and on the axis of this trundle is fixed the ratchet-wheel ‡ N, into the teeth of which the catch O falls. P is the lever, from which goes a rope QQ, over a pulley R, to the catch; one end of the rope being fixed to the lever, and the other end to the catch. S is an elaſtic bar of wood, one end of which is ſcrewed to the floor; and, from the other end goes a rope (out of ſight in the figure) to the farther end of the lever, beyond the pin or axis on which it turns in the upright ſupporter T. The uſe of this bar is to keep up the lever and prevent its rubbing againſt the edge of the wheel

* *Winch* or *winder*, an inſtrument with a crooked handle, the uſe of which is to turn any thing round.

† A ſmall wheel, which is turned round by the teeth of a large wheel, which derives various denominations from its various ſhapes. It is called pinion when it is oblong, and the teeth are longer than the inſide ſolid part; the teeth are then called the *leaves* of the *pinion*. When the ſmall wheel is ſhaped like that which is repreſented at H, it is then called a *trundle*, or ſometimes a *lantern*, and even a *drum*.

‡ *A ratchet-wheel* or *ratchet*, is a wheel generally having its teeth bent one way, wherein a ſolid piece, called a *catch* or *click*, falls, by which means the ratchet-wheel, when the catch bears upon it, can turn one way only, but not the contrary way.

V, and

V, and to let the catch keep in the teeth of the ratchet-wheel. But a weight hung to the farther end of the lever would do full as well as the elaſtic bar and rope.

When the lever is pulled down it lifts the catch out of the ratchet-wheel, by means of the rope QQ, and gives the weight F liberty to deſcend; but if the lever P be pulled a little farther down than what is ſufficient to lift the catch O out of the ratchet-wheel N, it will rub againſt the edge of the wheel V, and thereby hinder the too quick deſcent of the weight, and will quite ſtop the weight, if pulled hard. And if the man who pulls the lever ſhould inadvertently let it go, the elaſtic bar will ſuddenly pull it up, and the catch will fall down and ſtop the machine.

WW are two upright rollers above the axis or upper gudgeon * of the gib E. Their uſe is to let the rope C bend upon them, as the gib is turned to either ſide, in order to bring the weight over the place where it is intended to be let down.

N.B. The rollers ought to be ſo placed, that if the rope C be ſtretched cloſe by their utmoſt ſides, the half thickneſs of the rope may be perpendicularly over the centre of the upper gudgeon of the gib. For then, and in no other poſition of the

* The pins or extremities of an axle, which pins move in holes, &c. are called *gudgeons* in large works, and *pevets* or *pivots* in ſmall works.

rollers, the length of the rope between the pulley in the gib, and the axle of the great wheel, will be always the ſame, in all poſitions of the gib, and the gib will remain in any poſition to which it is turned.

When either of the trundles is not turned by the winch in working the crane, it may be drawn off from the wheel, after the pin near the axis of the trundle is drawn out, and the thick piece of wood is raiſed a little behind the outward ſupporter of the axis of the trundle. But this is not material; for, as the trundle has no friction on its axis but what is occaſioned by its weight, it will be turned by the wheel without any ſenſible reſiſtance in working the crane.

This engine is to be ſituated in a room with the gib E projecting out of it, ſo that the load may be raiſed from the ſtreet or other lower ſituation, by turning the winches of the trundles as at G.

This crane has four different powers. The three trundles H, I, K, are furniſhed with different numbers of ſtaves*; the largeſt has 24 ſtaves, the next 12, and the ſmalleſt 6. The great wheel A has 96 cogs †; therefore the largeſt trundle makes four revolutions for one revolution of the wheel; the next makes 8, and the ſmalleſt makes 16. A

* The ſticks or cylindrical bars of trundles, which perform the office of teeth, are called *ſtaves*.

† *Cogs* are the wooden teeth of a large wheel.

winch

winch G is occaſionally put upon the axis of either of theſe trundles, for turning it; that trundle being then uſed which gives a power beſt ſuited to the weight. The length of the winch is ſuch, that in every revolution its handle deſcribes a circle equal to twice the circumference of the axle B of the wheel. So that the length of the winch doubles the power gained by each trundle.

If the winch be put upon the axle of the largeſt trundle, and turned four times round, the wheel and axle will be turned once round; and the circle deſcribed by the power that turns the winch, being in each revolution double the circumference of the axle, when the thickneſs of the rope is added thereto, the power goes round 8 times as much ſpace as the weight riſes through; and therefore (making ſome allowance for friction) a man will raiſe 8 times as much weight by the crane as he would by his natural ſtrength without it; the power, in this caſe, being to the weight as 8 to 1.

If the winch be put upon the axis of the next trundle, the power will be to the weight as 16 to 1, becauſe it moves 16 times as faſt as the weight moves.

If the winch be put upon the axis of the ſmalleſt trundle, and turned round, the power will be to the weight as 32 to 1.

But, if the weight ſhould be too great even for this power to raiſe, the power may be doubled by drawing up the weight by one of the parts of a double

double rope, going under a pulley in the moveable block which is hooked to the weight below the arm of the gib. And ſuch is the actual repreſentation of the figure. Then the power will be to the weight as 64 to 1. Whilſt the weight is drawing up, the ratch-teeth of the wheel N ſlip round below the catch or click that falls ſucceſſively into them, and thus hinders the crane from turning backward, or detains the weight in any part of its aſcent, if the man, who works at the winch, ſhould accidently quit his hold, or chooſe to reſt himſelf before the weight be quite drawn up.

In order to let down the weight, a man pulls down the end Z of the lever, which lifts the catch out of the ratchet-wheel, and gives the weight liberty to deſcend. But, if the deſcent be too quick, he pulls the lever a little farther down, ſo as to make it rub againſt the outer edge of the round wheel V, by which means he lets down the weight as ſlowly as he pleaſes; and by pulling a little harder, he may ſtop the weight, if neceſſary, in any part of its deſcent. If he accidentally quits hold of the lever, the catch immediately falls, and ſtops both the weight and the whole machine.

In the conſtruction of machines in general, the queſtions which occur in the firſt place, relate to the choice of the power, and to the eſtimation of its quantity; viz. whether the force of wind or water, or of a man, &c. ſhould be preferred, and what

what is the value or quantity of any one of thofe powers?

Of the principal active powers which have been enumerated at the beginning of this chapter, we fhall briefly mention the common eftimation of their forces; but we fhall take more particular notice of the force of wind, of water, and of fteam, when we come to treat of the properties of air, of the fteam of water, &c.

The power which can be applied as the firft mover of a machine, in the eafieft manner, and whofe action is moft uniform, is the fimple weight, fuch as is applied to clocks, jacks, and other machines; but this fort of power requires to be renewed after a certain period; that is, it muft be wound up, or raifed, on which account it is moftly ufed for flow movements; efpecially when a very regular action is required.

The force of running water, and that of the wind, where the fituation of the place admits of their being ufed, are very powerful and advantageous movers of machines, fuch as mills, pumps, fawing engines, &c.—They may be applied to the working of the greateft engines. Running water is preferable to wind, on account of its acting with much more conftancy and uniformity.

The fteam of boiling water is likewife a moft powerful agent; and the recent improvements which have been made by feveral ingenious mechanics in this country have extended the application of it

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from the ſmalleſt to the largeſt engines. The application of this power requires a very nice conſtruction of the mechaniſm, and is attended with a conſiderable conſumption of fuel, which particulars are not to be obtained in every ſituation.

A ſpring is likewiſe a uſeful and commodious moving power; but a ſpring, like the weight, requires to be wound, or ſet up, after a certain time; viz. when it is quite unbent; on which account it is more commonly uſed for ſlow movements, ſuch as watches, table clocks, &c. But this ſort of power differs from the weight in a very remarkable circumſtance; which is, that its action is never uniform. It is ſtrongeſt when moſt bent, and it decreaſes in proportion as it unbends.

In order to remedy this defect, and to render the action of a ſpring uniform and effectual, a curious contrivance has been long in uſe, and is as follows: An hollow groove of a ſpiral form is made round a ſolid piece of metal, ſuch as is repreſented at fig. 13, Plate VII. which is furniſhed with an axis AB, round which it turns in the frame of the machine, and is connected with a wheel *g*, whoſe teeth act upon the other wheels of the machine. This ſpiral piece is called the *fuſee*, and ſerves to render the action of the ſpring equable or uniform. It is connected with the ſpring by means of a ſtring or chain F, one end of which is faſtened to the ſpring which is not ſeen in the figure, and the other end is faſtened to the loweſt part *d* of the ſpiral groove.

When

When the fuſee is turned ſo as to wind the ſtring or chain upon it, the ſpring is thereby ſet up, or bent, and when afterwards the machine is left to itſelf, the force of the ſpring will, by pulling the chain or ſtring, force the fuſee to turn round its axis in a direction contrary to that in which it was wound up. Now when the ſtring bears upon the ſmalleſt part *c* of the fuſee; viz. neareſt to the axis where a greater force is required to produce a certain effect, the ſpring pulls the chain with its greateſt force, becauſe it is then bent moſt; whereas when the ſtring bears upon the lower and larger part of the fuſee, where leſs force is required to produce the above-mentioned effect, there the ſpring pulls the ſtring or chain with leſs force, becauſe then it is bent leſs. Therefore the decreaſing force of the ſpring is compenſated by the increaſe of power with which the ſtring or chain acts on the axis AB; hence the teeth of the wheel *g* act always with the ſame degree of force upon the next wheel; and thus the motion of the mechaniſm is rendered uniform. This mechaniſm will be found almoſt univerſally applied to pocket watches and ſpring clocks.

The natural ſtrength of living animals is the laſt power that remains to be taken notice of; and here we ſhall not extend our obſervations beyond the force of men and horſes, concerning which the following particulars are deſerving of notice. The different writers on mechanics do not quite agree

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in the eſtimation of the mean ſtrength of a man; nor is it likely they ſhould, conſidering how the conſtitution of men varies according to the difference of climate, of nouriſhment, and of other circumſtances. Upon the whole, a man of ordinary ſtrength is reckoned capable of raiſing a weight of 600 pounds avoirdupoiſe ten feet high in one minute, and to be able to work at that rate for 10 hours out of 24; or to do any other work that may be equivalent to it. I am however inclined to think that this eſtimation is rather above than below the real fact.

By means of a judicious application of the human ſtrength, the effect may in ſome caſes be increaſed, and on the other hand an improper application of it will diminiſh the effect. Thus if two men work at a windlaſs, or axle, by means of handles or levers, they will be able to draw a weight of 70 pounds more eaſily than one man can a weight of 30 pounds, provided the handles or levers are at right angles to each other.

A man is able to draw horizontally not above 70 or 80 pounds; for in that caſe he can only employ half the weight of his body.

N. B. Here it is not to be underſtood that a man cannot draw a cart or carriage that weighs more than 80 pounds; for the weight of the cart is ſupported by the ground; but we mean, that a man will not be able to draw ſuch a cart as will require more than 80 pounds to move it along.

If a man weigh about 140 pounds, he can exert no greater force in thrusting horizontally at a height even with his shoulders, than what is equal to 27 pounds.

A horse, in general, is reckoned capable of doing as much work as five men.

A horse draws with the greatest advantage when the line of direction is a little elevated above the horizon; and the power acts against its breast.

A horse is reckoned capable of drawing against a resistance of 200 pounds at the rate of 2½ miles an hour, and to continue that exertion for eight hours out of 24.

The strength of the horse, like that of a man, may be rendered more or less efficacious, by means of a proper or improper application of it. In going up a steep ascent, five men can carry a much greater weight than a horse. And in certain ascents, men will be able to carry some weight, where a horse will not be able to carry himself. In mills and other machines, where the circular motion of a horse is employed, the diameter of the circular walk should not be less than 25 or 30 feet; otherwise, the motion is neither very advantageous, nor pleasant to the animal. With respect to the quantity of power, it must be observed, that in the practical application of a moving power to a machine of any sort, it is not enough to employ a power which is barely sufficient to overcome the obstacle, or to produce the effect; but such a power

power muſt be applied as will produce the deſired effect in the moſt advantageous manner poſſible; for inſtance if a power of a pound, applied to a machine, will produce a certain effect in one hour; whereas if a power of two pounds were applied to the ſame machine, it would produce the like effect in 20 minutes, it is evident, that the application of the latter power would be more advantageous than of the former; for though the latter power be double the former, yet the time of its performing the operation is leſs than half the time of the former powers performing the ſame operation. *So that the moſt advantageous power for moving a machine is that, which being multiplied by the time of performing a determined effect, produces the leaſt product**.

With reſpect to friction, two objects muſt be obſerved; viz. the loſs of power which is occaſioned by it, and the contrivances which have been made, and are in uſe, for the purpoſe of diminiſhing its effects.

A body upon an horizontal plane ſhould be capable of being moved by the application of the leaſt force; but this is not the caſe; and the principal cauſes which render a greater or leſs quantity

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* For a farther inveſtigation of the moſt advantageous application of powers to machines, ſee Graveſand's Mat. Elem. of Nat. Phil. B. I. chap. 21, and the following ſcholia; alſo almoſt all the writers on mechanics.

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of force neceſſary for it, are, 1ſt, the roughneſs of the contiguous ſurfaces; 2dly, the irregularity of the figure, which ariſes either from the imperfect workmanſhip, or from the preſſure of one body upon the other; 3dly, an adheſion or attraction which is more or leſs powerful according to the nature of the bodies in queſtion; and 4thly, the interpoſition of extraneous bodies; ſuch as moiſture, duſt, &c.

Innumerable experiments have been made for the purpoſe of determining the quantity of obſtruction, or of friction, which is produced in particular circumſtances*. But the reſults of apparently ſimilar experiments, which have been made by different experimenters, do not agree; nor is it likely they ſhould, ſince the leaſt difference of ſmoothneſs or poliſh, or of hardneſs, or in ſhort of any of the various concurring circumſtances, produces a different reſult. Hence no certain and determinate rules can be laid down with reſpect to the ſubject of friction.

If a body be laid upon another body, and ſoon after be moved along the ſurface of it, a leſſer force will be found ſufficient for the purpoſe, than if the body be left ſome time at reſt before it be moved. This ariſes principally from an actual

* See Mr. Coulomb's Eſſay in the tenth vol. of the Mémoires des Savants Etrangers. And M. de Prony's Architecture Hydraulique, § 1089, and following.

change of figure, which is produced in a longer or ſhorter time according to the nature of the bodies. Thus the maximum of adheſion between wood and wood takes place in a few minutes' time ; between metal and metal it takes place almoſt immediately. A hard and heavy body laid upon a ſofter one will ſometimes continue to increaſe its adheſion for days and weeks.

When a cubic foot of ſoft wood of eight pounds weight is to be moved upon a ſmooth horizontal plane of ſoft wood, at the rate of three feet per ſecond, the power which is neceſſary to move it, and which is equivalent to the friction, amounts to between $\frac{1}{4}$ and $\frac{1}{3}$ of the weight of the cube.— When the wood is hard the friction amounts to between $\frac{1}{7}$ and $\frac{1}{8}$ of the weight of the cube.

In general the ſofter or the rougher the bodies are, the greater is their friction. Yet when two pieces of metal, extremely well poliſhed, are laid one upon the other with an ample ſurface of contact, they adhere to each other much more forcibly than when they are not ſo well poliſhed.

Iron or ſteel moves eaſieſt in braſs. Other metals, acting againſt each other, produce more friction.

The friction, *cæteris paribus*, increaſes with the weight of the ſuperincumbent body, and almoſt in the ſame proportion.

The friction or obſtruction which ariſes from the bending of ropes about machines, is influenced by by a variety of circumſtances, ſuch as their peculiar quality, the temperature of the atmoſphere, and the diameter or curvature of the ſurface to which they are to be adopted. But when other circumſtances remain the ſame, the difficulty of bending a rope increaſes with the ſquare of its diameter, as alſo with its tenſion; and it decreaſes according as the radius of the curvature of the body to which it is adapted, increaſes.

Of the ſimple mechanical powers the lever is the leaſt ſubject to friction.

In a wheel, the friction upon the axis is, as the weight that lies upon it, as the diameter of the axis, and as the velocity of the motion. But upon the whole, this ſort of friction is not very great, provided the machine be well executed.—In common pulleys, eſpecially thoſe of a ſmall ſize, the friction is very great. It increaſes in proportion as the diameter of the axis increaſes, as the velocity increaſes, and as the diameter of the pulley decreaſes. With a moveable tackle, or block, of five pulleys, a power of 150 pounds will barely be able to draw up a weight of 500 pounds.

The ſcrew is ſubject to a great deal of friction; ſo much ſo that the power which muſt be applied to it, in order to produce a given effect, is at leaſt double that which is given by the calculation independent of friction. But the degree of friction in the ſcrew is influenced conſiderably by the nature of the conſtruction; for much of it is owing to the tightneſs of the ſcrew, to the diſtance between its threads, and to the ſhape of the threads; the ſquare threads, like thoſe of fig. 14, Plate VII. producing upon the whole leſs friction than thoſe which are ſharp, as in the figures 6 and 7 of the ſame plate.

The friction which attends the uſe of the wedge, exceeds, in general, that of any other ſimple mechanical power. Its quantity depends ſo much upon the nature of the body upon which the wedge acts, beſides other circumſtances, that it is impoſſible to give even an approximate eſtimate of it.

The friction of mechanical engines does not only diminiſh the effect, or, which is the ſame thing, occaſion a loſs of power; but is attended with the corroſion and wear of the principal parts of the machine, beſides producing a conſiderable degree of heat, and even actual fire; it is therefore of great importance in mechanics, to contrive means capable of diminiſhing, if not of quite removing, the effects of friction.

In compound engines, the obſtruction which ariſes from friction can be aſcertained only by means of actual experiments. An allowance, indeed, may be made for each ſimple component mechanical power; but the error in eſtimating the friction of any one ſingle power is multiplied and increaſed ſo faſt by the other parts, that the eſtimate generally turns out very erroneous. Beſides, much depends on the execution of the work; the quality

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of which cannot be learned but by experience. Novices are generally apt to expect too much or too little from any mechanifm.—In general it can only be faid, that in compound engines, at leaft one-third of the power is loft on account of the friction.

The methods of obtaining the important object of diminifhing the friction, are of two forts, viz. either by the interpofition of particular unctuous or oily fubftances between the contiguous moving parts; or by particular mechanical contrivances.

Olive-oil is the beft, and perhaps the only fubftance that can be ufed in fmall works, as in watches and clocks, when metal works againft metal. But in large works the oil is liable to drain off, unlefs fome method be adopted to confine it. Therefore for large works tallow is moftly ufed, or greafe of any fort; which is ufeful for metal, as well as for wood. In the laft cafe tar is alfo frequently ufed.

In delicate works of wood, viz. when a piece of wood is to flide into or over wood, and when a wooden axis is to turn into wood, the fine powder of what is commonly called *black-lead*, when interpofed between the parts, eafes the motion confiderably, and is at the fame time a clean and durable fubftance

Though olive-oil be the beft and the only fubftance that is ufed for delicate mechanifms; yet it is far

far from being free from objections. Oil, when in contact with braſs, is liable to grow rancid, in which ſtate it ſlowly corrodes the braſs. In different temperatures it becomes more or leſs fluid; but upon the whole it grows continually thicker, and of courſe leſs fit to eaſe the motion of the parts, &c. Trifling as thoſe defects may at firſt ſight appear, they are however of ſuch moment in delicate works, that in the greatly improved ſtate to which watch-work has been brought in this country, the changeable quality of the oil ſeems at preſent to be the principal, if not the only, impediment to the perfection of chronometers.

The mechanical contrivances which have been made, and are in uſe, for the purpoſe of diminiſhing the effects of friction, conſiſt either in avoiding the contact of ſuch bodies as produce much friction, or in the interpoſition of rollers, viz. cylindrical bodies, between the moving parts of machines, or between moving bodies in general. Such cylinders derive, from their various ſize and application, the different names of *rollers*, *friction-wheels*, and *friction-rollers*.

Thus in mill-work and other large machines, the wooden axes of large wheels terminate in iron gudgeons, which turn in wood, or more frequently in iron or braſs, which conſtruction produces leſs friction than the turning of wood in wood. In the fineſt ſort of watch-work the holes are jewelled, viz. many of the pivots of the wheels, &c. move in holes

holes made in rubies, or topazes, or other hard ſtone, which when well finiſhed are not liable to wear, nor do they require much oil.

In order to underſtand the nature of rollers, and the advantage with which their uſe is attended, it muſt be conſidered, that when a body is dragged over the ſurface of another body, the inequalities of the ſurfaces of both bodies meet and oppoſe each other, which is the principal cauſe of the friction or obſtruction; but when one body, ſuch as a caſk, a cylinder, or a ball, is rolled upon another body, the ſurface of the roller is not rubbed againſt the other body, but is only ſucceſſively applied to, or laid on, the other; and is then ſucceſſively lifted up from it. Therefore, in rolling, the principal cauſe of friction is avoided, beſides other advantages; hence a body may be rolled upon another body, when the ſhape admits of it, with incomparably leſs exertion than that which is required to drag it over the ſurface of that other body. In fact we commonly ſee large pieces of timber, and enormous blocks of ſtone, moved upon rollers, that are laid between them and the ground, with eaſe and ſafety; when it would be almoſt impoſſible to move them otherwiſe.

The form and diſpoſition of friction-wheels is repreſented by fig. 1. Plate VIII. which exhibits a front view of the axis *d* of a large wheel, which moves between the friction-wheels A, B, C. Here the end *d* of the axis (and the ſame thing muſt be understood of the opposite extremity of the axis) instead of moving in a hole, moves between the circumferences of three wheels, each of which is moveable upon its own axis, and is unconnected with the others. Now if the end *d* of the axis turned in a hole, the surface of the hole would stand still, and the surface of the axis would rub against it; whereas when the axis moves between the circumferences of the wheels A, B, C, its surface does not rub against, but is successively applied to the circumferences of those wheels; so that this sort of motion has the same advantage over the turning of the axis in a hole, that the moving of a heavy body upon rollers has over the simple method of dragging it upon the ground. In this construction the contact of the axis *d*, moves the wheels A, B, C, round their axes, where indeed some friction must unavoidably take place, but that friction is very trifling; for if the circumference of the axis *d* be to that of each wheel as one to 20, the axis must make 20 revolutions whilst the friction-wheels will turn round once only.

A few years ago the same principle was applied in a very ingenious manner, by Mr. John Garnett then of Bristol, to pulleys, and other sorts of circular motion round an axis, for which he obtained a patent. The use of this application has proved very advantageous, especially on board of ships, where it has been found, that with a set of Mr. Garnett's pulleys, three men were able to draw as much

much weight as five men were barely able to accomplish with a similar set of common pulleys.

One of those pulleys is represented by fig. 2, Plate VIII. where the shaded part BBB is the pulley, A is the axis, and *c*, *c*, *c*, *c*, *c*, *c*, are the cylindrical rollers, which are situated between the axis and the inside cavity of the pulley. The ends of the axis A, are fixed in a block, after the usual manner. Every one of the rollers has an axis, the extremities of which turn in holes made in two brass or iron flat rings, one of which is visible in the figure.

After having given a general explanation of the action of rollers, the advantage which Mr. Garnett's pulleys must have over those of the common sort, needs no farther illustration. I shall however only observe, that the friction of the pivots of each roller in the holes of the brass rings is very inconsiderable; for those holes are made rather large, the use of the axes to the rollers being only to prevent their running one against the other. Nor does the addition of weight upon the pulley increase that friction; for the addition of weight upon the pulley will press the rollers harder upon the axis A; but not upon their own axes, as may be easily understood by inspecting the figure.

CHAPTER XIV.

DESCRIPTION OF THE PRINCIPAL MACHINES, WHICH ARE NECESSARY TO ILLUSTRATE THE DOCTRINE OF MOTION, AND OF THEIR PARTICULAR USE.

THE doctrine of motion in all its extensive branches is derived, as we have already shewn, from a few general principles; and its application to particular circumstances requires only the knowledge of a few experimental facts, such as the natural descent of bodies towards the earth, the time of vibration of a pendulum of a determinate length, &c. Then whatever relates to other complicated movements may be derived, by means of strict and unequivocal reasoning, from those few principles, and few ascertained facts or natural laws. Yet notwithstanding the assent which a rational being must give to the clear and evident demonstrations that are derived from those principles, it must be allowed that an experimental confirmation of any theoretical proposition never fails to impress the mind with a pleasing, lasting, and satisfactory conviction. Though the unavoidable imperfections of machines render the result of experiments seldom so accurate as to coincide exactly with

with the theory, yet when the error is not very great, and at the ſame time it ſeems to be proportionate to the imperfections of the mechanical conſtruction and operation, the mind of the obſerver will always feel itſelf ſufficiently ſatisfied.

In this chapter the reader will find the deſcription of the principal machines which have been contrived for the purpoſe of confirming in an experimental manner the propoſitions which relate to motion.

The ſpace deſcribed, and the acceleration gained, by bodies which deſcend freely towards the earth has been often attempted to be proved by means of direct experiments; but the reſiſtance of the air which oppoſes a conſiderable and fluctuating impediment, and the difficulty of meaſuring the time of deſcent when falling bodies have acquired a great degree of velocity, which ſoon increaſes beyond the power of our ſenſes to eſtimate, have always rendered the reſult of ſuch experiments precarious and unſatisfactory. * But we are indebted to Mr. Atwood,

* Dr. Deſaguliers obſerved the time that a leaden-ball, of two inches in diameter, employed in deſcending from reſt through 272 feet; that is, from the inſide of the cupola of St. Paul's cathedral, wherein the experiment was tried, to the floor; and found it to be 4,5 ſeconds, whereas it ſhould have been 4,1 ſeconds; for in 4,5 ſeconds it ought to have deſcended through ſomewhat more than 325 feet. See his Courſe of Experimental Philoſophy.

Atwood, F.R.S. for a very curious machine of his contrivance, which obviates the abovementioned impediments, and exhibits the phenomena of accelerated and retarded motion in a commodious and ſatisfactory manner; or, in Mr. Atwood's words, "which will ſubject to experimental examination "the properties of the five mechanical quantities; "that is, the quantity of matter moved, the con-"ſtant force which moves it, the ſpace deſcribed "from reſt, the time of deſcription, and the velo-"city acquired."

The repreſentation of this machine in fig. 3, Plate VIII. is divided into two parts for the conveniency of the plate, which however can make no difference with reſpect to the explanation; for the reader needs only imagine that theſe two parts are placed one upon the other, and are joined at the places which are indicated by the ſame letters, E F G H, ſo as to form one entire figure.

The foot or pedeſtal of this machine is in the form of a croſs, with adjuſting ſcrews, which ſerve to ſet the machine in a ſteady and perpendicular ſituation. A ſtrong wooden pillar XGT, about five feet high, is firmly fixed upon the pedeſtal, and ſupports the wooden ſtage VD, which is ſecured upon it by means of the ſcrew at D. Upon this ſtage there is another ſtage or ſtand, to which the wheel-apparatus is fixed. This apparatus conſiſts of a braſs wheel *a*, *b*, *c*, whoſe ſteel horizontal axis moves upon four friction-wheels; viz. one end of the axis

reſts

reſts upon the circumferences of two friction-wheels, and the other end reſts on the circumferences of the two others. The axes of the four friction-wheels are ſupported by, and move in, holes made in the braſs frame which is faſtened to the upper ſtage, and whoſe ſhape is ſufficiently indicated by the figure.

There is a groove all round the circumference of the wheel *a*, *b*, *c*, for the reception of a fine flexible ſilk line, at the extremities of which the bodies A, B are ſuſpended. By this means the motion of the wheel *a*, *b*, *c*, with the ſilk line is rendered ſo very free, that when the bodies A and B are equal, if one of them be gently impelled upwards or downwards, both bodies will readily move in contrary directions; the friction of the axis being almoſt entirely removed by the application of the friction-wheels.

KEL is a ſcale or rod, divided into inches and tenths; and is ſo ſituated that one of the bodies, viz. A, may move very near the ſurface of it. C and I are two little ſtages, either of which may be fixed, by means of the lateral ſcrew M or N, on any part of the ſcale LEK. The former of thoſe ſtages ſerves to ſtop the body A, when deſcending, at any required height. The latter ſtage has a perforation ſufficiently large to permit the free paſſage of the body A; but its uſe is to ſupport occaſionally a weight in the form of a bar, like the one ſeen upon it.—The perſpective repreſentation of this ſtage I, is is conſiderably ſtrained for the purpoſe of rendering its conſtruction more intelligible.

Upon the pillar of this machine there is adapted a ſimple ſort of time-piece, conſiſting of a pendulum which vibrates ſeconds, and is kept in motion for a few minutes by means of a wheel and weight O. On the axis of the wheel there is a hand or index, which indicates the number of ſeconds on the dial Z. The uſe of this time-piece is to ſhew by the beats of the pendulum, the time which is employed by the body A in aſcending or deſcending through a given ſpace *.

The uſeful property of this machine is to diminiſh the force which acts upon, and occaſions the deſcent of bodies, in conſequence of which a body will deſcend much ſlower; hence the obſerver will be enabled to perceive the ſpace it moves through, as alſo its acceleration in a given time, &c. in a clear and commodious manner. I ſhall endeavour

* It hardly needs be obſerved that thoſe who have a common clock, that beats ſeconds, may have the machine conſtructed without the laſt-deſcribed appendage. Beſides this, I ſhall juſt mention that the abovementioned machine has been improved, or rather altered, by ſome philoſophical inſtrument makers; but as thoſe alterations are not of great importance either with reſpect to its conſtruction or to its performance, I have preferred Mr. Atwood's original conſtruction; ſuch as is deſcribed in his very valuable treatiſe on the rectilinear motion, and rotation of bodies.

to

to render the explanation of this property more intelligible, previously to the narration of the experiments.

When the weights or bodies A and B are exactly equal, they will balance each other, and of course will remain at rest. But if a body of little weight be added to one of those bodies, as for instance to A, then A will preponderate, and consequently will descend; the opposite weight B ascending at the same time. Now in this case both the bodies A, B, and the wheels are put in motion by the gravity of the small additional body; so that the sum of all those bodies, being moved by a smaller force, must move through a shorter space in a given time, than if the force were greater.

For instance, imagine that the weights A and B, together with the weight which is required to put the wheels in motion (which is equivalent to the *inertia* of the wheels) amounts to 4 ounces, and let the weight of the body which is added to A be half an ounce, then it is evident that a mass of matter of $4\frac{1}{2}$ ounces is put in motion by the gravity of a body of half an ounce; that is by the gravity of a body equal to the 9th part of the matter which is put in motion, which amounts to the same thing as if that mass of matter were attracted by the earth with the ninth part of its ordinary attraction. But it has been shewn in page 64, that the space which is described in a given time by a descending body, is proportionate to the force of gravity;

gravity; therefore if in the natural way a body descends from rest through 16,087 feet in the first second of time, in the above-stated circumstance the body A will descend through the ninth part of 16,087 feet, viz. through 21,4 inches, in the first second of time. Thus by adding a smaller weight to the body A, that body may be made to move as slowly as the observer pleases.

The other properties of descending bodies remain unaltered by this machine. Thus the spaces which are described by the descending body A will be found to be as the squares of the times; that is, if A describe 21,4 inches in the first second of time, it will describe 4 times 21,4 inches in the second second of time, 9 times 21,4 inches in the third, &c. Thus much may suffice with respect to the principal effect of this machine. I shall now add Mr. Atwood's computation, and general mode of conducting the experiments.

In the first place he ascertained the inertia of the wheels when the silk line with the bodies A, B was removed, and found it equivalent to 2¾ ounces (1).

"The

(1.) Having removed the weights A and B, with their silk line, Mr. A. affixed a weight of 30 grains to a silk line (the weight of which was not so much as ¼ of a grain, and consequently too inconsiderable to have any sensible effect in the experiment); this line being wound round the wheel *a b c*, the weight of 30 grains, by descending from rest, communicated

"The resistance to motion, therefore, arising from the wheel's inertia will be the same as if they were absolutely removed, and a mass of $2\frac{3}{4}$ ounces were uniformly accumulated in the circumference of the wheel *a b c*. This being premised, let the boxes A and B be replaced, being suspended by the silk line over the wheel *a b c*, and balancing each other.

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municated motion to the wheel, and by many trials it was observed to describe a space of about 38,5 inches in three seconds. From these data the inertia of the wheels may be determined in the following manner:

If the weight of 30 grains had descended through 9 times 193 inches in three seconds, as it would have done by itself, the inertia of the wheels would have amounted to nothing; but since it moved through 38,5 inches in three seconds, its retardation was occasioned by the inertia of the wheels. Let the quantity of this inertia be called x; then the attractive force of the earth upon the mass $x + 30$ must be less than upon the body of 30 grains alone; therefore $x + 30$ descends slower than the body of 30 grains would by itself; or, properly speaking, the spaces which are described in the same time, are inversely as the masses; for the quantity of force being the same, the effect upon $x + 30$ must be as much less than the effect upon 30, as 30 is less than $x + 30$; hence in the present experiment the space described by the body of 30 grains in three seconds, is to the space described by $x + 30$ in the same time, as $x + 30 : 30$, viz. $9 \times 193 : 38{,}5 :: x + 30 : 30$; therefore $x + 30 = \frac{30 \times 9 \times 193}{38{,}5} = 1353{,}5$ grains, and $x = 1323{,}5$ grains, or $2\frac{3}{4}$ ounces.

"To proceed in describing the construction of the ensuing experiments. In order to avoid troublesome computations in adjusting the quantities of matter moved, and the moving forces, some determinate weight of convenient magnitude may be assumed as a standard, to which all others are referred. This standard weight in the subsequent experiments is ¼ of an ounce, and is represented by the letter *m*. The inertia of the wheels being therefore equal to 2¾ ounces, will be denoted by 11*m*. A and B are two boxes constructed so as to contain different quantities of matter, according as the experiment may require them to be varied: the weight of each box, including the hook to which it is suspended, is equal to 1½ oz. or 6*m*: these boxes contain such weights as are represented by Q, each of which weighs an ounce, or 4*m*: other weights of ½ an ounce = 2*m*, ¼ = *m*, and aliquot parts of *m*, may also be included in the boxes, according to the conditions of the different experiments.

"If 4¾ oz. or 19*m*, be included in either box, this with the weight of the box itself will be 25*m*; so that when the weights A and B, each being 25*m*, are balanced in the manner above represented, their whole mass will be 50*m*, which being added to the inertia of the wheels, 11*m*, the sum will be 61*m*. Moreover, three circular weights, such as that which is represented by Y, are constructed; each of which is equal to ¼ oz. or *m*: if one of these be be added to A and one to B, the whole maſs will now become 63 *m*, perfectly in equilibrio, and moveable by the leaſt weight added to either (ſetting aſide the effects of friction) in the ſame manner preciſely as if the ſame weight or force were applied to communicate motion to the maſs 63 *m*, exiſting in free ſpace and without gravity."

OF THE MOVING FORCE.

"Since the natural weight or gravity of any given ſubſtance is conſtant, and the exact quantity of it eaſily eſtimated, it will be convenient in the ſubſequent experiments to apply a weight to the maſs A, as a moving force: thus when the ſyſtem conſiſts of a maſs = 63*m*, according to the preceding deſcription, the whole being perfectly balanced, let a weight of $\frac{1}{4}$ oz. or *m*, ſuch as Y, be applied to the maſs A, this will communicate motion to the whole ſyſtem." But ſince now the whole maſs is 64, and the moving force is the gravity of one of thoſe parts only; "therefore the force which accelerates the deſcent of A, is $\frac{1}{64}$ part of the accelerating force by which bodies deſcend freely towards the earth's ſurface."

Thus by varying the weights, the moving force may be altered without altering the maſs; or the moving force may be made to be in any required ratio to the maſs.

OF THE SPACE DESCRIBED.

" The method of estimating practically the space described from quiescence, is next to be considered. The body A descends in a vertical line, and a scale of about 64 inches in length, graduated into inches and tenths of an inch, is adjusted vertically, and so placed that the descending weight A may fall in the middle of the square stage, fixed to receive it at the end of the descent; the beginning of the descent is estimated from o on the scale, when the bottom of the box A is on a level with o. The descent of A is terminated when the bottom of the box strikes the stage, which may be fixed at different distances from the point o, so that by altering the position of the stage, the space described from quiescence may be of any given magnitude less than 64 inches."

CONCERNING THE TIME OF MOTION.

" The time of motion is observed by the beats of the pendulum which vibrates seconds: and the experiments intended to illustrate the elementary propositions may be easily so constructed, that the time of motion shall be a whole number of seconds; the estimation of the time therefore admits of considerable exactness, provided the observer take care to let the bottom of the box A begin its descent precisely at any beat of the pendulum; then

then the coincidence of the ſtroke of the box againſt the ſtage, and the beat of the pendulum at the end of the time of motion, will ſhew how nearly the experiment and the theory agree together. There might be various mechanical devices thought of for letting the weight A begin its deſcent at the inſtant of a beat of the pendulum; but the following method may perhaps be ſufficient: let the bottom of the box A, when at o on the ſcale, reſt on a flat rod held in the hand horizontally, its extremity being coincident with o; by attending to the beats of the pendulum, and with a little practice, the rod which ſupports the box A, may be removed at the inſtant the pendulum beats, ſo that the deſcent of A ſhall commence at the ſame inſtant."

OF THE VELOCITY ACQUIRED.

"It remains only to deſcribe in what manner the velocity acquired by the deſcending weight A, at any given point of the ſpace through which it has deſcended, is made evident to the ſenſes. The velocity of A's deſcent being continually accelerated, will be the ſame in no two points of the ſpace deſcribed: this is occaſioned by the conſtant action of the moving force; and ſince the velocity of A at any inſtant is meaſured by the ſpace which would be deſcribed by it, moving uniformly for a given time with the velocity it had acquired at that

that inſtant, this meaſure cannot be experimentally obtained, except by removing the force by which the deſcending body's acceleration was cauſed."

"In order to ſhew in what manner this is effected practically, let us ſuppoſe that, according to a former example, the boxes A and B = 25*m* each, ſo as together to be = 50*m*; this with the wheels inertia 11*m* will make 61*m*: now let *m* be added to A, and an equal weight *m* to B, thoſe bodies will balance each other, and the whole maſs will be 63*m*. If a weight *m* be added to A, motion will be communicated, the moving force being *m*, and the maſs moved 64*m*. In a former example, the circular weight, equal *m*, was made uſe of as a moving force; but for the preſent purpoſe of ſhewing the velocity acquired, it will be convenient to uſe a flat rod (like that which is ſhewn at I on the perforated ſtage) the weight of which is alſo equal to *m*. Let the bottom of the box A be placed on a level with o on the ſcale, the whole maſs being as deſcribed above, 63*m*, perfectly balanced in equilibrio. Now let the rod, the weight of which = *m*, be placed on the upper ſurface of A; this body will deſcend along the ſcale preciſely in the ſame manner as when the moving force *m* was applied in the form of a circular weight. Suppoſe the maſs A to have deſcended by conſtant acceleration of the force *m*, for any given time, or through a given ſpace: let the perforated ſtage

be

be ſo affixed to the ſcale contiguous to which the weight deſcends, that A may paſs centrally through it, and that this perforated ſtage may intercept the rod *m*, by which the body A has been accelerated from quieſcence. After the moving force *m* has been intercepted at the end of the given ſpace or time, there will be no force operating on any part of the ſyſtem, which can either accelerate or retard its motion ; this being the caſe, the weight A, the inſtant after *m* has been removed, muſt proceed uniformly with the velocity which it had acquired that inſtant : in the ſubſequent part of its deſcent, the velocity being uniform will be meaſured by the ſpace deſcribed in any convenient number of ſeconds."

OF RETARDED MOTION.

"The motion of bodies reſiſted by conſtant forces are reduced to experiment by means of the inſtrument above deſcribed, with as great eaſe and preciſion as the properties of bodies uniformly accelerated. A ſingle inſtance will be ſufficient : thus ſuppoſe the maſs contained in the weights A and B, and the wheels, to be 61*m*, when perfectly in equilibrio, as in a former example ; let a circular weight *m* be applied to B, and let two long weights or rods, each equal to *m*, be applied to A, then will A deſcend by the action of the moving force *m*, the maſs moved being 64*m* : ſuppoſe that when it it has described any given space by constant acceleration, the two rods *m* are intercepted by the perforated stage, while A is descending through it; the velocity acquired by that descent is known, and when the two rods are intercepted, the weight A will begin to move on with the velocity acquired, being now retarded by the constant force *m*; and since the mass moved is 62*m*, it follows, that the force of retardation will be $\frac{1}{62}$ part of the force whereby gravity retards bodies thrown perpendicularly upwards. The weight A will therefore proceed along the graduated scale in its descent with an uniformly retarded motion, and the spaces described, times of motion, and velocities destroyed by the resisting force, will be subject to the same measures as in the examples of accelerated motion above described."

Besides those properties, Mr. Atwood's machine may be easily adapted to other uses, such as the experimental estimation of the velocities communicated by the impact of bodies elastic and non-elastic; the quantity of resistance occasioned by fluids, &c. Mr. Atwood also shews its use in verifying practically the properties of rotatory motion*.

After the preceding sufficiently ample description of the general mode of using this instrument,

* See his Treatise on Motion, Sect. VIII.

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we ſhall by way of example ſubjoin three or four experiments, and ſhall leave its further application to particular caſes, for the exerciſe of the reader's ingenuity. It is however neceſſary, in the firſt place, to obviate ſome doubts which may naturally occur with reſpect to the performance of this machine; the accuracy of which may be diſturbed by three cauſes, viz. the friction of the axes of the wheels, the weight of the ſilk line, and the reſiſtance of the air.

"The effects of friction are almoſt wholly removed by the friction-wheels; for when the ſurfaces are well poliſhed and free from duſt, &c. if the weights A and B be balanced in perfect equilibrio, and the whole maſs conſiſts of 63*m*, according to the example already deſcribed, a weight of 1 ½ grain, or at moſt 2 grains, being added either to A or B, will communicate motion to the whole, which ſhews that the effects of friction will not be ſo great as a weight of 1 ½ or 2 grains. In ſome caſes, however, eſpecially in experiments relating to retarded motion, the effects of friction become ſenſible; but may be very readily and exactly removed by adding a ſmall weight of 1 ½ or 2 grains to the deſcending body, taking care that the weight added is ſuch as is in the leaſt degree ſmaller than that which is juſt ſufficient to ſet the whole in motion, when A and B are equal, and balance each other, before the moving force is applied."

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The silk line by which the weights are suspended is 72 inches long, and weighs about 3 grains: a quantity too small to affect sensibly the result of the experiments.

The effect of the resistance of the air is likewise insensible; for that resistance increases with the velocity, and in the experiments which are performed with this machine, the greatest velocity communicated to the bodies A and B, cannot much exceed that of about 26 inches in a second.

Experiment 1st. Let A and B, together with the $2\frac{3}{4}$ oz. (which are equal to the inertia of the wheels), amount to 16 oz. or $63m$; then add a weight of $\frac{1}{4}$ oz. that is m, to A, and A will descend and will describe from rest three inches in the first second; so that if the square stage be fixed even with the 3 inches on the scale, and A be permitted to descend from 0 on the scale just when the pendulum strikes, it will be found that exactly when the pendulum strikes the next stroke, the body A will strike against the stage. If the experiment be repeated with this variation only, viz. with the stage fixed even with the 12 inches on the scale, then the weight A will strike the stage exactly when the pendulum strikes the second stroke after the commencement of A's motion. And if the stage be fixed even with the 27 inches, the stroke of A on the stage will coincide with the third stroke of the pendulum; and so on.

Here

Here it is evident that the quantity of matter in motion is repreſented by 64 parts, and this quantity is put in motion by the gravity of one of thoſe parts; therefore the moving force being the 64th part of what the earth would otherwiſe exert upon the whole maſs, this maſs muſt move through the 64th part of that ſpace which, if deſcending freely, it would move through in the ſame time. But in the natural way deſcending bodies paſs through 16,087 feet, or 193 inches, in the firſt ſecond; therefore in this experiment the body A muſt deſcend through the 64th part of 193, viz. 3 inches nearly. In two ſeconds it muſt deſcend through 4 times 3, or 12 inches; in three ſeconds it muſt deſcend through 9 times 3, or 27 inches, &c. the ſpaces being as the ſquares of the times.

Experiment 2d. If the weight of A and B, together with the inertia of the wheels, be made equal to 62*m*, and a weight of 2*m* be added to A, then the whole maſs in motion will be 64*m*, and the moving force 2*m*, viz. $\frac{1}{32}$ of the maſs; therefore in the firſt ſecond of time A will be found to deſcend through a ſpace equal to the 32d part of 193, viz. 6 inches. In two ſeconds it will deſcend through four times 6, viz. 24 inches, and ſo on.—Thus the force may be varied at pleaſure, and the ſpace deſcribed by the deſcending body in a given time will be found proportionate to the force.

Experiment 3d. Let the quantity of matter be 63*m*, as in the firſt experiment; add a bar of the weight *m* to A, and place the perforated ſtage even with

with the 12 inches, and let the weight A commence its descent when its upper surface is even with the o on the scale. It will be found that in two seconds the bar on the body A will strike against, and remain on, the perforated stage, after which the body A, not being any longer acted upon by any accelerative force, will continue to descend with an equable motion, and will describe (according to the law which has been mentioned and proved in page 67.) a space equal to twice the above-mentioned descent in the same time, that is, 24 inches in 2 seconds. Thus the degree of velocity acquired after any other descent may be proved experimentally.

Experiment 4th. Let A be equal to $24\frac{1}{2}m$, and B equal $25\frac{1}{2}m$, and apply to the upper surface of A two rods, each of which is equal m, then will the weight A preponderate and descend by the action of a moving force equal to m; the whole mass moved being equal to $63m$. Fix the perforated stage at 26,44; then the weight A by descending from rest through 26,44 inches, will acquire a velocity equal to 18 inches per second: (viz. the square root of $\frac{4 \times 193 \times 26{,}44}{63}$) and at that instant the two rods, each of which is equal to m, being intercepted by the stage, the body A will continue to descend with an uniformly retarded motion; which will be precisely the same as if a mass of $61m$, without gravity, were projected with a vea velocity of 18 inches in a ſecond in free ſpace, and a force or reſiſtance equal to *m* were oppoſed to its motion; wherefore A (with the other parts of the ſyſtem) will loſe its motion gradually, and will deſcribe a ſpace equal to 25,6 inches (that is, $\frac{18 \times 18 \times 61}{4 \times 193}$) before its motion is entirely deſtroyed: A will therefore be obſerved in the experiment to deſcend as low as 52 inches, before it begins to aſcend by the ſuperior weight of B.

The next machine we ſhall deſcribe is called a *whirling-table*, and its uſe is for ſhewing, in an experimental way, the nature and properties of centripetal and centrifugal forces.

The machine itſelf is exhibited by fig. 1, Plate IX. and the apparatus is repreſented by the numbers 1, 2, 3, &c. adjoining to it*.

Upon the ſteady table *f f f*, the two ſtrong pillars *e*, *e* are immoveably fixed, which are alſo ſteadily ſcrewed to the croſs piece *a b*. Within this frame the two upright hollow axes are ſituated ſo that each of them may turn with a pointed pin in a hole on the table, and with its upper extremity through a hole in the croſs piece *a b*. The lower part of

* Whirling tables have been varied more or leſs in ſhape and ſize by almoſt all the different makers of thoſe inſtruments. That which I have preferred has conſiderable advantage in point of ſimplicity and durability. This machine was contrived and made by Mr. J. B. Haas.

each

each axis is immoveably connected with a doubly grooved wheel or pulley KH, CY parallel to the table. The grooved wheel B turns alſo parallel to the table, round a ſtrong pin or axis which is fixed to the table; and a catgut-ſtring is diſpoſed round the wheel B, and round the large or ſmall circumference of the wheel at the bottom of each axis, in the manner which is clearly indicated by the figure. In this diſpoſition it is eaſy to conceive that by applying the hand to the handle at A, and turning the wheel B, both the axes will be cauſed to turn round. A ſocket, or tube I, I is connected with a circular braſs plate FG, ED, and ſlides freely up and down each axis. From the inſide of each of thoſe tubes or ſockets a wire paſſes through an oblong ſlit, and projects within the cavity of the axis, where it is ſhaped like a hook; ſo that a ſtring may be tied to this hook, which paſſing upwards through the aperture of the axis, may be pulled or let down in ſuch a manner as to let the plate and ſocket move up or down the axis. Upon thoſe plates, ſemicircular leaden weights *o o* may be placed occaſionally.—Thoſe weights, being perforated, are ſlipped over two wires which proceed from the plate E D, or F G, as they are indicated by the figure; by which means the weights are prevented from falling off.

To the upper part of each axis (viz. to the part of it which projects above the croſs piece *ab*) a variety of different mechaniſms may be occaſionally

ally ſcrewed ſo faſt as to turn with the axis when the machine is in action.

The oblong pieces which are repreſented in the figure as being actually fixed to the axes, are called *bearers.*—Their conſtruction being exactly the ſame, we need deſcribe only one of them.

A perforated braſs plate with a ſtrong ſcrew which fits the ſcrew at the top of the axis, is fixed in the middle of the bearer M L; ſo that when the bearer is ſcrewed to the axis, the hole in it communicates with the cavity of the axis. On one ſide of this hole, a perpendicular projection T riſes above the ſurface of the bearer, and a ſimilar projection riſes above the end L of the bearer, which is on the other ſide of the central hole. Two ſmooth, ſtrong, and parallel wires are ſtretched between thoſe two projections by means of the ſcrew-nuts at W. A cylindrical heavy body V is perforated with two longitudinal holes, through which the abovementioned wires paſs, ſo that the body may be freely moved backwards and forwards upon thoſe wires. On that ſide of the cylinder V, which lies towards T, there is a hook, to which a ſtring is faſtened. This ſtring paſſes through a hole in the projection T; after which it goes round the grooved pulley S, which moves round an axis in an upright frame *h*, fixed to the bearer, and whoſe ſituation is ſuch that the ſtring in its deſcent at T, may paſs through the middle of the hole in the bearer, and of the cavity of the axis, ſo as to

be

be fastened with its extremity to the hook of the wire which proceeds from the socket of the plate E D. After this description it is easy to understand, that the cylinder V and the plate E D are connected together by means of the string, and that if V be drawn towards W, in which situation it appears in the figure, the plate E D with its superincumbent weights will be pulled towards the upper part of the axis; otherwise the weight of the plate E D will draw the cylinder V towards T, and will itself descend towards the lower part of the axis.

Either of those bearers may be removed from, and one of the following mechanisms may be screwed fast upon, the axis.

No. 1. represents a circular board, turned upside down, having a strong screw in its middle, which fits the screw at the top of either axis of the machine. There is a hole through the middle of this board and of its screw, which opens the communication with the cavity of the axis. But this hole in the middle of the board may be occasionally filled up with a piece of wood in the form of a stopple, which is furnished with a short pin, that, when the piece of wood is fixed in the hole, projects a little above the surface of the board.

No. 2. is an oblong bearer, which may be screwed, like any of the others, upon one of the axes of the machine. It has an upright projection at each end, and a strong and smooth wire is stretched be-

tween

tween thoſe projections by means of the ſcrew nuts, A, B. C and D are two perforated braſs balls, of unequal weights, which are connected together by means of a braſs tube, and are freely moveable upon the wire AB from one end to the other. On the outſide of the braſs tube which connects the two balls, there is fixed, exactly at the common centre of gravity of thoſe balls, a ſhort wire E as an index, which ſerves to ſhew when the common centre of gravity of thoſe balls is placed exactly againſt the middle of the bearer.

No. 3. repreſents a board having at its lower end C, a ſcrew which fits the ſcrew at the top of one of the axes of the machine, upon which it may be firmly ſcrewed; but this ſcrew is ſituated a little aſlant to the board, ſo that when placed upon the axis of the machine this board may ſtand inclined to the horizon, making an angle of 30 or 40 degrees with it.

On the upper ſide of this board are fixed two glaſs tubes, A G and B F, cloſe ſtopped at both ends; and each tube is about three-quarters full of water. In the tube B F is a little quickſilver, which, in conſequence of its weight, remains under the water at the end B. In the other tube A G is a piece of cork, which being lighter than water, floats upon it towards the end G, and is ſo ſmall as not to ſtick faſt within the cavity of the tube.

No. 4. is an axis or ſtrong wire fixed to a board, and having a ſcrew at its lower part beneath the board, which fits the ſcrew-hole at the top of one of the axes of the whirling table. Two circular braſs hoops, AC, BD, made very thin and pliable, ſoldered to each other, and ſoldered or ſcrewed to the axis, at I, have each a hole at the upper part through which the axis paſſes freely; ſo that if a hand be applied to the upper part E of thoſe hoops, they may be flattened down as far as the pin O, which is ſeen acroſs the axis. In this caſe the hoops will change their circular form into an elliptical one; but, being elaſtic, they will reſume their circular form as ſoon as the preſſure is removed.

No. 5. repreſents a hemiſphere, which is to be ſituated upon the board No. 1, when that is fixed upon one of the axes of the machine, in the following manner: A pin with a ſcrew *e* (which is not fixed to the hemiſphere) is ſcrewed in the middle of the board ſo as to project a little above it, and the hemiſphere A is laid upon it, there being a cavity *d i* on the flat part of the hemiſphere made on purpoſe to lodge the pin; but this cavity is an oblong groove, as is pretty well indicated by the dotted line on the figure of the hemiſphere; and it is made ſo that the hemiſphere by ſliding over the pin the whole length of that groove, may be placed either concentric with the board, or out of centre with it. The lateral wire C, with the ſmall ball B, may be ſcrewed occaſionally on the

ſide

ſide of the hemiſphere A, in a direction oppoſite to that of the abovementioned groove.

No. 6. repreſents a forked wire with a ſcrew at its lower part, which fits the ſcrew in the middle of the board No. 1. This fork ſerves to ſupport the wire C with the two unequal balls A and B; but this wire being in no way connected with the forked wire, muſt be balanced upon it, that is, it muſt be laid with the common centre of gravity upon the fork; which is eaſily done by trial.

No. 7. repreſents a ball of about two inches in diameter, having a hole from ſide to ſide, through which a wire BA paſſes quite freely. This wire out of the ball at A is ſhaped like the letter T, each of whoſe projections is longer than the radius of the ball, and has a blunt termination. On the other ſide B the wire terminates in a ring, to which a ſtring is tied.—We ſhall now proceed to deſcribe the experiments which are to be made with the whirling table and its apparatus.

Experiment 1. Fix the board No. 1. upon one of the axes of the machine, and put the piece of wood or ſtopper with the pin, in the middle of it. Take the ball apparatus, No. 7, make a loop on the end C of the ſtring, taking care that the length CA be not greater than the radius of the board. Put the loop of the ſtring over the pin in the middle of the board, and leave the ball upon the board. Then apply a hand at A, and turn the wheel B of the machine, which will give the board a whirling motion. It will be found that the ball does not immediately begin to move with the board; but, on account of its *inertia*, it endeavours to continue in its ſtate of reſt, in which it ſtood before the machine was put in motion. But, by means of the friction on the board, that *inertia* is gradually overcome; ſo that by continuing to whirl the board, the ball's motion will become equal to that of the board; after which the ball will remain upon the ſame part of the board, it being then relatively at reſt upon the board. But if you ſtop the board ſuddenly, by applying a hand to it, the ball will be found to go on in virtue of its *inertia*, and continue to revolve, until the friction of the board, by gradually diminiſhing its velocity, finally ſtops it. This ſhews that matter is as incapable of ſtopping itſelf when in motion, as it is incapable of moving itſelf from a ſtate of reſt.

Experiment 2. Remove the piece of wood with the pin from the middle of the board. Inſtead of the ſtring with the loop, put a longer ſtring to the ring B of the ball No. 7. Let this ſtring down through the hole in the middle of the board, and through the cavity of the axis, and faſten it to the ring of the wire which proceeds from the ſocket of the plate FG; the weight of which will draw the ball towards the centre of the board. Care muſt be had to let the ſtring be of ſuch a length as that when the plate FG is quite down, the ring B of

B of No. 7. be about an inch from the hole in the middle of the circular board. The weight of the plate FG muſt be very little more than ſufficient to draw the ball to the abovementioned ſituation; for which purpoſe the leaden weights muſt be removed from over the plate FG, its own weight alone being ſufficient for the purpoſe.

Having placed the ball ſo that the ring B may be about one inch diſtant from the centre of the board, put the machine in motion by turning the wheel B; and it will be found that the ball by going round and round with the board, will gradually fly off to a greater and greater diſtance from the centre of the board, raiſing up the plate FG at the ſame time; which ſhews that all bodies which revolve in circles, have a tendency to fly off, ſo that a certain power from the centre muſt act upon them in order to prevent their flying off. If the machine be ſtopped ſuddenly, the ball will continue to revolve for ſome time longer; but the friction of the board gradually diminiſhing its velocity, its tendency to fly off will alſo decreaſe, and the weight of the plate FG will gradually draw it nearer and nearer the centre, until its motion ceaſes entirely.

Experiment 3d. Let the apparatus remain as in the preceding experiment, excepting only that the ſtring, being diſengaged from the plate FG muſt be let out of the ſlit in the axis, and the operator muſt hold its extremity in his hand. With his other hand the operator muſt throw the ball upon the circular board as it were in a direction perpendicular to the ſtring, by which means the ball will make ſeveral revolutions upon the board (the machine being in this experiment at reſt). But if whilſt the ball is revolving you gradually pull the lower end of the ſtring below the board, you will find that the ball, in proportion as it comes nearer to the centre of motion, and of courſe it performs its revolutions in ſmaller circles, will revolve faſter; which ſhews, as far as ſuch a machine can do it, that the ſame moving force will enable a revolving body to deſcribe a circular orbit faſter when the circle is ſmaller, and ſlower when the circle is larger. (See chap. VIII.)

Experiment 4th. Remove the circular board, and inſtead of it, put the bearer on the axis; ſo that both the bearers may be upon the machine, as is repreſented in the figure.

Let the cylinders R V, be of equal weights; place equal weights upon the plates FG, ED; and adjuſt the lengths of the ſtrings which connect thoſe cylinders with thoſe plates, ſo that when the plates are quite down the cylinders may ſtand at equal but ſmall diſtances from the centres of their reſpective bearers. The catgut ſtring muſt be put either over both the large, or over both the ſmall circular grooves at the bottom of the axes (on which account it is neceſſary to have two catgut ſtrings, viz. one longer than the other) then put the machine

machine in motion, and the cylinders R, V will be ſeen to recede from the centres of the bearers, and to advance towards the ends X and W, raiſing at the ſame time, and to an equal height, the plates FG, ED. This experiment proves that when equal bodies revolve in equal circles, with equal velocities, their centrifugal forces are equal.

Experiment 5th. Inſtead of the cylinder R, place another cylinder of half its weight, viz. equal to half the weight of V, on the wires of the bearer PN; adjuſt the ſtrings ſo that when the plates FG, ED are quite down, the diſtance of the cylinder V from the centre of the bearer ML may be half the diſtance of the other cylinder from the centre of its bearer, which is eaſily ſhewn by the diviſions which are marked upon the bearers; and leave the reſt of the apparatus as in the preceding experiment. Now when the machine is put in motion, there will be two bodies revolving, one of which is half the weight of the other, but the former revolves in a circle which is as large again as the circle in which that other revolves. And it will be found that the equal weights of the plates FG, ED will be equally raiſed; which ſhews that the centrifugal forces of the revolving bodies (which raiſe the plates FG ED) are equal as long as the products of the bodies multiplied each by its velocity, viz. the momentums, are equal.

The proportion of the weights of the two bodies may be varied at pleaſure; but the ſtrings muſt be adjusted so that their distances from the centres of the bearers may be inversely as those weights; and the plates FG, ED, which are loaded with equal weights, will always be lifted to equal heights; the products of the bodies by their respective velocities being always equal.

Experiment 6th. Repeat the preceding experiment, with this difference, that the cylinders be left at equal distances from the centres of their respective bearers; also that the weights on the plates FG, ED be in the proportion of the weights of the cylinders, viz. when V weighs as much again as R, the weight of the plate ED must be double the weight of the plate FG, &c. On putting the machine in motion it will be found that the plates FG, ED are raised to the same height; which proves that when revolving bodies move with the same velocity, their centrifugal forces are proportionate to their respective quantities of matter.

Experiment 7th. Put cylinders of equal weights on the wires of the bearers PN, ML. Place the catgut string round the wheel B, the wheel Y, and round the small wheel H, which is exactly the disposition represented by the figure. Also, adjust the strings between the cylinders and the axes, so that when the plates FG, ED are quite down, the cylinders may lie at equal distances from the centres of the bearers. Farther, if the circumference of the wheel Y be equal to twice that of the wheel H, you must put four times as much weight on the

the plate FG, as upon the plate ED. If the circumference of the wheel Y be equal to three times that of the wheel H, you muſt put nine times as much weight on FG as upon ED. In ſhort the weights on the plates muſt be inverſely as the ſquares of the circumferences of the wheels Y and H. On putting the machine in motion it will be found that the plates FG, ED are raiſed to an equal height; which ſhews that when equal bodies revolve in equal circles with unequal velocities, their centrifugal forces are as the ſquares of the velocities.

Experiment 8th. Let the catgut remain in the ſame ſituation as in the laſt experiment, and let the circumference of the wheel Y be to that of the wheel H as two to one (in which proportion the circumferences of thoſe wheels of whirling machines are generally conſtructed). Alſo let the cylinders R and V be of equal weights, but adjuſt the ſtrings ſo that when the plates FG, ED are quite down, the diſtance of the cylinder R from the centre of the bearer PN be two inches, whilſt that of the cylinder V from the centre of the bearer ML be $3\frac{1}{6}$ inches*. The circumference of the wheel Y being equal to twice

* Inſtead of inches, the diſtances may be of any other denomination; provided they be in that proportion. The bearers are generally divided into equal parts which are longer than inches; ſo that the diſtances may be made equal to 2 and $3\frac{1}{6}$ of thoſe diviſions.

the

the circumference of the wheel H, it follows that when the machine is put in motion, V muſt make one revolution whilſt R makes two; therefore their periodical times are as one to two, and the ſquares of thoſe times are 1 and 4; the former of which is contained four times in the latter. But the diſtance of R is 2, the cube of which is 8; and the diſtance of V is $3\frac{1}{6}$, the cube of which is 32 nearly, in which 8 is contained 4 times; therefore the ſquares of the periodical times are as the cubes of the diſtances. Now let the weight of the plate ED be 4 ounces, equal to the ſquare of the diſtance 2; and the weight of the plate FG be 10 ounces, nearly equal to the ſquare of the diſtance $3\frac{1}{6}$; then on turning the wheel B, which will put the axes in motion, it will be found that the plates FG, ED are raiſed to an equal height.

This experiment proves that when equal bodies revolve in unequal circles, and the ſquares of the times of their going round are as the cubes of their diſtances from the centres of the circles, then their centrifugal forces are inverſely as the ſquares of their diſtances.

Experiment 9th. Remove one of the bearers from the machine, and place the mechaniſm No. 3, upon the axis. (See the deſcription of this mechaniſm in p. 305.) On turning the wheel B, it will be found that the contents of the glaſs tubes AG, BF will, in conſequence of their centrifugal forces, run towards the outward and uppermoſt ends of of thofe tubes. And fince with equal velocities the heavieft bodies have the greateft centrifugal force, therefore the quickfilver in the tube BF will go quite to the end F of the tube; its weight being greater than that of an equal bulk of water; but in the tube AG the piece of cork will be found at the bottom of the water; the water being much heavier than an equal bulk of cork.

Experiment 10th. Remove the apparatus No. 3, and place No. 4. upon the axis. On putting the machine in motion, the upper part E of the hoops will defcend towards the pin O, and the quicker the machine is whirled, the nearer will the hoops come to the pin, their middle parts receding at the fame time from the axis; fo as to affume an elliptical form. This effect arifes from the different centrifugal forces of the different parts of thofe hoops; the centrifugal forces of thofe parts which are farther from the axis of motion being greater than of thofe which are nearer to it.

It appears therefore, that when globular bodies, whofe matter is fufficiently yielding, revolve round their axes, their figure cannot be perfectly fpherical, but it is that of an *oblate fpheroid.*

Experiment 11th. Remove the preceding apparatus from the axis of the whirling table; place the board No. 1. upon it; fix the pin *e* of the machine No. 5. in the middle of the board, with the hemifphere A upon it, but without the wire C. If the hemifphere A be placed concentric with the board,

board, on whirling the machine, the hemiſphere will be found to remain in its place upon the board; the centre of gravity of the hemiſphere coinciding with the centre of motion. But if the centre of the hemiſphere be placed a little on one ſide of the centre of motion, then on whirling the machine, the larger portion of the hemiſphere, which lies on one ſide of the centre of motion, will acquire a greater centifugal force, and conſequently will draw the hemiſphere that way; ſo that the pin ſliding through the groove *d i*, the hemiſphere will at laſt be found with the part *d* upon the pin.

If the wire C with the little body B be ſcrewed on the ſide of the hemiſphere, and the latter be placed upon the pin, concentric with the board; on whirling the machine, the ſame thing as the laſt mentioned effect will take place; for though the hemiſphere be placed concentric with the board, yet when the body B is affixed to it, their common centre of gravity is different from the centre of gravity of the hemiſphere alone.

Experiment 12th. Nearly the ſame thing is ſhewn by means of the apparatus No. 2. For when this is ſcrewed upon the axis of the whirling table, (the preceding mechaniſm being removed) if the index E, viz. the centre of gravity of the two bodies C,D, be placed exactly over the centre of the bearer AB, the whirling of the machine will not move the ſaid bodies upon the wire AB; but if the centre of gravity E be placed ever ſo little

little on one ſide of the centre of motion, on whirling the machine, the two bodies will move towards that ſide, as far as the upright projection A or B.

Experiment 13th. The ſame thing may be ſhewn by means of the mechaniſm No. 6. Place the circular board No. 1. upon one of the axes of the whirling machine; fix the forked wire D, of No. 6. in the middle of it; balance the bodies A,B, with their connecting wire, upon the fork; then put the machine in motion by turning the wheel B, and the bodies A,B will remain balanced upon the fork and will turn with it.

A vaſt variety of machines have been invented for the purpoſe of illuſtrating other branches of the doctrine of motion and equilibrium; but as the propoſitions which relate to ſuch other branches are very eaſy and evident, the particular deſcription of thoſe machines would render the work voluminous, without proving of much advantage to the reader. I ſhall therefore only add a ſhort account of the manner of ſhewing, by means of pendulums, the principal phenomena which attend the colliſion of bodies; and the deſcription of a machine which ſerves to ſhew a few of the caſes, which relate to the compoſition and reſolution of forces, with which this chapter will be concluded.

The phenomena which attend the direct colliſion of bodies, viz. when their centres of gravity lie in the direction of their motion, may be very commodiouſly exhibited by means of pendulums, ſuch are represented in the figures 4, 5, 6 of Plate VIII; for if one of the pendulums as A in fig. 5, be removed to a certain distance from the perpendicular, as to the situation BC, and be then let go, the impulse which its ball gives to the next pendulum D, will force the latter to move from its state of rest, and to describe a certain arch, which will be longer or shorter according to the quantity of matter of the body which is struck, and according to the momentum of the striking body, which momentum may be increased or diminished by elevating the striking body to a greater or lesser angle, and by varying its weight. The pendulums may also be made to strike against each other after having been both put in motion, either the same way or contrary ways.

The effects of the collision, viz. the directions of the bodies after the stroke, and their velocities, may be estimated by observing the arches which they describe after the impact.

In this manner the experiments may be performed on elastic, as well as non-elastic, bodies. When the bodies are required to be elastic, ivory balls are suspended to the threads; but when the bodies are to be non-elastic, the balls are made of soft wax*, or of moist clay. And though the former be

* White wax may be rendered sufficiently soft for this purpose by melting it over a gentle fire and incorporating it with about one quarter of its weight of olive-oil; it may afterwards,

be not perfectly elastic, nor the latter perfectly non-elastic; yet the difference which arises from their imperfect properties, is so trifling, that it may be safely neglected in these experiments. And here it is proper to observe that in performing such mechanical experiments, wherein some allowance must be made on account of friction, of resistance of the air, of imperfect elasticity, &c. the result must be reckoned conclusive as long as the effect is somewhat less than what it ought to be according to the theory; but if the effect is greater than that which is determined by calculation; then some defect in the machinery, or error in the theory, must be suspected.

In the abovementioned figures the threads of the pendulums are represented as being fixed to common nails; but a machine, or stand, may be easily contrived (and many machines of this sort are described in almost all the books of mechanical philosophy *) upon which two or more pendulums may be easily suspended; where the lengths of the pendulums might be accurately adjusted, and where a graduated arch, as in fig. 5, might be easily applied,

afterwards, when cooled, be easily formed into balls, and the figure of the balls may be easily restored, after being altered in the course of the experiments.

* The best description of the construction and use of such a machine, is, in my opinion, that which is given at large in the second Book of s'Gravesande's Mat. Elem. of Nat. Phil. edited by Desaguliers.

for

for the purpose of measuring the arches from which the pendulums are permitted to descend, or those to which they ascend.

The facility with which such machines may be contrived and constructed renders the particular description of any of them in this place superfluous; one particular mechanism concerning it is however deserving of notice; and such is represented by fig. 7, Plate VIII. When a pendulous body, suspended by a single string, is raised to a certain height in order to give it motion, the least jerk or irregularity of the hand is sufficient to make it deviate from the proper plane of vibration, in which case the stroke on the other pendulous body will not be given in the direction of its centre of gravity; hence the effect will not turn out conformable to the theory. Now the suspension which is represented in fig. 7, avoids the possibility of that deviation; and therefore such suspension has been generally adopted for experiments of the abovementioned nature. DE is a slip of brass, the form of which is sufficiently indicated by the figure. It is fastened to the ball by means of a screw, and the thread BDEC, whose two extremities are fastened at B and C to a bracket, or horizontal arm of the machine, passes through two holes in the projections D,E of the brass slip.—It is evident that this pendulum must vibrate in a plane perpendicular to the plane BD EC of the figure, without any possible deviation.

Fig. 5.

Fig. 5. repreſents the caſe when the bodies are equal, and with this apparatus one of the bodies may be made to ſtrike againſt the other at reſt; or they may be made to ſtrike againſt each other when they are both in motion. The ſame variety of experiments may be performed with the pendulums fig. 4. but in theſe the quantities of matter are unequal.

Fig. 6. repreſents the caſe in which three equal elaſtic bodies lie contiguous to each other, where if one of the outer bodies, as F, be lifted up to G, and then be permitted to deſcend againſt E, the ſtroke will be communicated from E to D; ſo that E will remain at reſt, and D will be impelled up to H.—For the various caſes of colliſion which may be exhibited by means of pendulums, ſee chap. VII.

Various machines have been contrived for the purpoſe of illuſtrating the compoſition and reſolution of forces; and the weights ſuſtained by oblique powers*. One of the cleareſt methods of ſhewing the compoſition of forces is the following:

Suſpend two pendulums ACI, BD, as repreſented by fig. 8. Plate VIII. ſo that their balls may be very little above the ſurface of a flat and ſmooth

* Such machines are particularly deſcribed in moſt of the works on mechanics and natural philoſophy, eſpecially Graveſande's Elem. of Natural Phil. and Muſſchenbroek's Philoſophy.

table LMRK. Place an ivory ball, E, upon the table, in contact with both the balls of the pendulums; then if you draw the pendulum A a certain way out of the perpendicular direction, and then let it fall againſt the ball E, in the direction EO, the ball E will be forced to move from E to O.—Replace the ball E in its former ſituation; raiſe the pendulum B ſo as to make an angle with the perpendicular, equal to that made by the other pendulum, then let it fall upon the ball E, whereby this ball will be forced to move from E to H. Laſtly, put the ball E once more in its former ſituation; raiſe both the pendulums at the ſame time, and to the ſame angle to which they were before raiſed ſeparately; then let them go both at the ſame inſtant, ſo that they may both ſtrike the ball E at the ſame time; and the ball E will thereby be forced to move ſtraight from E to G, which is the diagonal of the parallelogram GHEO, whoſe ſides are the directions of the ſeparate impulſes EO, and EH.

The effect may be varied by increaſing or diminiſhing one of the impelling forces, which muſt be done by increaſing or diminiſhing the weight of one of the balls A or B.

In the uſe of ſuch a machine, care muſt be had to let the two pendulums ſtrike the ball E at the very ſame inſtant; which requires a conſiderable degree of dexterity. Mechanical means might indeed be eaſily contrived for the purpoſe of diſ-

charging

charging both the pendulums at the proper time; but it is hardly worth while to conſtruct a complicated machine for illuſtrating ſo evident a propoſition.—In this machine the two pendulums are ſuſpended nearly after the manner of fig. 7, viz. each pendulum is ſuſpended by two threads, but the ſlip of braſs is omitted.

CHAPTER XV.

CONTAINING THE APPLICATION OF SOME PARTS OF THE FOREGOING DOCTRINE OF MOTION; WITH REMARKS ON THE CONSTRUCTION OF WHEEL CARRIAGES.

OF all the branches of mechanics the properties of the centre of gravity occur moſt frequently, and are of the greateſt conſequence, to the human being.

Whatever body reſts upon another body muſt have its centre of gravity ſupported by that other body, viz. the line drawn from its centre of gravity ſtraight to the centre of the earth; or, which is the ſame thing, the line which falls from its centre of gravity perpendicularly to the horizon, muſt be intercepted by, or fall upon, the other body; otherwiſe the former will not be ſupported by the latter.

 The

The abovementioned line, that is, a line drawn from the centre of gravity of a body, or fyftem of bodies that are connected together, perpendicularly to the plane of the horizon, is called the *line of direction*; it being in fact the line along which the body will direct its courfe in its defcent towards the centre of the earth; and, of courfe, in order to be fupported, it muft meet with an obftacle in that line. Thus in fig. 9. Plate VIII. the body CDOG will reft very well with its bafe upon the ground or other horizontal plane, becaufe its line of direction IF, drawn from its centre of gravity I, perpendicular to the plane of the horizon, falls within the bafe YGO, every point of which is fupported by the ground; but if another body ABCD be laid upon it, the whole will fall to the ground, for in the latter cafe the centre of gravity of the whole will be higher up, as at K, and the line of direction KH falls out of the bafe. Thus alfo in fig. 10. Plate VIII. the body D will roll down the inclined plane AB, becaufe its line of direction falls without its bafe; whereas the body C, whofe line of direction falls within its bafe, will only flide down that plane, unlefs the friction prevents it, in which cafe it will remain at reft; but friction will not prevent the rolling down of the body D.

It is therefore evident that the narrower the bafe is, the eafier a body may be moved, and, on the contrary, the broader the bafe is, and the nearer the line of direction is to the middle of it, the more firmly

firmly does the body ſtand. Hence it appears that a ball, or a circular plane figure ſtanding upright, ſuch as a wheel, is moved upon a plane with greater facility than any other figure; for the leaſt change of poſition is ſufficient to remove the line of direction of a ſpherical or circular body, out of the baſe. Hence alſo it is that bodies with narrow terminations, ſuch as an egg or a ſtick, &c. cannot be made to ſtand upright upon a plane, at leaſt not without the utmoſt difficulty.

The application of the properties of the centre of gravity to animal œconomy is eaſy and evident. If the line of direction falls within the baſe of our feet, we remain erect; and the ſteadieſt, when that line falls in the middle of that baſe; otherwiſe we inſtantly fall to the ground.

On account of the great importance which the preſervation and management of that centre is to animal motion, the infinite wiſdom of the Creator has implanted in all animals a natural propenſity to balance themſelves in almoſt every circumſtance. Many animals acquire the habit of keeping themſelves upon their legs within a few hours after their birth, and ſuch is particularly the caſe with calves.

It is wonderful to reflect, and to obſerve, how a child begins to try and improve his ſtability. He generally places his feet at a conſiderable diſtance from each other, by which means he enlarges the baſe, and diminiſhes the danger of a lateral fall:—he endeavours to ſtand quite erect, and with his body im-

immoveable, ſo as to prevent as much as poſſible a fall backwards or forwards;—and when he lifts up one foot, he inſtantly replaces it upon the ground, finding his inability to reſt upon ſo ſmall a baſe as that of one foot. Farther advanced in years, he adopts farther methods of preſerving and uſing the centre of gravity, and that without the leaſt knowledge of the mechanical principle upon which he acts. Thus a man naturally bends his body when he riſes from a chair, in order to throw the centre of gravity forwards. He leans forwards when he carries a burden on his back, in order to let the line of direction (which in that caſe deſcends from the common centre of gravity of his body and burden) fall within the baſe of his feet. For the ſame reaſon he leans backwards when he carries a burden before him; and leans on one ſide when he carries ſomething heavy on his oppoſite ſide.

Human art improved by conſtant exerciſe and experience, goes far beyond thoſe common uſes of the centre of gravity, and line of direction. We ſee, for inſtance, men who can balance themſelves ſo well as to remain erect with one foot upon a very narrow ſtand, or upon a rope, and even with their heads downwards and their feet uppermoſt.—Their art entirely conſiſts in quickly counterpoiſing their bodies, the moment that the line of direction begins to go out of the narrow baſis upon which they reſt. Thus, if they find themſelves

ſelves falling towards the right, they ſtretch out the left arm or the left leg, and *vice verſa*; for though the weight of the arm be much leſs than the weight of the body; yet by being extended farther from the fulcrum, its momentum may be rendered equal to that of the reſt of the body, which lies vaſtly nearer to the fulcrum, or to the line of direction.— *See chap.* VI.

This explanation likewiſe ſhews the great uſe of a long horizontal pole in the hands of a rope-dancer; for as the extremities of the pole, which are generally loaded with leaden weights, lie far from the rope, which is the fulcrum; when the pole is moved a little one way, the momentum of that extremity of it which lies that way, is increaſed conſiderably, and ſo as to counterpoiſe the body of the man, when he finds himſelf going the other way.

Notwithſtanding the uſe of the centre of gravity which mankind acquires naturally or merely by experience; yet in many caſes people are ſeen to act contrary to the laws of nature; and the conſequences are ſometimes quite fatal. Thus we frequently find that when a boat or carriage is overſetting, the perſons in it riſe ſuddenly from their ſeats; by which means they remove the centre of gravity of the whole higher up, and thereby accelerate the fall, (exactly like the caſe which has been repreſented in fig. 9, Plate viii.) which they might probably prevent, either by remaining on their ſeats, or rather by lowering themſelves down as cloſe as they could to the bottom of the boat or carriage.

The natural application of the mechanical laws might be inſtanced in almoſt every occurrence of life; for whatever moves muſt move conformably to thoſe laws. But, to avoid prolixity, we ſhall only mention a few more particular inſtances: whence the attentive reader may eaſily learn how to apply the foregoing doctrine of motion to a variety of occurrences.

The man, or the horſe, that runs in a circular path, naturally leans towards the centre of the circle, or towards the concave part of the curvilinear pathway; and that in order to counteract the effect of the centrifugal force, which would otherwiſe throw him out of the perpendicular. And the ſwifter he runs, the more he leans towards the concave ſide; the centrifugal force encreaſing with the velocity.

When a man is to hold a great weight in his hand, he naturally places the hand near the body; for if he extend his arm, the momentum of the weight which is placed at the end of it, as if it were at the end of a long lever, becomes too great for his power; conſidering that the arm becomes a lever where the power and the fulcrum lie near one extremity, viz. near the ſhoulder, and the weight lies at the oppoſite extremity.

Perſons

Perſons who are accuſtomed to uſe a hammer generally hold it by the loweſt part of the handle, for the purpoſe of rendering the ſtroke as powerful as poſſible; for in that caſe the head of the hammer, by being fartheſt from the centre of motion, moves with the greateſt velocity, and, of courſe, ſtrikes with the greateſt momentum. But ſuch perſons as are not ſufficiently accuſtomed to uſe a hammer, generally place their hand near the head of the inſtrument, by which means they render the ſtroke of very little effect.

The like obſervation might be made with reſpect to the uſe of almoſt all other tools and inſtruments, including thoſe which are commonly in uſe, ſuch as ſciſſars, knives, razors, &c. And it it is by the different management of ſuch inſtruments that a mechanical hand is diſtinguiſhed from an unmechanical or clumſy one; and that a perſon poſſeſſed of uſeful experience, uſeful habits and uſeful knowledge, is diſtinguiſhed from one of the contrary deſcription; excepting indeed when the aukward management of tools, &c. is wilfully adopted, under the refined idea, and for the purpoſe of ſhewing, that a perſon has never been under the diſgraceful neceſſity of handling any mechanical inſtrument.

Of the different machines of luxury or convenience, that are in uſe amongſt civilized nations, none have been more generally adopted, and more univerſally uſed, than wheel carriages; and yet it

is

is very remarkable, that, notwithſtanding the preſent greatly improved ſtate of mechanical knowledge, thoſe machines are by no means conſtructed or uſed in the moſt advantageous manner poſſible.

Could the ſurfaces of bodies be rendered perfectly ſmooth, flat, and deſtitute of adheſion, it would be as eaſy to drag a body upon a plane, as it would be to move it upon wheels; but as this is far from being the caſe, the advantage which ariſes from the uſe of wheel carriages, is too evident to need any particular demonſtration; and, in fact, we almoſt every day obſerve, that a ſingle horſe is able to carry upon a cart ſuch a load as ten horſes would perhaps have not ſtrength ſufficient to move on the bare ground.

When a heavy body is dragged upon the ground, the friction is very great, becauſe the ground ſtands ſtill and the body moves upon it, ſo that all the inequalities of the ground, the accumulation of dirt, and ſtones, the ſinking of the ground, &c. form ſo many obſtables to the moving body, which obſtacles muſt be overcome by the power which is applied to draw it. But when the body is carried upon wheels, as upon a cart, or waggon, &c. the ſurface of the rims of the wheels does not rub on, but is ſucceſſively applied to the ground, (agreeably to what we have ſaid above with reſpect to rollers; ſee page 279) and the obſtacles ariſing from ſinking, from ſtones, ſand, &c. offer an oblique

lique oppoſition to the wheels, which is overcome vaſtly eaſier than a direct oppoſition, and the more ſo the larger the wheels are in diameter; ſo that by the uſe of large wheels the friction againſt the ground is almoſt entirely removed. There is however another ſort of friction introduced by the uſe of wheels, viz. the friction of the wheel upon the axle; but this friction, when the parts are properly ſhaped, and oiled or greaſed, is not very material, eſpecially when the wheels are large; for when a wheel turns upon an axle, the force neceſſary to overcome the friction is diminiſhed in the ratio of the diameter of the wheel to the diameter of the axle.

A wheel carriage is drawn with the leaſt power, when the line of draught paſſes through the centre of gravity of the carriage, and in a direction parallel to the plane on which it moves. It therefore follows that the height of the carriage ſhould be regulated by the nature of the power which is to draw the carriage; viz. whether it is to be drawn by high or low horſes, by bullocks, &c. It muſt however be obſerved, that, from the make of his body, a horſe draws with the greateſt advantage when the traces, or the ſhaft, makes a ſmall angle with the plane which paſſes through the axles of the wheels. But this angle muſt not exceed a few degrees, otherwiſe part of the power of the horſe is employed in lifting up the fore wheels from the ground;

ground; conſequently the power is not entirely employed in drawing the carriage forward.

Four-wheeled carriages are almoſt always made with the two fore wheels ſmaller than the hind wheels. The fore wheels are made ſmaller than the others for the conveniency of turning, as they require leſs room for that purpoſe. The ſmallneſs of their ſize does alſo prevent their rubbing againſt the traces: but, thoſe objects excepted, ſmall wheels are by no means ſo advantageous as thoſe of a larger diameter, as has been already mentioned, and as will be confirmed by the following illuſtration.

In fig. 2. Plate IX. let the hollows BGC, and DFO be equally large, and equally deep in the ground. It is evident that the large wheel A will not go ſo far into the hollowing, as the ſmall wheel R. Beſides, even ſuppoſing that they deſcend equally deep into thoſe hollowings, the large wheel, by the power acting far above the impediment, may be eaſily drawn out of it; whereas the ſmall wheel can hardly be drawn out by means of an horizontal draught, unleſs indeed when the ground gives way before it, which is not always to be expected.

The idea of the two large wheels helping to drive the fore ſmall ones, is a vulgar error, which has not the leaſt foundation in truth. The abſurdity of this idea might be proved various ways, but by none more ſatisfactorily than by the following experiment.

Take

Take a real carriage, or the model of a large one, having two large and two ſmall wheels. Faſten a rope at each of its ends, but equally high from the ground; then extending one of thoſe ropes horizontally, let it go over a pulley, which muſt be placed at ſome diſtance from the carriage, and tie as much weight to the deſcending extremity of the rope, as may be juſt ſufficient to move the carriage. This done, diſcharge this rope; turn the carriage with its other end towards the pulley, and, in ſhort, repeat the experiment with the other end of the carriage foremoſt. It will be found that preciſely the ſame weight will be required to draw the carriage, and to draw it with equal velocity, whether the large or the ſmall wheels be placed foremoſt.

The figure, or rather the breadth of the rims of the wheels, influences conſiderably the motion of the carriage. Upon a ſmooth and hard road no advantage is derived from the uſe of broad wheels; but upon a ſoft road the broad wheels are much more advantageous than narrow ones; the latter cutting and ſinking into the ground; on which account they muſt be conſidered as always going up hill, beſides their ſuffering a great deal of friction againſt the ſides of the ruts that are made by themſelves; whereas the broad wheels produce nearly the ſame effect as a garden-roller; that is, they ſmooth and harden the road, beſides their moving with great freedom. It muſt however be obſerved, that upon ſand, as alſo upon ſtiff clayey

roads,

roads, leſs force is required to draw a cart with narrow, than one with broad, wheels. Upon ſand the broad wheels form their own obſtacles, by driving and accumulating the ſand before them. Upon clayey roads they gather up the clay upon their ſurfaces, and become in a great meaſure clogged by it.

Some perſons imagine that the broad wheels, by touching the ground in a great many more points than narrow wheels, muſt meet with proportionably greater obſtruction. But it ſhould be conſidered, that though the broad wheels touch the ground with a larger ſurface, yet they preſs upon it no more than narrow wheels do. Let, for inſtance, two carts be equal in every reſpect and equally loaded, excepting that the wheels of one of them be 3 inches broad, whilſt thoſe of the other be 12 inches in breadth. It is evident that the latter wheels reſt upon the ground with a ſurface which is equal to four times the ſurface upon which the former wheels reſt. But ſince an equal weight is ſupported by the wheels of both carts, every three inches breadth on the ſurface of the broad wheels ſuſtains a quarter of that weight; whereas the three inches breadth of the narrow wheels ſuſtain the whole weight; ſo that the broad wheels touch the ground as much lighter as they are broader than the narrow wheels. It is for the ſame reaſon that if a heavy body in the form of a parallelepipedon, viz. like a brick, be dragged upon a plane ſurface, the ſame power will be required to draw it along, whether its broad or its narrow ſide be laid upon the plane.

" If the wheels were always to go upon ſmooth " and level ground, the beſt way would be to " make the ſpokes perpendicular to the naves; " that is, to ſtand at right angles to the axles; " becauſe they would then bear the weight of the " load perpendicularly, which is the ſtrongeſt way " for wood. But becauſe the ground is generally " uneven, one wheel often falls into a cavity or rut " when the other does not; and it bears much " more of the weight than the other does; in " which caſe, concave or diſhing wheels are beſt, " becauſe when one falls into a rut, and the other " keeps upon high ground, the ſpokes become " perpendicular in the rut, and therefore have the " greateſt ſtrength when the obliquity of the load " throws moſt of its weight upon them; whilſt " thoſe on the high ground have leſs weight to " bear, and therefore need not be at their full " ſtrength. So that the uſual way of making the " wheels concave is by much the beſt."

" The axles of the wheels ought to be perfectly " ſtraight, that the rims of the wheels may be " parallel to each other; for then they will move " eaſieſt, becauſe they will be at liberty to go on " ſtraight forwards. But in the uſual way of prac- " tice, the axles are bent downward at their ends; " which brings the ſides of the wheels next the

" ground

" ground nearer to one another, than their oppo-
" ſite or higher ſides are : and this not only makes
" the wheels drag ſidewiſe as they go along, and
" gives the load a much greater power of cruſhing
" them than when they are parallel to each other,
" but alſo endangers the overturning of the car-
" riage when any wheel falls into a hole or rut, or
" when the carriage goes in a road which has one
" ſide lower than the other, as along the ſide of a
" hill ;"* for on that conſtruction the carriage ſtands upon a narrower baſe, than when the rims of the wheels are parallel to each other.

Upon level ground a carriage with four equal wheels may be drawn by the ſame power with equal facility, whether the load be placed on any particular part of the carriage, or it be ſpread equally all over it.

Upon a two wheeled carriage the moſt advantageous diſpoſition of the load is, when the centre of gravity of the weight coincides with the middle of the axle, or with a perpendicular line which paſſes through that middle.

A carriage having the two hind wheels large, and the two fore wheels ſmall, when going upon an horizontal plane, ſhould have the principal part of the load laid towards its hind part ; but when going upon uneven roads, up and down hill, and when the load cannot be eaſily ſhifted, the beſt

* Ferguſon's Lectures, lecture iv.

way

way is to lay the load principally in the middle, or to ſpread it equally all over the carriage.

The common practice of carriers, who place the principal part of the load upon the fore axle of the waggon, is evidently very abſurd; for by that means they preſs that axle with greater force upon the wheels, and the fore wheels deeper into the ground, in conſequence of which thoſe wheels, which turn oftener round than the large wheels, will not only wear much ſooner, but require a greater power to draw them along, and eſpecially over any obſtacle.

The lower the centre of gravity of the load is ſituated, the leſs apt is the carriage to be overturned. The following obſervations are of Profeſſor Anderſon of Glaſgow.

" In Glaſgow and its neighbourhood, a ſingle " horſe, on a level turnpike road, draws 25 cwt. " in a cart which weighs about 10 cwt. having " wheels ſix feet high, and its axle paſſing through " the centre of gravity of the load and cart, but, " in a common cart, he draws only the half of " that load. Two horſes yoked in a line, " in a common cart eaſily draw 30 cwt. upon " an even road. And ſix horſes, yoked two a- " breaſt, draw 80 cwt. in a common waggon."

" Six horſes, in ſix carts, with high wheels, can " draw 150 cwt. on a level road; and ſix horſes, " in three common carts, with two horſes in each, " can draw, upon an uneven road, 90 cwt. that "is 10 cwt. more than they can do in a waggon; "the weight, tear and wear, and the eaſe in drawing a waggon, or three carts, being, it is ſaid, "nearly equal; and the price of the three carts "being leſs than that of the waggon." *

CHAPTER XVI.

OF PROJECTILES.

WHATEVER body is impelled by any power, and is afterwards left to proceed by itſelf, is called a *projectile*, which denomination is derived from a Latin word, the meaning of which is to throw, to hurl. Thus the bullets which are thrown out of fire arms, ſtones that are thrown by the hand, or by a ſling, or by any other projecting inſtrument, &c. are called *projectiles*.

It has been already ſhewn (in chap. IX.) that projectiles, unleſs they be thrown perpendicularly upwards or downwards, muſt deſcribe a curve line; becauſe they are acted upon by two forces, one of which, viz. the impelling force, produces an

* Inſtitutes of Phyſics: Mech. ſect. xvii.

equable

equable motion; whilſt the other, viz. the attraction of the Earth, produces an accelerated motion.

It has likewiſe been ſhewn that projectiles deſcribe ſuch curves as are called by the mathematicians *parabolas*; or rather that they would deſcribe ſuch curves, if they were not influenced by certain fluctuating circumſtances, which cauſe the paths of projectiles to deviate more or leſs from true parabolic curves.

Thus much might have ſufficed with reſpect to the motion of projectiles. But the great uſe which is made of them both in peace and in war, obliges us to conſider this branch of the doctrine of motion in a more particular manner, and to derive from the theory ſuch rules as may be of uſe in the practical management of projectiles.

There are three cauſes, which force the projectile to deviate from the parabolic path: viz. 1ſt, the force of gravity's not acting in directions perpendicular to the horizon; 2dly, The decreaſe of the force of gravity according to the ſquares of the diſtances from the centre of the Earth; and 3dly, the reſiſtance of the air.

The effects which are produced by the firſt and ſecond of thoſe cauſes, are too ſmall and trifling; for the centre of the Earth is at ſo great a diſtance from the ſurface, that both the height and the diſtance to which we are able to throw projectiles, are exceedingly ſmall in proportion to it.

But the resistance of the air offers a very considerable opposition to the motion of projectiles, and its action is so very fluctuating, that, notwithstanding the endeavours of several able philosophers and mathematicians, the deviation of projectiles from the parabolic path has not yet been subjected to any determined and practical rules.

After the preceding remarks it will be easily allowed, that the only method we can follow is to lay down the theory on the supposition that the projectiles move in parabolas, and then to subjoin a concise statement of the result of the principal experiments, which have been made for the purpose of shewing the deviation of the real path of a projectile from the parabolic curve.

When a body is projected obliquely from any kind of engine, such as a ball from the cannon A, fig. 3, Plate IX. in the direction A C, the force of gravity, acting upon it in directions nearly perpendicular to the horizontal plane A B, forces it to deviate from the straight direction A C, and to describe the parabola ADB, lastly falling upon the horizontal plane at B; whence it evidently follows, that a ball or any other projectile cannot move even for a moment in a straight line; but that it must deviate more or less from the straight line of its initial direction, and must immediately begin to incline towards the ground; excepting however some particular cases; namely, when the shot has acquired a rotatory motion

tion round its axis, or when its ſhape is ſomewhat oblong and bent; for in thoſe caſes it may deviate not only ſideways, but even upwards for a ſhort time.

The diſtance A B between the mouth of the projecting engine and the place where the ſhot falls upon the horizontal plane, is called the *range of the ſhot*, or the *amplitude of projection*; D E is the *height* of its path, or of the parabola ADB. The angle CAB, which the direction of the projection, or of the cannon, makes with the horizontal plane, is called the *angle* of *elevation*. The time during which the ſhot performs the path ADB, is called *the time of flight*, and the force with which it ſtrikes an object at B, is its *momentum*.

It will be found demonſtrated in the note that thoſe particulars, viz. the range, the height, the angle of elevation, &c. bear a certain determinate proportion to each other, ſo that when two of them are known, the others may thereby be found out. It is demonſtrated likewiſe, that, *cæteris paribus*, the greateſt range or greateſt diſtance to which a ſhot may be thrown upon an horizontal plane, takes place when the angle of elevation is equal to half a right angle, or 45 degrees (1.); we ſhall

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(1.) Propoſition I. *A body which is projected in a direction not perpendicular, but oblique, to the horizon, will deſcribe a parabola; and its velocity in any point of that parabo-* *la* ſhall therefore proceed in this place to ſhew a practical method of determining the moſt uſeful of thoſe particulars, when the angle of elevation, and the greateſt range the cannon is capable of, are known.—This greateſt range, viz. the diſtance to which the cannon when charged with the uſual quantity of powder, and elevated to an angle of 45°. is capable of throwing the ſhot, muſt be aſcertained by means of actual experiment and menſuration in every piece of artillery, eſpecially with large cannons, and mortars; ſince the balls from thoſe pieces deflect much more from the ſtraight direction.—The angle of elevation is aſcertained by means of a graduated circular inſtrument, and a plummet, or a level.

Let

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la is the ſame as it would acquire by deſcending perpendicularly through a ſpace equal to the fourth part of the parameter belonging to that point as a vertex; ſuppoſing that the force of gravity is uniform, and that it acts in directions perpendicular to the horizontal plane; alſo that the air offers no reſiſtance to the motion of projectiles.

Let a body be projected from A, fig. 5. Plate IX. in the direction AE, and let AE repreſent the ſpace, through which the projecting force alone would carry it with an equable motion in the time T. Alſo let AB repreſent the ſpace through which the force of gravity alone would cauſe it to deſcend in the ſame time T. Complete the parallelogram ABEC, and it is evident that the body, being impelled both by the projecting force, and by the force of gravity, Let the greatest horizontal range of a cannon, or mortar, be 6750 yards, and let the actual angle of elevation be 25°; the other particulars may be found by delineating this case upon paper; for which purpose the instruments that are generally put

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gravity, must, at the end of the time T, be found at C. Now AE is as the time T, because it represents the space described uniformly; but AB is as the square of the time T; therefore AE or its equal BC, is as the square of AB. And the same reasoning may be applied to any other contemporary distances, as AH, AF, or FG, AF. But AE is a tangent to the curve at the point A, AF is a diameter at the point A, and BC, FG, &c. being parallel to the tangent AE, are ordinates to the diameter AF; and since the squares of those ordinates have been demonstrated to be as the respective abscissas AB, AF, &c.; therefore the curve ACGD is the parabola.

The velocity of the projectile at any point, as A, in the curve is such, that the space AE would be described uniformly by it, in the same time that the body would employ in descending perpendicularly by the force of gravity from A to B. Also (see p. 66.) the velocity acquired by the perpendicular descent AB is such as would carry the body equably through twice AB in the same time, (that is, in the same time that AE is described;) therefore the velocity which is acquired by the perpendicular descent AB, is to the velocity with which AE is described, as twice AB is to AE. But the velocity acquired by the perpendicular descent through AB, is to the velocity acquired by the perpendicular descent through a quarter of the parameter belonging to the vertex A of the parabola ACG, also as twice put in a common caſe of drawing inſtruments, are quite ſufficient, viz. a pair of compaſſes, a ruler with a ſcale of equal parts, and a protractor.

Draw an indefinite right line AK, fig. 4. Plate IX. to repreſent an horizontal plane, paſſing through

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twice AB is to AE; for that parameter is (by conics) equal to $\frac{\overline{AE}^2}{AB}$, and a quarter of it is $\frac{\overline{AE}|^2}{4\,AB}$, and, by the laws of gravity, the velocity acquired by the perpendicular deſcent AB, is to the velocity acquired by the perpendicular deſcent $\frac{\overline{AE}|^2}{4\,AB}$, as the ſquare roots of thoſe ſpaces; viz. as $\overline{AB}|^{\frac{1}{2}}$ is to $\frac{AE}{2\,\overline{BA}|^{\frac{1}{2}}}$; or as twice AB is to AE. Therefore, ſince the like reaſoning may be applied to any other point of the parabolic path, we conclude that, univerſally, the velocity of the projectile in any point of its path is the ſame as would be acquired by a perpendicular deſcent through a ſpace equal to the fourth part of the parameter belonging to that point as a vertex.

Corollary 1. It is evident that the projectile muſt move in the plane of the two forces, viz. in the plane which paſſes through AE, AB, and is, of courſe, perpendicular to the horizon.

Cor. 2. It follows from the laws of compound motion, that the projectile will deſcribe the arch AC, in the ſame time in which it would deſcend by the force of gravity from A to B, or in which it would deſcribe uniformly the ſpace AE.

Cor. 3. When a body is projected from A in the direction AE, if the parameter which belongs to the vertex A, be through the point of projection A. Make AB perpendicular to it, and equal to twice the greatest horizontal range, viz. equal to 13500 yards; which is done by making it equal to 13500 divisions of the

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be equal to $\frac{\overline{AE}^2}{EC}$, the parabola must pass through the point C.

Cor. 4. Either in the same, or in different parabolas, the parameters belonging to different points are to each other as the squares of the velocities of the projectile at those points (see p. 65); whence it follows, that at the vertex of the parabola the velocity or the momentum of the projectile is the least, and at equal distances from that vertex the velocities or the momentums are equal.

Proposition II. *The initial velocity being given, to find the direction in which a body must be projected in order to hit a given point.*

Let A, fig. 6, Plate IX, be the projecting point, and C the object, or point which is required to be hit.

The velocity of projection being given, the parameter of the parabola which must pass through the point C will easily be found by means of the preceding proposition; viz. by finding the space, through which a body must fall from rest, in order to acquire the given velocity; for that space is equal to the fourth part of the parameter belonging to the point A.

Join AC, draw the horizontal line AL, and at A erect AP perpendicular to the horizontal line AL, and equal to the above-mentioned parameter. Divide AP into two equal parts at G, and through G draw an indefinite right line KGH parallel to the horizontal line AL. Through A draw the scale of equal parts, for those parts must represent yards. Upon AB, as a diameter, describe the semicircle AFB. At A, by means of the protractor, draw the line of projection AF, making an angle of 25° with the horizontal line AK. Through

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draw AK perpendicular to the direction AC of the object, which AK will meet KH in a point K. With the centre K and radius KA draw the circular arch PHEA. Through the point or object C draw BCI perpendicular to the horizon, and if this perpendicular meets the circular arch, as at E and I, draw AE, AI; and either of those directions will answer the desired purpose.

Join PI and PE; and the triangles PAE, EAC are similar; the angle PAE being equal to the angle AEC (Eucl. p. 29, B. I.) and the Angle APE equal to the angle EAC (Eucl. p. 32, B. III.) Hence PA : AE : : AE : EC; therefore $PA = \frac{\overline{AE}|^2}{EC}$. Farther, the triangles PAI, AIC are also similar; the angle PAI being equal to the angle AIC (Eucl. prop. 29, B. I.) and the angle API equal to IAC (Eucl. p. 32, B. III.) Hence PA : AI : : AI : IC; and $PA = \frac{\overline{AI}|^2}{IC}$. Therefore since PA is the parameter belonging to the point A of the parabola, which is to be described by the projectile, &c. the said parabola (by cor. 3 of the preceding prop.) must pass through the point C.

Corollary 1. The angular distance CAP between the object and the zenith, is divided into two equal parts by the line AH; for KH being equal to KA, the angle AHK is equal to the angle HAK, and likewise equal (on account of

Through the point F, where AF cuts the semi-circle, draw OF perpendicular to the horizon AK. Divide AO into two equal parts, and at the point of division C erect CD perpendicular to the horizontal line AK; also make CD equal to a quarter

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of the parallelism of KH, AB) to the Angle HAB. But KAC is equal to GAB; for they are both right angles; therefore, subtracting the angle GAC from both, there remains KAG equal to CAB; consequently GAH is equal to HAC.

Cor. 2. The two directions AI, AE are equidistant from the direction AH; for KH being perpendicular to PA and to IE, the arch AH is equal to HP. and EH is equal to HI.

Cor. 3. When the directions AI, AE coincide with AH, then the distance AC is the greatest distance to which the projectile can be thrown upon the plane AO with the given velocity of projection. Hence it appears that when the object C is placed upon the horizontal plane, as in fig. 7, Plate IX. where AC coincides with AB, AK coincides with AG, PHA becomes a semicircle, and HAC half a right angle; then the greatest distance to which the projectile can be thrown, takes place, viz. when the angle of elevation HAB is half a right angle.

Cor. 4. The velocity of projection being known, the greatest distance AL, to which the projectile can be thrown upon the horizontal plane, or greatest range, is likewise known; it being equal to half the parameter AP; for AL is equal to the radius KH, or AK, which is the half of AP.

Cor. 5. When the point of projection A, and object C, are both upon the same horizontal plane, as in fig. 7; then ter of OF; then the path of the ſhot is repreſented by a curve line, which paſſes through the points A,D,O. Take the diſtance AO in your compaſſes, and, applying it to the ſame ſcale of equal parts as was uſed before, you will find it equal to 5170, which repreſent yards. If you apply the diſtance CD

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then the diſtance AC of the object is as the ſine of twice the angle of elevation CAE, or CAI; for (Eucl. p. 32, B. III.) CAE is equal to APE, and likewiſe equal (Eucl. p. 20, B. III.) to half AKE, whoſe ſine is EN; and EN is equal to AC. EN is likewiſe the ſine of double the angle CAI; for CAI = API = ½ AKI; and IS = NE, is the ſine of the angle AKI.

Cor. 6. If AE be the direction of the projectile, the greateſt height of the parabolic path above the horizon, is equal to a quarter of AC, and is as the verſed ſine of twice the angle of elevation CAE. For divide AC into two equal parts at T, and erect TR perpendicular to it. Divide TR into two equal parts at V; then TV is equal to half TR, and to a quarter of EC. It is evident that AC is an ordinate to the axis TR of the parabolic path; and that V muſt be the vertex of that parabola; for the direction AE being a tangent to the parabolic curve, the part VT of the axis is (by conics) equal to the part VR. Farther, CE is equal to AN, which is the verſed ſine of the angle AKE, viz. of twice the angle of elevation EAC, or IAC; therefore TV is equal to a quarter of EC, and is as the verſed ſine of twice the angle of elevation CAE.

Cor. 7. The greateſt height, to which the projectile will aſcend, when the direction of the projection is perpendicular

CD to the ſame ſcale, it will be found equal to 603 diviſions, or yards; which ſhews that with the angle of elevation equal to 25°, the cannon in queſtion will throw the ſhot to the horizontal diſtance of 5170 yards, and that the vertex, or greateſt

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dicular to the horizon, is equal to a quarter of the parameter AP; for in that caſe AE, EC, and AP coincide; conſequently a quarter of CE becomes the ſame thing as a quarter of AP.

Cor. 8. In the caſe of the preceding corollary, the time of the projectile's remaining in the air, is the ſame that a body would employ in deſcending from P to A, merely by the force of gravity; for the projectile will be as long in aſcending, as in deſcending along a quarter of AP; viz. it will employ twice the time which is required to deſcend along one quarter of PA, and which is equal to the time that is required to deſcend from P to A; the ſpaces deſcribed by deſcending bodies being as the ſquares of the times.

Cor. 9. The time of flight when a body is projected in any direction, as for inſtance AE, is as the ſine of the angle of elevation EAC; for it is as the chord AE, or as half AE, which is the ſine of the angle APE, or of half AKE, which is equal to the angle of elevation CAE.

From the abovementioned two propoſitions, with their corollaries, the moſt uſeful properties of projectiles have been derived, and are conciſely expreſſed together with the reſults of experiments, in the following practical rules, which every gunner ſhould impreſs in his mind; for when the greateſt range that a piece of artillery is capable of with the uſual charge of powder, is known, and which muſt be learned eſt height of its path ADO will be equal to 603 yards.

By the like means the range anſwering to any other angle of elevation may be aſcertained; and the

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learned from actual experiment; thoſe rules will anſwer all the neceſſary caſes in gunnery; excepting the obſtruction which ariſes from the reſiſtance of the air.

Rules for Shooting.

1. Horizontal ranges as right ſines of twice the angles of elevation.
2. Heights as verſed ſines of twice the angles of elevation.
3. Times of flight, or times in the air, as right ſines of the angles of elevation.
4. The time of flight at an elevation of 45°, is equal to the time of perpendicular deſcent through a ſpace equal to the horizontal range.
5. The impetus is equal half the horizontal range at 45° of elevation.
6. The height is equal to a quarter of the horizontal range at 45° of elevation.
7. In aſcents or deſcents, (viz. when the point of projection and the object are not both upon the ſame horizontal plane) for the beſt elevation take the complement of half the angular diſtance between the object and the zenith.
8. The charges of powder in the ſame piece are nearly as the horizontal ranges.

The inſtruments which are required for the practical application of thoſe rules, are a graduated circular inſtrument with a plummet or a level, and a table of ſines and verſed ſines;

the reaſon of this practice will be found demonſtrated in the note.

If, on the other hand, the diſtance of the object, and the cannon's greateſt range being known, the angle of elevation neceſſary to hit that object be required; you muſt proceed by the reverſe of the preceding method. Let, for inſtance, the diſtance of the object from the cannon be 5170 yards, and the cannon's greateſt range, 6750 yards. Draw a right line AO, fig. 4. Plate IX. equal to 5170 diviſions of the ſcale of equal parts. Make AB perpendicular to AO, and equal to twice the cannon's greateſt range; viz. to 13500 diviſions of the ſame ſcale. Upon AB, as a diameter, deſcribe the ſemicircle AFB. At O erect the line OG perpendicular to the horizontal line AO, and the perpendicular OG will meet the ſemicircle either in one, or in two points, or not at all. Should this line meet the ſemicircle in one point, it muſt be at Y, its middle, and then the required angle of elevation is YAO, viz. of 45°. If OG

ſines; but there is an inſtrument in uſe, called *the gunner's callipers*, which anſwers every purpoſe relative to the application of thoſe rules; as it contains a graduated circle, and is ſuſceptible of the application of a plummet, &c. A table of ſines and verſed ſines, together with many other tables and meaſures are likewiſe marked upon it.—See a very good deſcription, and account of the various uſes of the gunner's callipers, in Robertſon's Treatiſe on the Uſe of Mathem. Inſtrum.

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meets the ſemicircle in two points, as at F and G, which is the caſe in the preſent inſtance; then either the direction AF or AG will anſwer the purpoſe; and if the angles which thoſe directions make with the horizon AO, be meaſured by means of the protractor, the former will be found equal to 25°, and the latter to 65°. But when the perpendicular OG does not meet the ſemicircle, then we muſt conclude that the given power, or force of the cannon in queſtion, is not ſufficient to throw the ſhot to the propoſed diſtance.

It follows from the foregoing conſtruction, that the higher the direction line cuts the ſemicircle, the longer is the horizontal range; not exceeding however the middle point Y; for SY, which is equal to the radius of the circle, is longer than any other line that can be drawn in the ſemicirle AYB, parallel to the horizon; hence the horizontal range is the greateſt when the angle of elevation is 45°. It is alſo evident, that at equal diſtances from the point Y, as at F and G, the horizontal range will be the ſame; the height of the path only being different. When the angle of elevation is 90°, then the line of direction, becoming perpendicular to the horizon, coincides with the line AB, and the horizontal range becomes equal to nothing, viz. the ſhot will aſcend to a quarter of AB, and will then deſcend again to A.

When the angle of elevation is half a right angle, that is 45°. the time of flight, viz. of the ſhot's ſhot's remaining in the air, is equal to the time that a body would employ in deſcending perpendicularly, by the force of gravity, from a height equal to the horizontal range, which may be found by the rules given in chap. V.—In order to find the time of flight, at any other inclination, as for inſtance, the inclination OAF; ſay as AY (meaſured on the ſcale of equal parts) is to AF (alſo meaſured on the ſame ſcale), ſo is the time that a body would employ in deſcending perpendicularly through a ſpace equal to the greateſt horizontal range, to the time in queſtion, which will be known by the common rule of three.

Thus much may ſuffice with reſpect to the ſuppoſed parabolic paths of projectiles. We ſhall now ſubjoin a ſhort account of the reſult of ſuch experiments as have been made for the purpoſe of determining how far the parabolic theory may be depended upon.

In the common practice of directing cannons, an arbitrary allowance is made for the deviation of the ſhot from the ſtraight line. Practice indeed renders ſome gunners very expert; but their practical accuracy cannot be reduced to certain rules; viz. ſuch as may be of uſe to other perſons.

Mortars, which throw the ſhots in general with leſs velocity than cannons, have heretofore been directed by means of a graduated inſtrument; preferring, of the two directions which produce the ſame horizontal range, that which may be thought preferable according to the nature of the object.

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But of late another method has been found more advantageous in practice.—The mortar is steadily fixed upon its bed or carriage, at an elevation of 45°; but it is loaded with more or less powder according as the shell is required to go farther or nearer; not exceeding, however, the greatest horizontal range the piece is capable of; the errors of amplitude having been found to be less with an elevation of 45°, than with any other elevation; and the horizontal ranges having been found to be, *cæteris paribus*, nearly as the charges of powder; viz. half the weight of the full charge will throw the shell nearly to half the greatest horizontal range; a quarter of the weight of the full charge will throw the shell to a quarter of the greatest range, &c.

It has been found that a 24 pounder (viz. a cannon whose ball weighs 24 pounds) when charged with 16 pounds of gunpowder, and elevated to an angle of 45°, will generally range its shot upon an horizontal plane 20250 feet, which is not above one fifth of the range assigned by the theory, viz. of what it ought to be, if the air could be removed. —The opposition which the shot meets with from the air in this case has been estimated by the ingenious Mr. Robins, equivalent to 400 pounds.* Hence it appears that the path of a shot is far different from the parabola.

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* Robins's Essays on Gunnery.

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The resistance of the air varies principally according to its fluctuating qualities, viz. temperature, gravity, &c. according to the shape of the shot, and according to the velocity with which the shot is impelled, viz. the initial or incipient velocity.—When that velocity is small, the resistance of the air is very trifling; but when the initial velocity is very considerable, the resistance of the air becomes so great as to render the theory quite inapplicable to practice. The resistance sometimes amounts to 20 or 30 times the weight of the shot, and the horizontal range frequently is much less than the tenth part of what it ought to be, according to the parabolic theory.

It has been found that with the same angle of elevation, the horizontal ranges are in proportion to one another as the square roots of the initial velocities, and that the times of flight are as the ranges; whereas, according to the theory, the times ought to be as the velocities, and the ranges as the squares of the initial velocities.

Mr. Robins likewise found that very little advantage was gained by projecting a body with a velocity greater than 1200 feet per second. When a 24 pound shot is projected with the velocity of 2000 feet in a second, it will meet with so great an opposition from the air, that when it has advanced not more than 1500 feet, viz. in about one second, its velocity will be reduced to that of about

1200 feet per ſecond. In conſequence of this quick reduction of velocity, Mr. R. concluded that a certain projectile velocity at the ſame angle, might carry a ſhot farther than a greater velocity; for the body projected with the greater velocity, when its velocity becomes equal to that of the other projection, has a leſs angle of elevation, on which account it may go not ſo far from that point, ſo as to make the whole diſtance ſhorter.

No gun to carry far ſhould be charged with powder whoſe weight exceeds one ſixth, or at moſt one fifth part of the weight of the ſhot; for in field-pieces that quantity of powder will impel the ſhot with the initial velocity of about 1200 feet per ſecond. In a battering piece, when the object is near, the weight of the powder ſhould be about one third part of the weight of the ſhot.

When the initial velocity is greater than about 1100, or 1200 feet per ſecond, the reſiſtance of the air ſeems to be three times greater than it ought, if it varied only as the ſquare of the velocity. "The velocity at which the variation of the law of reſiſtance takes place, is nearly the ſame as that with which ſound moves. Indeed if the treble reſiſtance in the greater velocities is owing to a vacuum being left behind the reſiſted body, it is not unreaſonable to ſuppoſe that the celerity of ſound is the very laſt degree of celerity with which a projectile can form this vacuum, and

" can in ſome ſort avoid the preſſure of the at-" moſphere on its hinder parts. It may perhaps" confirm this conjecture to obſerve, that if a" bullet, moving with the velocity of ſound, does" really leave a vacuum behind it, the preſſure of" the atmoſphere on its fore part is a force about" three times as great as its reſiſtance, computed" by the laws obſerved for ſlow motions.*"

A ſhot, beſides its being drawn downwards from the line of direction, is ſometimes deflected ſideway; which takes place when the ſhot by rubbing againſt one ſide of the cavity of the piece, acquires a rotatory motion round its axis, and proceeds through the air with that motion; for in that caſe the ſide of the ſhot, which in its courſe through the air turns forwards, meets with greater reſiſtance than the oppoſite ſide, whoſe motion coincides with that of the air. It is eaſy to conceive that when the axis of rotation happens to be parallel to the horizon, then the rotation will contribute to the ſhot's deflection, not ſideways, but upwards or downwards.

We ſhall laſtly obſerve as a ſtrong inſtance of

* Robins's Eſſays on Gunnery.—The reader may derive conſiderable information from Dr. Hutton's Paper *on the Force of fired Gunpowder, and the initial Velocities of Cannon Ball*, &c. in the 68th vol. of the Phil. Trans. And from the Chev. de Borde's Memoirs in the Hiſt. of the Acad. of Scienc. for 1769.

the difficulty which attends the practical management of projectiles, that even with the very same piece of ordnance, like shots, equal weights of the same sort of gunpowder, and the same angle of elevation, the places on which the shots strike the horizontal plane frequently differ by several yards from each other.

END OF THE FIRST VOLUME.

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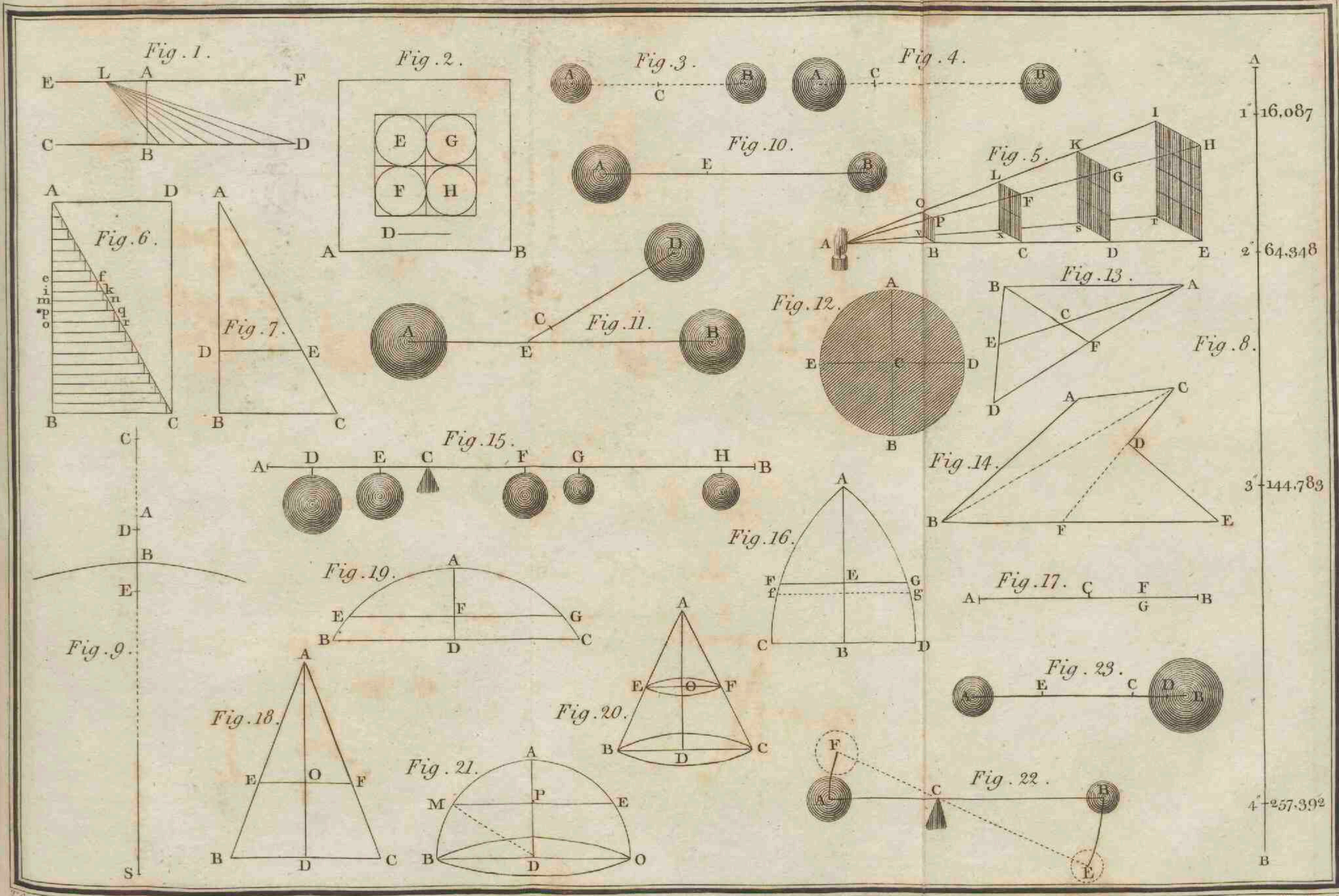

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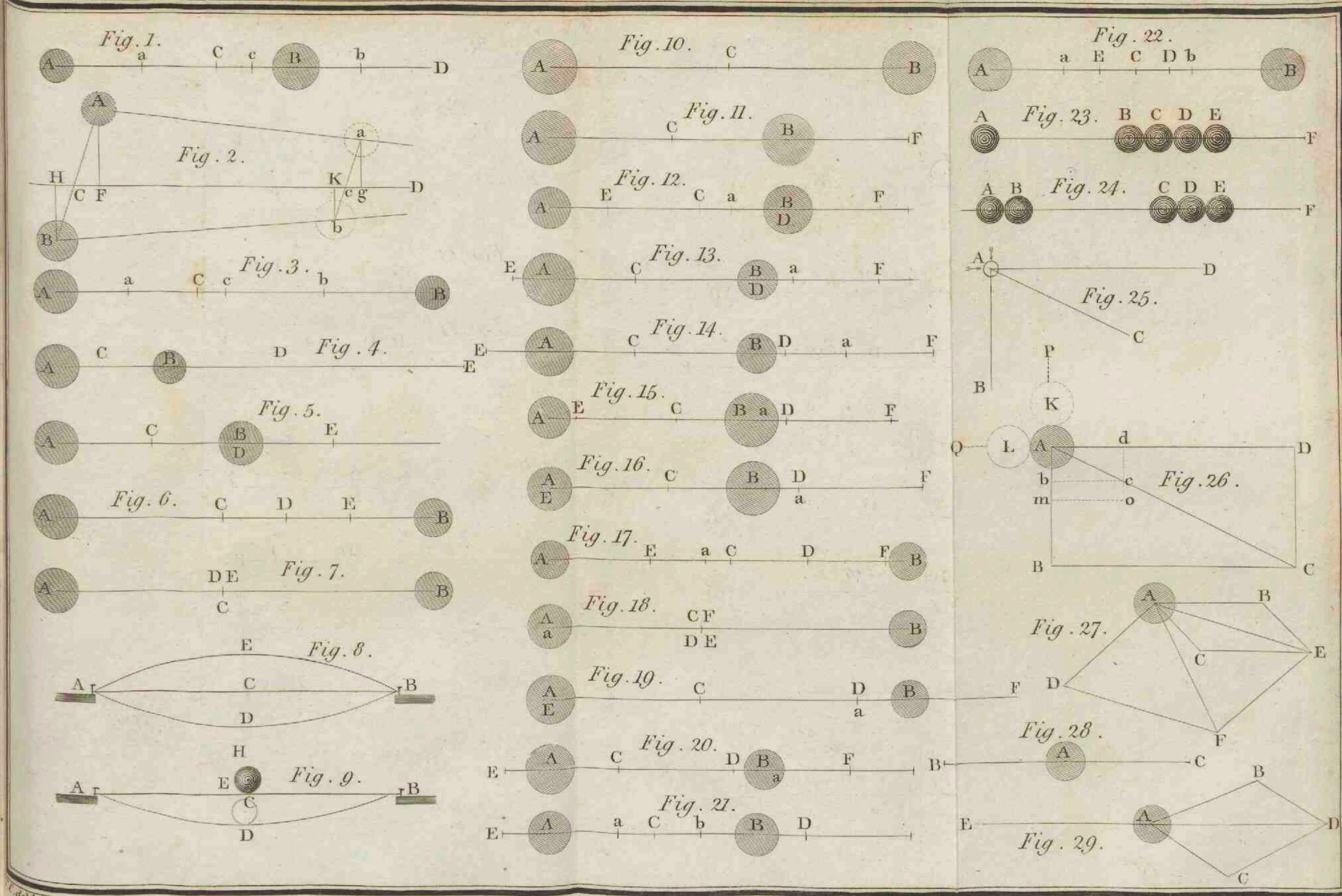

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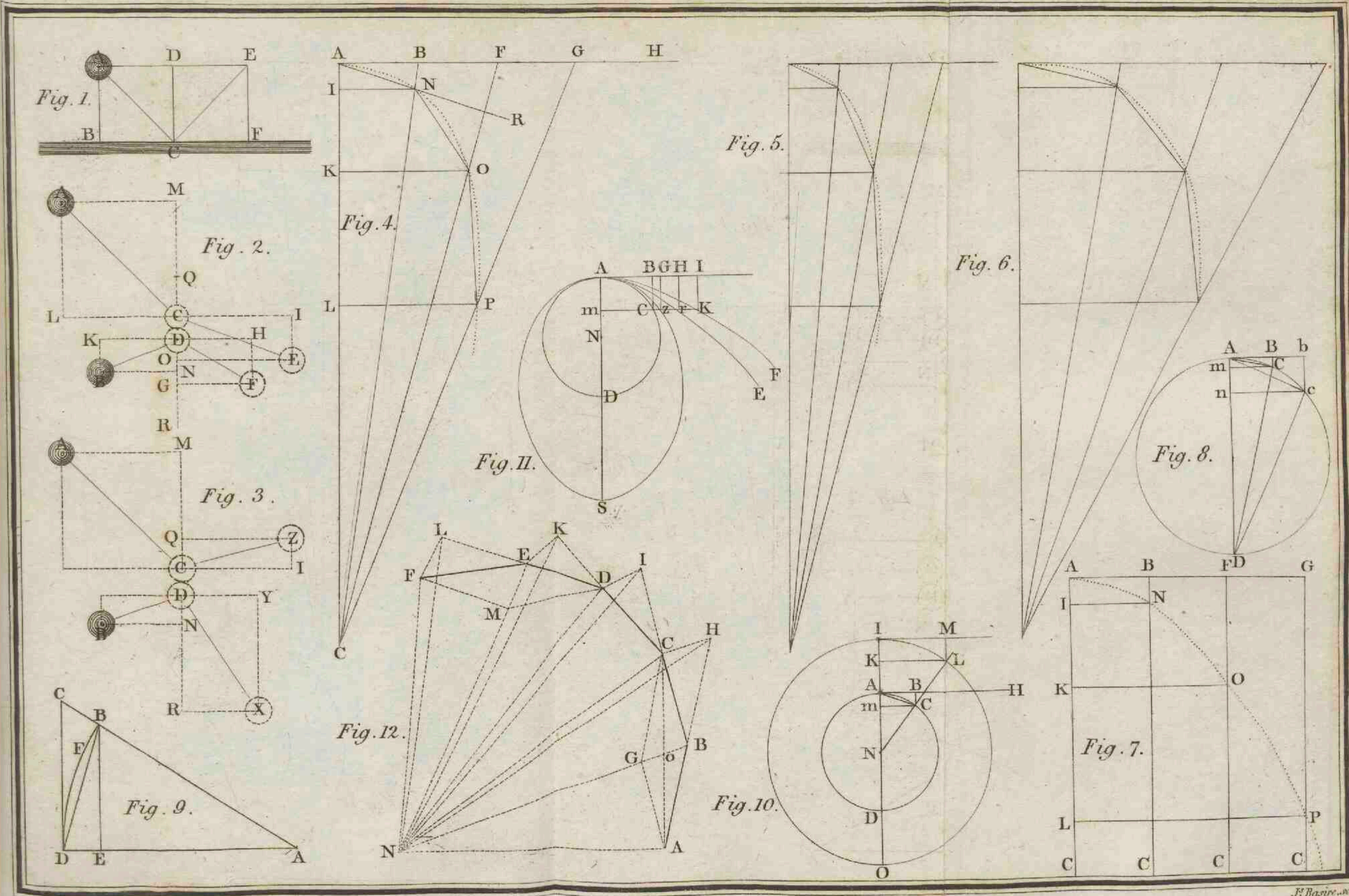

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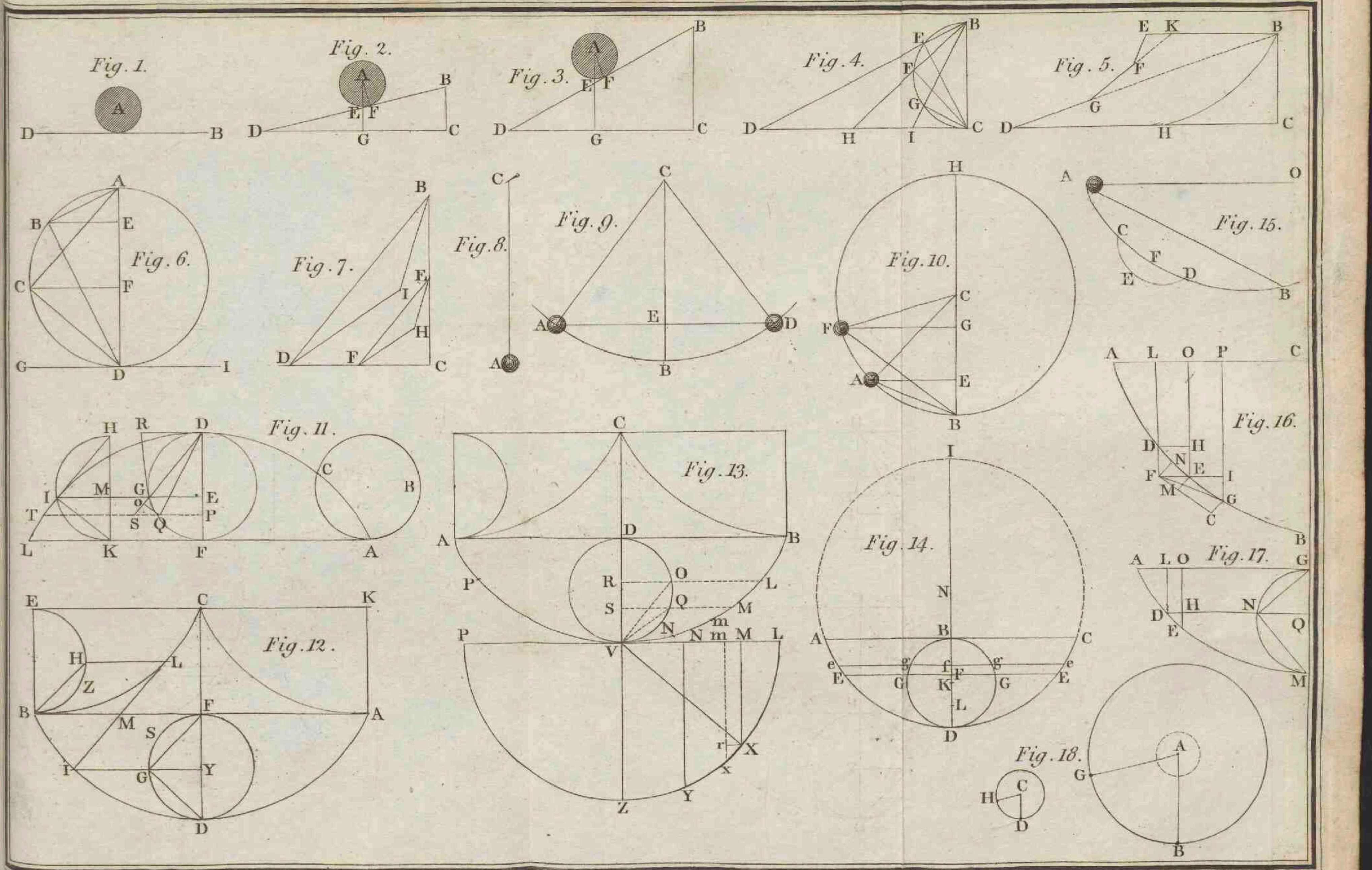

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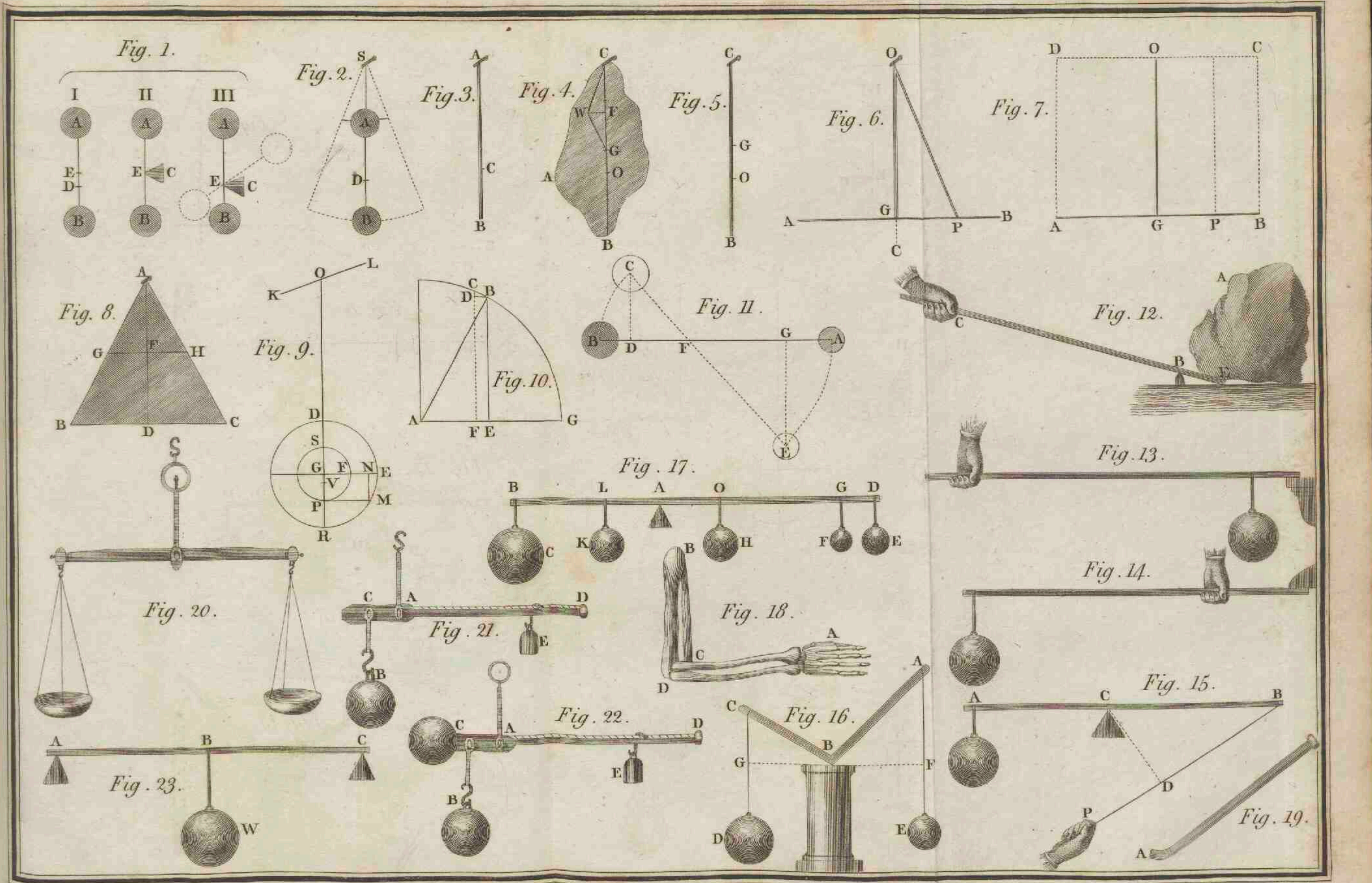

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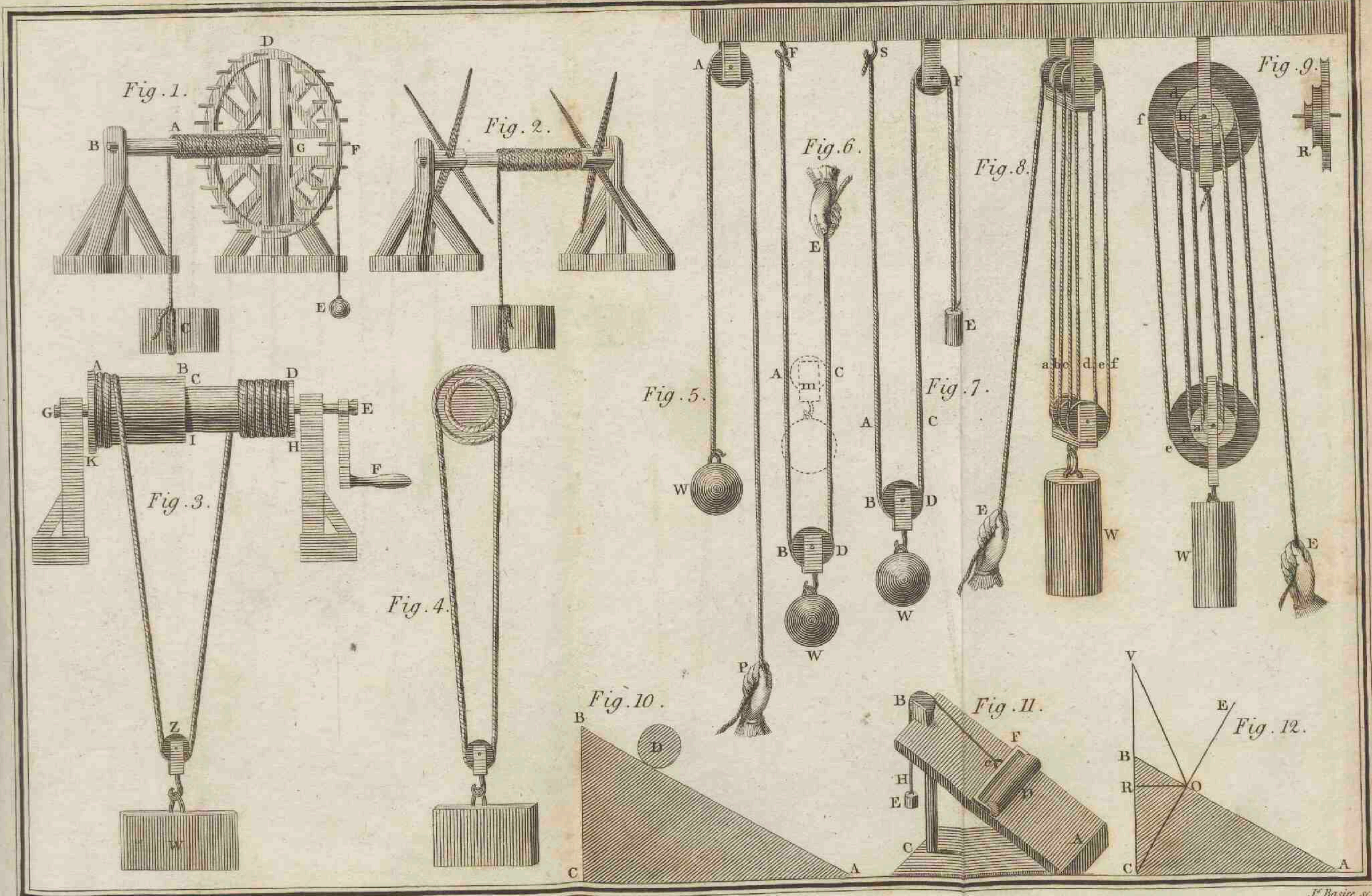

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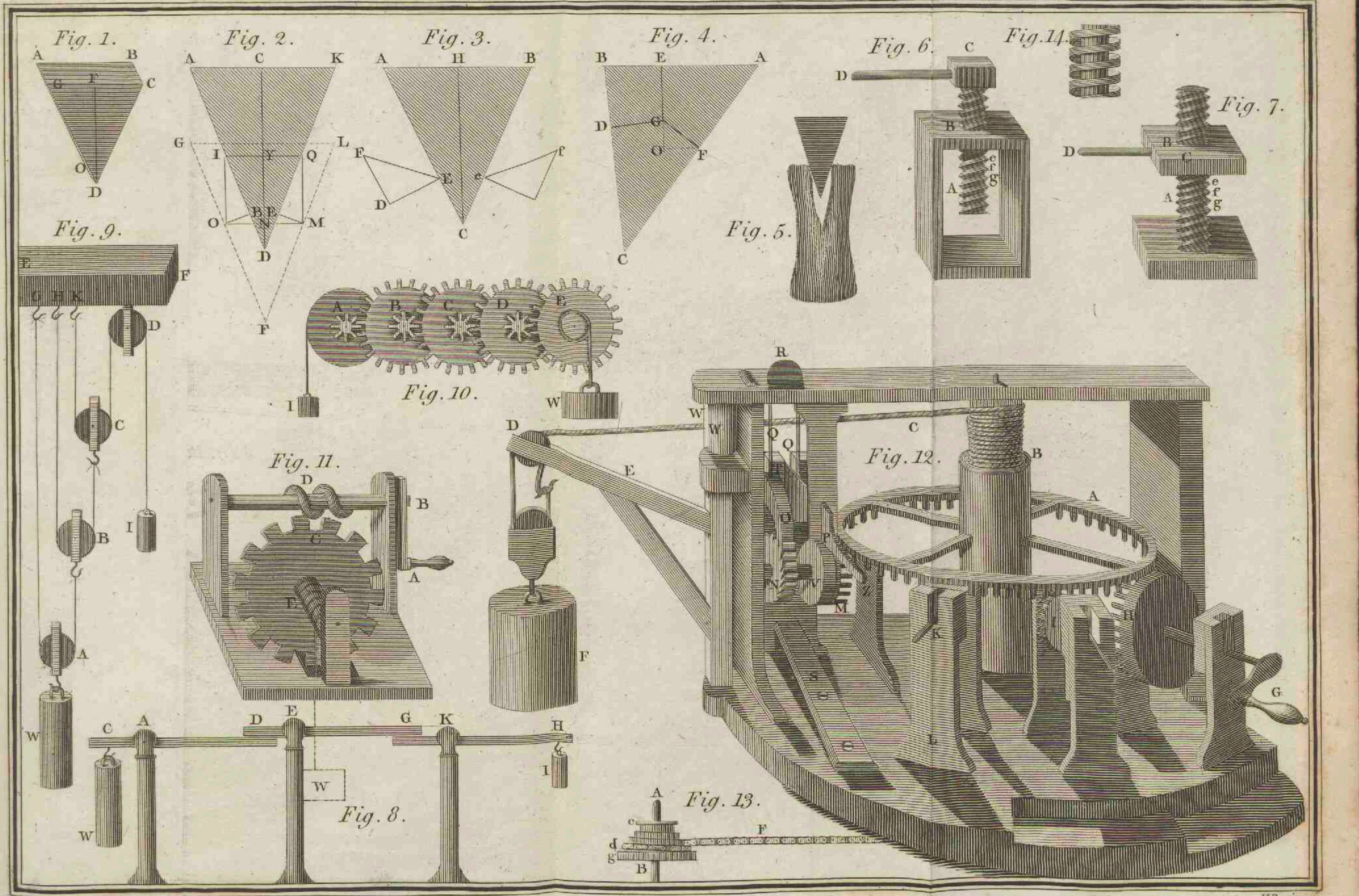

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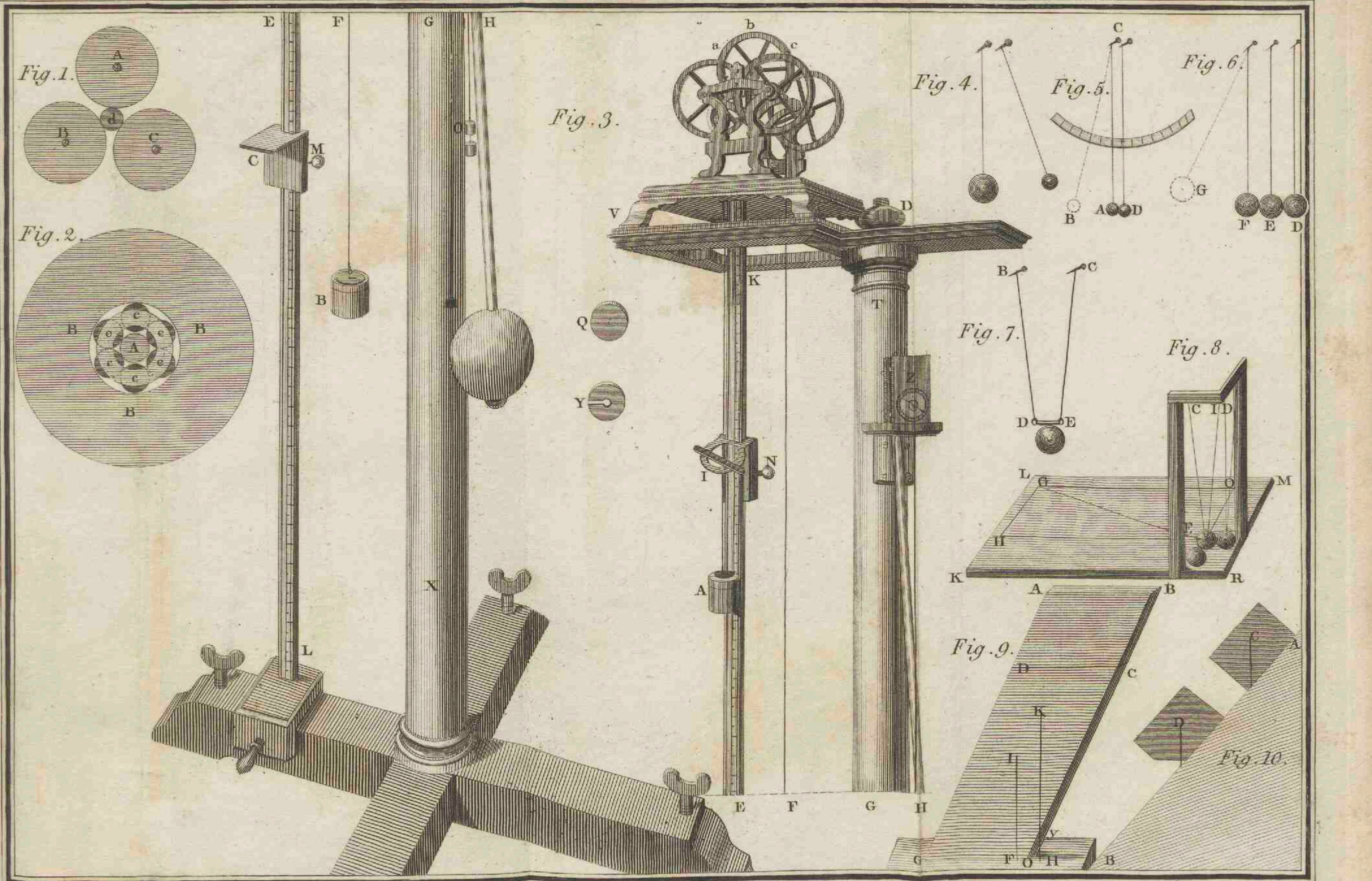

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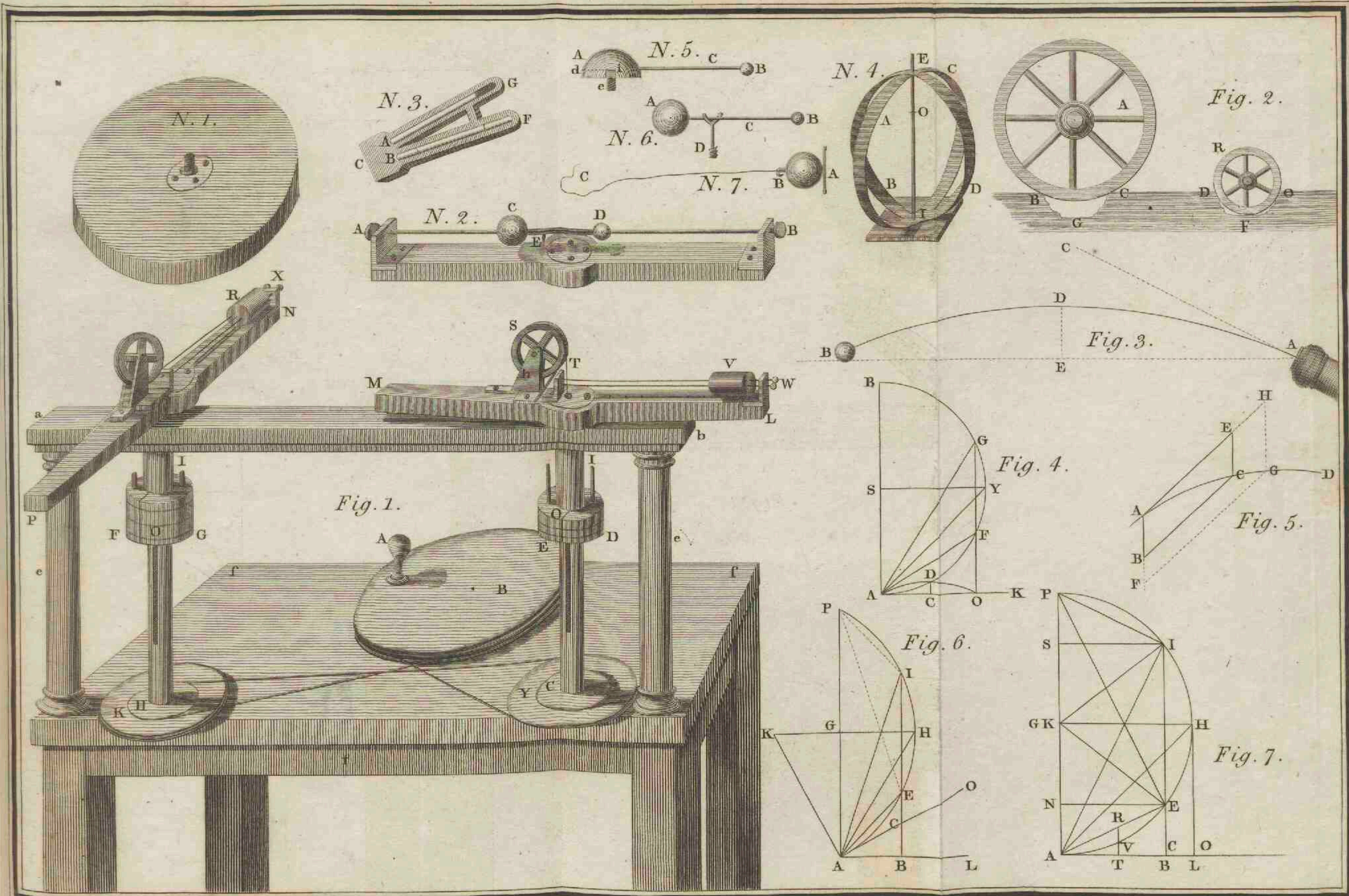

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